ECE 656: Fall 2011
Lecture 17 Homework
Due FRIDAY, October 14, 2011

This homework exercise will help you become familiar with how B –fields affect transport

Consider the equation of motion for an average electron,

$$\vec{F}_e = -q\vec{\mathcal{E}} - q\vec{\upsilon} \times \vec{B} = \frac{d\vec{p}}{dt} \ . \qquad (1)$$

Assume that the electron moves for a time, xx, then scatters, returning the average momentum to zero, so

$$\frac{dp}{dt} = \frac{p}{\tau_m} \, . \qquad (2)$$

Assuming that $\vec{p} = m^* \vec{\upsilon}$, we find an equation for the average velocity as

$$\vec{\upsilon} = -\frac{q\tau_m}{m^*}\vec{\mathcal{E}} - \frac{q\tau_m}{m^*}\vec{\upsilon} \times \vec{B} \, . \qquad (3)$$

This equation can be solved exactly for the velocity (see prob. 4.18, Lundstrom, Fundamentals of Carrier Transport, 2000), but let's take a different approach.

1) Assume carriers move in 2D and that only a z-directed B-field is present. Evaluate eqn. (3) and find two coupled equations for υ_x and υ_y.

2) Solve the two equations for υ_x and υ_y in terms of the electric field and the B-field.

3) Write the current densities as

$$J_x = -n_S q \upsilon_x \qquad (4a)$$

$$J_y = -n_S q \upsilon_y \qquad (4b)$$

and use the results of problem 2) to find the current densities as

$$J_x = \frac{\sigma_n}{1+\left(\mu_n B_z\right)^2}\left(\mathcal{E}_x - \mu_n B_z \mathcal{E}_y\right) \tag{5a}$$

$$J_y = \frac{\sigma_n}{1+\left(\mu_n B_z\right)^2}\left(\mathcal{E}_x + \mu_n B_z \mathcal{E}_y\right), \tag{5b}$$

which can also be written as

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_n}{1+\left(\mu_n B_z\right)^2}\begin{bmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{bmatrix}\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} \tag{6a}$$

or as

$$J_i = \sigma_{ij}\left(B_z\right)\mathcal{E}_j. \tag{6b}$$

Note that the magnetic field affects both the diagonal and off-diagonal components of the magnetoconductivity tensor. **Explain** why there is no Hall factor, r_H, in the result.

4) Show that for small B-fields, eqn. (6a) can be written as

$$\vec{J}_n = \sigma_n \vec{\mathcal{E}} - \left(\sigma_n \mu_n\right)\vec{E}\times\vec{B} \tag{7}$$

5) Solve eqn. (6a) for the electric field and show that

$$\begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} = \frac{1}{\sigma_n}\begin{bmatrix} 1 & \mu_n B_z \\ -\mu_n B_z & 1 \end{bmatrix}\begin{pmatrix} J_x \\ J_y \end{pmatrix} \tag{8}$$

According to eqn. (8), the longitudinal magnetoresistivity is independent of the *B*-field (while the longitudinal magnetoconductivity depends on *B* as shown in eqn. (6a). Equation (8) shows that the Hall voltage is proportional to *B*.

Note that while this analysis is simpler than solving the BTE, by beginning with an average electron with an average momentum, $\vec{p}$, we have missed the averaging of the distribution of momenta which leads to a non-unity Hall factor, r_H.