1. 2.4-8

   Generally, $t(x,y)=\exp(-jnk_0 d)$

   In this case, $t(x,y)=\exp(-jk_0 n_1 d_1)\exp(-jk_0 n_2 d_2)\cdots\exp(-jk_0 n_N d_N)=\exp(-jk_0\sum_{q=1}^{N} n_q d_q)$

   $\sum_{q=1}^{N} n_q d_q=\int_A^B n(r)ds=d=$ optical path length

2. $d(x,y)=\frac{1}{2}d_0[1+\Pi(\frac{2\pi x}{\Lambda})]$

   $t(x,y)=\exp(-jknd(x,y))\cdot\exp(-jk(d_0-d(x,y)))$

   $=\exp(-jkn\frac{1}{2}d_0)\exp(-jkn\frac{1}{2}d_0\Pi(\frac{2\pi x}{\Lambda}))\exp(-jk\frac{1}{2}d_0)\exp(jk\frac{1}{2}d_0\Pi(\frac{2\pi x}{\Lambda}))$

   $=\exp(-\frac{1}{2}jk(n+1)d_0)\exp(-\frac{1}{2}jk(n-1)d_0\Pi(\frac{2\pi x}{\Lambda}))$

   Let, $h_0=\exp(-\frac{1}{2}jk(n+1)d_0)$ then

   $t(x,y)=h_0\exp(-\frac{1}{2}jk(n-1)d_0\Pi(\frac{2\pi x}{\Lambda}))=h_0\sum_{q=-\infty}^{\infty}c_q\exp[j2\pi qx/\Lambda]$

   Generally Fourier coefficient is given by

   $c_q=\frac{1}{T}\int_0^T f(x)\exp(-jk_0 qx)dx=-j\sin c(\frac{q\pi}{2})\sin((n-1)k_0\frac{d_0}{2})$

   Plane wave after t(x,y) $A\exp(-jk_0 z)t(x,y)=Ah_0\sum_{q=-\infty}^{\infty}c_q\exp[-jk_0 z+j2\pi qx/\Lambda]$

   $\tan\theta=\frac{2\pi q}{\Lambda}\frac{\lambda}{2\pi}=q\frac{\lambda}{\Lambda}$, if $\lambda\ll\Lambda$ then $\theta\approx q\frac{\lambda}{\Lambda}$

   $\left|E_q\right|=A\left|\sin c(\frac{q\pi}{2})\right|\left|\sin((n-1)k_0\frac{d_0}{2})\right|$

3. Farfield diffraction pattern = FT[t(x,y)]

   $FT[t(x,y)]=FT[\Lambda(x)]=\sin c^2(\xi)=\sin c^2(\frac{x}{\lambda z})$

   $I(x)=\sin c^4(\frac{x}{\lambda z})$

4. $FT[\delta(x+\frac{a}{2})+\delta(x-\frac{a}{2})]=2\cos(\pi a\xi)=2\cos(\pi a\frac{x}{\lambda z})$

   $I(x)=4\cos^2(\pi a\frac{x}{\lambda z})$

5. (a)

$$f(x) * \exp(-j\pi x^2 / \lambda d) \to \left(\int_{-\infty}^{\infty} f(x') \exp(\frac{-j\pi x'^2}{\lambda d}) \exp(-j2\pi v_x x') dx' \right) \exp(\frac{-j\pi x^2}{\lambda d})$$

$$= \int_{-\infty}^{\infty} f(x') \exp(\frac{-j\pi (x-x')^2}{\lambda d}) dx' = f(x) * \exp(-j\pi x^2 / \lambda d)$$

(b)

Let $f(x)$ be the image before lens and $g(x)$ be the image after lens.

f(x) propagated by distance f, $f(x) * h_0 \exp(\frac{-j\pi x^2}{\lambda f})$

upon crossing the lens multiply by lens phase factor $\left(f(x) * h_0 \exp(\frac{-j\pi x^2}{\lambda f}) \right) \exp(\frac{j\pi x^2}{\lambda f})$

again propagate a distance of f,

$$\left(f(x) * h_0 \exp(\frac{-j\pi x^2}{\lambda f}) \right) \exp(\frac{j\pi x^2}{\lambda f}) * h_0 \exp(\frac{-j\pi x^2}{\lambda f})$$

Then using (a),

$$g(x) = \left(f(x) * h_0 \exp(\frac{-j\pi x^2}{\lambda f}) \right) \exp(\frac{j\pi x^2}{\lambda f}) * h_0 \exp(\frac{-j\pi x^2}{\lambda f})$$

$$= h_0^2 \{FT[f(x) \exp(\frac{-j\pi x^2}{\lambda f})] \exp(\frac{-j\pi x^2}{\lambda f}) \exp(\frac{j\pi x^2}{\lambda f})\} * \exp(\frac{-j\pi x^2}{\lambda f})$$

$$= h_0^2 FT[FT[f(x) \exp(\frac{-j\pi x^2}{\lambda f})] \exp(\frac{-j\pi x^2}{\lambda f})] \exp(\frac{-j\pi x^2}{\lambda f})$$

$$= h_0^2 \{FT[F[\frac{x}{\lambda f}] * FT[\exp(\frac{-j\pi x^2}{\lambda f})]] * FT[\exp(\frac{-j\pi x^2}{\lambda f})]\} \exp(\frac{-j\pi x^2}{\lambda f})$$

$$= h_0 \{[f(x) \exp(\frac{-j\pi x^2}{\lambda f})] * FT[h_0 \exp(\frac{-j\pi x^2}{\lambda f})]\} \exp(\frac{-j\pi x^2}{\lambda f})$$

$$= h_0 \{[f(x) \exp(\frac{-j\pi x^2}{\lambda f})] * H_0 \exp(\frac{j\pi x^2}{\lambda f})\} \exp(\frac{-j\pi x^2}{\lambda f})$$

$$= h_0 H_0 FT[f(x) \exp(\frac{-j\pi x^2}{\lambda f}) \exp(\frac{j\pi x^2}{\lambda f})] = h_0 H_0 F[\frac{x}{\lambda f}] = h_l F[\frac{x}{\lambda f}]$$

6. $p(x, y) = \sum_{m=-L}^{L} \delta(x - ma)$

$$FT[p(x,y)] = P(\xi,\eta) = \frac{1}{\sqrt{2\pi}}(1+\sum_{m=0}^{L}\cos(ma\xi)), \xi = \frac{x}{\lambda d}$$
$$= \frac{1}{\sqrt{2\pi}}(1+\sum_{m=0}^{L}\cos(m\frac{10x}{d})) = \frac{1}{\sqrt{2\pi}}(1+\sum_{m=0}^{L}\cos(10m\theta))$$

7. Aperture function $p(x,y)$ is multiplied by an oblique ray $\exp(-jkz)\exp(-jk\theta_x x), \theta_x \approx \tan(\theta_x)$

$$FT[p(x,y)\exp(-jk\theta_x x)] = P(\xi,\eta) * \delta(-\frac{\theta_x}{\lambda},0) = P(\xi+\frac{\theta_x}{\lambda},\eta)$$
$$I \propto \left|P(\xi+\frac{\theta_x}{\lambda},\eta)\right|^2$$

8. Fresnel diffraction(see 4.1-17 & 4.1-20)

$$g(x,y) = h_0\iint_{-\infty}^{\infty} p(x',y')\exp[-j\pi\frac{(x-x')^2+(y-y')^2}{\lambda d}]dx'dy'$$
$$= H_0\iint_{-\infty}^{\infty} P(v_x,v_y)\exp(j\pi\lambda d(v_x^{\ 2}+v_y^{\ 2}))\exp(-j2\pi(v_x x+v_y y))dv_x dv_y$$

Fraunhofer diffraction with aperture function $p(x,y)\exp(-j\pi\frac{x^2+y^2}{\lambda d})$

$$g(x,y) = h_0 FT(p(x,y)\exp(-j\pi\frac{x^2+y^2}{\lambda d}))$$
$$= P(\frac{x}{\lambda d},\frac{y}{\lambda d}) * H_0\exp(j\pi(\frac{x^2}{\lambda d}+\frac{y^2}{\lambda d}))$$
$$= H_0\iint_{-\infty}^{\infty} P(\frac{x'}{\lambda d},\frac{y'}{\lambda d})\exp(j\pi(\frac{(x-x')^2}{\lambda d}+\frac{(y-y')^2}{\lambda d}))dx'dy'$$
$$= H_0\exp(j\pi(\frac{x^2}{\lambda d}+\frac{y^2}{\lambda d}))\iint_{-\infty}^{\infty} P(\frac{x'}{\lambda d},\frac{y'}{\lambda d})\exp(j\pi\lambda d(v'^{\ 2}_x+v'^{\ 2}_y))\exp(-j2\pi(v'_x x+v'_y y))dx'dy'$$
$$\exp(j\pi(\frac{x^2}{\lambda d}+\frac{y^2}{\lambda d})) \sim 1$$
$$= H_0\iint_{-\infty}^{\infty} P(v_x,v_y)\exp(j\pi\lambda d(v_x^{\ 2}+v_y^{\ 2}))\exp(-j2\pi(v_x x+v_y y))dv_x dv_y$$