Quantum Transport: Atom to Transistor

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Lecture 3: The Quantum of Conductance

Ref. Chapter 1.3

Network for Computational Nanotechnology

nanoHUB
online simulations and more

Retouch on Concepts

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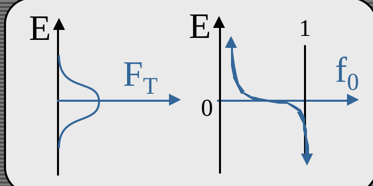

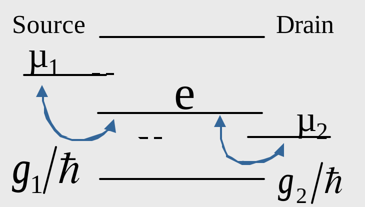

The expression that we've derived for current is only true if the bias is small.

- F_T: Thermal Broadening Function

$$F_T = -\,{}^{df}\!\big/_{dE} = \frac{1}{4k_BT} \bullet \frac{1}{\cosh^2(E/2k_BT)}$$

- f_0: Fermi Function

$$f_0 = 1\big/ e^{E/k_BT} + 1$$

- Small device with voltage applied, current flows when a level lies between μ_1 and μ_2

- $$I = \frac{2q}{\hbar}\frac{g_1 g_2}{g_1 + g_2}[f_1 - f_2]$$
$$= V\frac{2q^2}{\hbar}\frac{g_1 g_2}{g_1 + g_2}\left[\frac{-df_0}{dE}\right]_{E=e-E_f}$$

- Since,

$$f_0(E-m_1) - f_0(E-m_2) = (m_1 - m_2)\left[\frac{-df_0}{dE}\right]_{E=e-E_f}$$
$$= qV\left[\frac{-df_0}{dE}\right]_{E=e-E_f}$$

The Thermal Broadening Function

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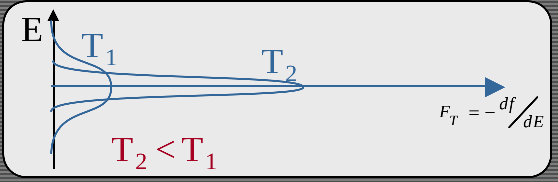
Two thermal broadening functions at temperatures T_1 and T_2



- F_T is the thermal broadening with peak value $1/(4k_BT)$ $F_T = -{}^{df}/_{dE} = \frac{1}{4k_BT} \bullet \frac{1}{\cosh^2(E/2k_BT)}$
- Area under curve is 1
- As temperature lowers, F_T becomes taller, at very low temperatures it tends to a delta function: $\lim_{T \to 0} F_T(E) = d(E)$
- Inserted into the current equation:

G: Conductance

$$I = V \overbrace{\frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2}}^{G} F_T(e - E_f)$$

No Upper Limit?

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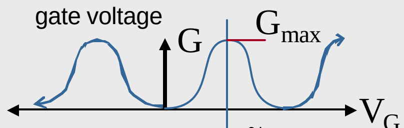
- For $I = V \frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2} F_T(e - E_f)$
take conductance to be:

$$G = \frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2} F_T(e - E_f)$$

$\therefore I = VG$

Conductance as a function of gate voltage



$$e = E_f = \tilde{e} - a\, qV_G$$

- Peak conductance occurs when e $=E_f$
- Let $\tilde{e}$ be the original unbiased level energy, thus $e = \tilde{e} - aqV_G$

- a is a fractional compensation component,
0< a <1, since an applied V_G component does not actually lower the channel energy levels by $(qV_G)eV$
(i.e. 1V will not lower the levels by 1eV)
- Conductance depends on how many levels we have between μ1 and μ2
- Maximum conductance for 1 level:

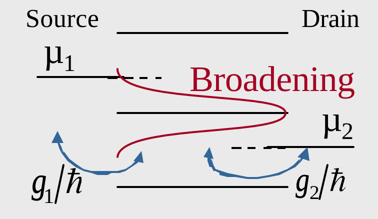
$$G_{\max} = \frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2} \frac{1}{4k_B T}$$

$\frac{2q^2}{\hbar}$=siemens; $\frac{g_1 g_2}{g_1 + g_2}$ =Joules; $\frac{1}{4k_B T}$ =1/Joules

- It appears that G can increase indefinitely with respect to the ratio: $\frac{g_1 g_2}{g_1 + g_2} \frac{1}{4 k_B T}$
- This is not true because of **broadening** which we have ignored so far.

Broadening and Density of States

Broadening of a level

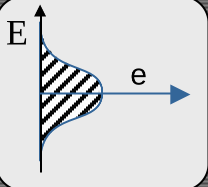


- When we couple to a contact we broaden the energy level in the channel.
- Level loses discreteness and a broadened continuous density of states D(E) results.

$$D(E)=\frac{g/2p}{(E-e)^2+(g/2)^2}, g=g_1+g_2$$

- Density of states tells you the availability of states, not whether they are occupied or not.

Example of a Lorentzian Curve



- D(E) is a Lorentzian
- Lorentzian characteristics: peak value of 2/p?; which depends on ?; 1 level has an area of 1 for 1 electron

- $\int_{-\infty}^{\infty} D(E)dE = 1$
- If ? is small, Lorentzian approaches delta function

- Fourier transforming D(E) we obtain: $e^{(-iet)/h}\ e^{-|t|/2t}$ where $t = \frac{\hbar}{g}$ can be viewed as the life time of the particle.
- Broadening in energy
  Fourier Transform
  Life time in Time Domain

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Current Expression including Broadening

• Current through a density of states is:

$$I = \int_{-\infty}^{\infty} dE \cdot D(E) \frac{2q}{\hbar} \frac{g_1 g_2}{g_1 + g_2} [f_1(E) - f_2(E)]$$

and for low bias:

$$I = V \frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2} \int_{-\infty}^{\infty} dE \cdot D(E) F_T [E - E_f]$$

Note: By symmetry $F_T(E-E_F) = F_T(E_F-E)$

• At low temperature broadening of D(E) is much greater than F_T, F_T approaches a delta function: $F_T(E_f-E) = d(E_f-E)$

$$\therefore I = V \frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2} \int_{-\infty}^{\infty} dE \cdot D(E) F_T (E_F - E)$$

$$= V \frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2} D(E_f)$$

• Conductance depends on density of states at the Fermi energy so…

$$G_{max} = \frac{2q^2}{\hbar} \frac{g_1 g_2}{g_1 + g_2} \cdot \frac{2}{pg}$$

Where, $\frac{2}{pg} = \frac{2}{p(g_1 + g_2)} = D(E_f)_{max}$

• $G_{max} = \frac{q^2}{p\hbar} \frac{4 g_1 g_2}{(g_1 + g_2)^2}$

When will this quantity reach a maximum?

Answer: When $\frac{4 g_1 g_2}{(g_1 + g_2)^2} = 1$

$$\therefore G_{max} = \frac{q^2}{p\hbar}$$

$$= \frac{2q^2}{\hbar} \cong 77.4 mS \cong 1/12.9 k\Omega$$

Ohm's Law

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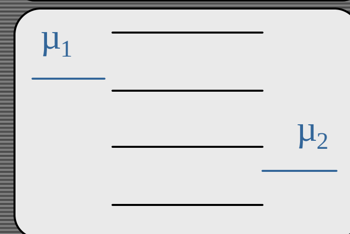
- For short conductors consider placing levels in series and in parallel
- Parallel: Conductance $= \frac{2q^2}{h} M$, where M is the number of levels in parallel
- Series: Not so simple as parallel, series combinations are not ballistic, and electron scattering occurs. L_0 is known as the mean free path (distance an electron travels before encountering an impurity).

Therefore, Series Conductance = $\frac{2q^2}{h}[\frac{L_0}{L+L_0}]$ where L is the total length of the conductor.

- Parallel Series Combination: $G = \frac{2q^2}{h} M [\frac{L_0}{L+L_0}]$

Note: for $L \gg L_0$ we get $G = \frac{2q^2}{h}[\frac{width}{length}]$ ohms law dependence.

Levels in Parallel



Levels in Series