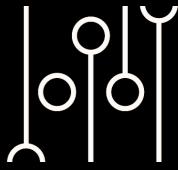


How physics and modern computers have revolutionized imaging

P Scott Carney

Beckman Institute for Advanced Science and Technology
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Biophotonics Summer School 2012



Acknowledgements

Prof. P. Scott Carney

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Post-Doctoral Fellows

Brynmor Davis

Post-Doctoral Fellows

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Graduate Students

Jin Sun (PhD 2008)

Graduate Students

Tyler Ralston

Other collaborators

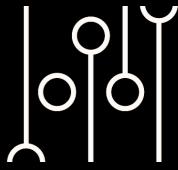
John Schotland (Michigan)

Vadim Markel (Penn)

Sergey Bozhevolnyi (Copenhagen)

Rainer Hillenbrand (nanoGUNE)

Sasha Govyadinov (nanoGUNE)



Understanding the math today: form and rhythm

Зимняя ночь.

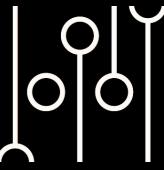
*Мело, мело по всей земле
Во все пределы.
Свеча горела на столе,
Свеча горела.*

*Как летом роем мошкора
Летит на пламя,
Слетались хлопья со двора
К оконной раме.*

*Метель лепила на столе
Кружки и стрелы.
Свеча горела на столе,
Свеча горела.*

*На озаренный потолок
Ложились тени,
Скрещенья рук, скркщенья ног,
Судьбы скрещенья.*

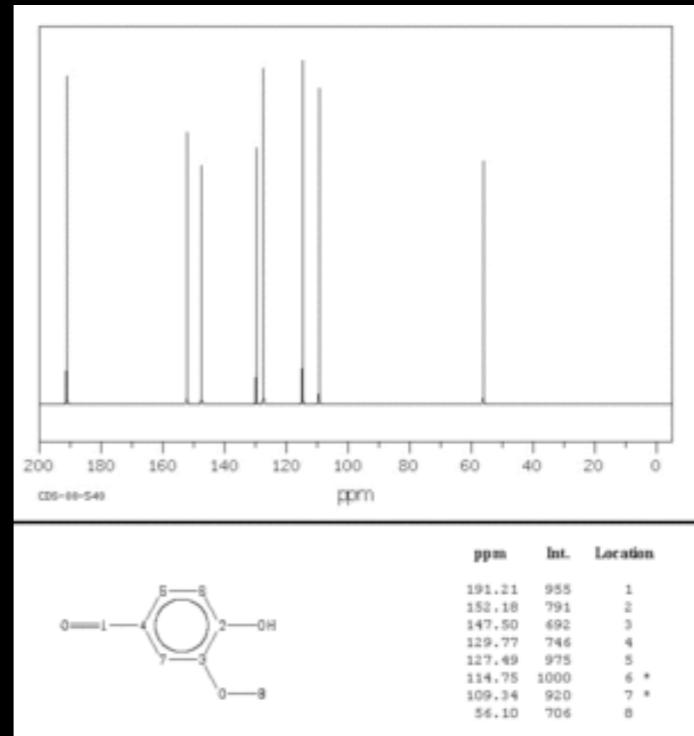
... Boris Pasternak



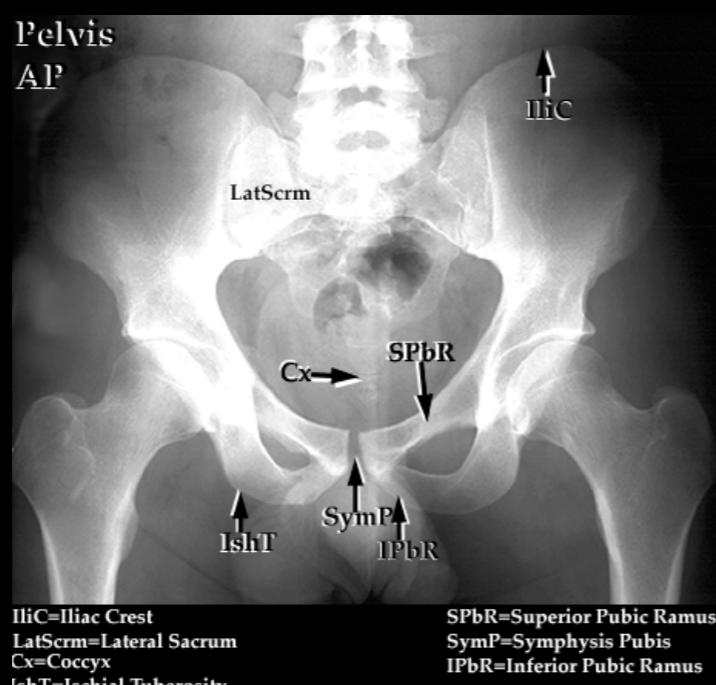
Inverse problems



RADAR

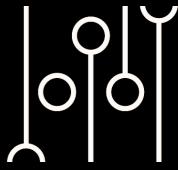


NMR spectroscopy



X-ray projection

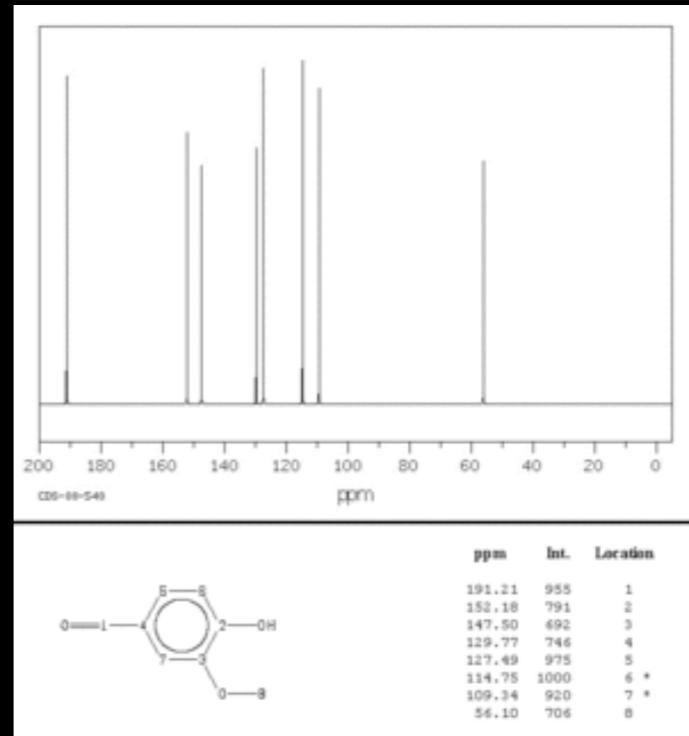
Optical Coherence Tomography
*Spectral Domain



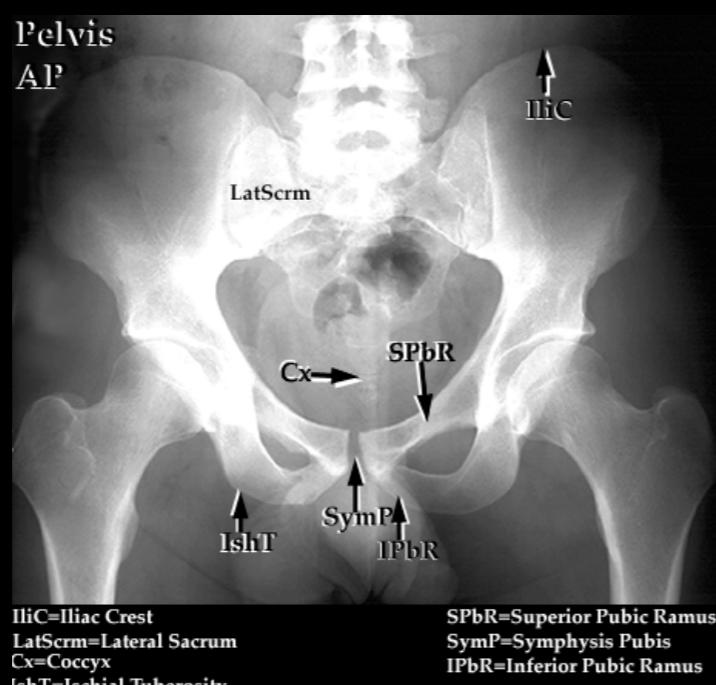
Inverse problems



RADAR

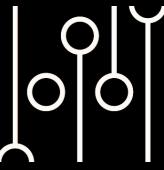


NMR spectroscopy



X-ray projection

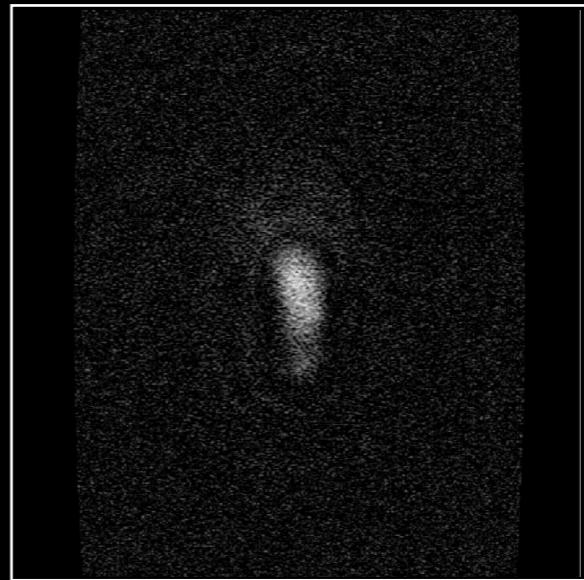
Optical Coherence Tomography
*Spectral Domain



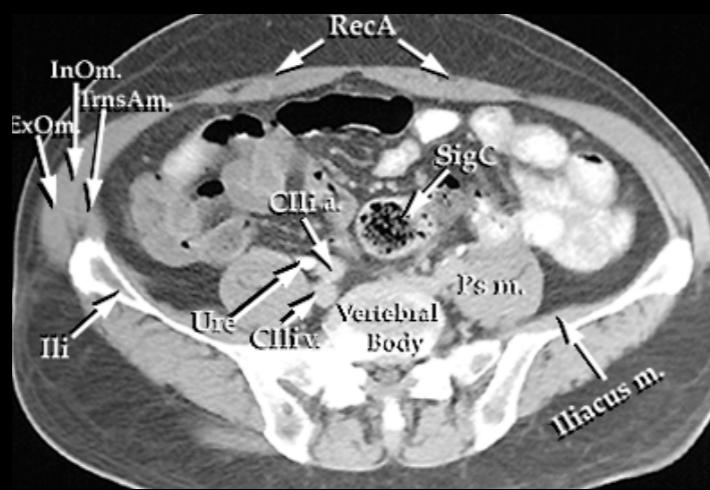
Inverse problems



Synthetic Aperture Radar
*Rob Morrison

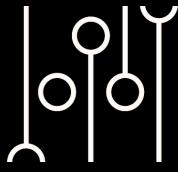


Magnetic Resonance Imaging
*Ian Atkinson

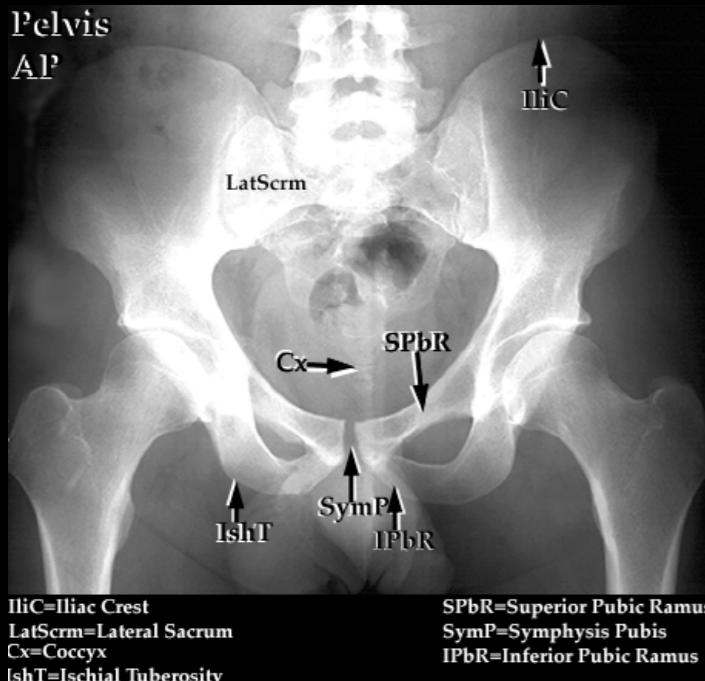


CT

Optical Coherence Tomography
*Spectral Domain



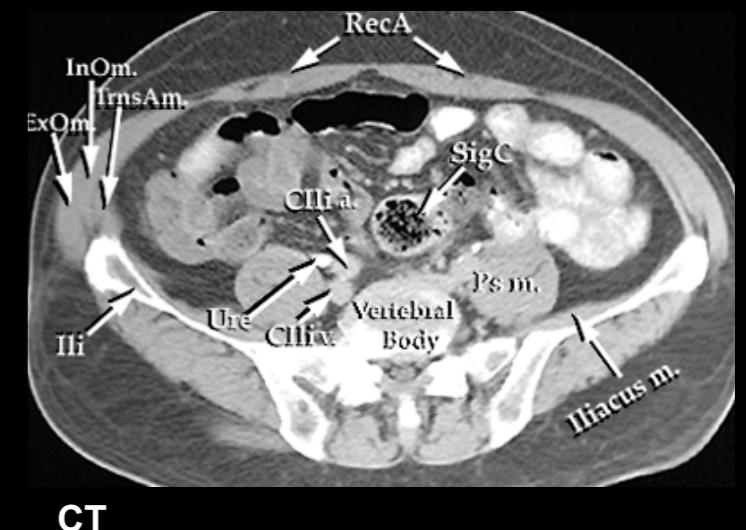
Shadowgrams to structure



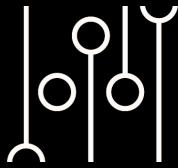
X-ray projection

Physics

$$I(P) = I(P_0)e^{-\int_{P_0}^P \alpha(\mathbf{r})dl}$$



Maxwell Eq. → Geometrical Optics → Beers law



The discovery of x-rays

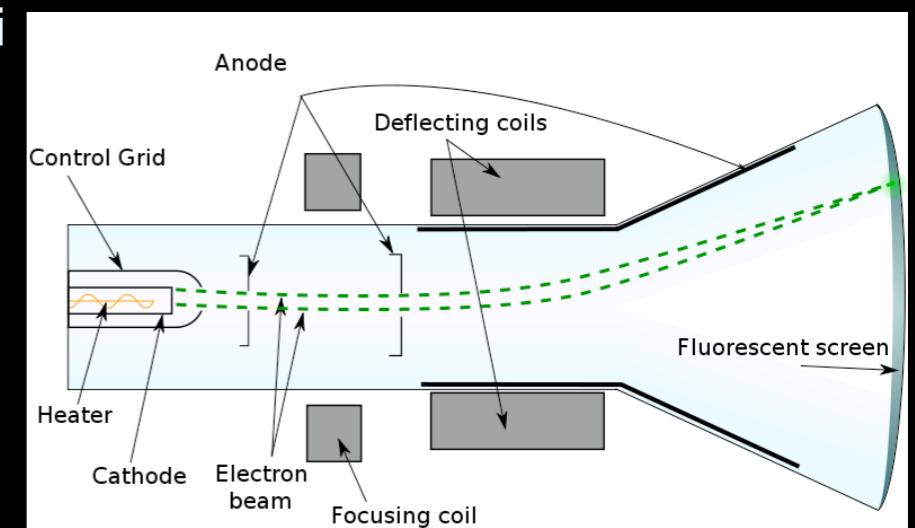


22 December, 1895

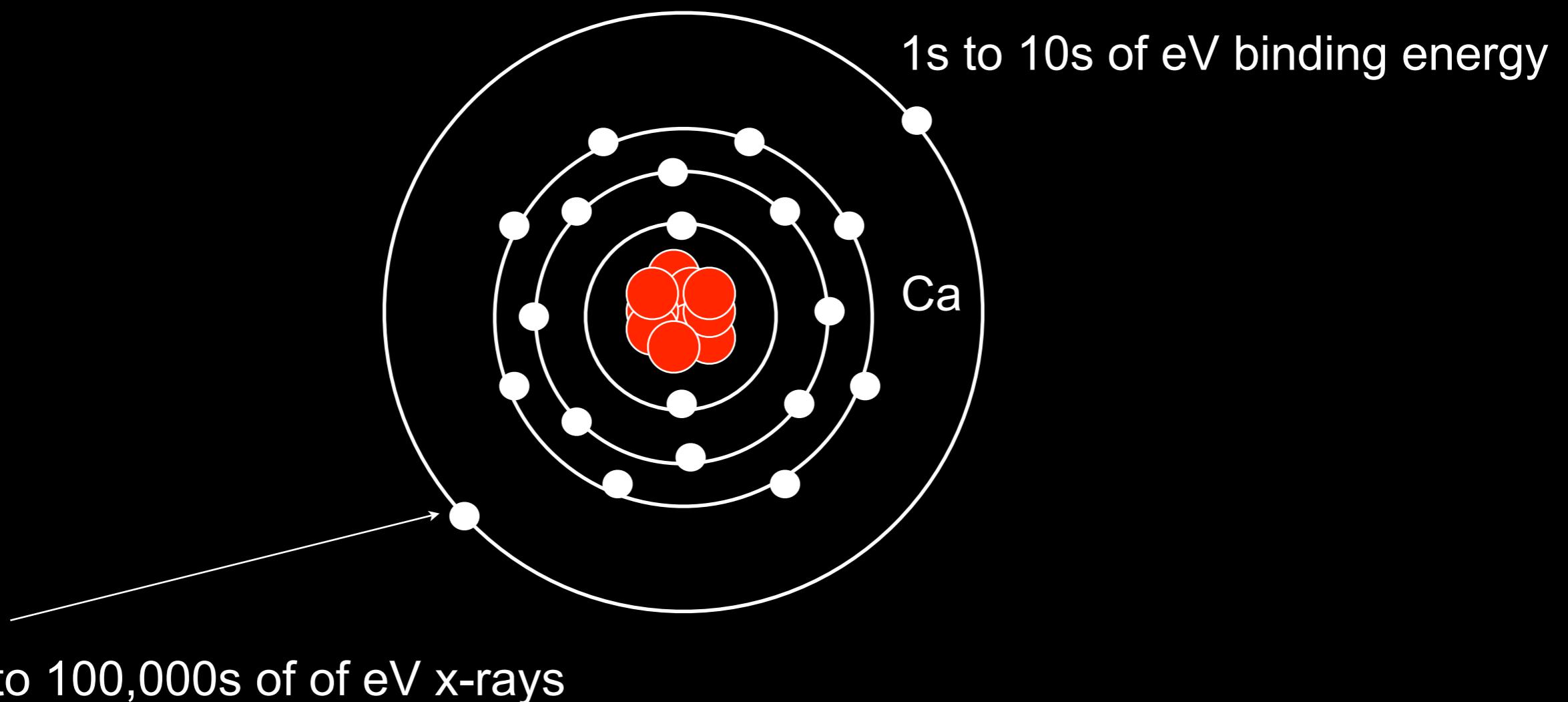


1st Nobel Prize in physics
Units named (the rontgen)
Father of diagnostic radiology
Institutes
Devices
Rays
etc...

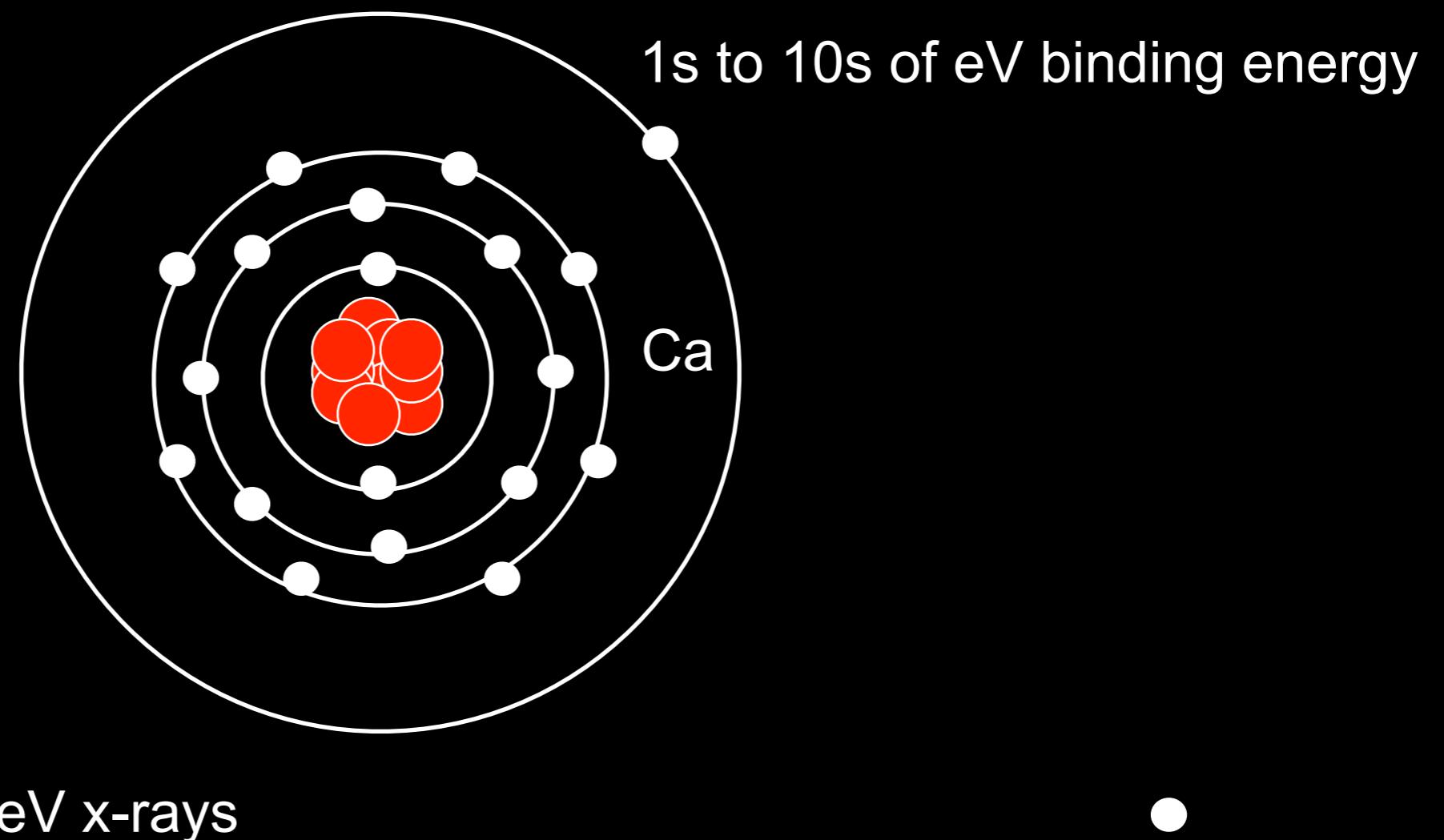
Cathode rays
1905 Nobel
Nazi



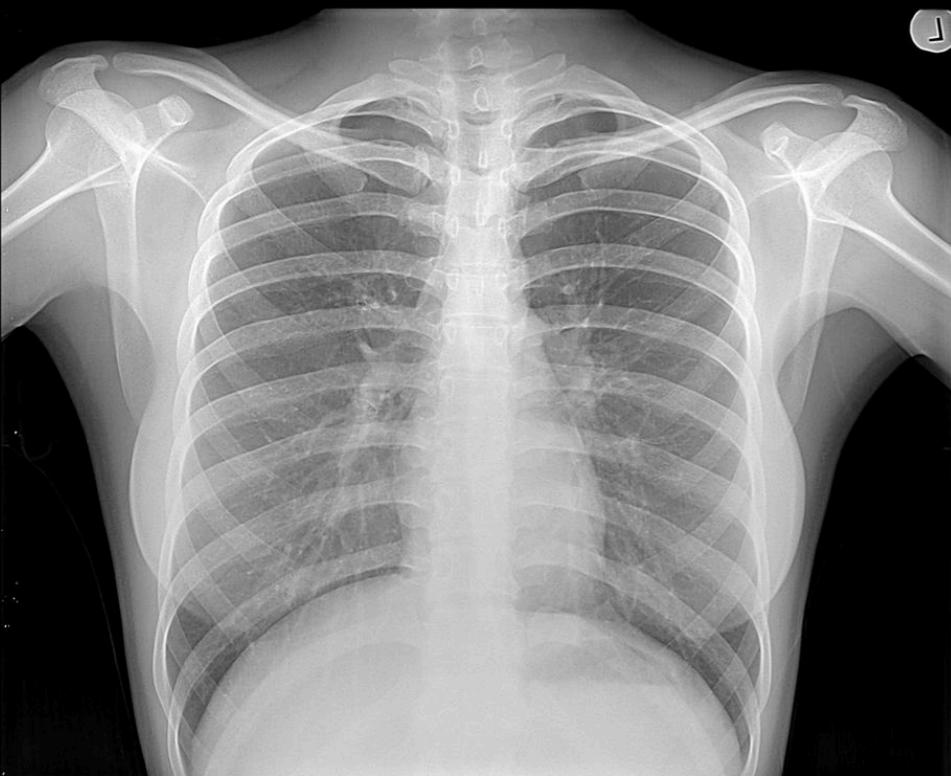
X-ray absorption, ionization



X-ray absorption, ionization



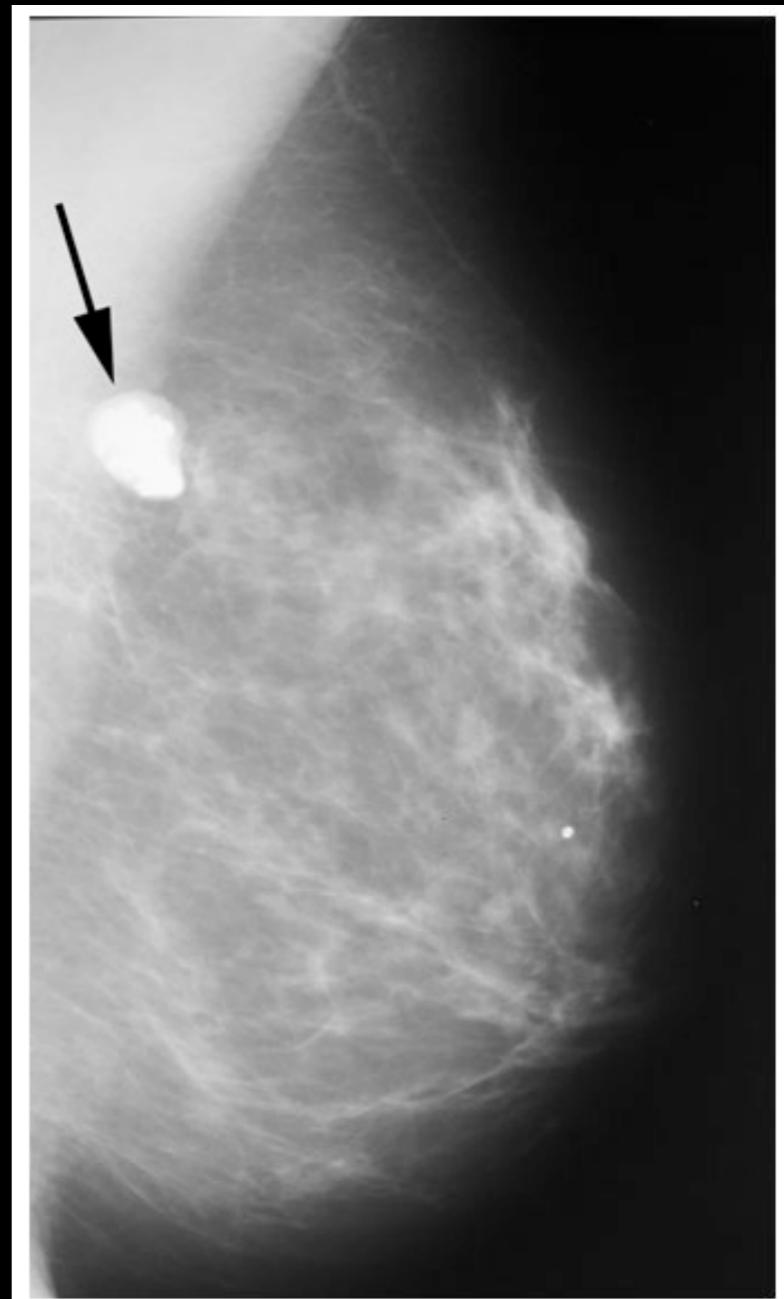
Electrons provide contrast through metals like Ca, Mg, Ba



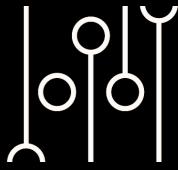
bones



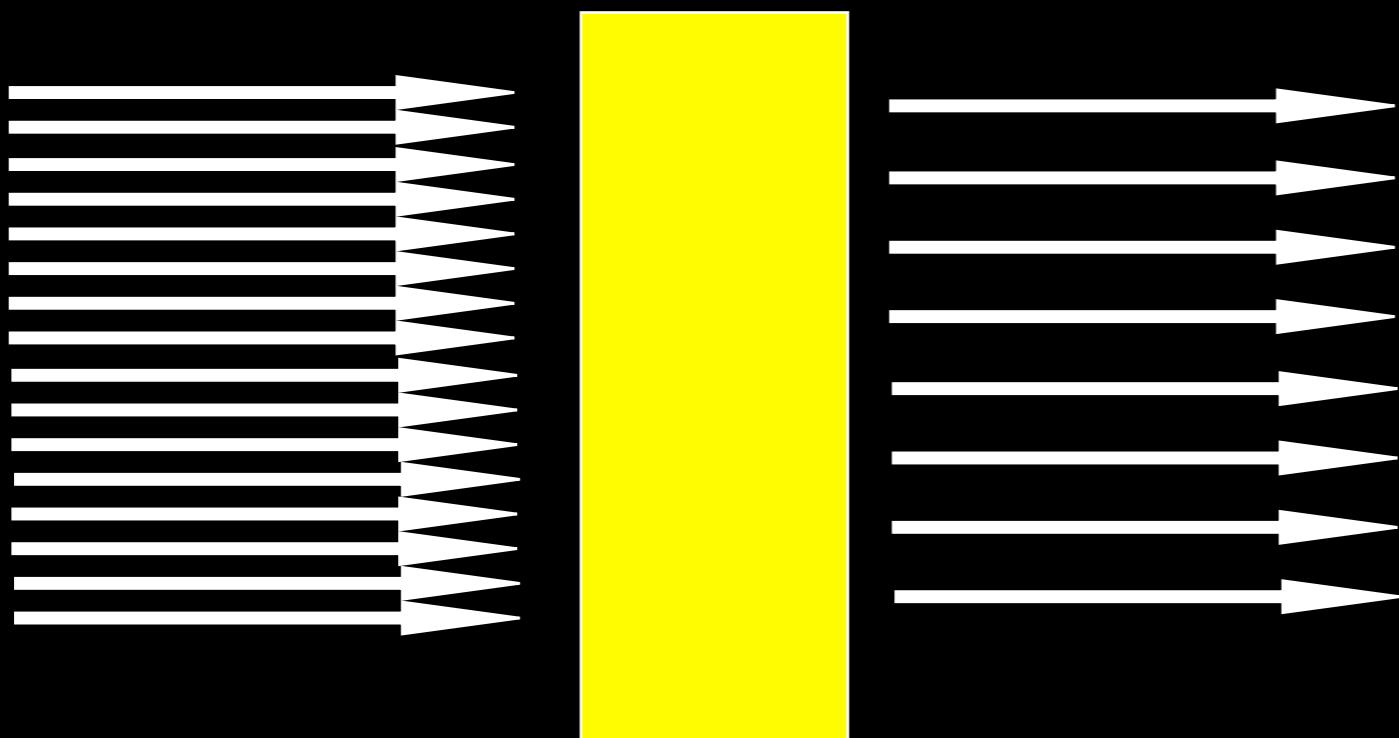
contrast agents



soft tissue

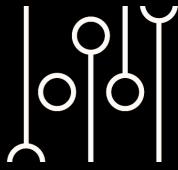


Beers law

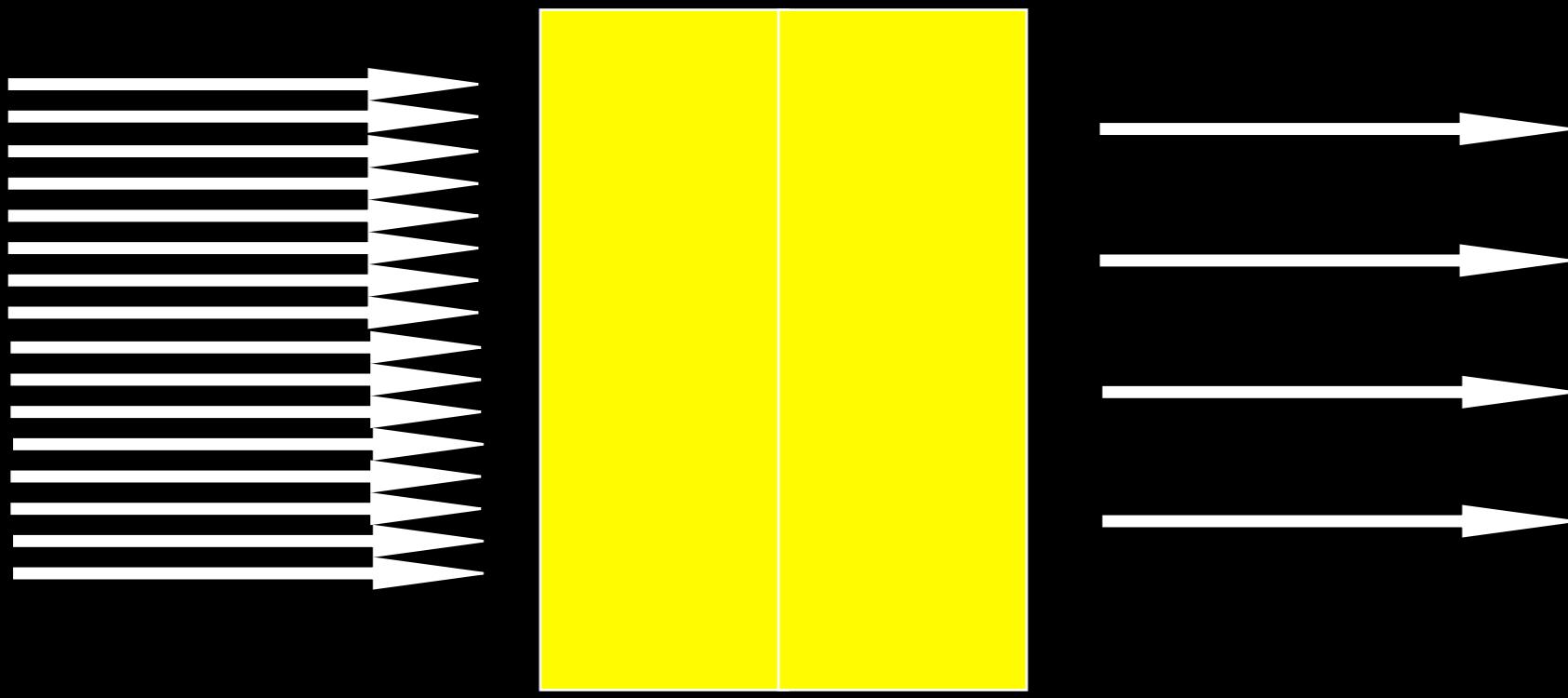


1/2

$$I = I_0 2^{-\alpha \ell}$$

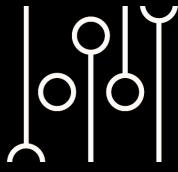


Beers law

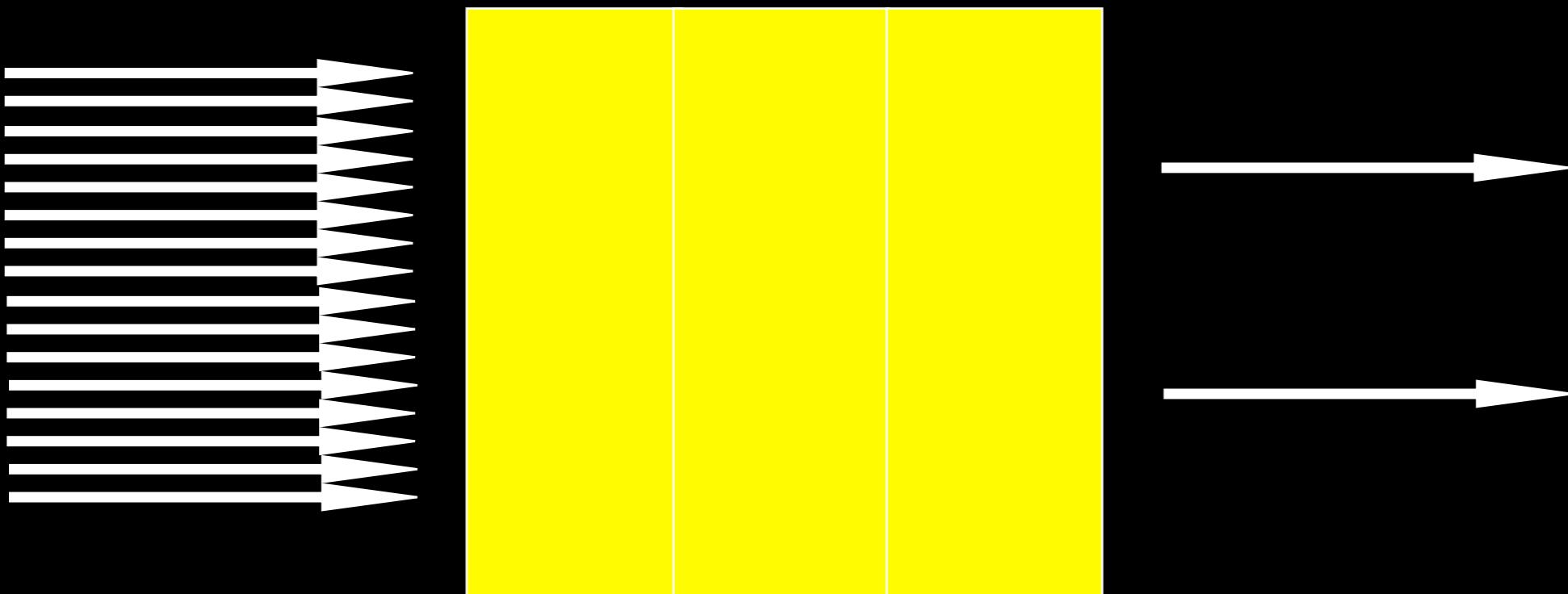


1/4

$$I = I_0 2^{-\alpha \ell}$$

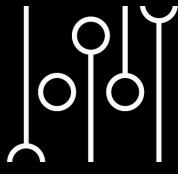


Beers law

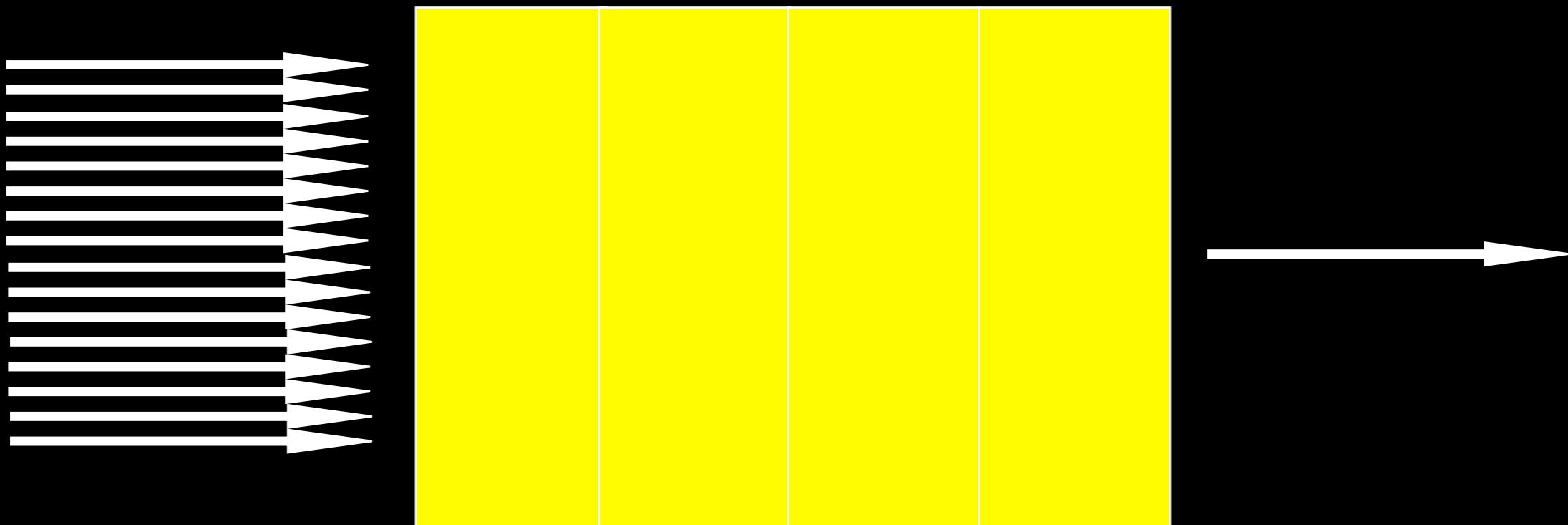


1/8

$$I = I_0 2^{-\alpha \ell}$$

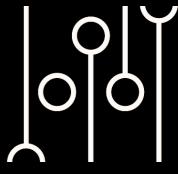


Beers law

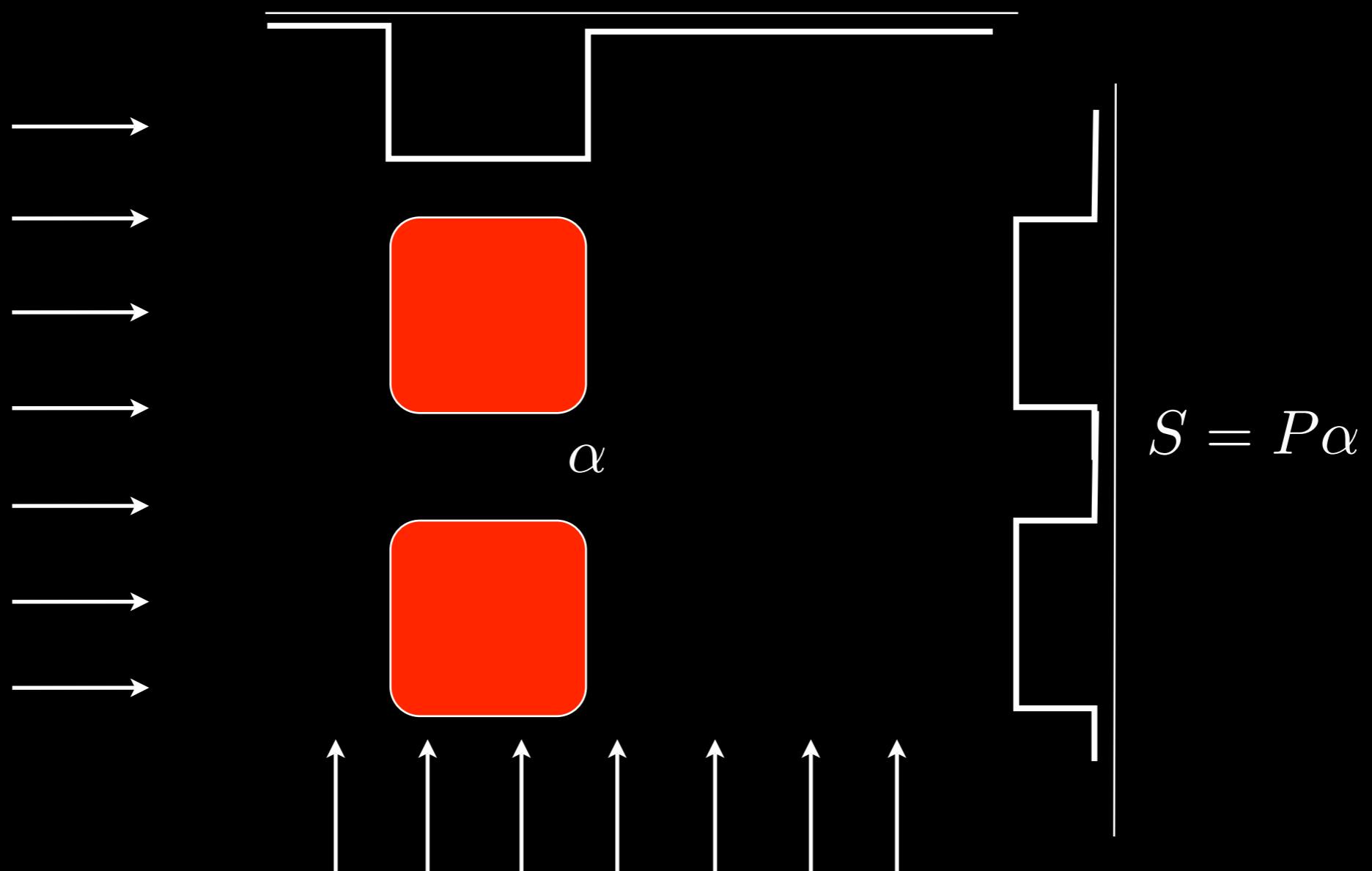


1/16

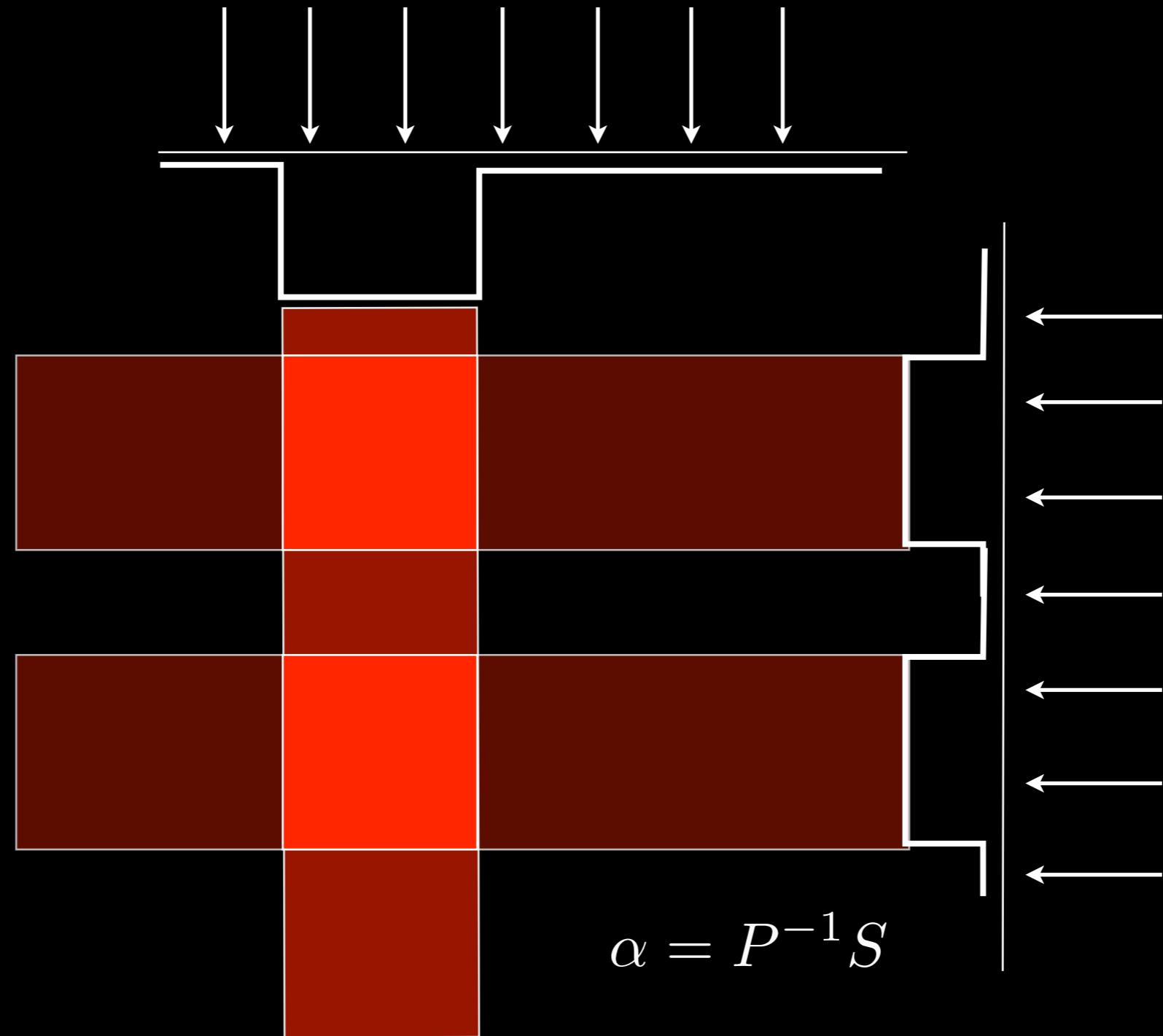
$$I = I_0 2^{-\alpha \ell}$$

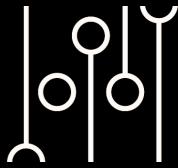


Projections



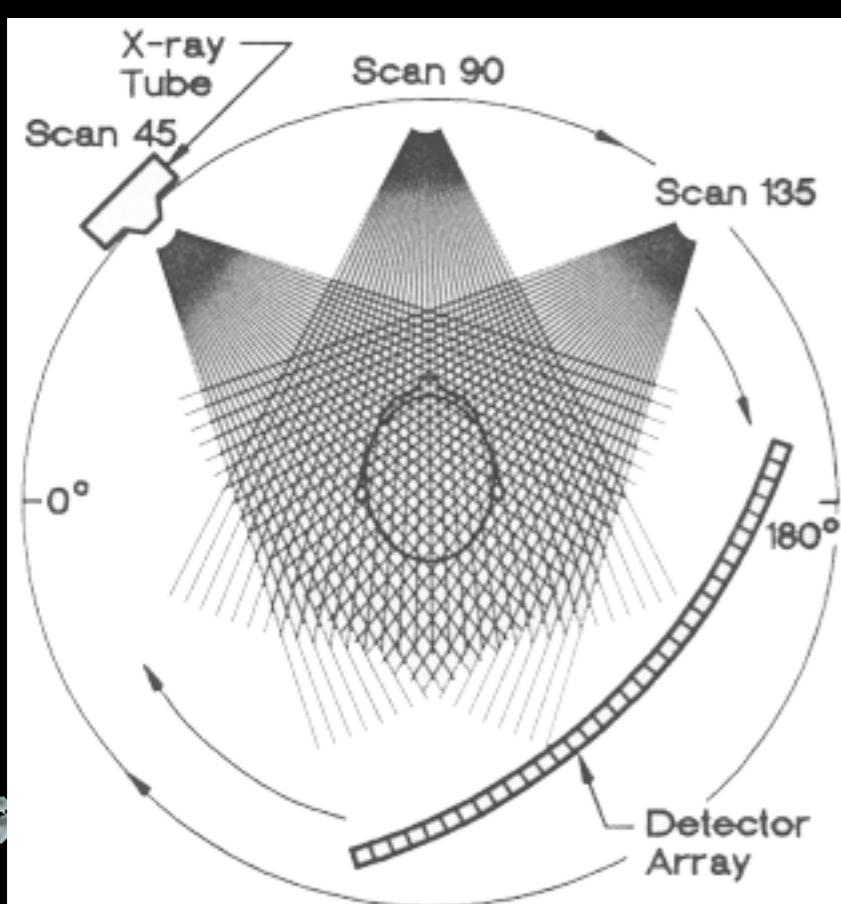
Back-Projections

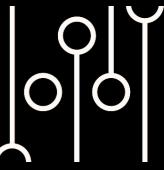




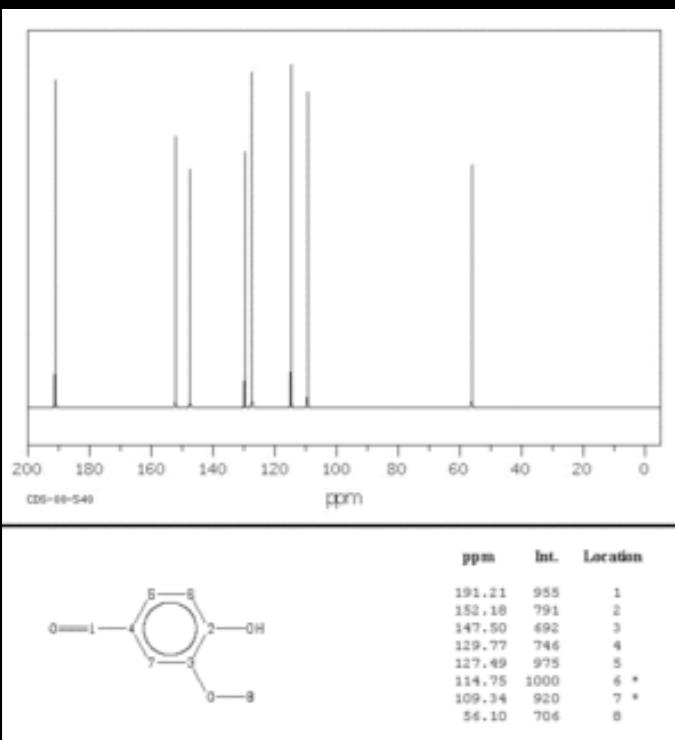
Computed tomography

- Johan Radon 1917
 - mathematics
- Bracewell 1956
 - filtered back propagation for astronomy
- Cormack 1963
 - reinvented Radon transform
- Hounsfield 1967
 - experimental implementation of CT





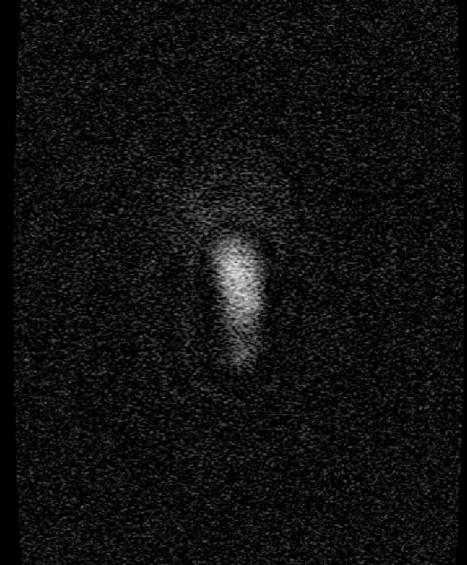
Spectra to structure



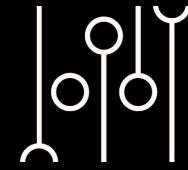
NMR spectroscopy

Physics

$$\omega = 2B\mu/\hbar$$



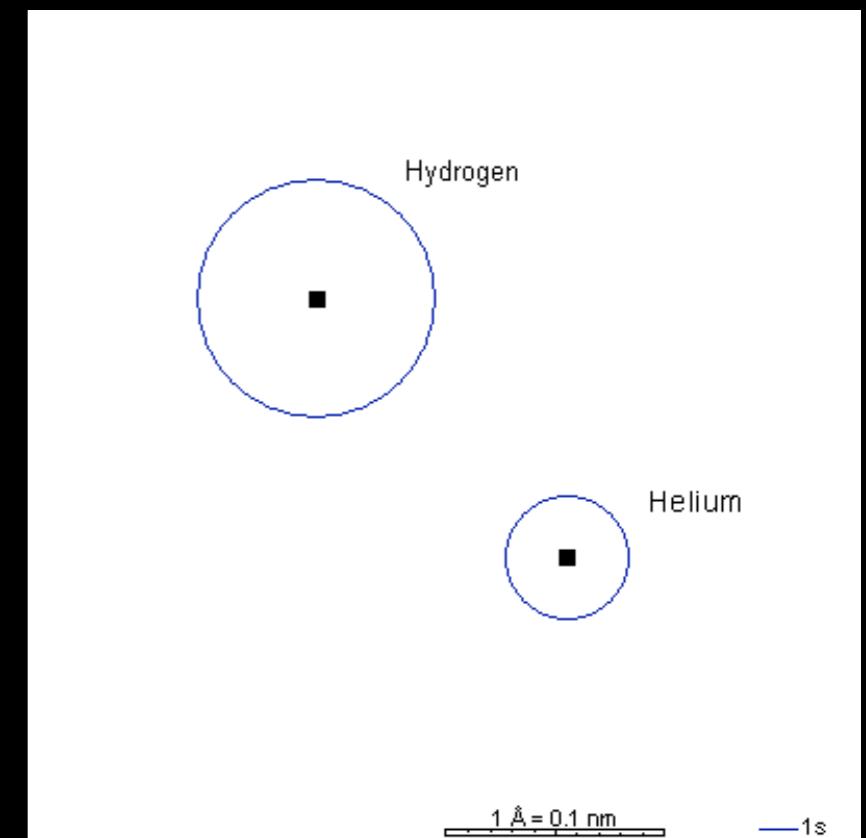
Maxwell Eq. + Q.M. w/ spin → Larmor freq.
+ Stat mech. → τ_1, τ_2

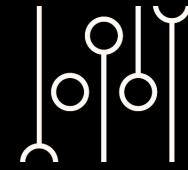


The discovery of spin Wolfgang Pauli 1924



Paraphrase of the scientific question: Why is helium a noble element and not hydrogen?





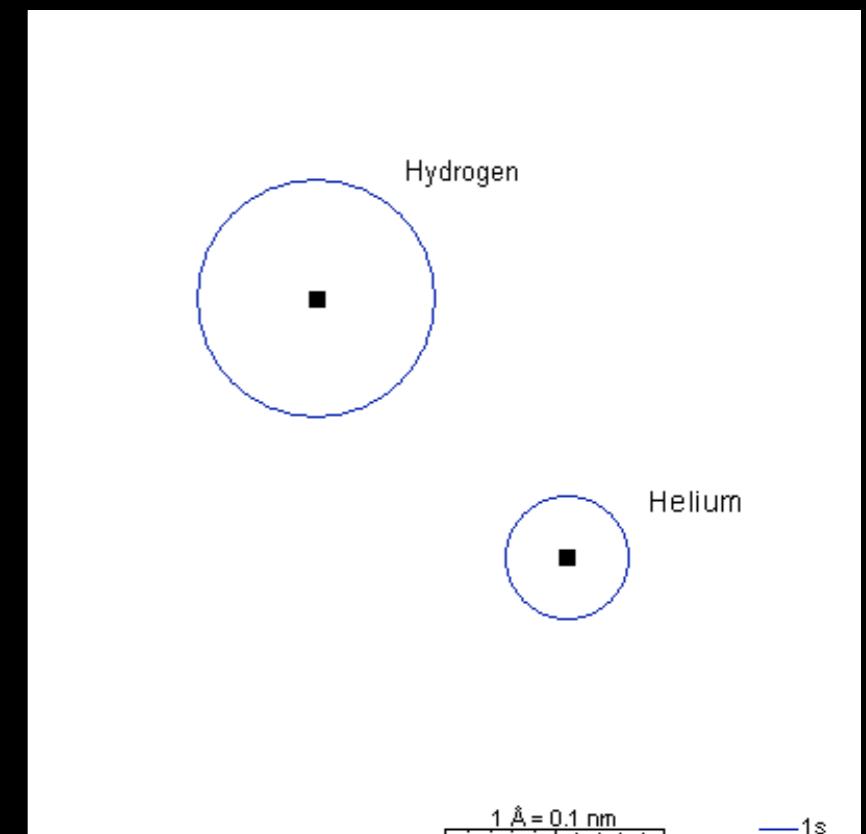
The discovery of spin Wolfgang Pauli 1924

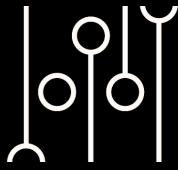


Paraphrase of the scientific question: Why is helium a noble element and not hydrogen?

Exclusion principle
(Nobel prize 1945):

$$P\Psi_{1,2} = \Psi_{2,1} = -\Psi_{1,2}$$





The discovery of spin Wolfgang Pauli 1924



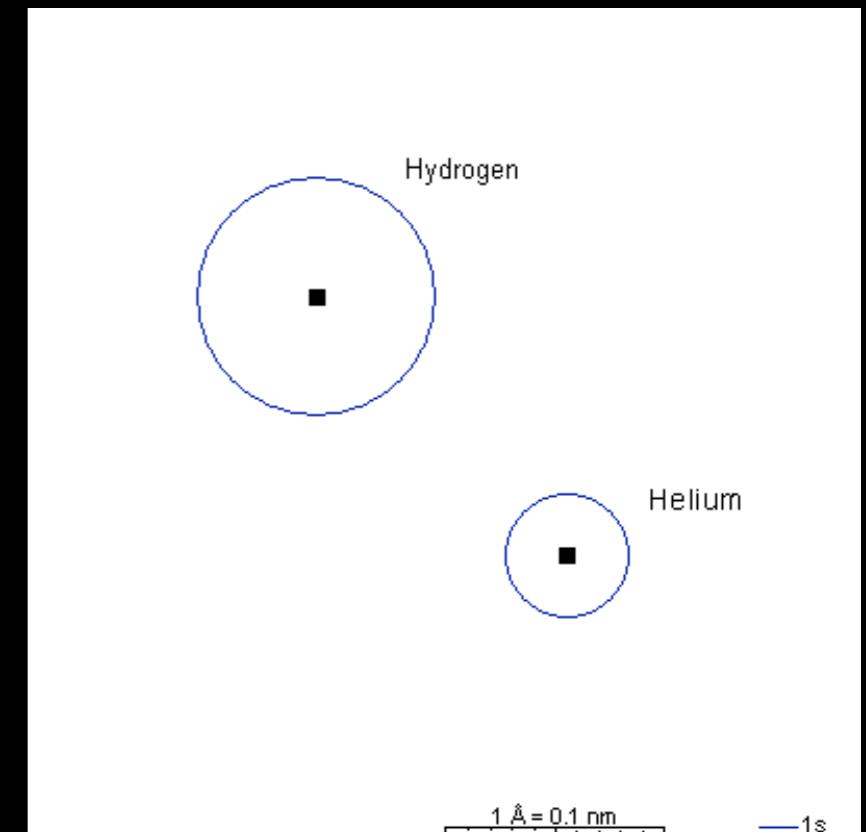
Paraphrase of the scientific question: Why is helium a noble element and not hydrogen?

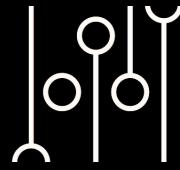
Exclusion principle
(Nobel prize 1945):

$$P\Psi_{1,2} = \Psi_{2,1} = -\Psi_{1,2}$$

BUT... it would seem

$$\Psi_{2,1} = \Psi_{1,2}$$





The discovery of spin Wolfgang Pauli 1924



Paraphrase of the scientific question: Why is helium a noble element and not hydrogen?

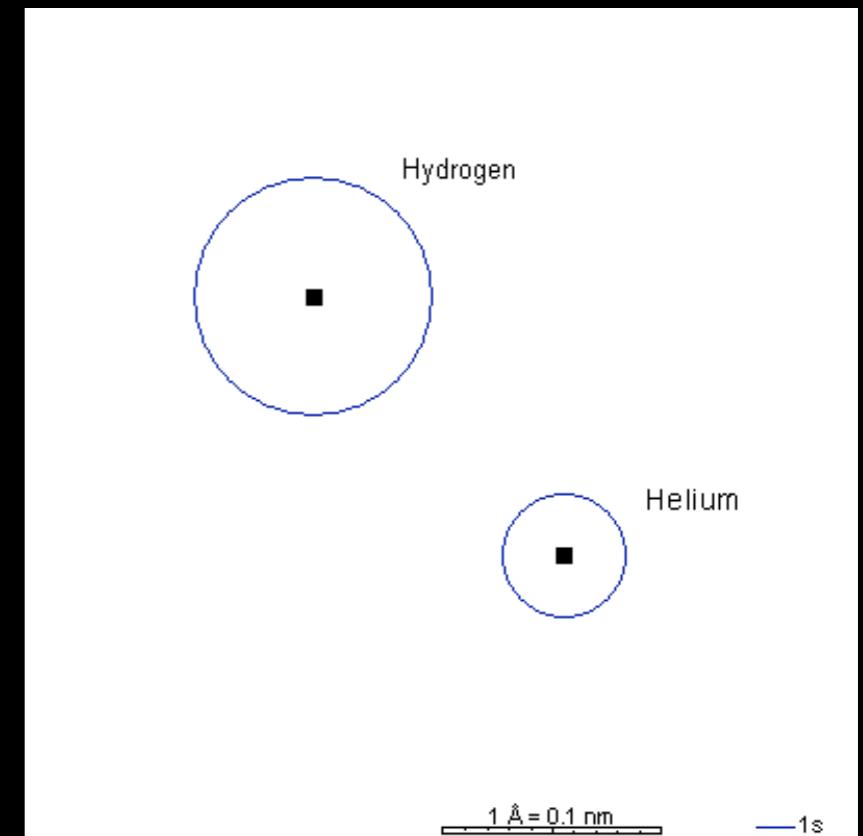
Exclusion principle
(Nobel prize 1945):

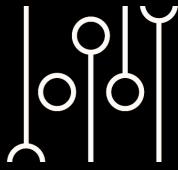
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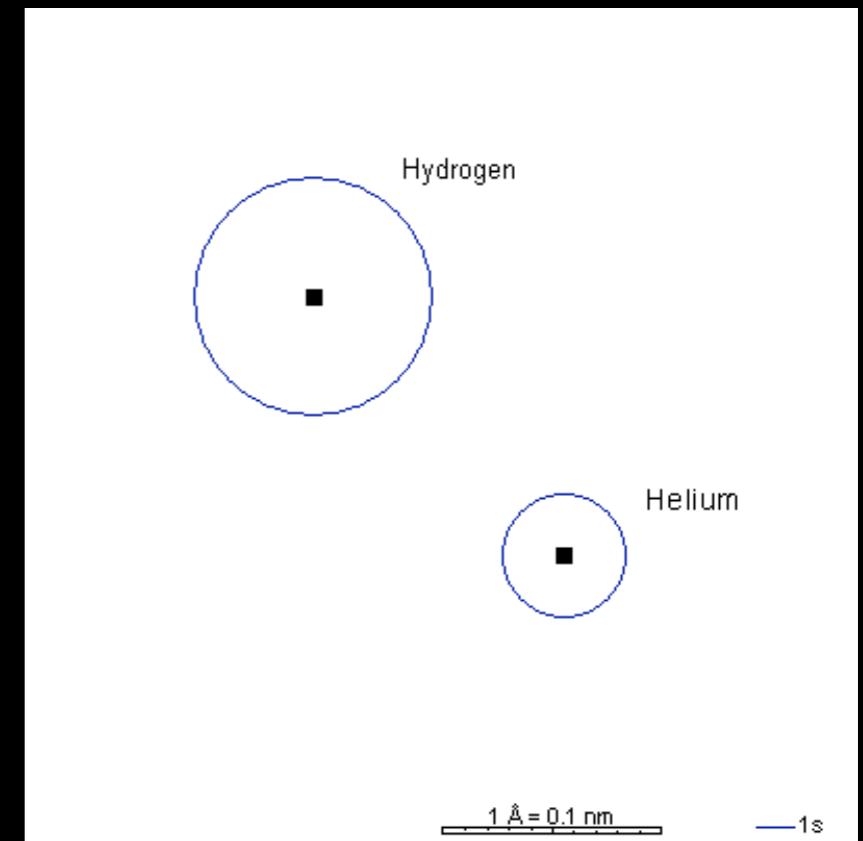
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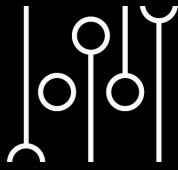
BUT... it would seem

$$\Psi_{2,1} = \Psi_{1,2}$$

So $\Psi_{1,2} = 0$



Pull an accounting trick: $\Psi_{1,2} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} - \begin{pmatrix} \psi_2 \\ \psi_1 \end{pmatrix}$



The discovery of spin Wolfgang Pauli 1924



Paraphrase of the scientific question: Why is helium a noble element and not hydrogen?

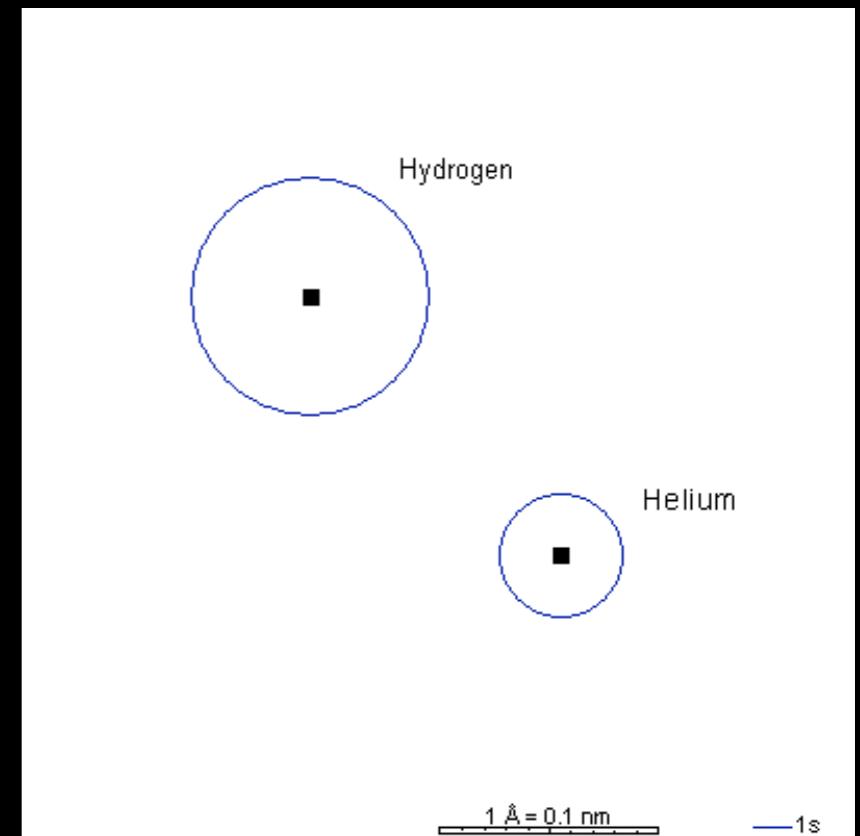
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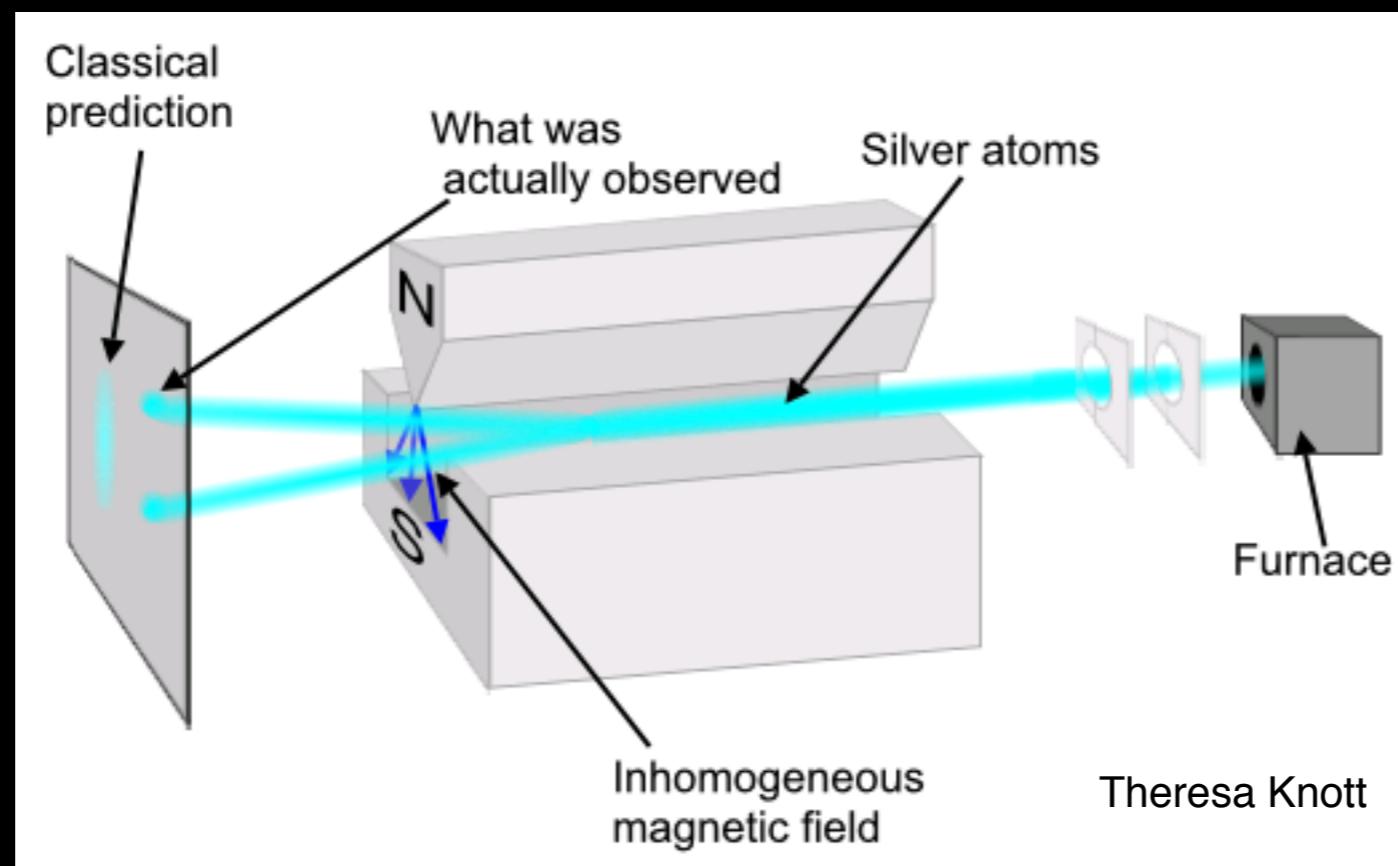
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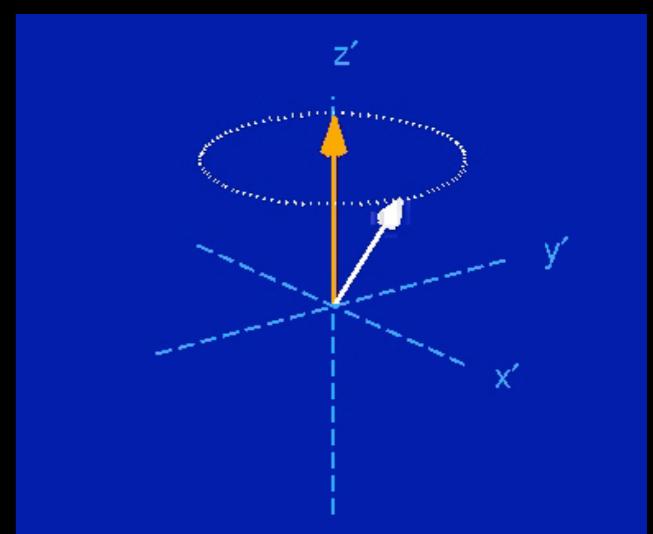
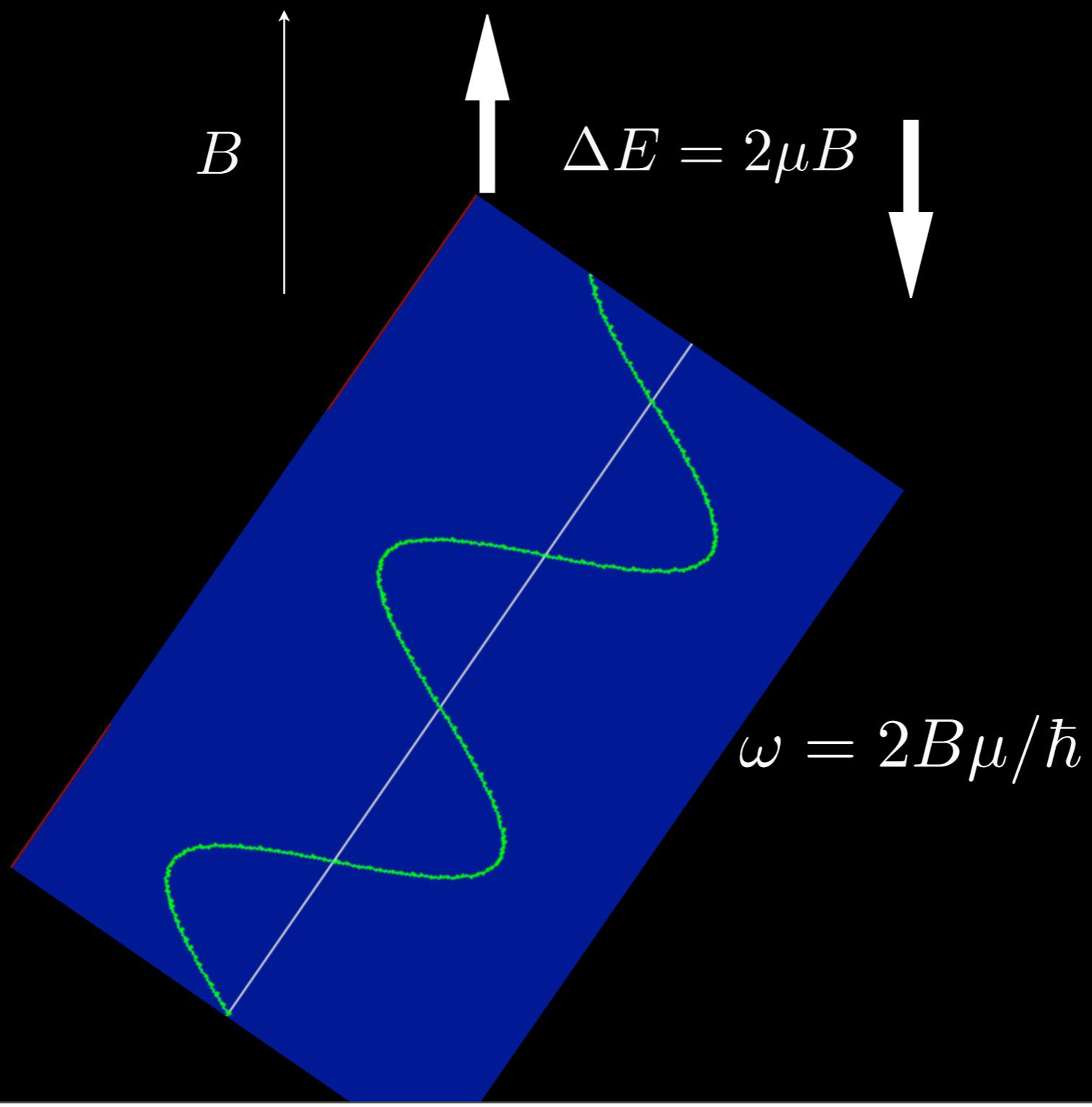
spin up
spin down

Spin as a little quantum magnet: The Stern-Gerlach experiment 1922

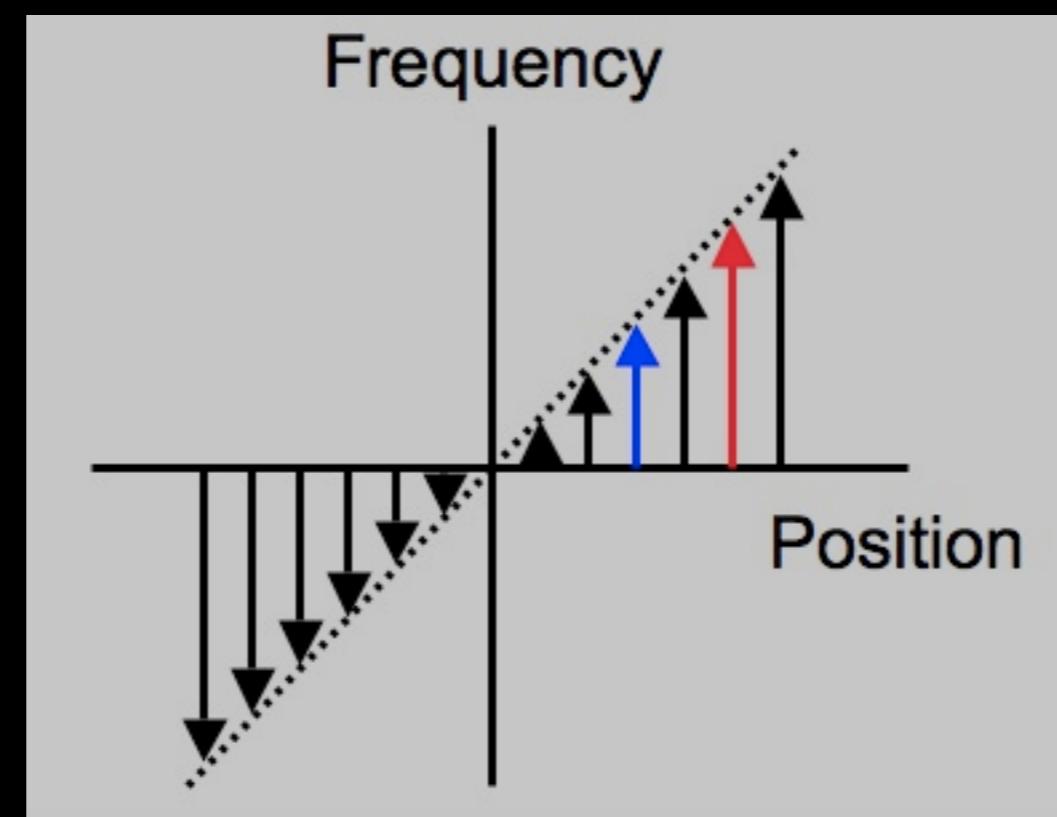


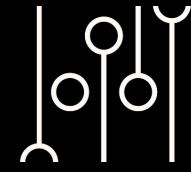
<http://phet.colorado.edu/en/simulation/stern-gerlach>

Nuclear magnetic resonance

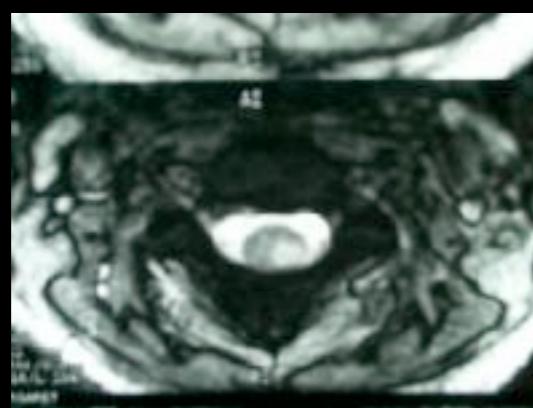


Field gradient





MRI images



Nobel Prizes in Magnetic Resonance

1944



Isidor Isaac Rabi

Nobel Prize in Physics

"For his resonance method for recording the magnetic properties of atomic nuclei."

1952

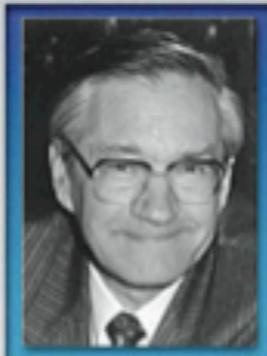


Felix Bloch and Edward Mills Purcell

Nobel Prize in Physics

"For their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith."

1991



Richard R. Ernst

Nobel Prize in Chemistry

"For his contributions to the development of the methodology of high resolution nuclear magnetic resonance (NMR) spectroscopy."

2002



Kurt Wüthrich

Nobel Prize in Chemistry

"For his development of nuclear magnetic resonance spectroscopy for determining the three-dimensional structures of biological macromolecules in solution."

2003



Paul C. Lauterbur and Sir Peter Mansfield

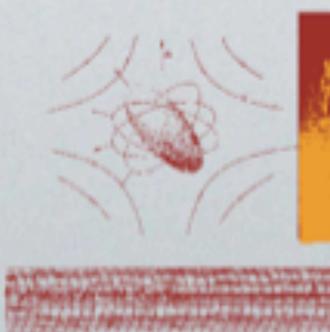
Nobel Prize in Physiology or Medicine

"For their discoveries concerning magnetic resonance imaging."

1930s

Isidor Isaac Rabi

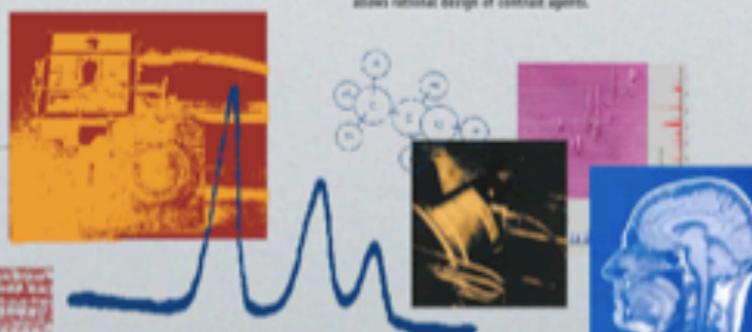
Development of molecular beam magnetic resonance by passing a beam of lithium through a magnetic field and then bombarding the beam with radio waves.



1940s

Felix Bloch
Edward Mills Purcell

Independent demonstration of the phenomenon known as "nuclear magnetic resonance (NMR) in condensed matter."



1950s

Nicolaas Bloembergen
Robert Pound
Edward Mills Purcell

Development of a dipole-dipole theory of magnetic relaxation, revealing the mechanism that creates contrast in vivo and allows rational design of contrast agents.

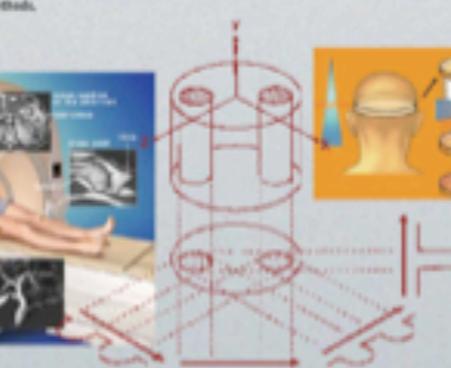
Discovery of the spin echo phenomena for nuclear magnetic resonance measurements.



1960s

Erwin Hahn
Richard R. Ernst
Weston A. Anderson

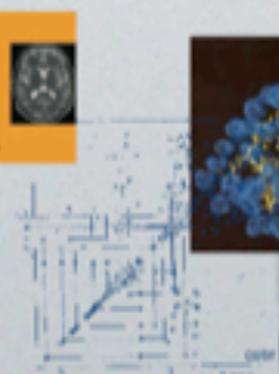
Proof that Fourier analysis of pulsed NMR signals provides increased sensitivity and flexibility over continuous wave NMR methods.



1970s

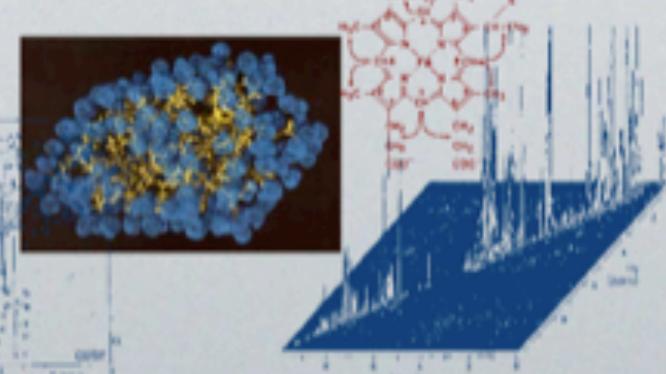
Allan M. Cormack
Godfrey N. Hounsfield

Construction of the first CT scanner; the foundation of nearly every sophisticated imaging system in use today.



Paul C. Lauterbur

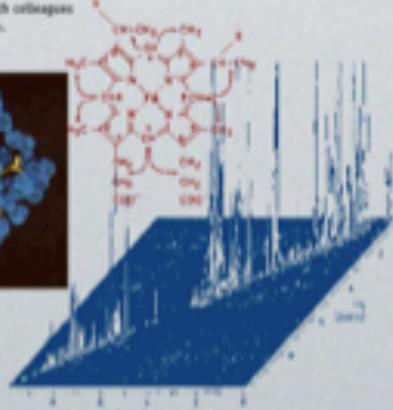
Coupling of the gradient concept to the CT scanner concept of multiple projections and reconstruction to obtain the first MRI.



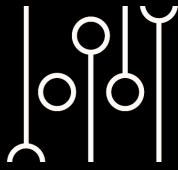
1980s

Sir Peter Mansfield

Use of magnetic field gradients to obtain a 1D projection from layers of voxels. Conception of echo planar imaging which yields complete 2D images from a single excitation. Optimization of slice-selective excitation and publication with colleagues of the first MRI from a human.



Timeline of the Chain of Research that Led to the Development of MRI



Echograms to structure



Physics

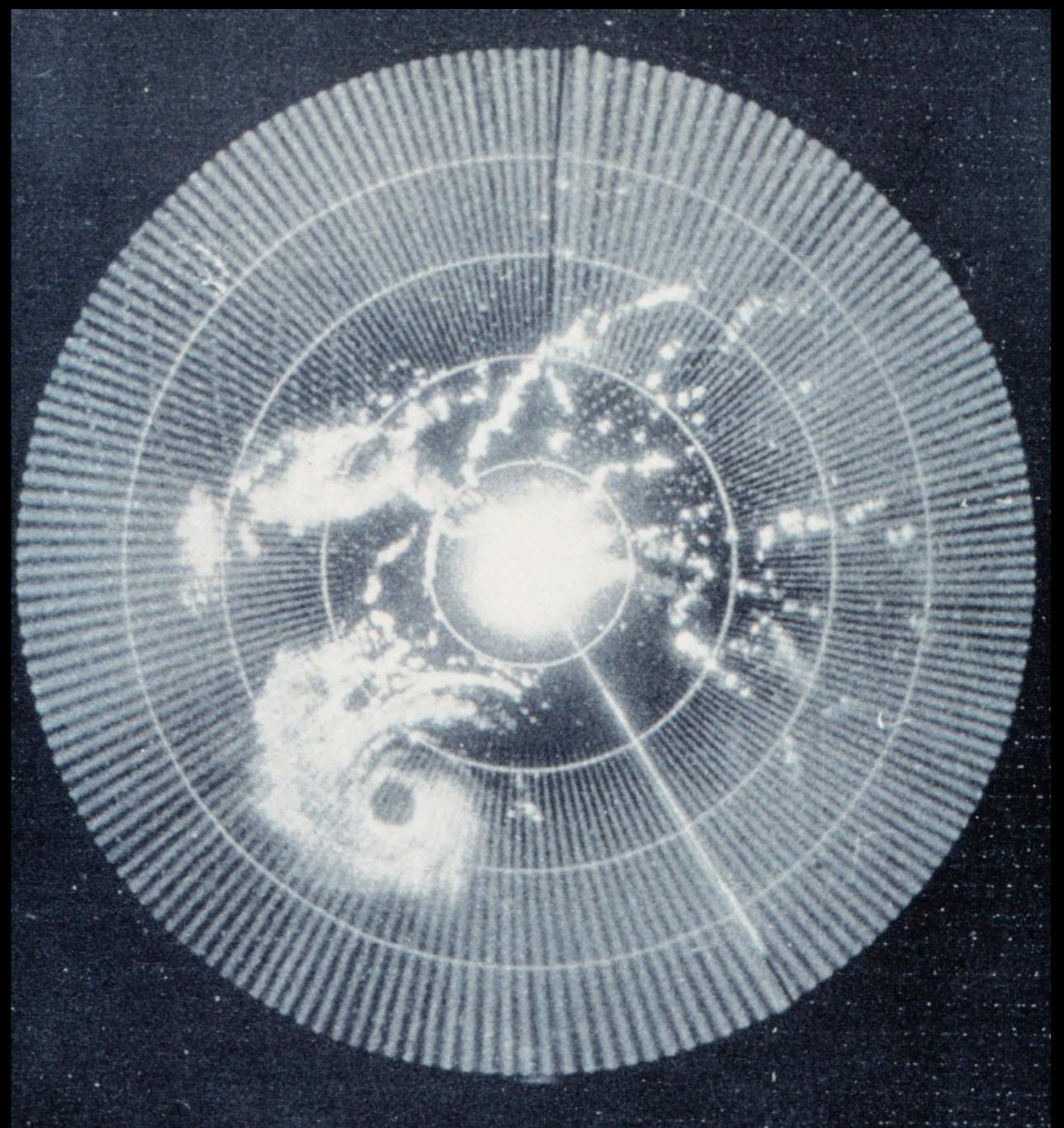


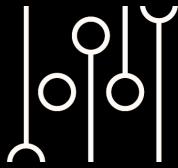
$$S(\mathbf{r}, \omega) = A(\omega) \int d^3 r' h(\mathbf{r}' - \mathbf{r}, \omega) \eta(\mathbf{r})$$

Maxwell Eq. + Reflectance → Stolt mapping

Radar

- J. C. Maxwell 1861 - Light is a disturbance of the electromagnetic field
- Heinrich Hertz 1886
- Marconi 1899
- Taylor and Young 1922





SAR

- Carl Wiley 1951
- Chalmers Sherwin (UIUC) 1952

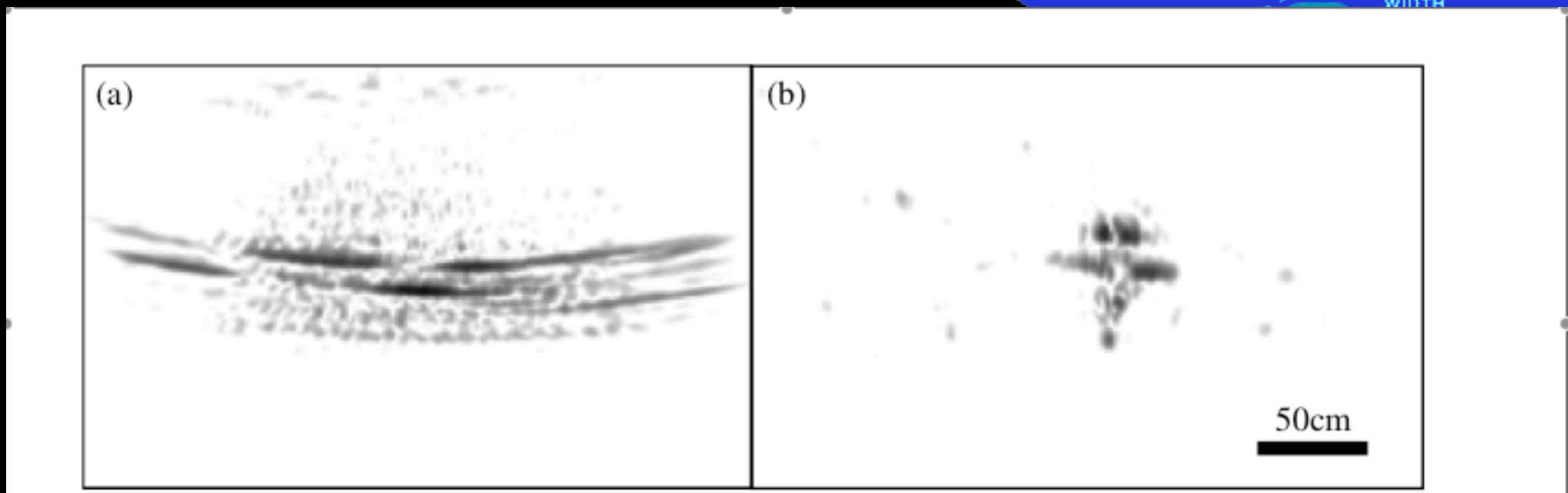
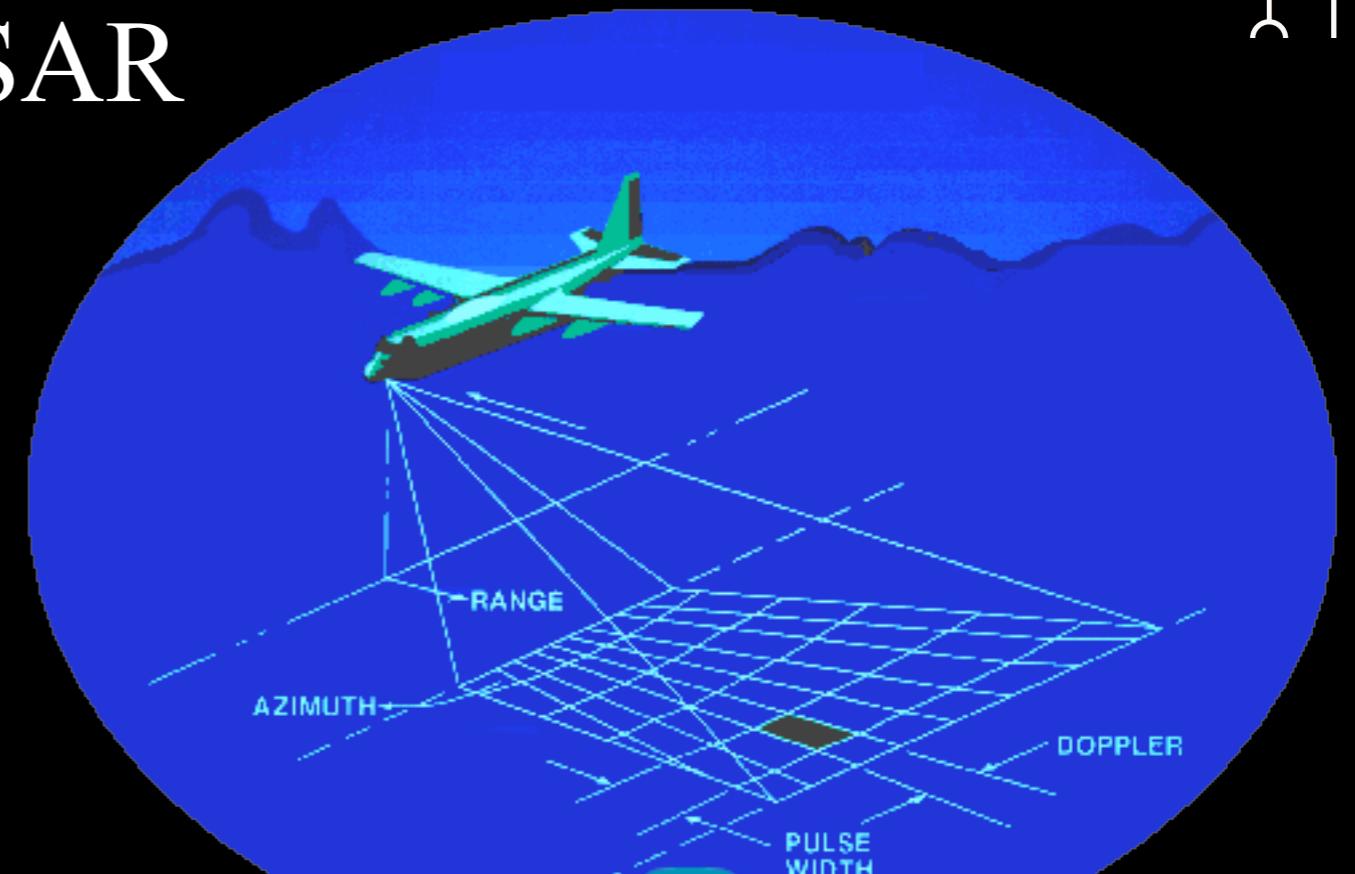
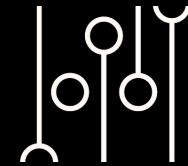


Figure 9. Raw strip-map radar image of a 1:32 scale model of a F14 fighter aircraft before Stolt Fourier resampling (a), and after Stolt Fourier resampling (b).

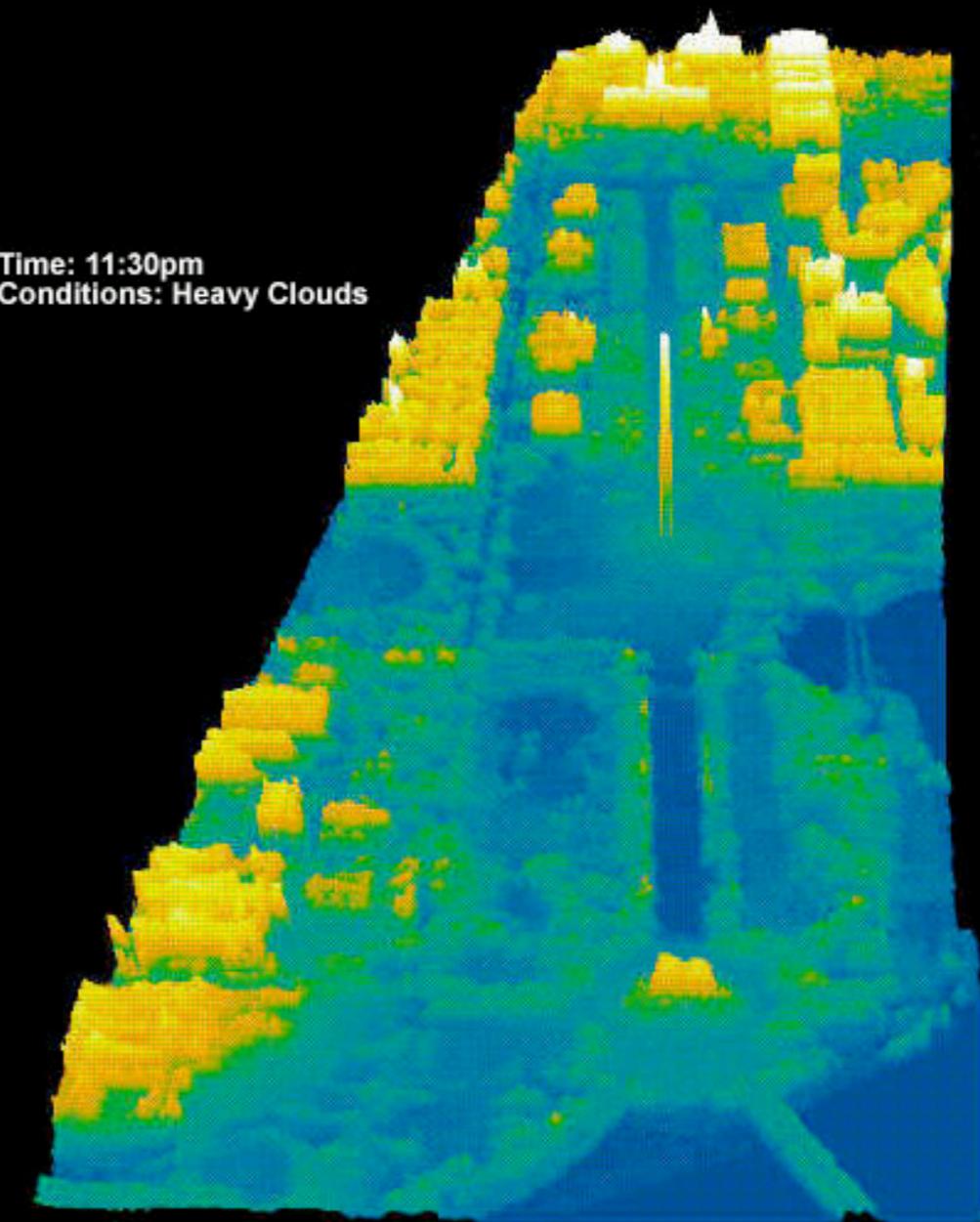
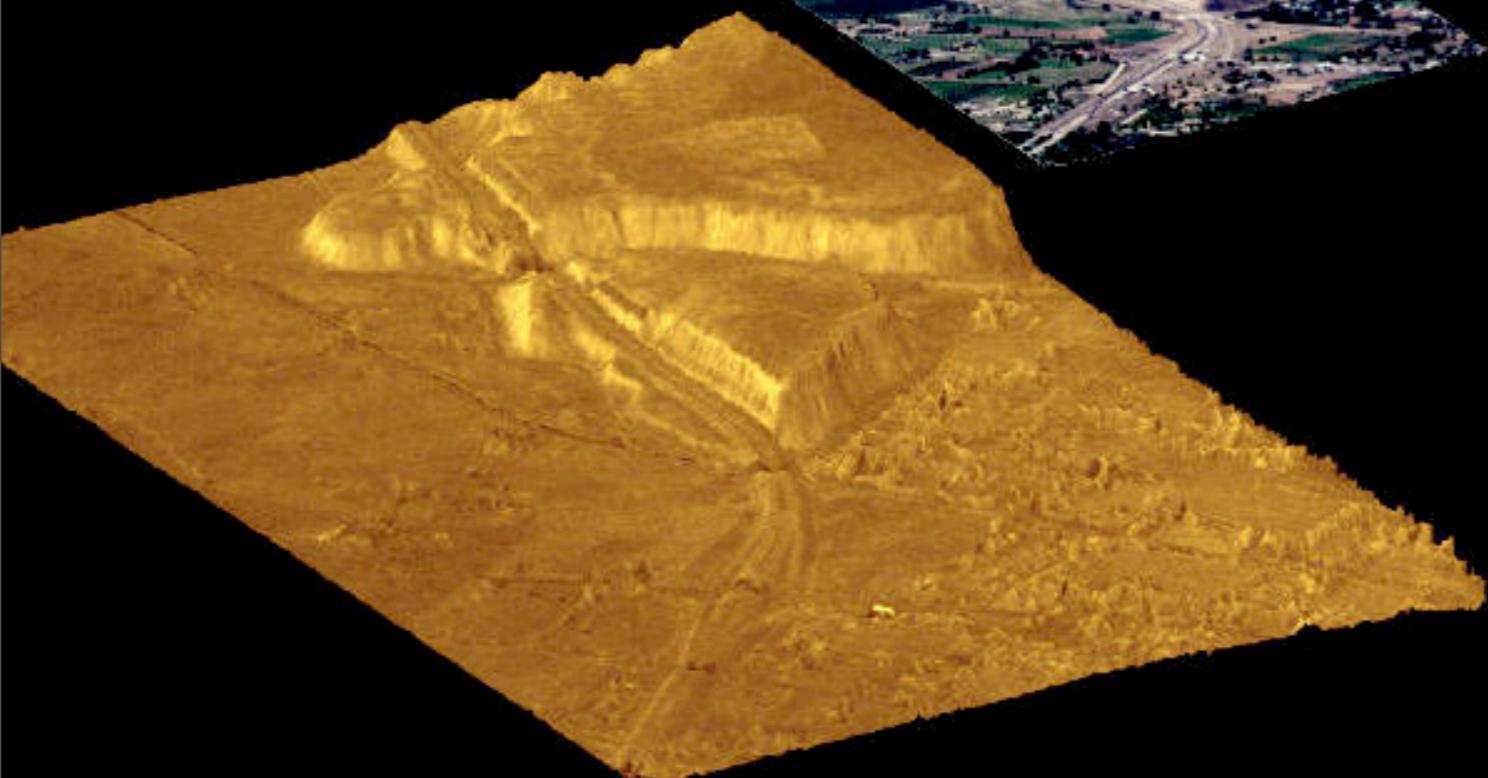


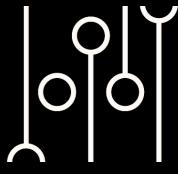
SAR images

**Black Mesa Area, I-25 and US-85
South of Albuquerque, NM - 22 December, 1994**



Time: 11:30pm
Conditions: Heavy Clouds





The common approach

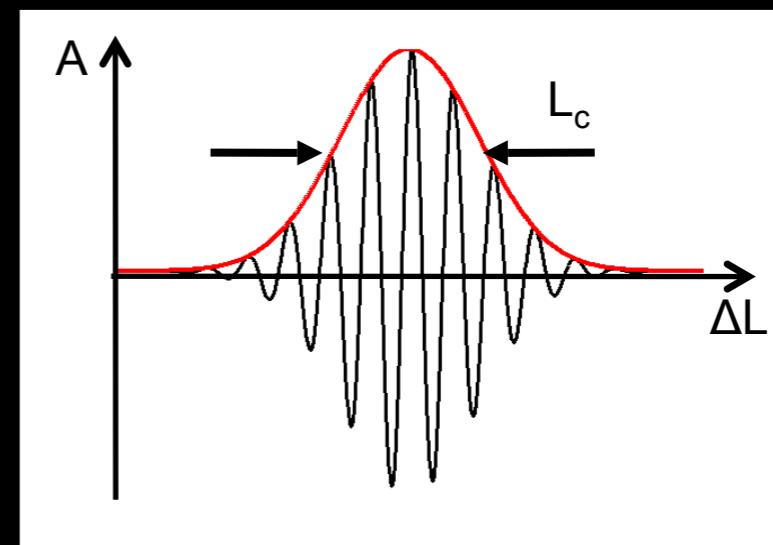
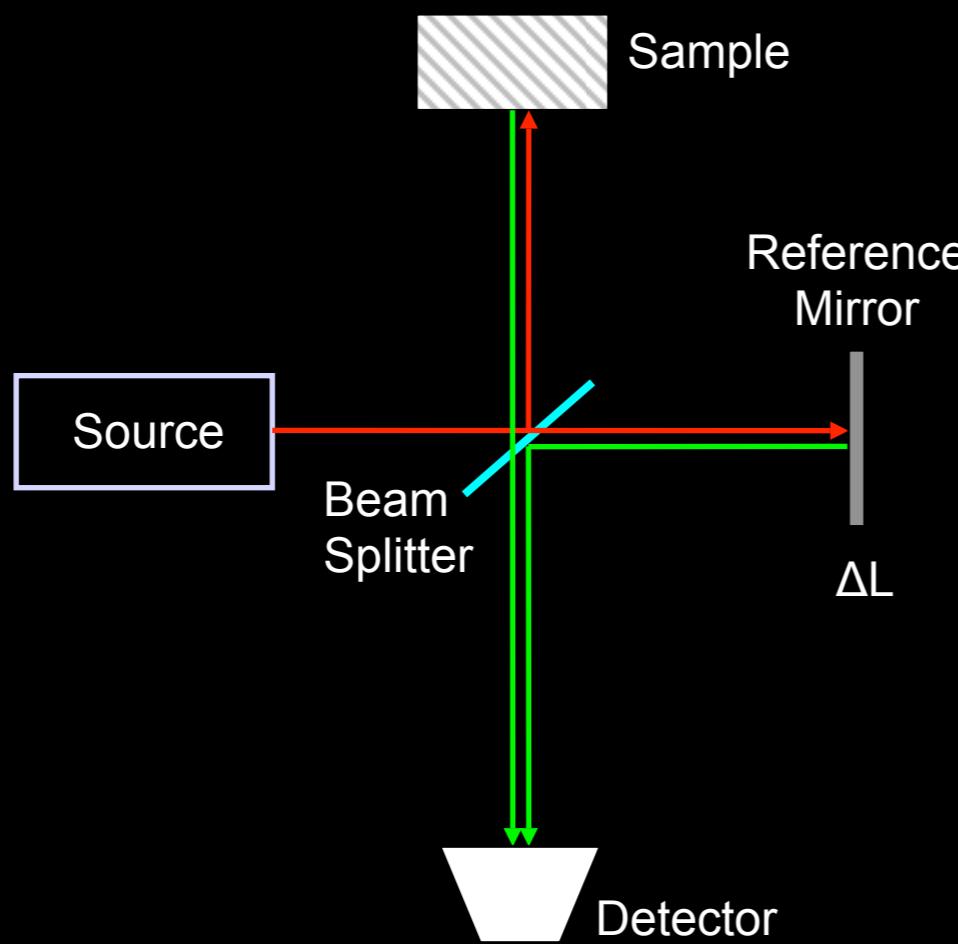
$$S = K\eta$$

A linearizable forward model

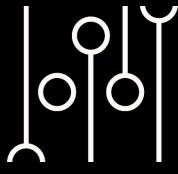
$$\hat{\eta} = K^+ S$$

A stable psudo-inverse

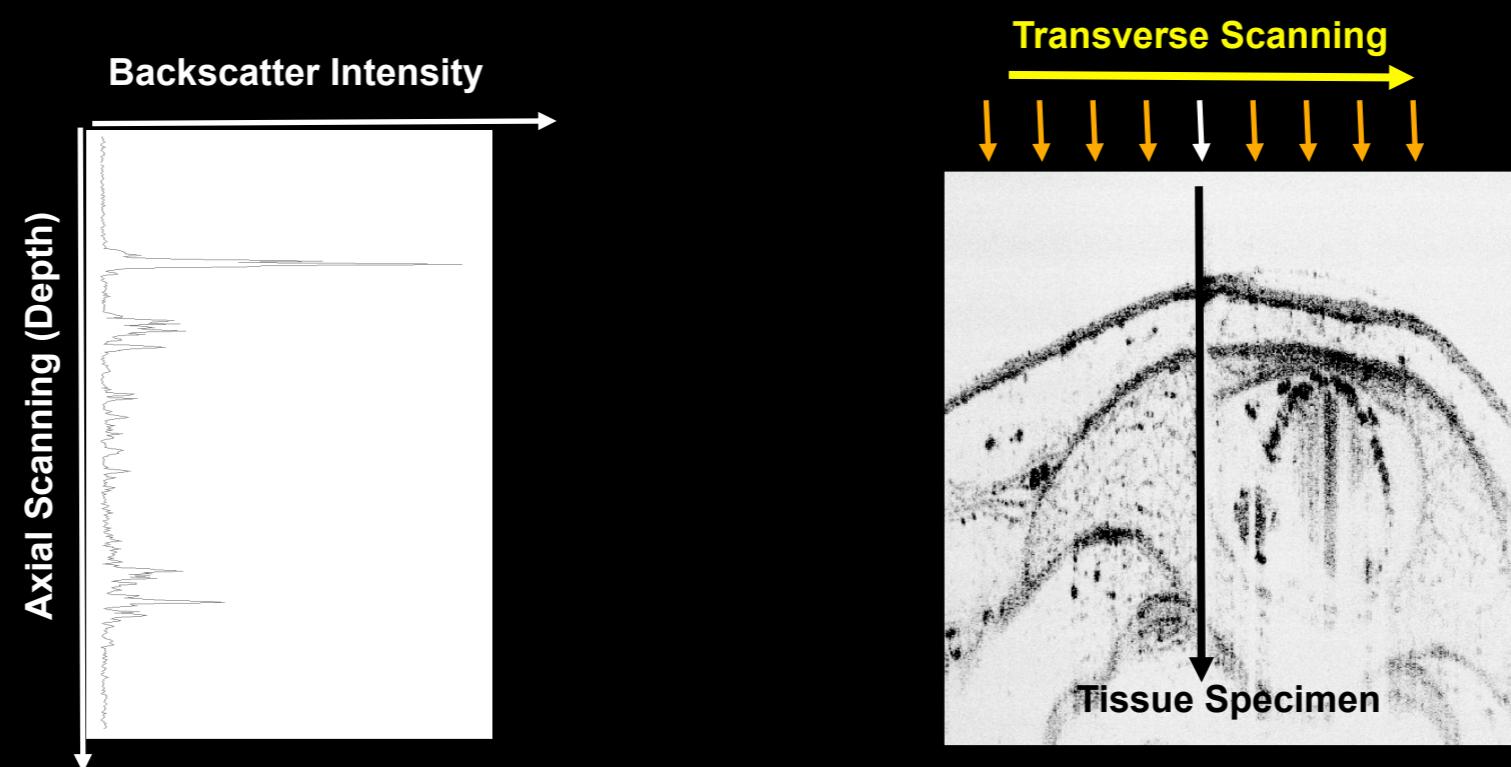
Optical Coherence Tomography: range finding with low- coherence interferometry



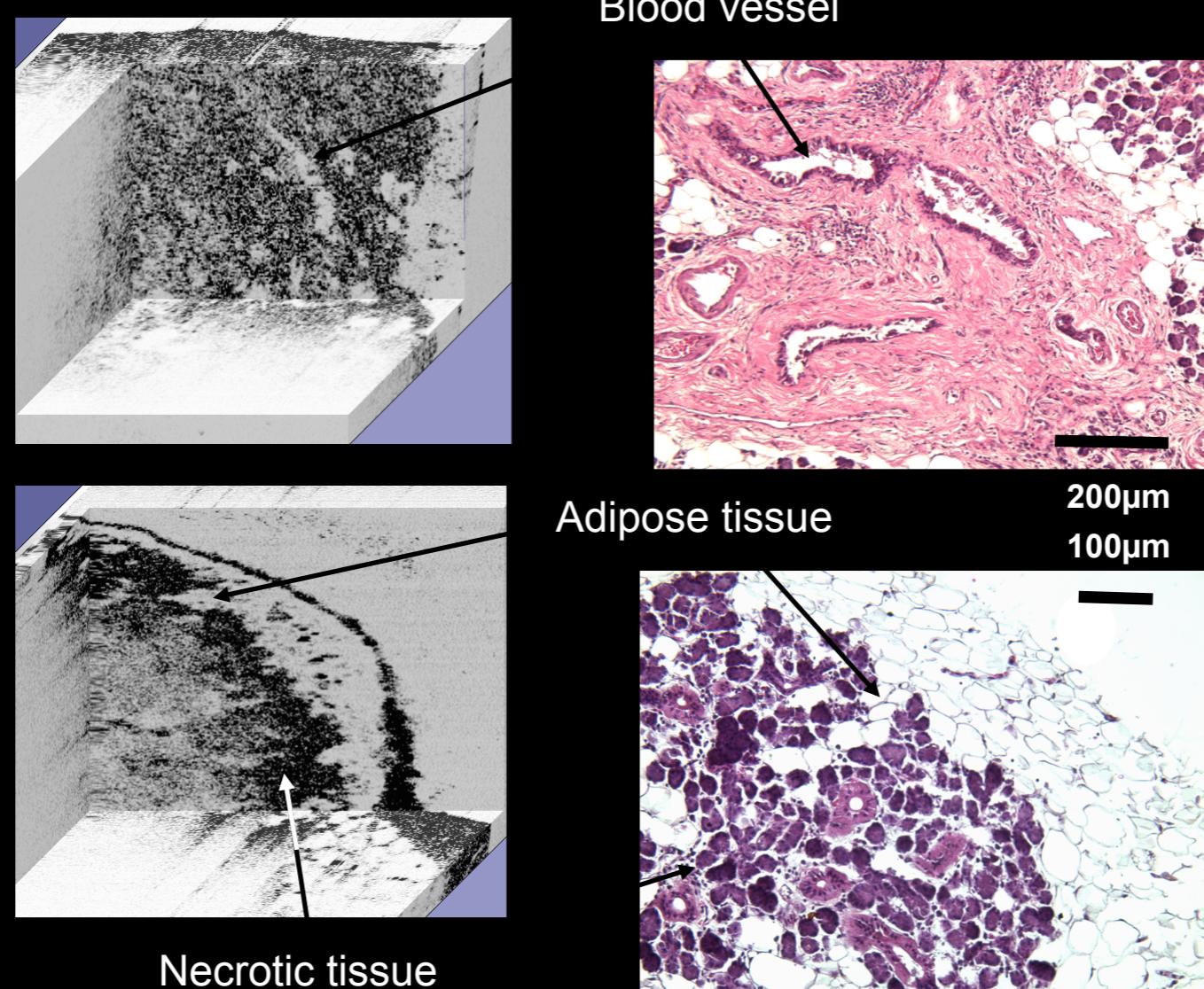
Signal processing:
speckle reduction,
numerical dispersion
correction



Transverse scanning in OCT imaging



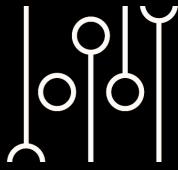
OCT vs histology - human tumor



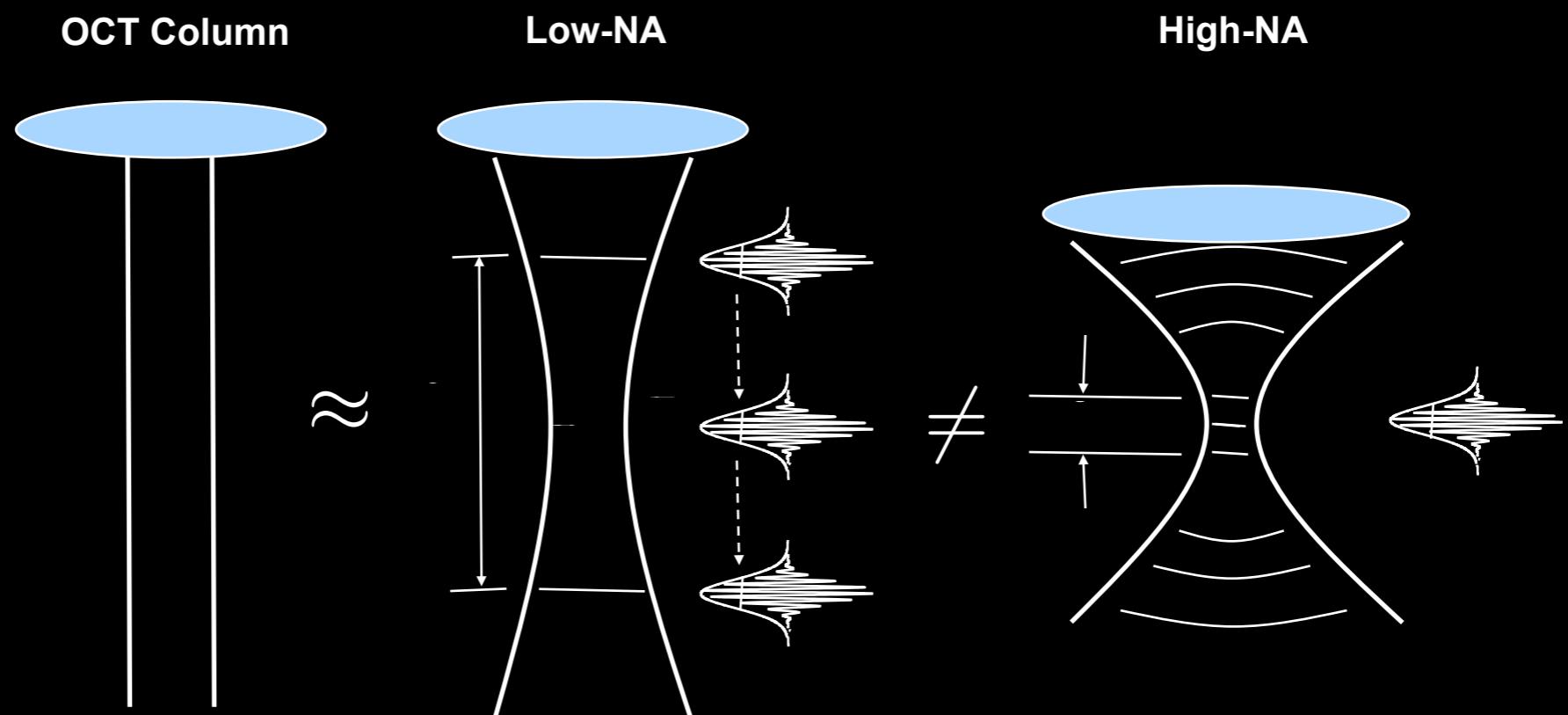
Luo W, Nguyen FT, Zysk AM, Ralston TS, Brockenbrough J, Marks DL, Oldenburg AL, Boppart SA, "Optical Biopsy of Lymph Node Morphology using Optical Coherence Tomography." *Technology in Cancer Research and Treatment*, 4 (5), 539-547, October 2005.

The problem



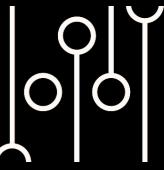


Beam Focusing



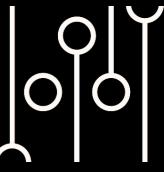
B. Hermann, et.al., "Adaptive-optics ultrahigh-resolution optical coherence tomography," Opt. Let., vol. 29, pp. 2142-2144, 2004.

W. Drexler et. al., "In vivo ultrahigh-resolution optical coherence tomography," Opt. Let. Vol.24 No.17 pp. 1221-1223

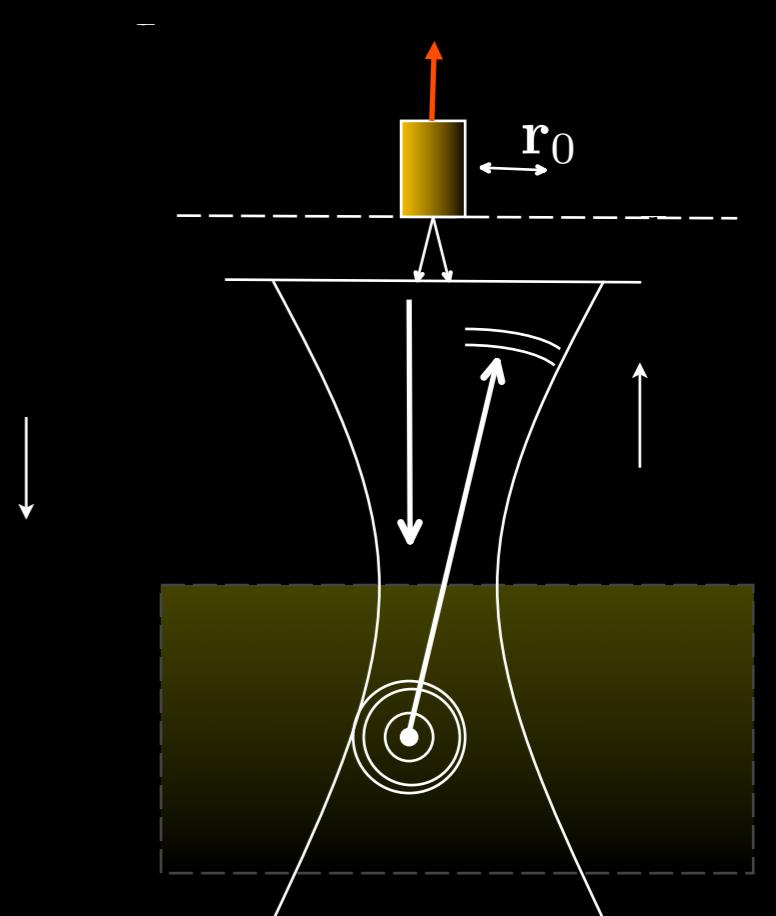


Strategy

- Develop forward model for illumination, light-sample interaction, and detection
- Simulate and compare to reality
- Apply appropriate approximations
- Invert, analytically if possible



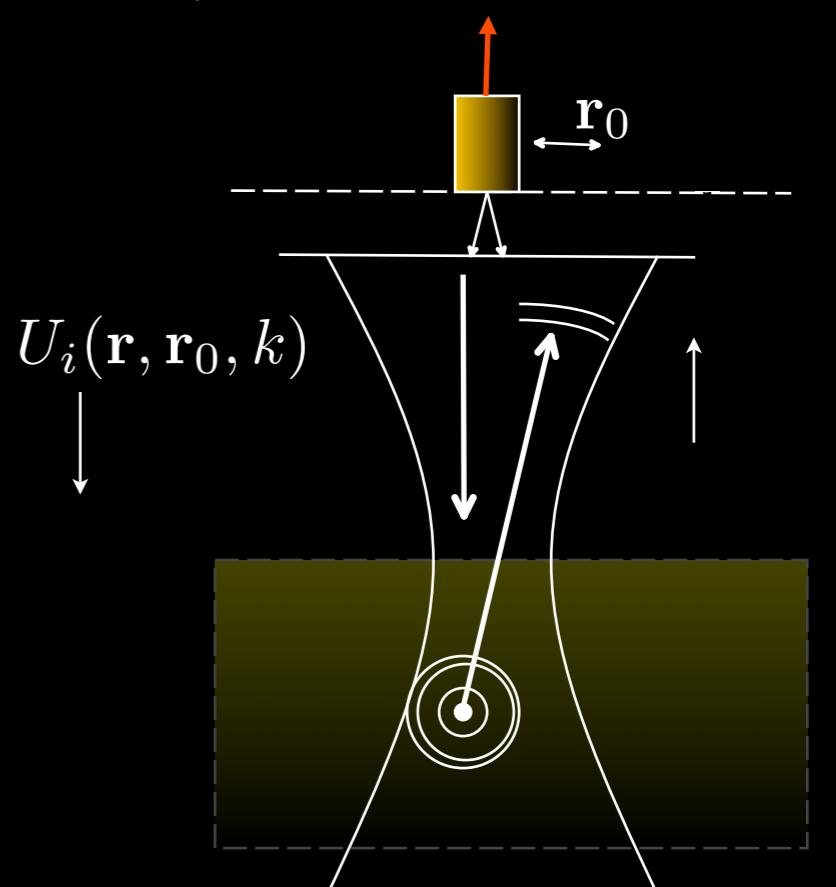
Mathematics





Mathematics

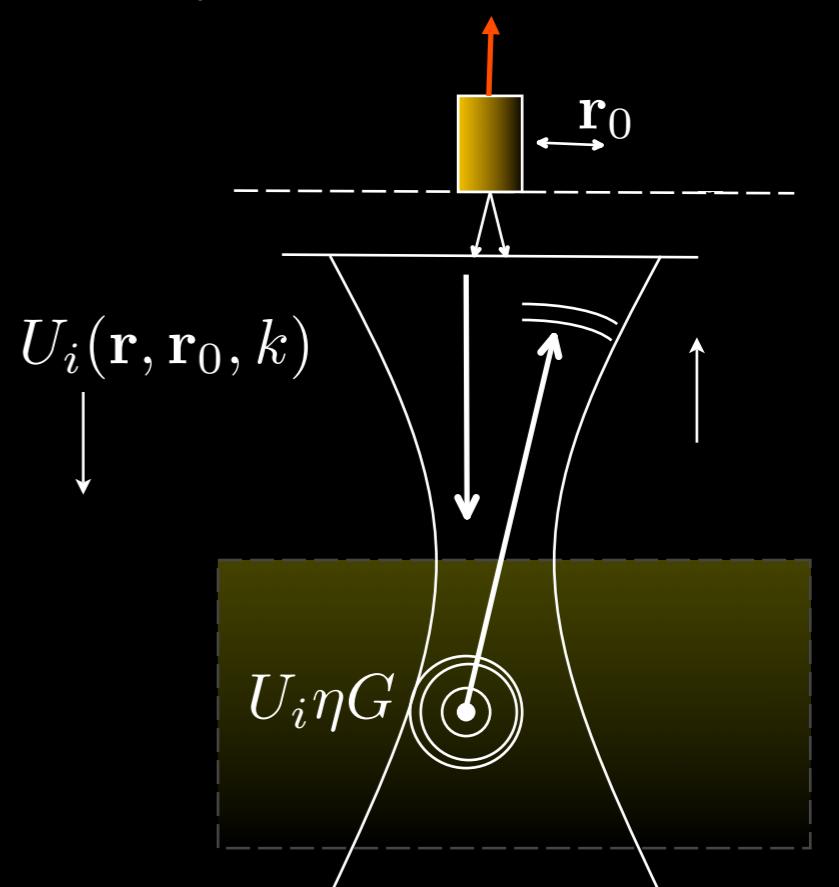
$$U_i(\mathbf{r}, \mathbf{r}_0, k) = A(k)g(\mathbf{r} - \mathbf{r}_0)$$





Mathematics

$$U_i(\mathbf{r}, \mathbf{r}_0, k) = A(k)g(\mathbf{r} - \mathbf{r}_0)$$

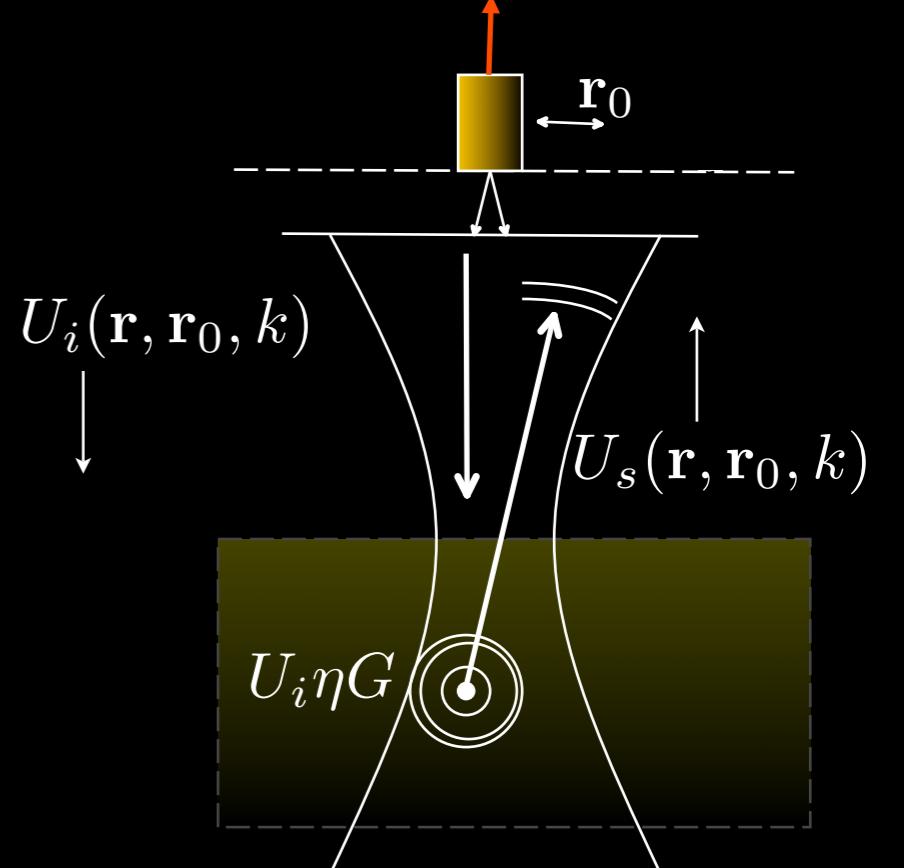




Mathematics

$$U_i(\mathbf{r}, \mathbf{r}_0, k) = A(k)g(\mathbf{r} - \mathbf{r}_0)$$

$$U_s(\mathbf{r}, \mathbf{r}_0, k) = \int d^3r' G(\mathbf{r}', \mathbf{r}, k)\eta(\mathbf{r}')U_i(\mathbf{r}', \mathbf{r}_0, k)$$



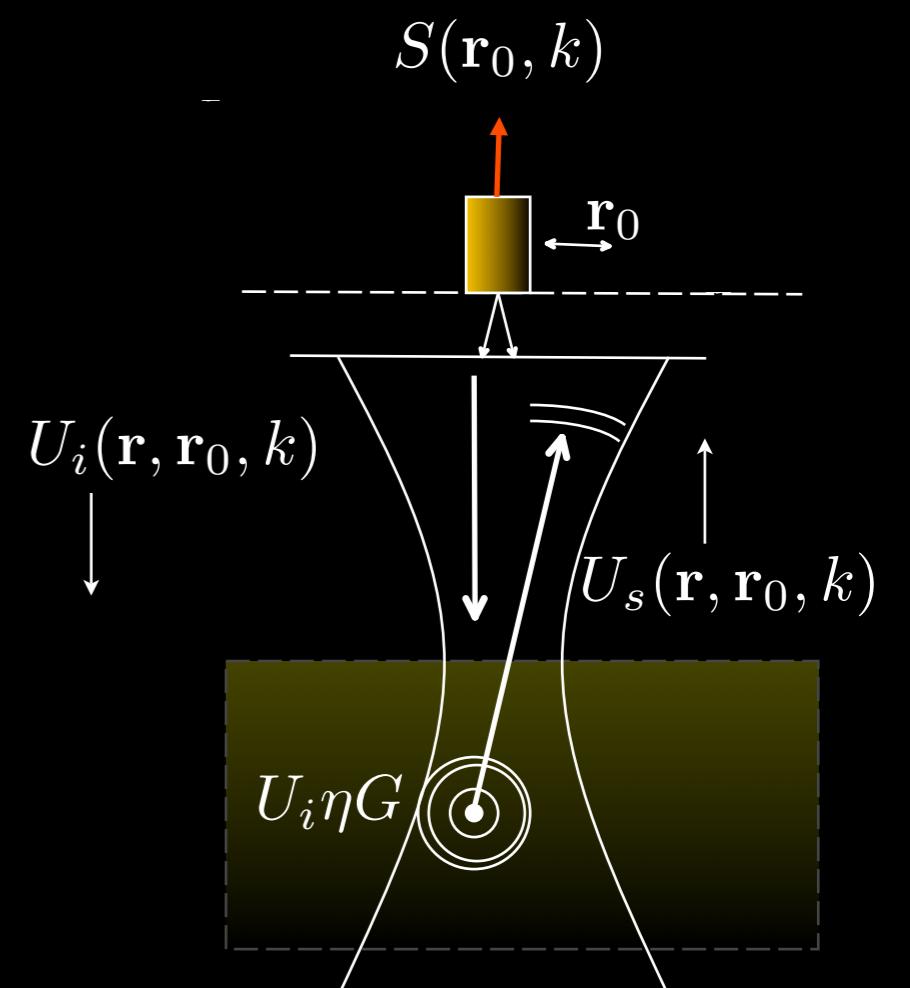


Mathematics

$$U_i(\mathbf{r}, \mathbf{r}_0, k) = A(k)g(\mathbf{r} - \mathbf{r}_0)$$

$$U_s(\mathbf{r}, \mathbf{r}_0, k) = \int d^3r' G(\mathbf{r}', \mathbf{r}, k)\eta(\mathbf{r}')U_i(\mathbf{r}', \mathbf{r}_0, k)$$

$$S(\mathbf{r}_0, k) = \int_{z=0} d^2r U(\mathbf{r}, \mathbf{r}_0, k)g(\mathbf{r} - \mathbf{r}_0, k)$$





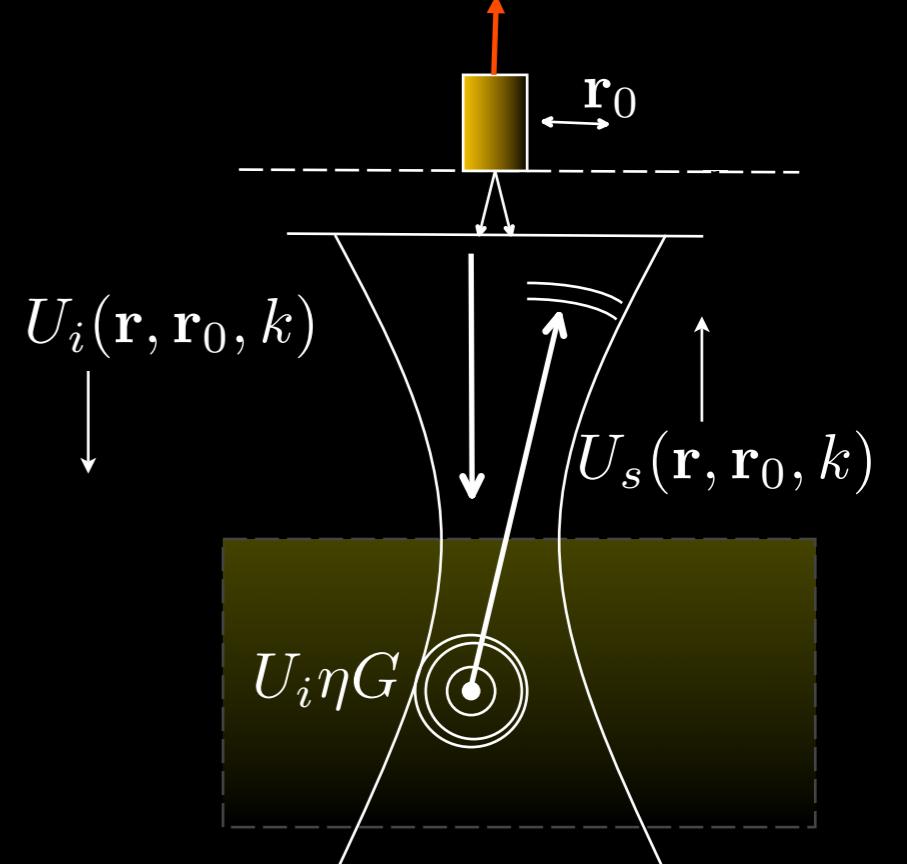
Mathematics

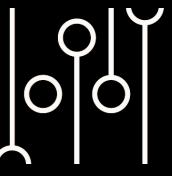
$$U_i(\mathbf{r}, \mathbf{r}_0, k) = A(k)g(\mathbf{r} - \mathbf{r}_0)$$

$$U_s(\mathbf{r}, \mathbf{r}_0, k) = \int d^3r' G(\mathbf{r}', \mathbf{r}, k)\eta(\mathbf{r}')U_i(\mathbf{r}', \mathbf{r}_0, k)$$

$$S(\mathbf{r}_0, k) = \int_{z=0} d^2r U(\mathbf{r}, \mathbf{r}_0, k)g(\mathbf{r} - \mathbf{r}_0, k)$$

$$S(\mathbf{r}_0, k) = A(k) \int_{z=0} d^2r \int d^3r' G(\mathbf{r}', \mathbf{r}, k)g(\mathbf{r}' - \mathbf{r}_0, k)g(\mathbf{r} - \mathbf{r}_0, k)\eta(\mathbf{r}').$$



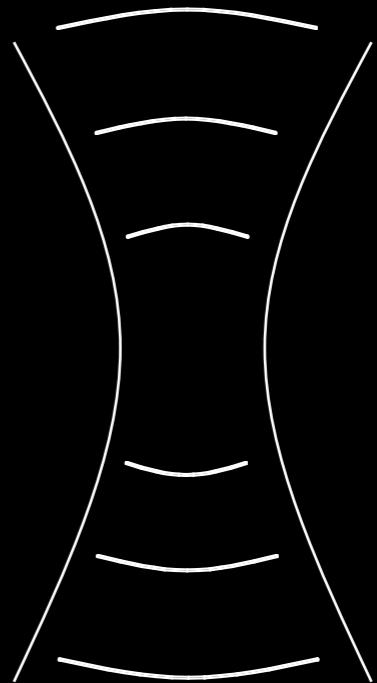


More math

$$g(\mathbf{r}, k) = \frac{1}{2\pi W_0^2(k)} e^{-r^2/2W_0^2(k)}$$

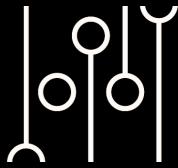
$$\alpha = \pi/NA \qquad \qquad W_0(k) = \alpha/k$$

$$\tilde{g}(\mathbf{q}, k) = e^{-q^2 W_0^2/2} = e^{-q^2 \alpha^2/(2k^2)}$$

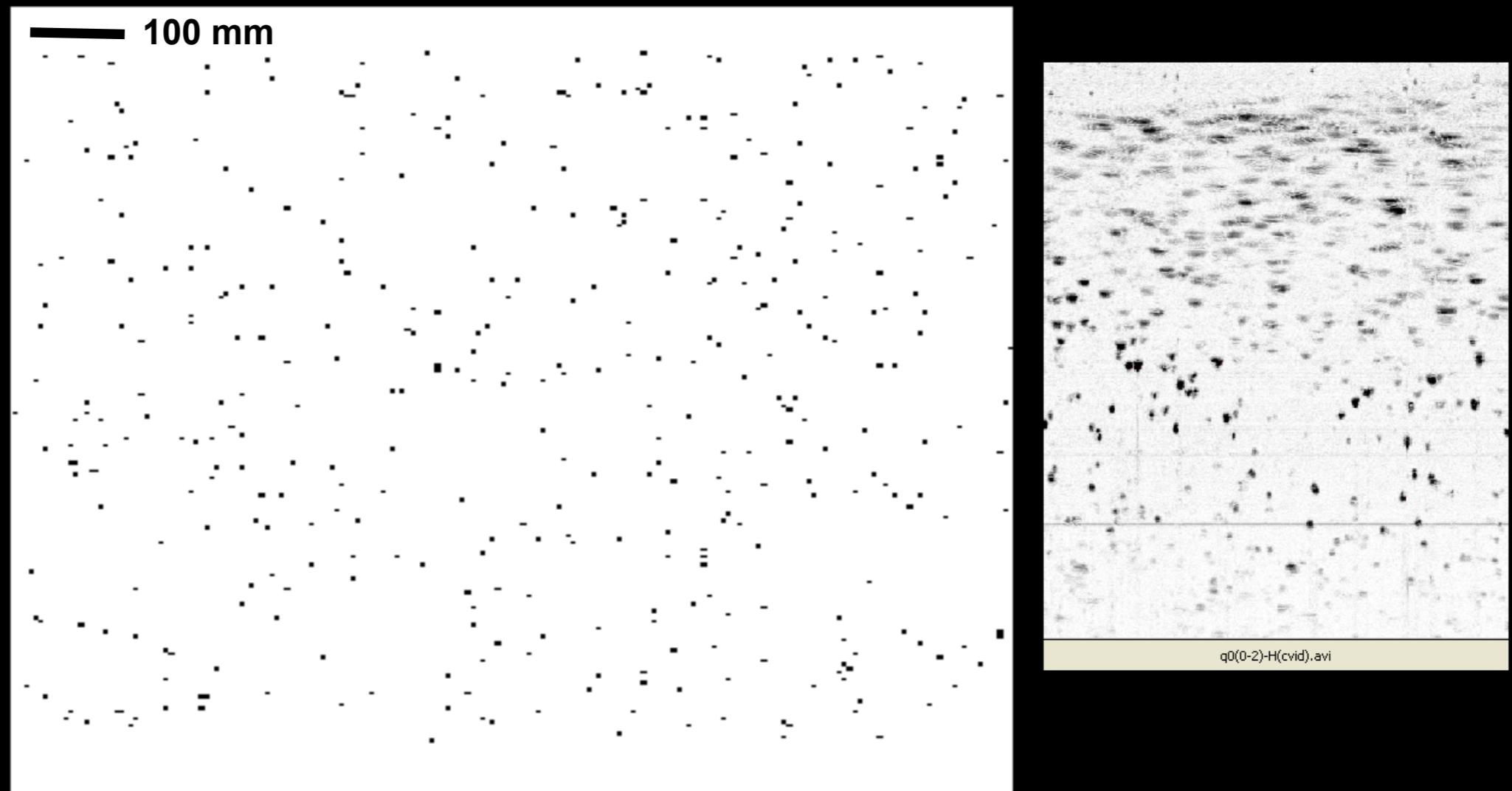


$$G(\mathbf{r}', \mathbf{r}, k) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} = \frac{i}{2\pi} \int d^2 q \ e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \frac{e^{-ik_z(\mathbf{q})(z-z')}}{k_z(\mathbf{q})}$$

$$k_z(\mathbf{q}) = \sqrt{k^2 - q^2}$$



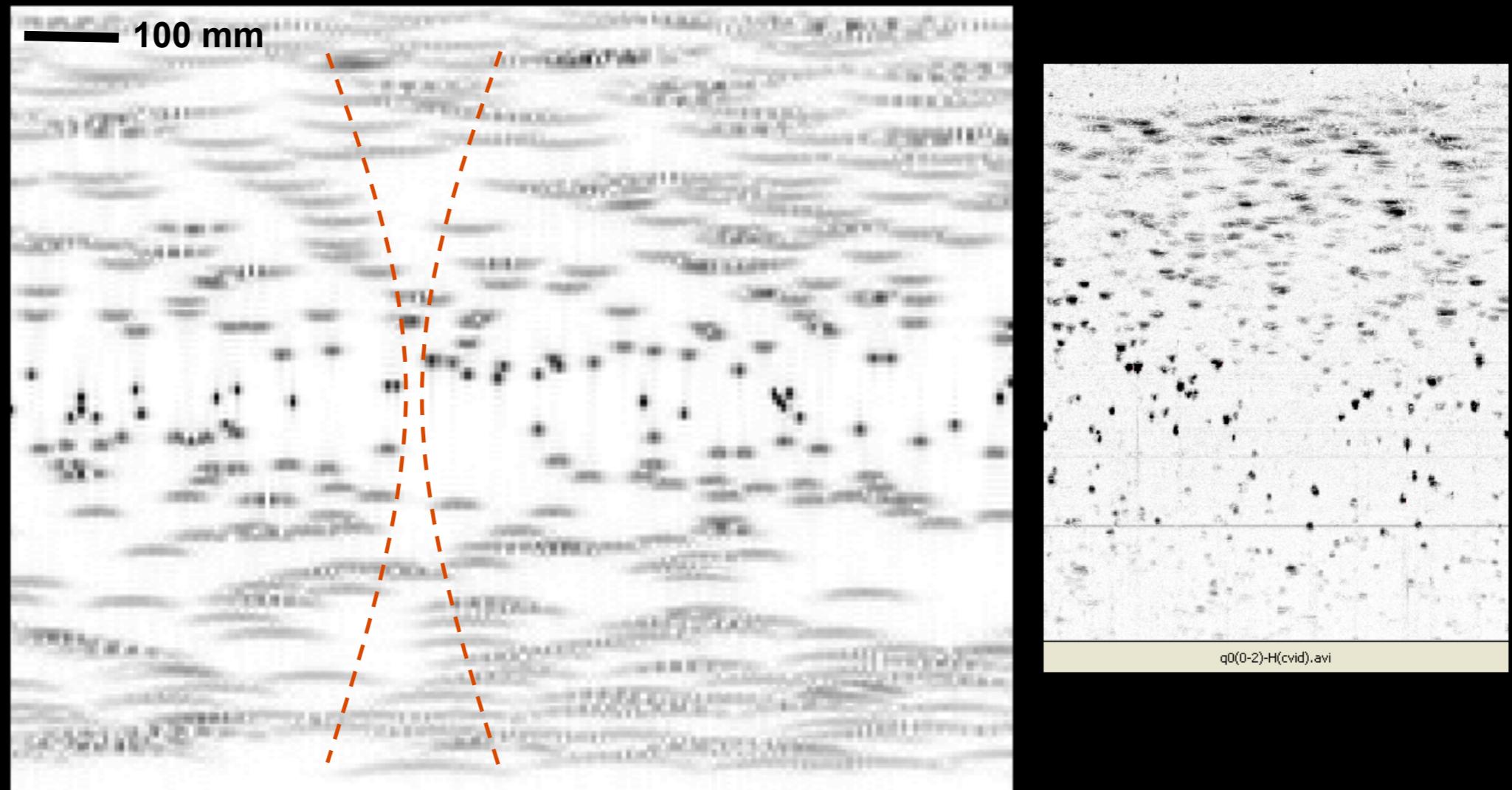
Forward simulation



T.S. Ralston, D.L. Marks, F. Kamalabadi, S.A. Boppart. "Deconvolution methods for mitigation of transverse blurring in optical coherence tomography." IEEE Trans. Image Proc. Special Issue on Molecular and Cellular Bioimaging, vol.14, no. 9, September 2005.

T.S. Ralston, D.L. Marks, P.S. Carney, S.A. Boppart, "Inverse scattering for optical coherence tomography," JOSA A, in press.

Forward simulation



T.S. Ralston, D.L. Marks, F. Kamalabadi, S.A. Boppart. "Deconvolution methods for mitigation of transverse blurring in optical coherence tomography." IEEE Trans. Image Proc. Special Issue on Molecular and Cellular Bioimaging, vol.14, no. 9, September 2005.

T.S. Ralston, D.L. Marks, P.S. Carney, S.A. Boppart, "Inverse scattering for optical coherence tomography," JOSA A, in press.



The inverse problem

$$S(\mathbf{r}_0, k) = A(k) \int_{z=0} d^2 r \int d^3 r' G(\mathbf{r}', \mathbf{r}, k) g(\mathbf{r}' - \mathbf{r}_0, k) g(\mathbf{r} - \mathbf{r}_0, k) \eta(\mathbf{r}').$$



The inverse problem

$$S(\mathbf{r}_0, k) = A(k) \int_{z=0} d^2 r \int d^3 r' G(\mathbf{r}', \mathbf{r}, k) g(\mathbf{r}' - \mathbf{r}_0, k) g(\mathbf{r} - \mathbf{r}_0, k) \eta(\mathbf{r}').$$

$$\begin{aligned} \tilde{S}(\mathbf{Q}, k) &= i2\pi A(k) \int d^2 q \int dz' \frac{1}{k_z(\mathbf{q})} e^{ik_z(\mathbf{q})(z' - z_0)} e^{ik_z(\mathbf{q} - \mathbf{Q})(z' - z_0)} \\ &\quad \times e^{\frac{-\alpha^2 Q^2}{4k^2}} e^{\frac{-\alpha^2 |\mathbf{q} - \mathbf{Q}/2|^2}{k^2}} \tilde{\eta}(\mathbf{Q}, z') \end{aligned}$$

The inverse problem

$$S(\mathbf{r}_0, k) = A(k) \int_{z=0} d^2 r \int d^3 r' G(\mathbf{r}', \mathbf{r}, k) g(\mathbf{r}' - \mathbf{r}_0, k) g(\mathbf{r} - \mathbf{r}_0, k) \eta(\mathbf{r}').$$

$$\tilde{S}(\mathbf{Q}, k) = i2\pi A(k) \int d^2 q \int dz' \frac{1}{k_z(\mathbf{q})} e^{ik_z(\mathbf{q})(z' - z_0)} e^{ik_z(\mathbf{q} - \mathbf{Q})(z' - z_0)} \\ \times e^{\frac{-\alpha^2 Q^2}{4k^2}} e^{\frac{-\alpha^2 |\mathbf{q} - \mathbf{Q}/2|^2}{k^2}} \tilde{\eta}(\mathbf{Q}, z')$$

asymptotic expansion

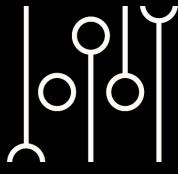
The inverse problem

$$S(\mathbf{r}_0, k) = A(k) \int_{z=0} d^2 r \int d^3 r' G(\mathbf{r}', \mathbf{r}, k) g(\mathbf{r}' - \mathbf{r}_0, k) g(\mathbf{r} - \mathbf{r}_0, k) \eta(\mathbf{r}').$$

$$\begin{aligned} \tilde{S}(\mathbf{Q}, k) &= i2\pi A(k) \int d^2 q \int dz' \frac{1}{k_z(\mathbf{q})} e^{ik_z(\mathbf{q})(z' - z_0)} e^{ik_z(\mathbf{q} - \mathbf{Q})(z' - z_0)} \\ &\quad \times e^{\frac{-\alpha^2 Q^2}{4k^2}} e^{\frac{-\alpha^2 |\mathbf{q} - \mathbf{Q}/2|^2}{k^2}} \tilde{\eta}(\mathbf{Q}, z') \end{aligned}$$

asymptotic expansion

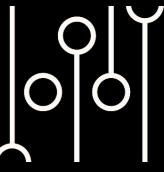
$$\tilde{S}(\mathbf{Q}, k) = \frac{k^2}{\alpha^2} i2\pi^2 A(k) \frac{e^{-2ik_z(\mathbf{Q}/2)z_0}}{k_z(\mathbf{Q}/2)} e^{\frac{-\alpha^2 Q^2}{4k^2}} \tilde{\eta}[\mathbf{Q}, -2k_z(\mathbf{Q}/2)]$$



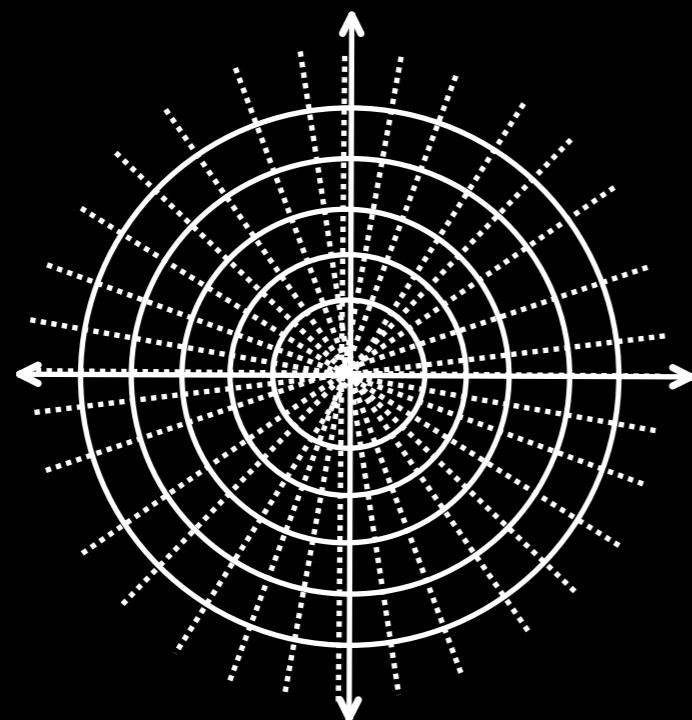
Solution and regularization

$$\tilde{S}(\mathbf{Q}, k) = K(\mathbf{Q}, k) \tilde{\eta} [\mathbf{Q}, -2k_z(\mathbf{Q}/2)]$$

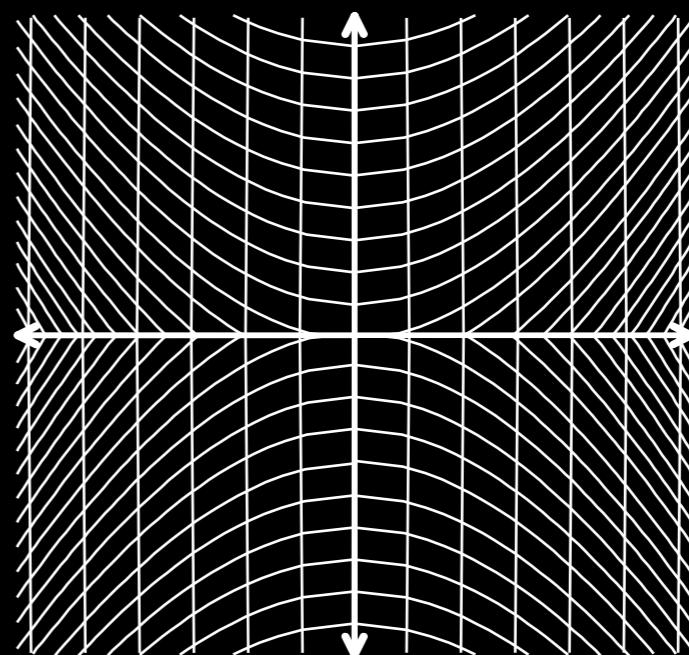
Interferometric synthetic
aperture microscopy



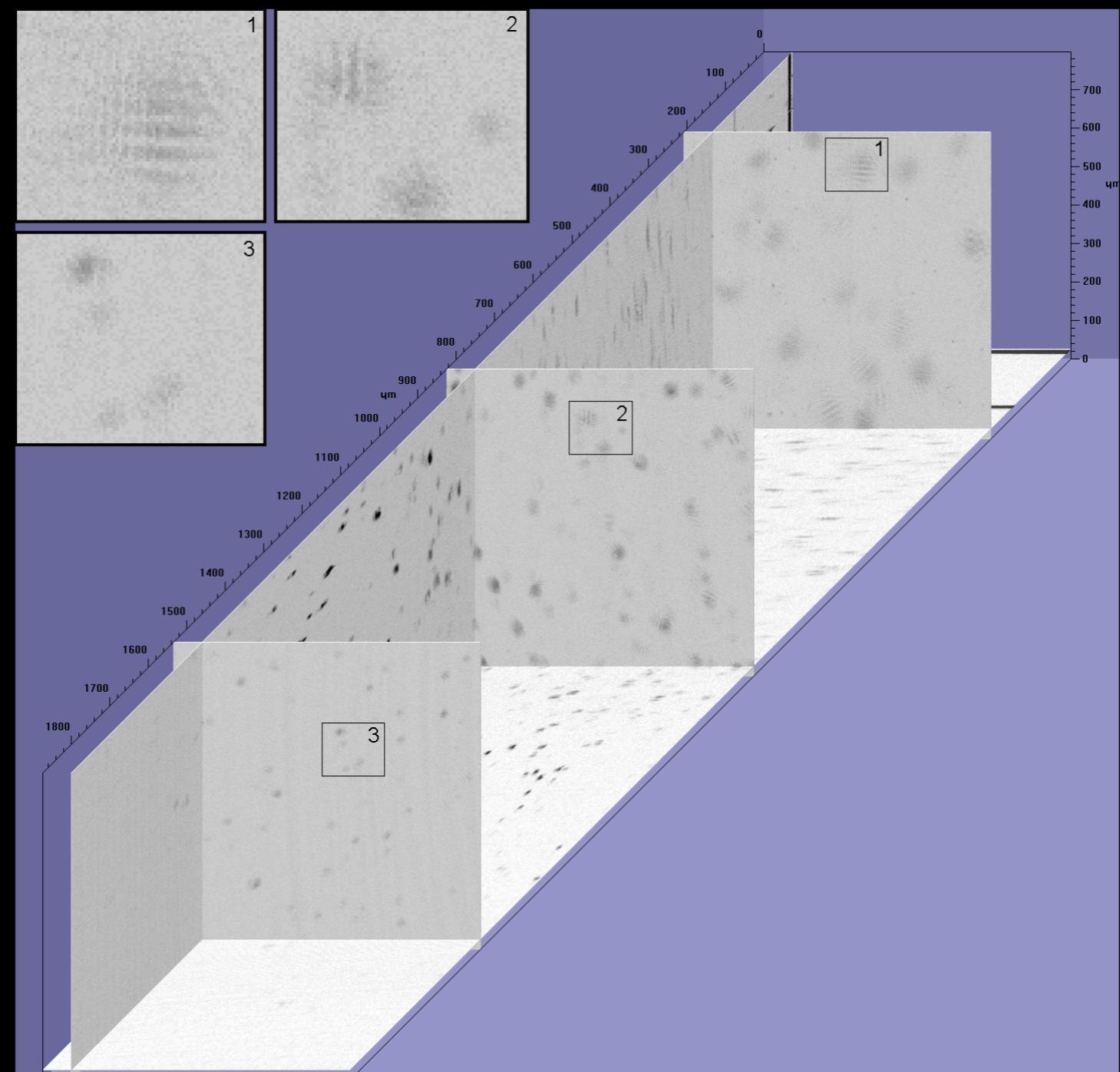
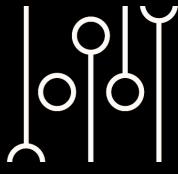
Polar vs. Hyperbolic Resampling

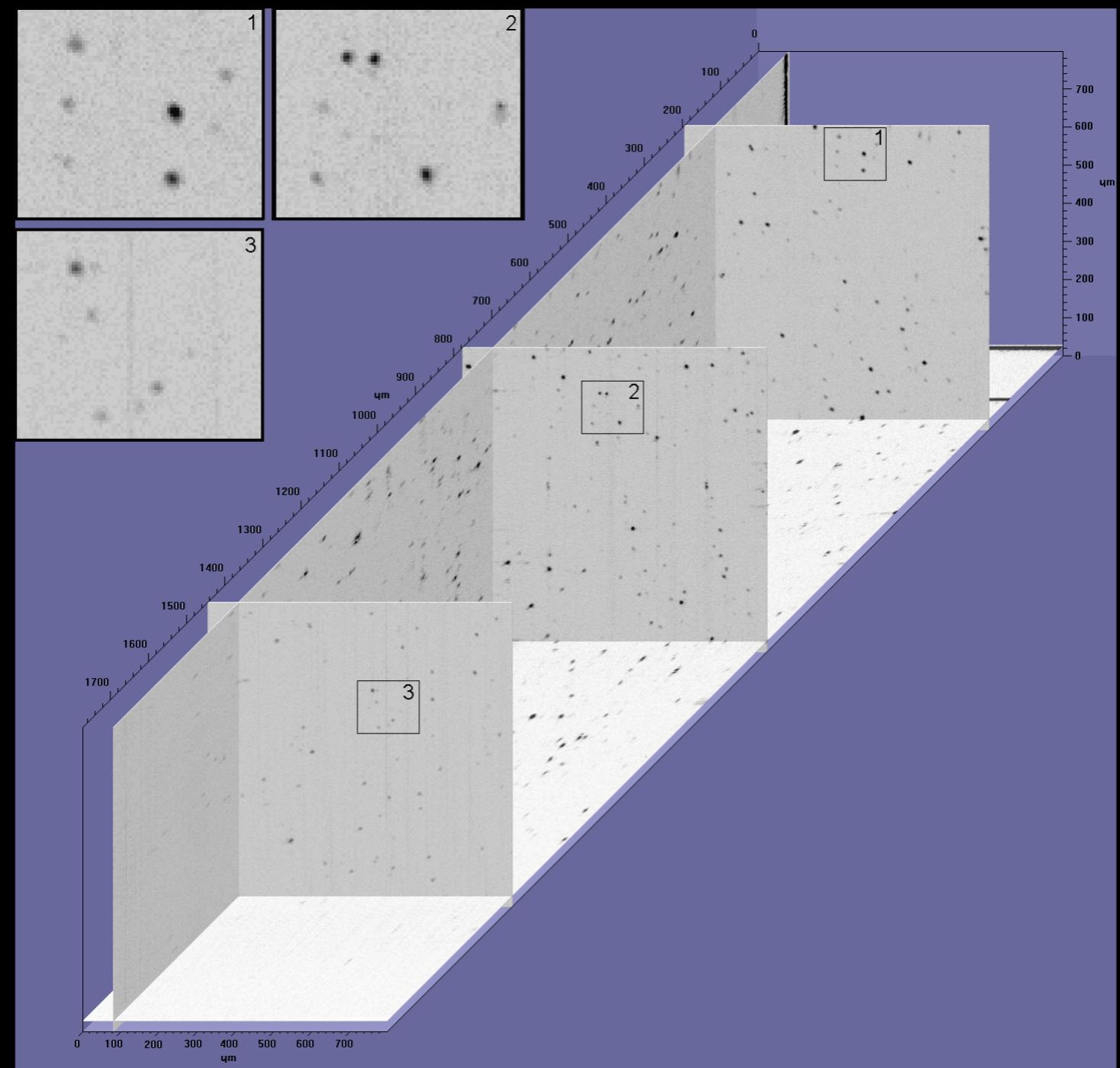
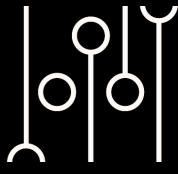


Radon
MRI
CT



ISAM





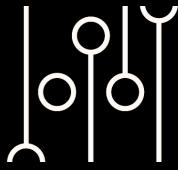
3D Rendered



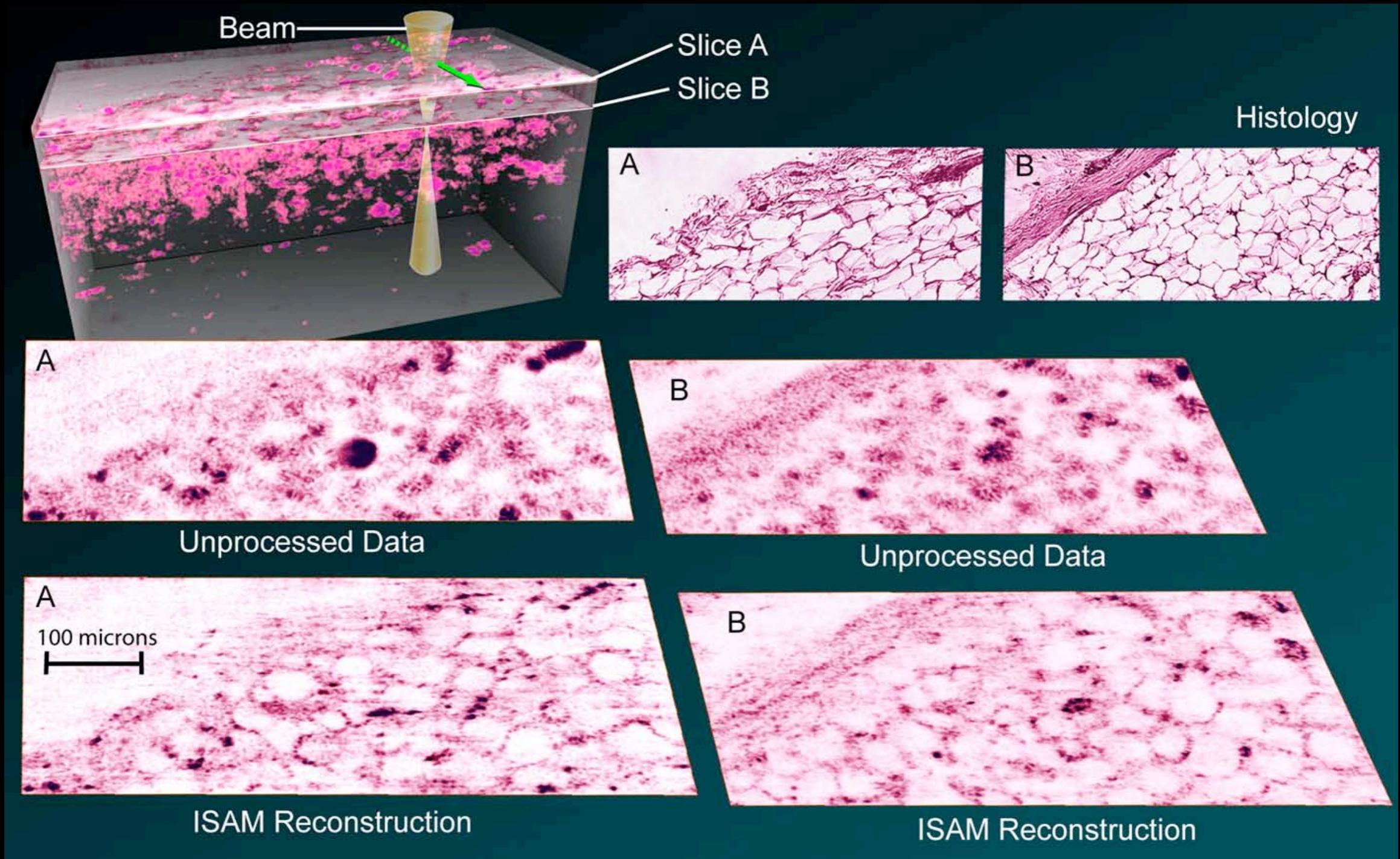


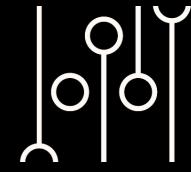
3D Rendered





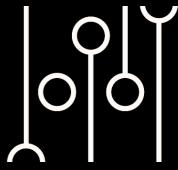
ISAM vs Histology





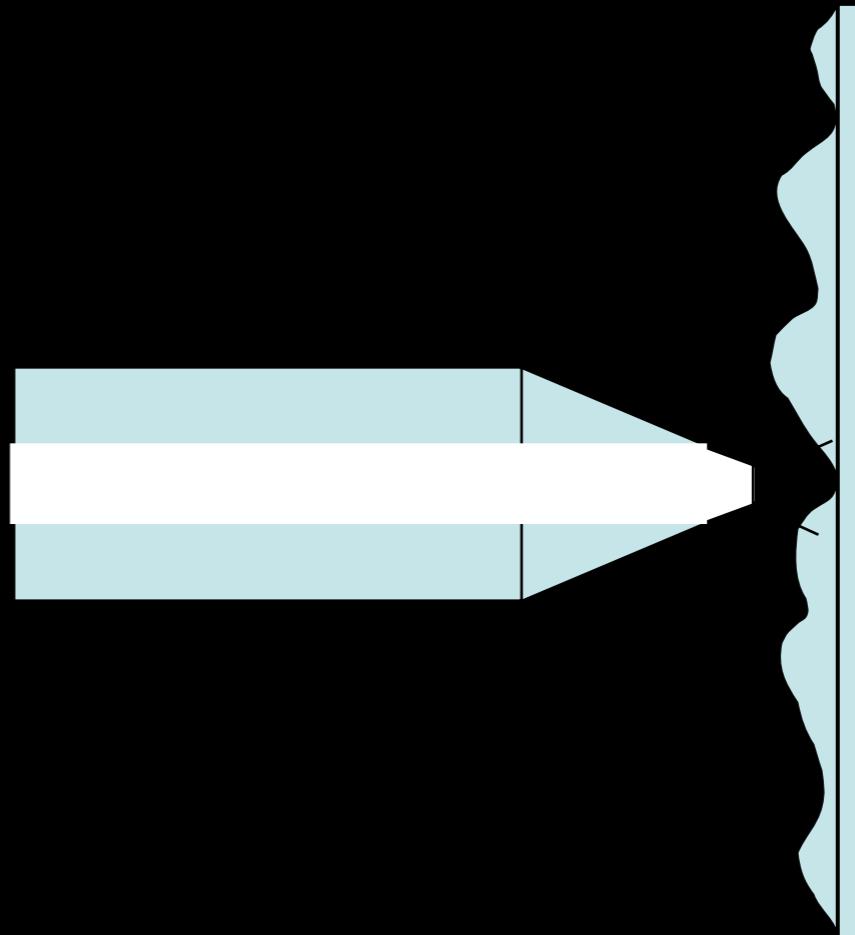
ISAM into the clinic





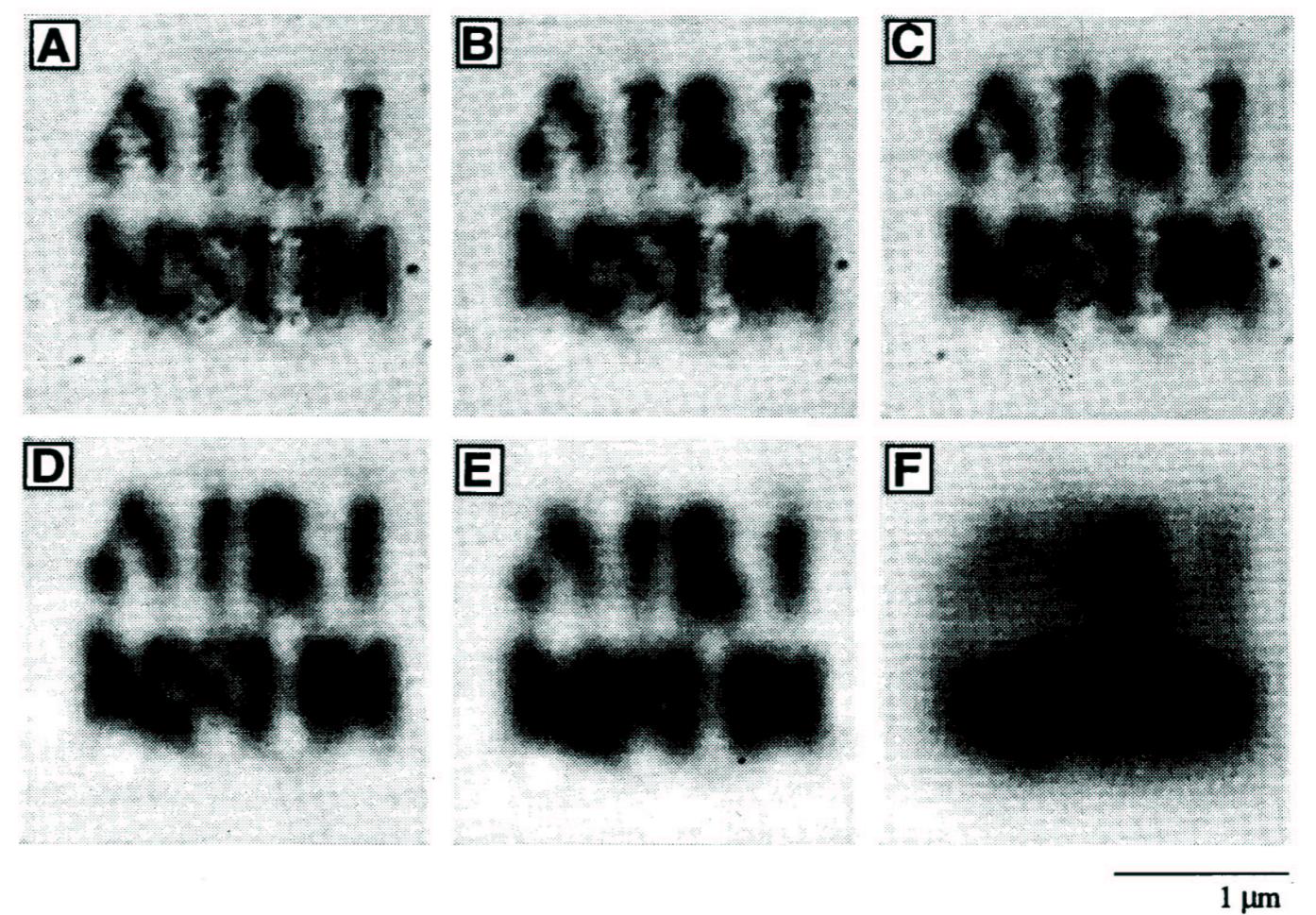
And now for something completely different

- Near-field
- Also scanning
- Also interferometric
- Harder



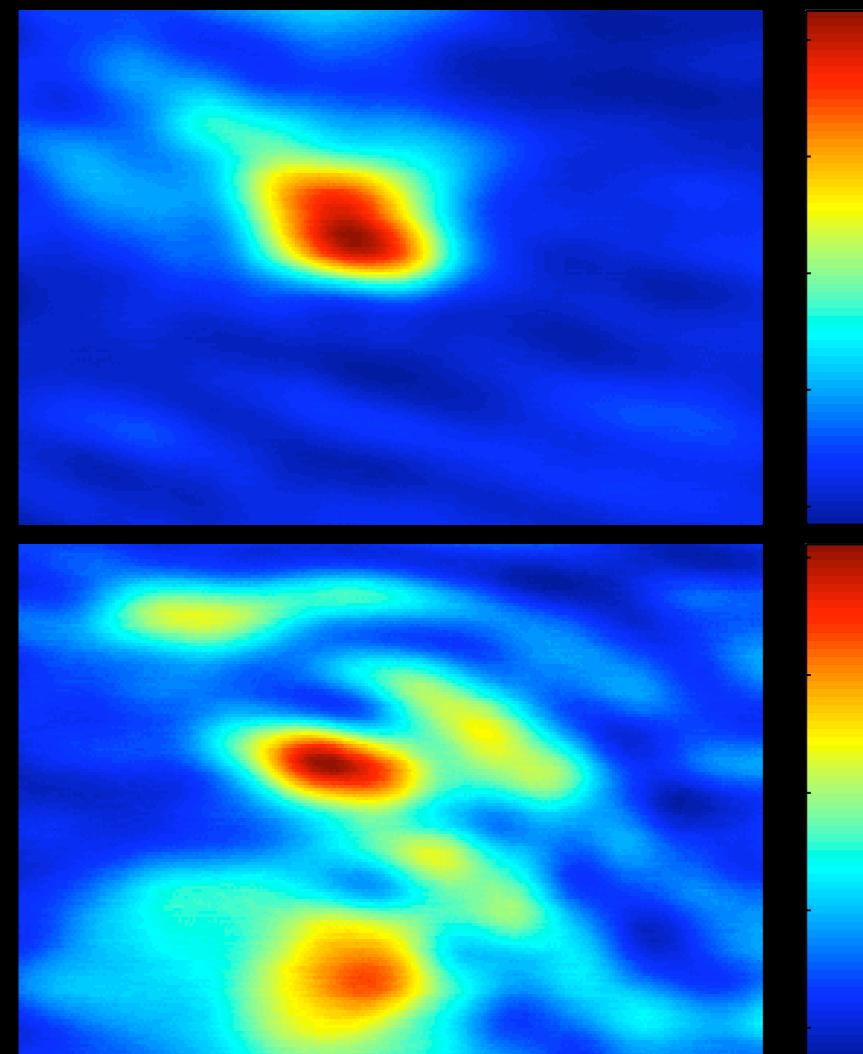
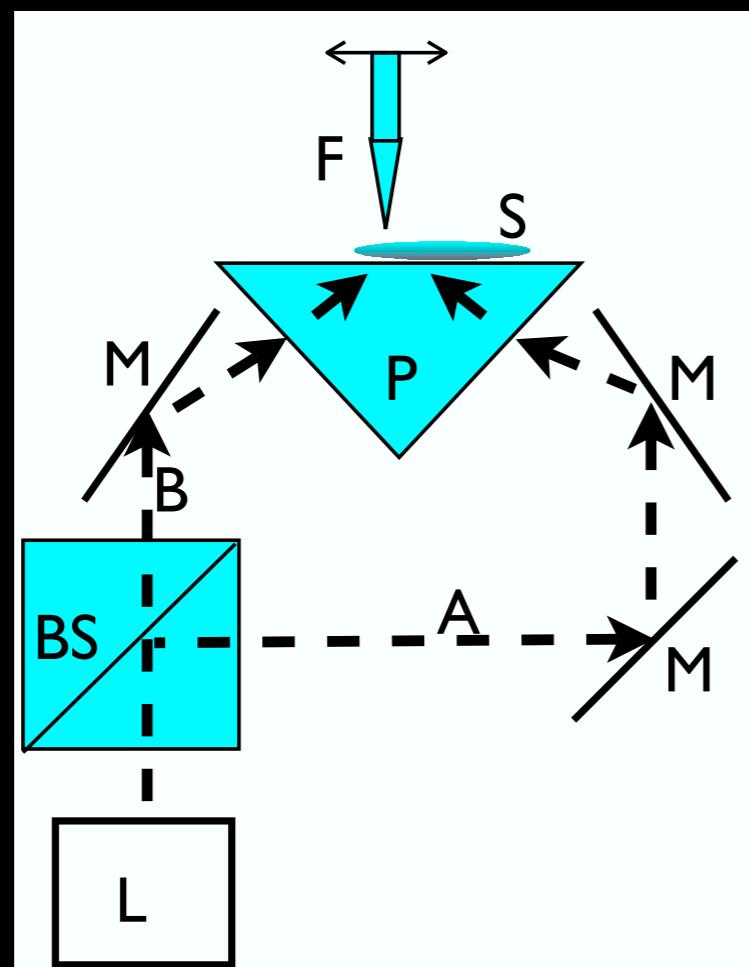
NSOM images

Fig. 4. Resolution in NSOM as a function of the probe-to-sample separation. The separation in each case is (A) near contact; (B) 5 nm; (C) 10 nm; (D) 25 nm; (E) 100 nm; and (F) 400 nm.



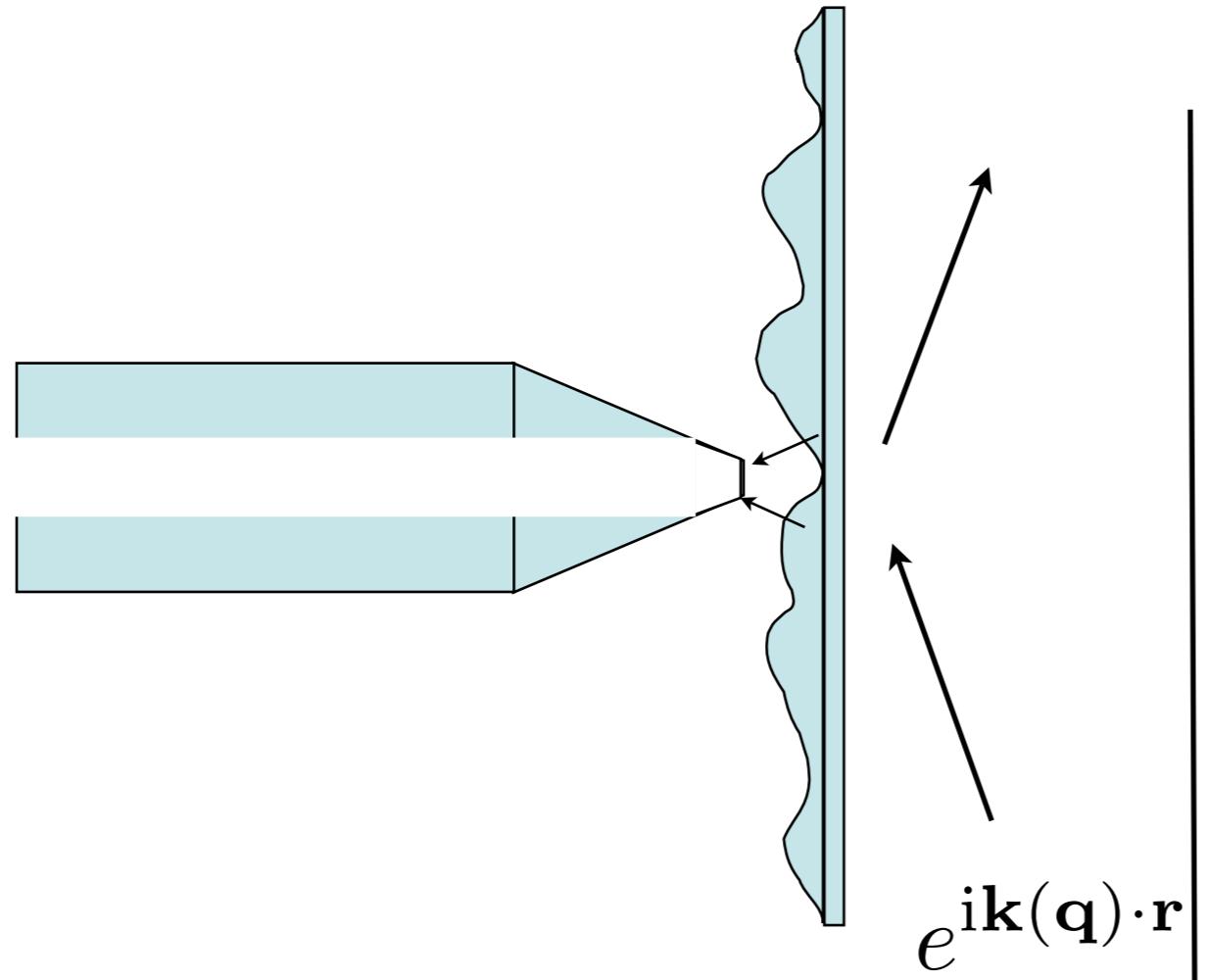
Betzig and Trautman, Science 257, 189-195 (1992).

PSTM holograms



$3\mu\text{m} \times 5\mu\text{m}$

PSTM/NSOM simple model



Monochromatic field $\omega_0 = ck_0$

$$U_{\mathbf{q}}(\mathbf{r}) = \int d^3r' e^{i\mathbf{k}(\mathbf{q}) \cdot \mathbf{r}'} G(\mathbf{r}, \mathbf{r}') V(\mathbf{r}')$$

$$K_{\mathbf{q}}(\mathbf{r}; \mathbf{r}') = \int d^2Q \lambda_{\mathbf{Q}}^{\mathbf{q}} g_{\mathbf{Q}}^{\mathbf{q}}(r) f_{\mathbf{Q}}^{\mathbf{q}*}(\mathbf{r}')$$

$$g_{\mathbf{Q}}^{\mathbf{q}}(r) = \frac{e^{i(\mathbf{Q} + \mathbf{q}) \cdot \mathbf{r}}}{2\pi}$$

$$f_{\mathbf{Q}}^{\mathbf{q}*}(\mathbf{r}') = \frac{-ie^{i\mathbf{Q} \cdot \mathbf{r}' - i|z'|k_z^*(\mathbf{Q} + \mathbf{q}) - izk_z^*(\mathbf{q})}}{4\pi k_z^*(\mathbf{Q} + \mathbf{q}) \lambda_{\mathbf{Q}}^{\mathbf{q}}}$$

$$\mathbf{k}(\mathbf{q}) = (\mathbf{q}, \sqrt{k_0^2 - q^2})$$

Calculation of the pseudo-inverse

$$M_{\mathbf{q}\mathbf{q}'}(\mathbf{Q})\delta^{(2)}(\mathbf{Q} - \mathbf{Q}') = \langle f_{\mathbf{Q}}^{\mathbf{q}} | f_{\mathbf{Q}'}^{\mathbf{q}'} \rangle \lambda_{\mathbf{Q}}^{\mathbf{q}} \lambda_{\mathbf{Q}'}^{\mathbf{q}'}$$

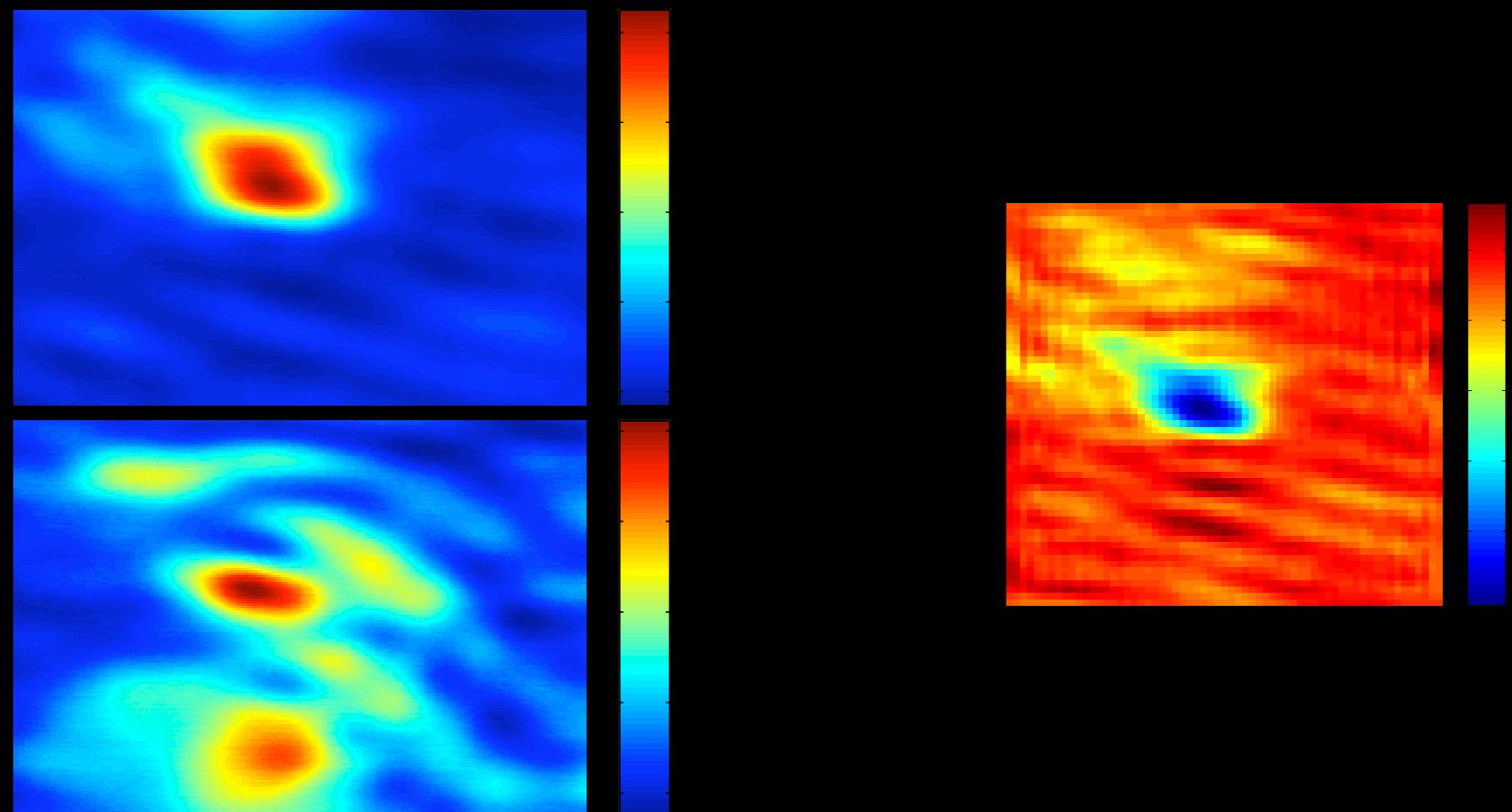
$$M(\mathbf{Q})\mathbf{c}_{\ell}(\mathbf{Q}) = \Lambda_{\mathbf{Q}\ell}\mathbf{c}_{\ell}(\mathbf{Q})$$

$$\psi_{\mathbf{Q}\ell}(\mathbf{r}') = \Lambda_{\mathbf{Q}\ell}^{-1} \sum_{\mathbf{q}} c_{\ell\mathbf{q}}(\mathbf{Q}) \lambda_{\mathbf{Q}}^{\mathbf{q}} f_{\mathbf{Q}}^{\mathbf{q}}(\mathbf{r}')$$

$$\phi_{\mathbf{Q}\ell\mathbf{q}'}(\mathbf{r}) = c_{\ell\mathbf{q}'}(\mathbf{Q}) g_{\mathbf{Q}}^{\mathbf{q}'}(\mathbf{r})$$

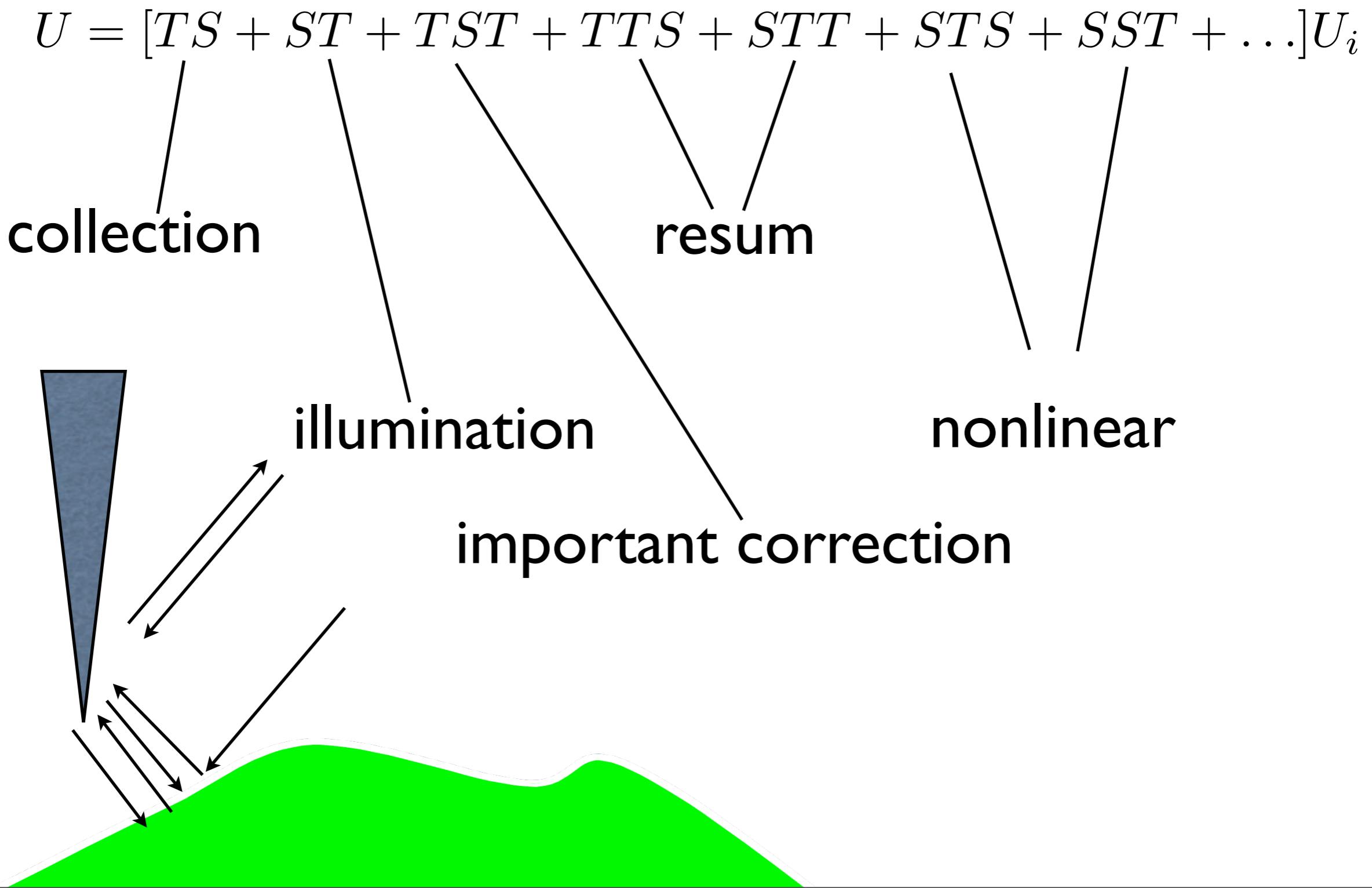
$$K^+(\mathbf{r}'; \mathbf{r}, \mathbf{q}) = \int d^2Q \sum_{\ell} \Lambda_{\mathbf{Q}\ell}^{-1} \psi_{\mathbf{Q}\ell}(\mathbf{r}') \phi_{\mathbf{Q}\ell\mathbf{q}}^*(\mathbf{r})$$

And the results are ...

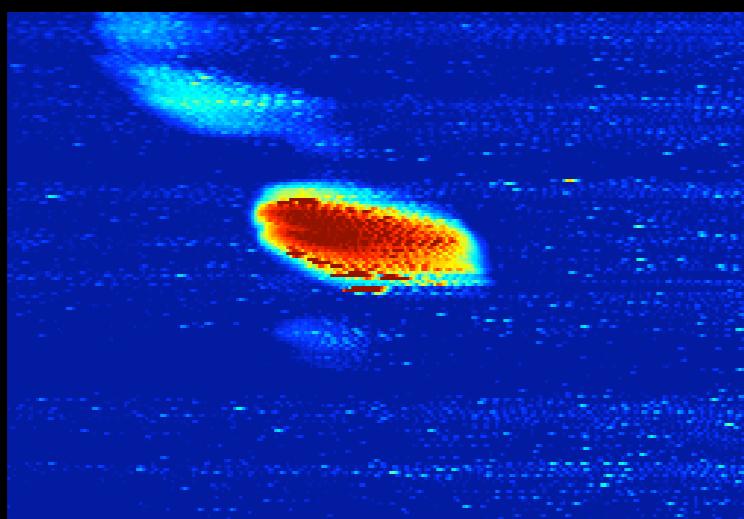
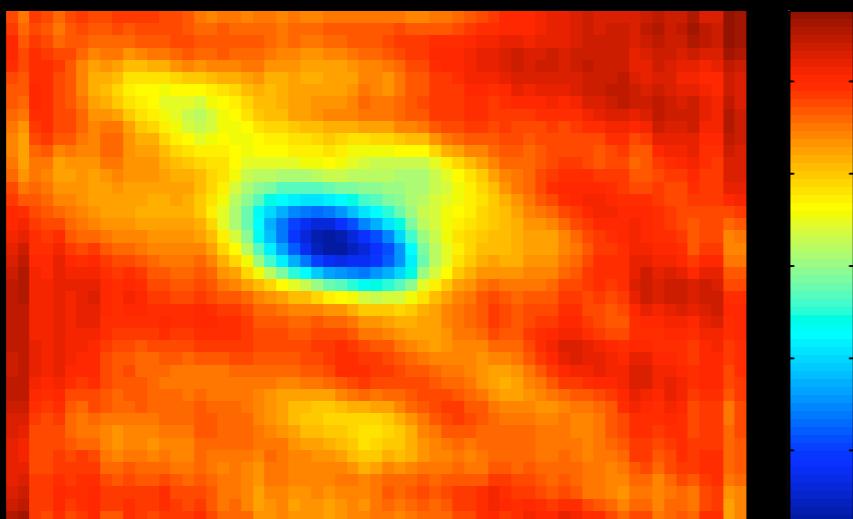


terrible.

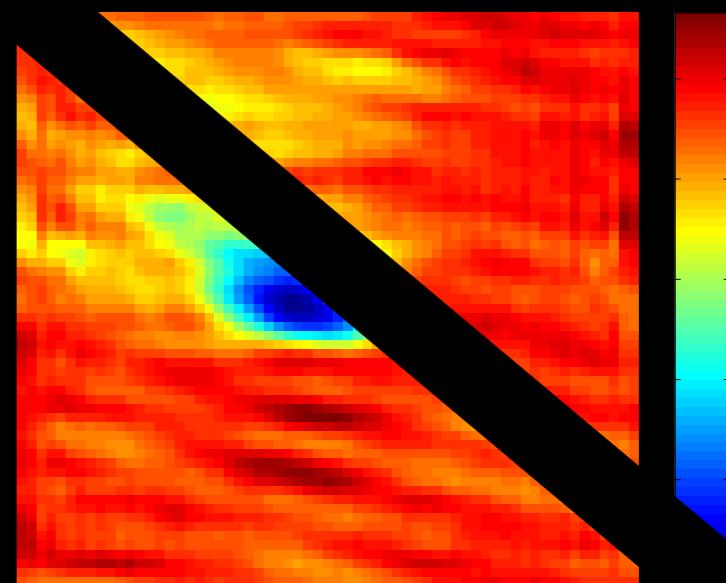
Fix the physics: Multiple interactions with the probe



Strong tip corrections



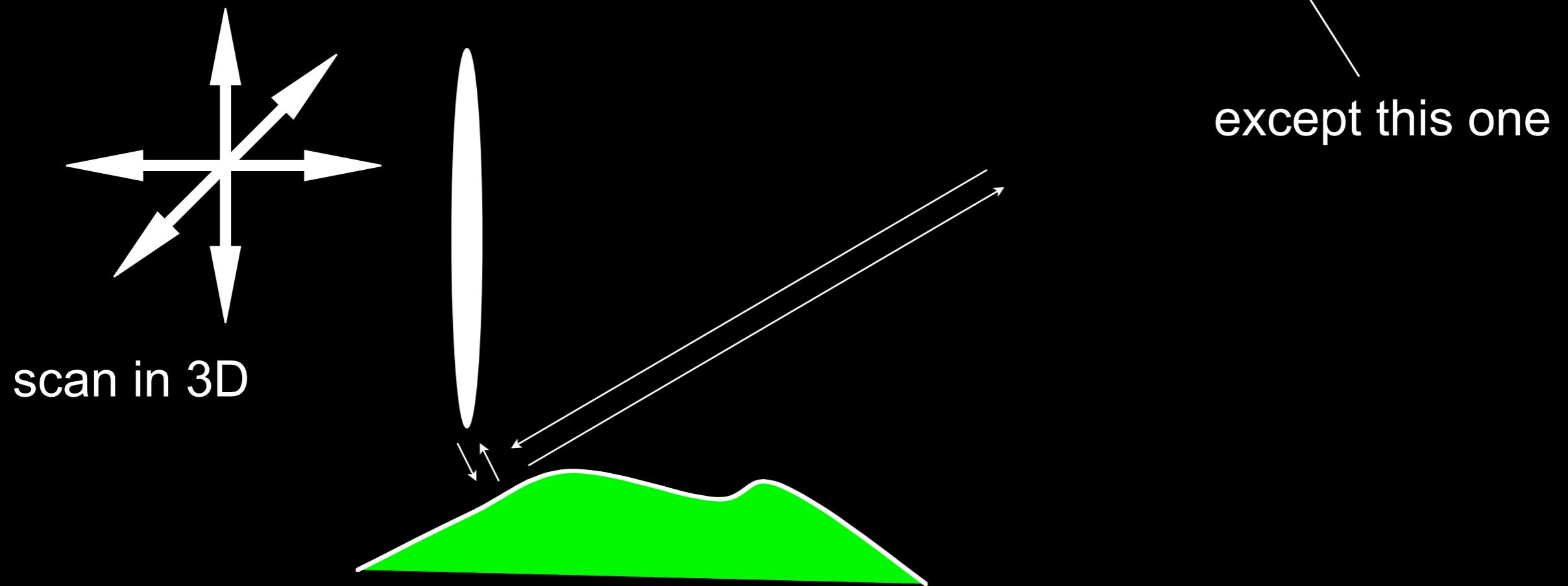
AFM



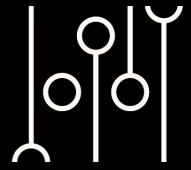
The next thing: vNFOT

$$U = [S + \bar{T} + S\bar{T} + \bar{T}S + \bar{T}S\bar{T}] U_i$$

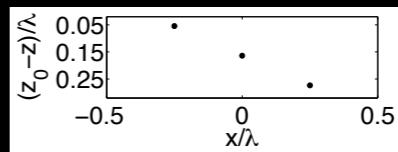
data from adjacent planes are linearly dependent



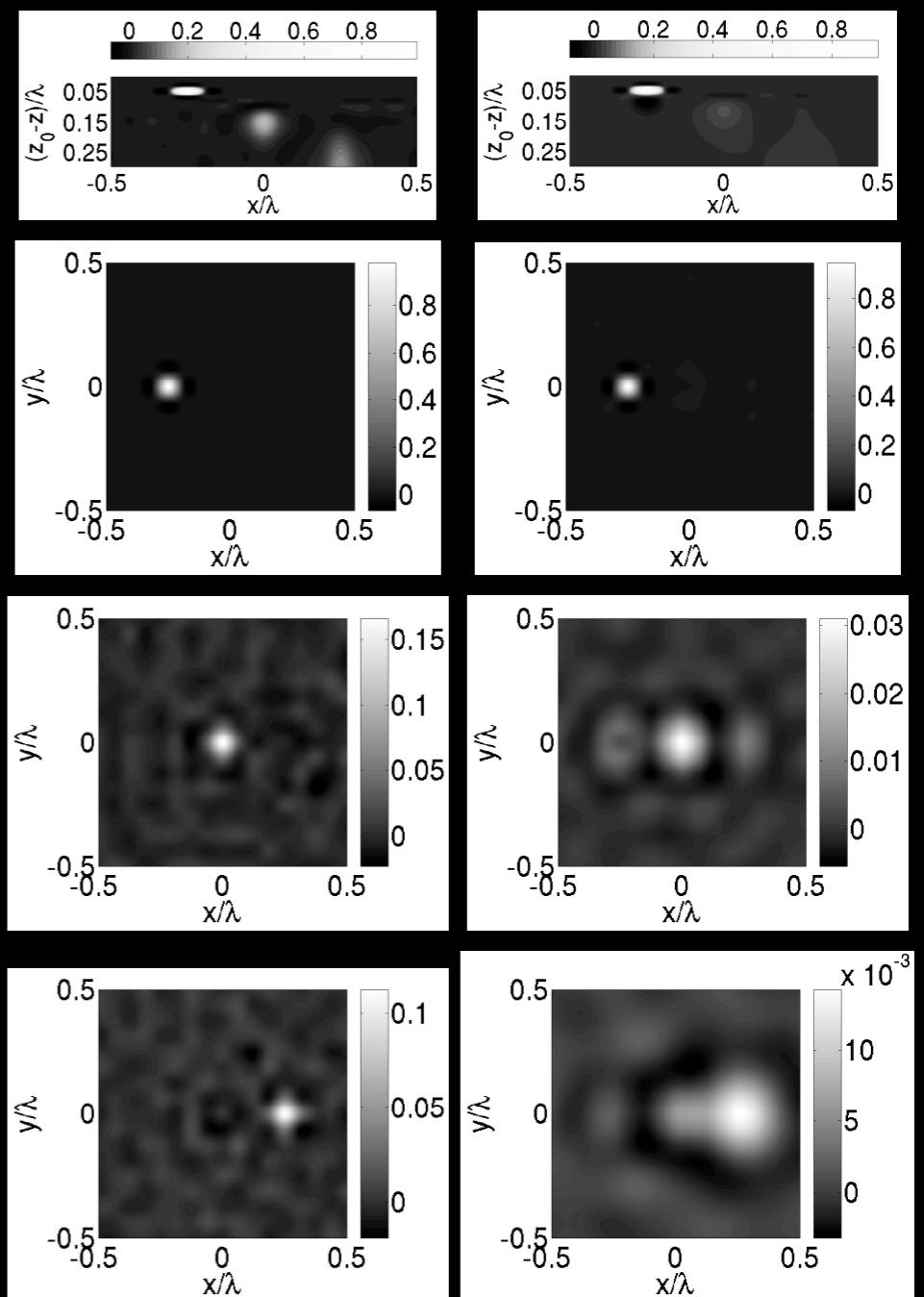
except this one

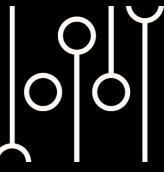


What we know about vNFOT now



- Simulations show 3-D reconstructions are in principle possible
- Experiments verify a dependence of the data on depth (the forward model works).
- Resolution of reconstructions is noise dependent
- Resolution is spatially inhomogeneous for fundamental reasons





Conclusions

- Inverse scattering and computed imaging extend the utility and scope of data collection methods
- Physics makes it go.