Lecture 2: IV Characteristics: traditional approach

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MOSFET IV characteristic

(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)
current is charge per unit time

\[ I_D = -WQ_n(x)\langle \nu_x(x) \rangle \]

\[ C \equiv \frac{Q}{V} \quad \text{F} \]

\[ Q_n \approx -C_{ox} \left( V_{GS} - V_T \right) \quad \text{C/m}^2 \]

\[ Q_n \approx 0 \quad \left( V_{GS} < V_T \right) \]

\[ C_{ox} = \frac{\kappa_{ox} \varepsilon_0}{t_{ox}} \quad \text{F/cm}^2 \]
MOSFET IV: low $V_{DS}$

$$Q_n(x) \approx -C_{ox}(V_{GS} - V_T)$$

$$I_D = -WQ_n(x)\langle \nu(x) \rangle$$

$$Q_n = -C_{ox}(V_{GS} - V_T)$$

$$\langle \nu(x) \rangle = -\mu_{eff} \mathcal{E}_x$$

$$\mathcal{E}_x = -\frac{V_{DS}}{L}$$

$V_{GS} > V_T$

$V_D$

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MOSFET IV: “pinch-off” at high $V_{DS}$

$$Q_n(x) = -C_{ox} \left( V_{GS} - V_T - V(x) \right)$$

$$V(x) = (V_{GS} - V_T)$$

$$Q_n(x) \approx 0$$

Note: thickness of channel illustrates the number of electrons – not the actual thickness.
MOSFET IV: high $V_{DS}$

$V(x) = (V_{GS} - V_T)$

$Q_i(x) = -C_{ox}(V_{GS} - V_T - V(x))$

$I_D = -W Q_i(x) \langle \nu_x(x) \rangle = W Q_i(0) \langle \nu_x(0) \rangle$

$Q_n(0) = -C_{ox}(V_{GS} - V_T)$

$\langle \nu(0) \rangle = -\mu_{\text{eff}} \mathcal{E}_x(0)$

$\mathcal{E}_x(0) \approx -(V_{GS} - V_T)/L$

$\mu_{\text{eff}} C_{ox}(V_{GS} - V_T)^2$

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velocity saturation

\[ \frac{V_{DS}}{L} \approx \frac{1.0 \text{ V}}{30 \text{ nm}} \approx 3 \times 10^5 \text{ V/cm} \]

\[ \nu = \mu E \]

\[ \nu = \nu_{sat} \]
MOSFET IV: velocity saturation

\[ I_D = -WQ_n(x)\langle v_x(x)\rangle \]

\[ Q_n = -C_{ox}(V_{GS} - V_T) \]

\[ \langle v_x \rangle = v_{sat} \]

\[ I_D = WC_{ox}v_{sat}(V_{GS} - V_T) \]

(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)
MOSFET: IV (re-cap)

\[
I_{DLIN} = \frac{W}{L} \mu_{\text{eff}} C_{ox} (V_{GS} - V_T) V_{DS}
\]

\[
I_{DSAT} = W C_{ox} \nu_{\text{sat}} (V_{GS} - V_T)
\]

\[
V_T(V_{DS}) = V_{T0} - \delta V_{DS}
\]

We have developed a 2-piece approximation to the MOSFET IV characteristic.
2-piece model for $I_D(V_G, V_D)$

Can we do better?

$V_D \leq V_{DSAT} : I_D = I_{DLIN}$

$V_D > V_{DSAT} : I_D = I_{DSAT}$

$V_{DSAT} = \frac{v_{SAT} L}{\mu_{eff}}$

$I_{DLIN} = \frac{W}{L} \mu_{eff} C_{ox} (V_G - V_T) V_D$

$I_{DSAT} = W C_{ox} v_{sat} (V_G - V_T)$
model for \( I_D(V_G, V_D) \)

Small \( V_{DS} \)

\[
I_{DLIN} = \frac{W}{L} \mu_{\text{eff}} C_{ox} (V_G - V_T) V_D
\]

\[
I_{DLIN}/W = C_{ox} (V_G - V_T) \left( \mu_{\text{eff}} \frac{V_D}{L} \right)
\]

\[
\langle v(V_D) \rangle = \left( \mu_{\text{eff}} \frac{V_D}{L} \right)
\]

Large \( V_{DS} \)

\[
I_{DSAT} = W C_{ox} \nu_{sat} (V_G - V_T)
\]

\[
I_{DSAT}/W = C_{ox} (V_G - V_T) \nu_{sat}
\]

\[
\langle v(V_D) \rangle = \nu_{SAT}
\]
model for $I_D(V_G, V_D)$

$$I_D/W = Q_S(V_G) \langle \nu(V_D) \rangle$$

$V_G \geq V_T$: $Q_n(V_G) = -C_{ox}(V_G - V_T)$

$V_D \leq V_{DSAT}$: $\langle \nu(V_D) \rangle = \left( \mu_{eff} \frac{V_D}{L} \right)$

$V_G < V_T$: $Q_n(V_G) = 0$

$V_D > V_{DSAT}$: $\langle \nu(V_D) \rangle = \nu_{SAT}$

If we can make the average velocity go smoothly from the low $V_D$ to high $V_D$ limits, then we will have a smooth model for $I_D(V_G, V_D)$. 
empirical model for $I_D(V_G, V_D)$

\[
\langle \nu(V_D) \rangle = F_{SAT}(V_D) \nu_{SAT}
\]

\[
F_{SAT}(V_D) \equiv \frac{V_D / V_{DSAT}}{1 + (V_D / V_{DSAT})^\beta}^{1/\beta}
\]

$V_D << V_{DSAT}$ : $F_{SAT}(V_D) \rightarrow \frac{V_D}{V_{DSAT}}$

$\langle \nu(V_D) \rangle \rightarrow \frac{V_D}{V_{DSAT}} \nu_{SAT}$

$V_D >> V_{DSAT}$ : $F_{SAT}(V_D) \rightarrow 1$

$\langle \nu(V_D) \rangle \rightarrow \nu_{SAT}$

$\langle \nu(V_D) \rangle \rightarrow \frac{V_D}{\nu_{SAT} L / \mu_{eff}} \nu_{SAT}$

$\langle \nu(V_D) \rangle \rightarrow \mu_{eff} \frac{V_D}{L}$

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understanding the saturating function: $F_{SAT}(V_D)$

$$\frac{1}{\langle \nu(V_D) \rangle} = \frac{1}{\mu_{eff} V_D / L} + \frac{1}{\nu_{SAT}} \rightarrow \langle \nu(V_D) \rangle = \left[ \frac{V_D / V_{DSAT}}{1 + V_D / V_{DSAT}} \right] \nu_{SAT}$$

$$\langle \nu(V_D) \rangle = F_{SAT}(V_D) \nu_{SAT} \quad F_{SAT}(V_D) = \frac{V_D / V_{DSAT}}{\left[1 + (V_D / V_{DSAT})^\beta\right]^{1/\beta}}$$

The extra parameter, $\beta$, is empirically adjusted to fit the IV characteristic. Typically, $\beta \approx 1.6 - 1.8$
saturating function: \( F_{\text{SAT}}(V_D) \)

\[
\langle \nu(V_D) \rangle = F_{\text{SAT}}(V_D) \nu_{\text{SAT}}
\]

\[
F_{\text{SAT}}(V_D) = \frac{V_D / V_{\text{DSAT}}}{\left[ 1 + \left( V_D / V_{\text{DSAT}} \right)^\beta \right]^{1/\beta}}
\]

Although this is just an empirical method to produce smooth curve that properly goes between the small and large \( V_D \) limits, it works very well in practice, which suggests that it captures something important about MOSFETs.
level 0 “Virtual Source model”

1) \[ I_D / W = Q_n(V_G)\langle \nu(V_D) \rangle \]

2) \[ V_{GS} \leq V_T : Q_n(V_{GS}) = 0 \]
   \[ V_{GS} > V_T : Q_n(V_{GS}) = C_{ox}(V_{GS} - V_T) \]

3) \[ \langle \nu(V_D) \rangle = F_{SAT}(V_D)\nu_{SAT} \]

4) \[ F_{SAT}(V_D) = \frac{V_D / V_{DSAT}}{\left[1 + \left(V_D / V_{DSAT}\right)^\beta\right]^{1/\beta}} \]

5) \[ V_{DSAT} = \frac{\nu_{SAT}L}{\mu_{eff}} \]

With this simple model, we can compute reasonable MOSFET IV characteristics, and the model can be extended step by step to make it more and more realistic.

There are only 5 device-specific input parameters to this model:

\[ C_{ox}, V_T, \nu_{SAT}, \mu_{eff}, L \]
The model we have outlined is based on physics that is valid for long channel MOSFETs from the 1960’s, 70’s, and early 80’s. For nanotransistors, we need to make some changes. Surprisingly, the changes are minor:

1) The saturation velocity for high-field transport in bulk semiconductors is replaced by the so-called “injection velocity”.

2) The effective mobility for carriers in the inversion layer of a long channel MOSFET is replaced by the “apparent mobility”.

(Effects such as subthreshold conduction, DIBL, quantum capacitance also need to be considered.)
Our goal is a simple, clear understanding of the physics of nanoscale MOSFETs along with a simple quantitative model (even if semi-empirical) to analyze and design nanotransistors.
For a discussion of velocity saturation in bulk semiconductors, see:


The virtual source model is described in: