**F**: Thermal Broadening Function
\[ F_T = -\frac{df_T}{dE} = \frac{1}{4k_B T} \cdot \frac{1}{\cosh^2(E/2k_B T)} \]

\( f_0 \): Fermi Function
\[ f_0 = \frac{1}{e^{E/k_B T} + 1} \]

* Small device with voltage applied, current flows when a level lies between \( \mu_1 \) and \( \mu_2 \)

The expression that we've derived for current is only true if the bias is small.

\[ I = \frac{2q}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \left[ f_1 - f_2 \right] \]

\[ = \frac{2q}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \left[ -\frac{df_0}{dE} \right] \delta_{E=-E_f} \]

* Since,
\[ f_0(E-\mu_1) - f_0(E-\mu_2) = (\mu_1 - \mu_2) \frac{-df_0}{dE} \delta_{E=-E_f} \]

\[ = qV \left[ -\frac{df_0}{dE} \right] \delta_{E=-E_f} \]
Two thermal broadening functions at temperatures $T_1$ and $T_2$

- $F_T$ is the thermal broadening with peak value $1/(4k_B T)$: 
  \[ F_T = -\frac{d^2}{dE^2} = \frac{1}{4k_B T} \cdot \frac{1}{\cosh^2(E/2k_B T)} \]

- Area under curve is 1

- As temperature lowers, $F_T$ becomes taller, at very low temperatures it tends to a delta function: 
  \[ \lim_{T \to 0} F_T(E) = \delta(E) \]

- Inserted into the current equation: 
  \[ I = \frac{2q^2}{h} \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} F_T(\epsilon - E_f) \]

**G: Conductance**

The Thermal Broadening Function
• For
  \[ I = V \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} F_f (\varepsilon - E_f) \]
  take conductance to be:
  \[ G = \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} F_f (\varepsilon - E_f) \]
  \[ \therefore I = VG \]

• Peak conductance occurs when \( \theta = E_f \)
• Let \( \tilde{\varepsilon} \) be the original unbiased level energy, thus \( \varepsilon = \tilde{\varepsilon} - \alpha q V_G \)

• Conductance as a function of gate voltage

• It appears that \( G \) can increase indefinitely with respect to the ratio:
  \[ \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \frac{1}{4kT} \]

• No Upper Limit?

• a is a fractional compensation component, \( 0 < \alpha < 1 \), since an applied \( V_G \) component does not actually lower the channel energy levels by \( (qV_G)eV \) (i.e. \( 1V \) will not lower the levels by \( 1eV \))
• Conductance depends on how many levels we have between \( \mu_1 \) and \( \mu_2 \)
• Maximum conductance for 1 level:
  \[ G_{\text{max}} = \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \frac{1}{4kT} \]

\( \frac{2q^2}{h} = \text{siemens}; \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \frac{1}{4kT} = \text{Joules} \)

• This is not true because of \textit{broadening} which we have ignored so far.
When we couple to a contact we broaden the energy level in the channel. Level loses discreteness and a broadened continuous density of states \( D(E) \) results.

Density of states tells you the availability of states, not whether they are occupied or not.

\[
D(E) = \frac{\gamma_1}{\pi (E - \gamma_1)^2 + (\gamma_2/2)^2}, \quad \gamma = \gamma_1 + \gamma_2
\]

- When we couple to a contact we broaden the energy level in the channel.
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Example of a Lorentzian Curve

- \( D(E) \) is a Lorentzian
- Lorentzian characteristics: peak value of \( 2/\pi \gamma \); which depends on \( \gamma \); 1 level has an area of 1 for 1 electron

\[
\int_{-\infty}^{\infty} D(E) dE = 1
\]

- If \( \gamma \) is small, Lorentzian approaches delta function

- Fourier transforming \( D(E) \) we obtain:
  \( e^{-\pi(\gamma t)^2/\hbar^2} \) where \( \tau = \frac{\hbar}{\gamma} \) can be viewed as the life time of the particle.

- Broadening in energy
  - Fourier Transform
  - Life time in Time Domain
• Current through a density of states is:

\[ I = \int dE \cdot D(E) \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_{1}(E) - f_{2}(E)] \]

and for low bias:

\[ I = V \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \int_{-\infty}^{\infty} dE \cdot D(E) F_T (E - E_f) \]

Note: By symmetry \( F_T(E-E_f) = F_T(E_f-E) \)

• At low temperature broadening of \( D(E) \) is much greater than \( F_T \), \( F_T \) approaches a delta function: \( F_T(E_f-E) = \delta(E_f-E) \)

\[ \therefore I = V \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \int_{-\infty}^{\infty} dE \cdot D(E) \delta(E_f-E) \]

\[ = V \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} D(E_f) \]

• Conductance depends on density of states at the Fermi energy so…

\[ G_{\text{max}} = \frac{2q^2}{h} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \frac{2}{\pi} \]

Where, \( \frac{2}{\pi} = \frac{2}{\pi (\gamma_1 + \gamma_2)} = D(E_f)_{\text{max}} \)

\[ G_{\text{max}} = \frac{q^2}{\pi h (\gamma_1 + \gamma_2)^2} \]

When will this quantity reach a maximum?

Answer: When \( \frac{4 \gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2} = 1 \)

\[ \therefore G_{\text{max}} = \frac{q^2}{\pi h} \]

\[ = \frac{2q^2}{h} \approx 77.4 \mu S \approx \frac{1}{12.9} k\Omega \]
• For short conductors consider placing levels in series and in parallel
  • Parallel: Conductance = \( \frac{2q^2}{h} M \), where M is the number of levels in parallel
  • Series: Not so simple as parallel, series combinations are not ballistic, and electron scattering occurs. \( L_0 \) is known as the mean free path (distance an electron travels before encountering an impurity).

Therefore, Series Conductance = \( \frac{2q^2}{h} \left( \frac{L}{L + L_0} \right) \) where L is the total length of the conductor.

• Parallel Series Combination: \( G = \frac{2q^2}{h} M \left[ \frac{L_0}{L + L_0} \right] \)

Note: for \( L \gg L_0 \) we get Ohm’s Law dependence.

\[ G = \frac{2q^2}{h} \left[ \frac{width}{length} \right] \] ohms law