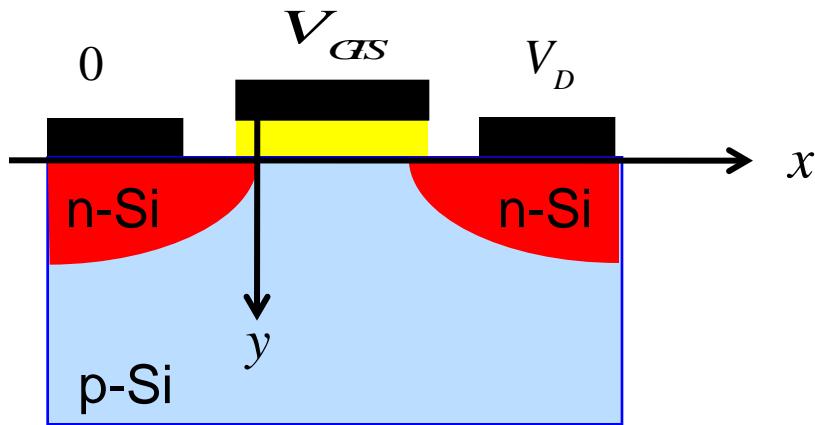


Lecture 4: MOS Electrostatics

Mark Lundstrom

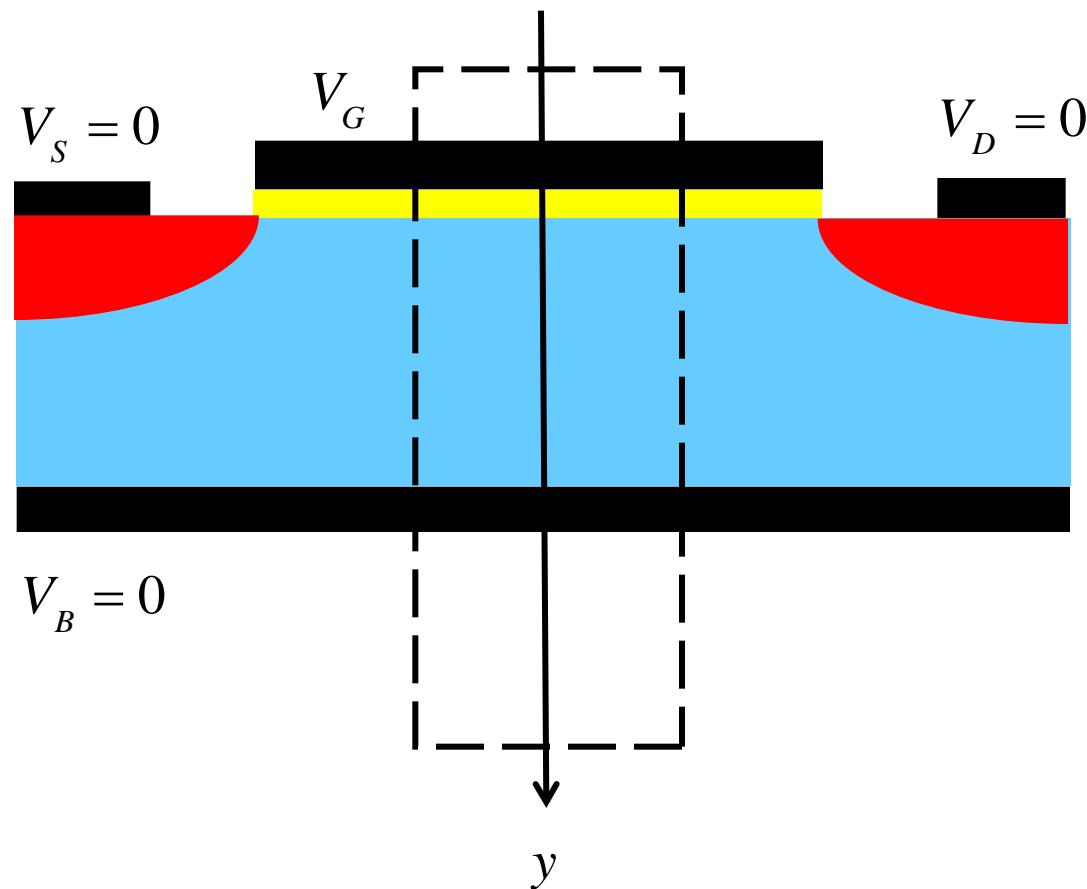
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understanding MOSFETs

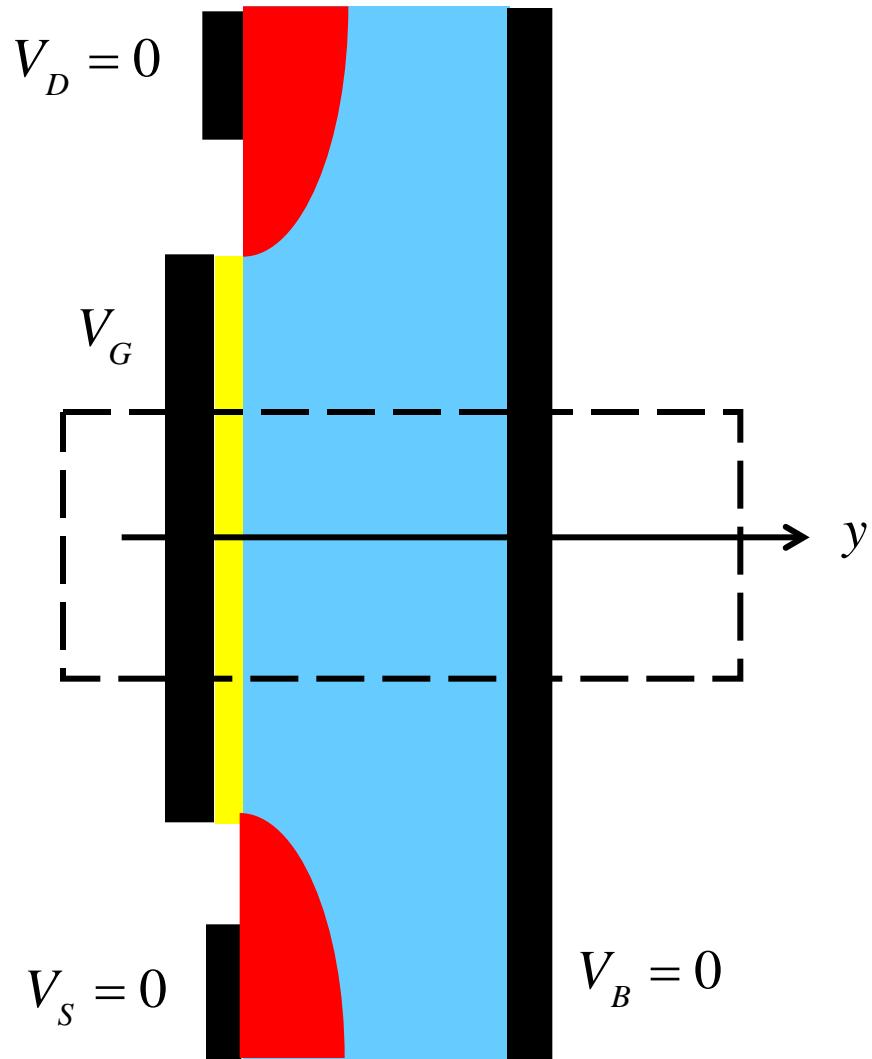


The operation of a MOSFET is determined by the 2D energy bands, which vary in space according to the 2D electrostatic potential: $\psi(x, y)$

1D MOS electrostatics



1D MOS electrostatics

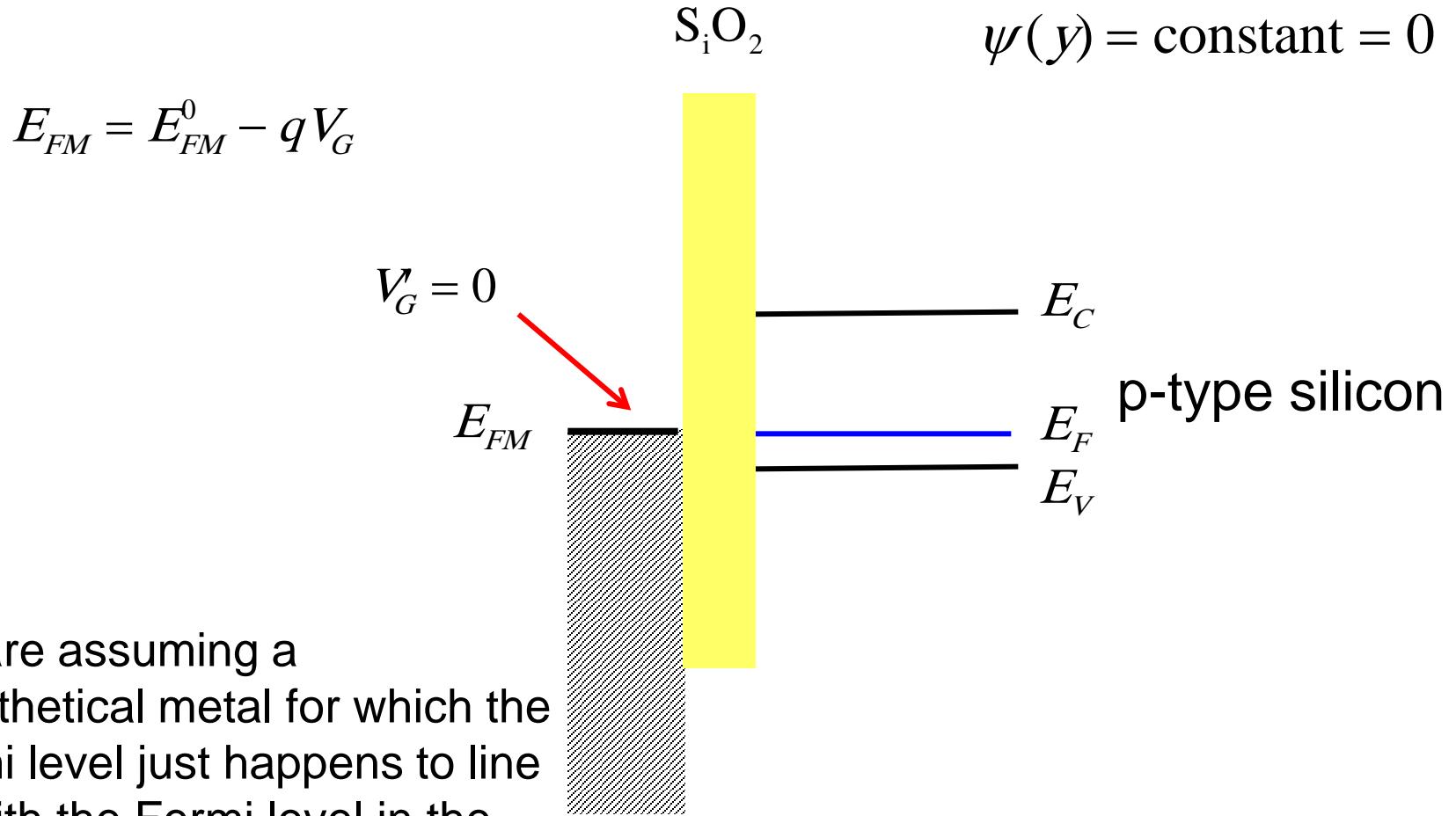


Goal: to understand:

$$Q_n(V_{GS})$$

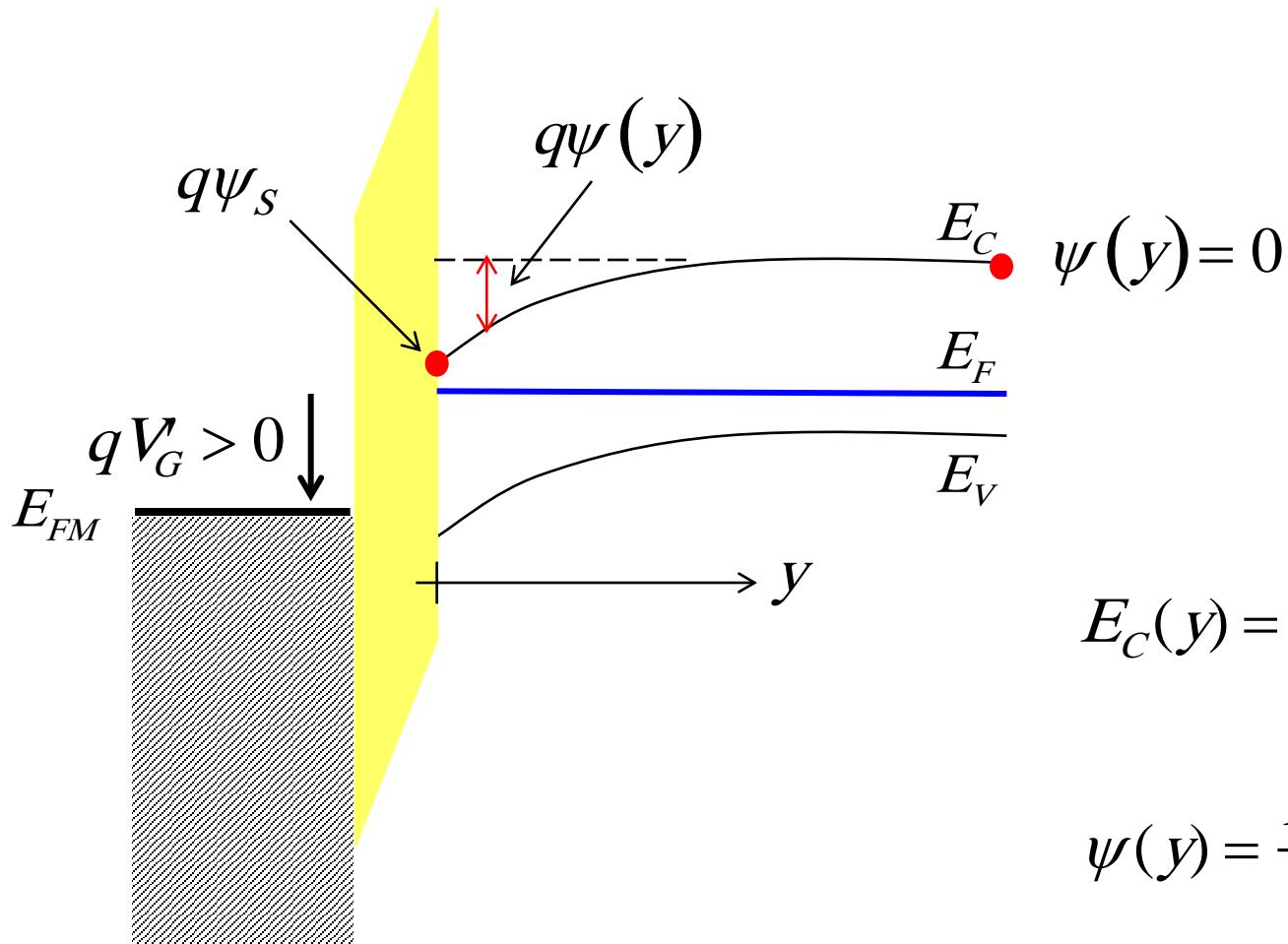
above and below threshold.

“flat band conditions”



We are assuming a hypothetical metal for which the Fermi level just happens to line up with the Fermi level in the semiconductor at $V_G = 0$.

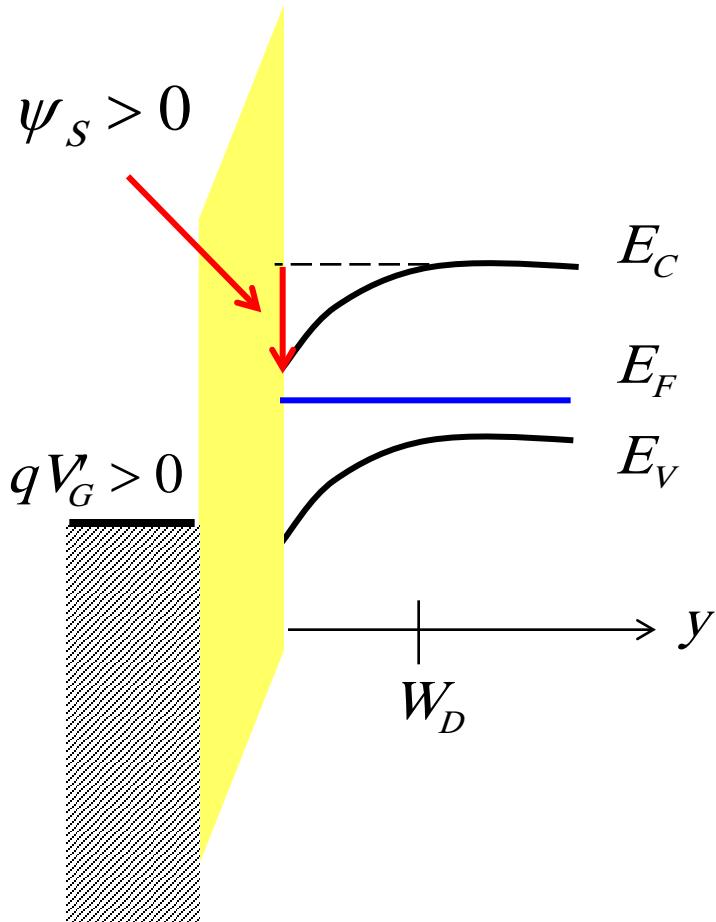
positive gate voltage



$$E_C(y) = \text{constant} - q\psi(y)$$

$$\psi(y) = \frac{E_C(\infty) - E_C(y)}{q}$$

depletion charge



$$p_0(y) = N_V e^{(E_V(y) - E_F)/k_B T_L} \approx 0$$

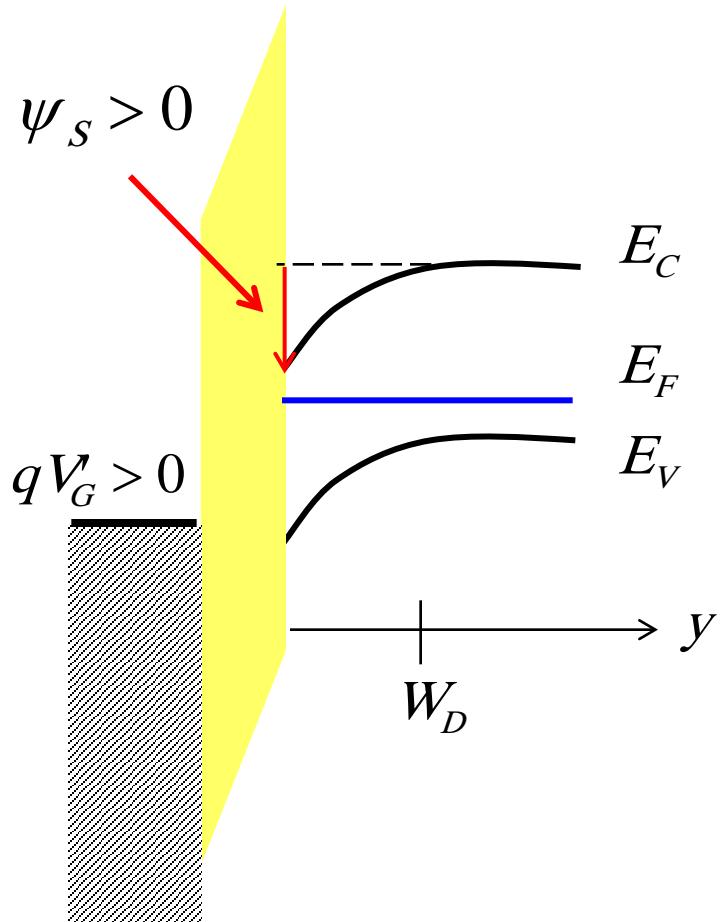
$$\rho(x) \approx -qN_A \quad (x < W_D) \quad \text{C/cm}^3$$

$$W_D = \sqrt{2\kappa_S \epsilon_0 \psi_S / qN_A}$$

$$Q_D = -qN_A W_D = -\sqrt{2qN_A \kappa_S \epsilon_0 \psi_S}$$

(charge in C/cm²)

mobile charge



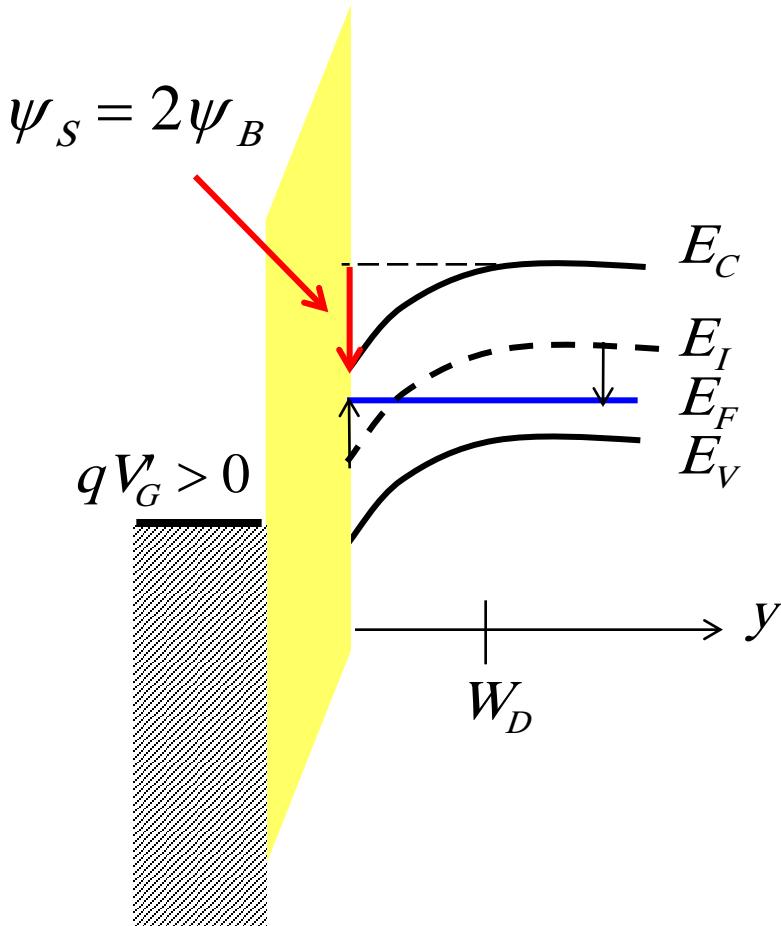
$$n_0(y) = N_C e^{(E_F - E_C(y))/k_B T_L}$$

$$Q_n = -q \int_0^{\infty} n(y) dy$$

$$Q_n = -qn_B e^{q\psi_s/k_B T_L} \left(\frac{k_B T_L / q}{\mathcal{E}_s} \right)$$

$$n_B = \frac{n_i^2}{N_A}$$

onset of inversion



$$\psi_B \equiv \frac{E_I(\infty) - E_F}{q}$$

$$p_B = n_i e^{(E_I(\infty) - E_F)/k_B T_L} \approx N_A$$

$$\psi_B = \frac{k_B T_L}{q} \ln \left(\frac{N_A}{n_i} \right)$$

“inversion” occurs when the surface is as n-type and the bulk is p-type.

$$\psi_s = 2\psi_B$$

gate voltage – mobile charge relation (i)

$$Q_s = Q_d + Q_n$$

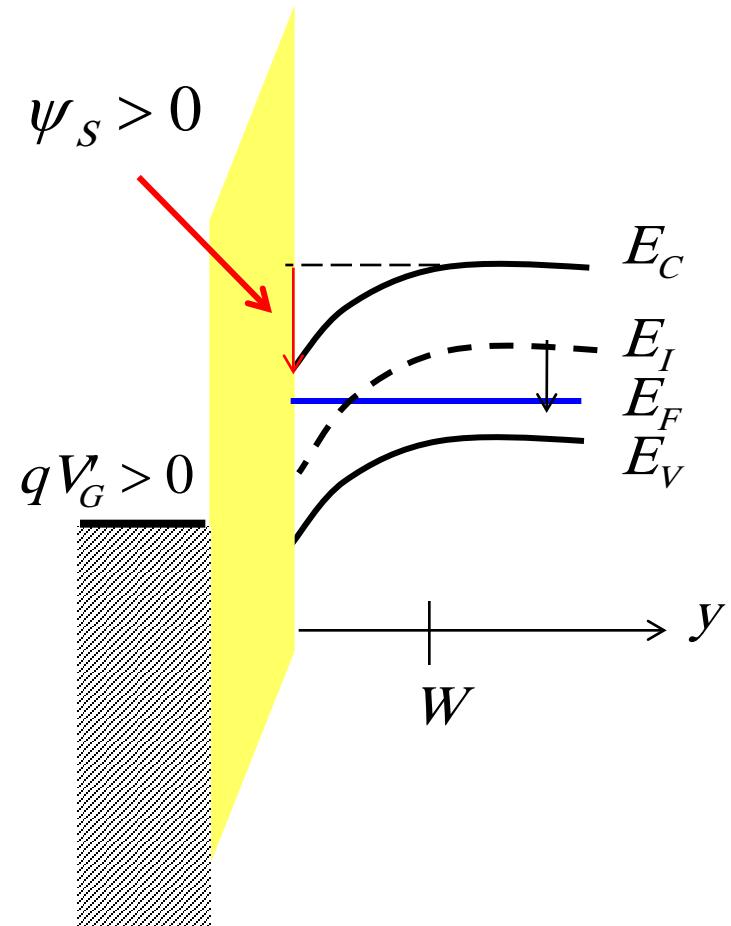
i) below threshold:

$$Q_s \approx Q_d$$

$$Q_n(\psi_s) = -q n_B e^{q\psi_s/k_B T_L} \left(\frac{k_B T_L / q}{E_s} \right)$$

$$Q_n(V_G) = ?$$

$$qV_G = \Delta V_{ox} + \psi_s$$



subthreshold charge vs. gate voltage

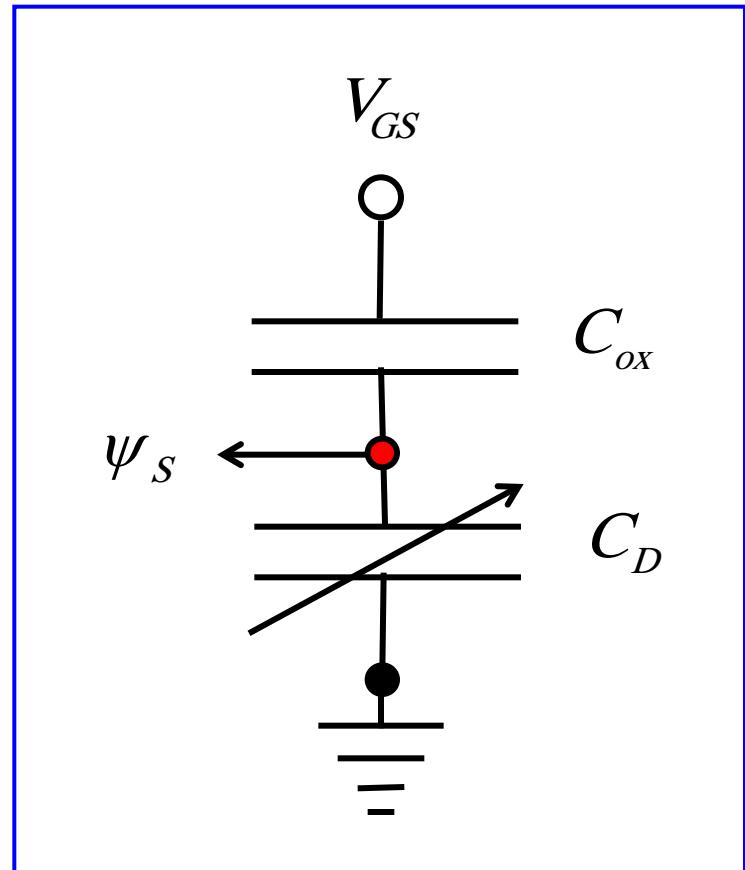
$$Q_n(\psi_s) = -qn_B e^{q\psi_s/k_B T_L} \left(\frac{k_B T_L / q}{\mathcal{E}_s} \right)$$

$$\psi_s = V_{GS} \frac{C_{ox}}{C_{ox} + C_D} = \frac{V_{GS}}{1 + C_D / C_{ox}}$$

$$\psi_s = \frac{V_{GS}}{m}$$

$$m = 1 + C_D / C_{ox}$$

$$Q_n(\psi_s) = -qn_B e^{qV_{GS}/mk_B T_L} \left(\frac{k_B T_L / q}{\mathcal{E}_s} \right)$$



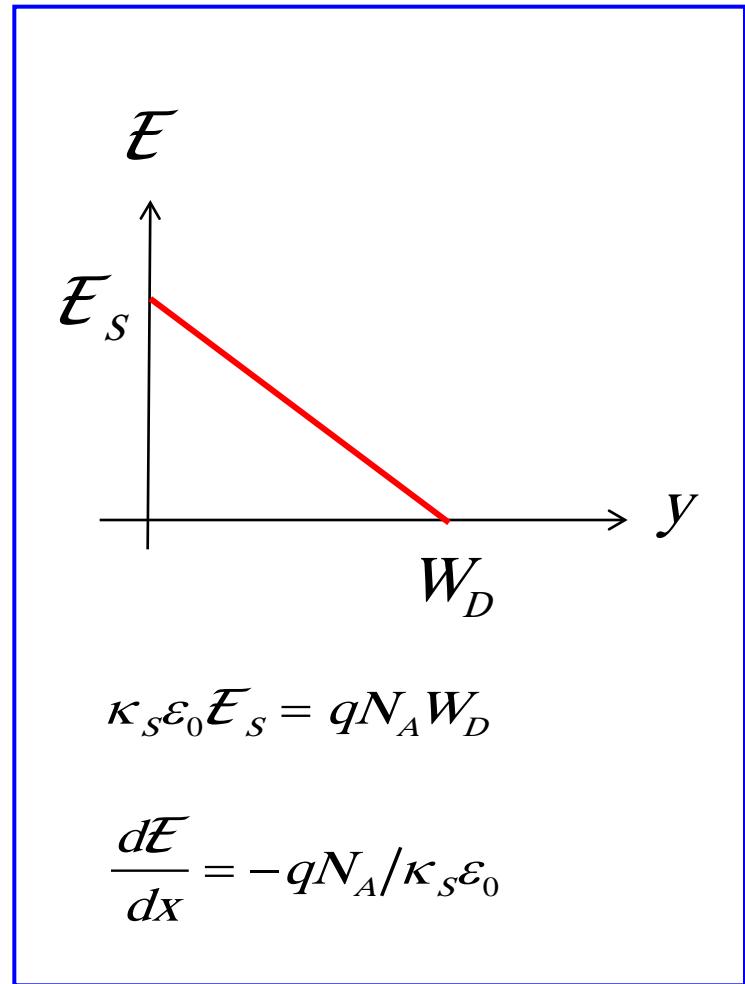
subthreshold charge vs. gate voltage

$$Q_n(\psi_s) = -qn_B e^{qV_{GS}/mk_B T_L} \left(\frac{k_B T_L / q}{\mathcal{E}_s} \right)$$

$$\mathcal{E}_s = \frac{qN_A W_D}{\epsilon_{Si}} = \frac{qN_A}{C_D}$$

$$m = 1 + C_D / C_{ox} \quad (m-1)C_{ox} = C_D$$

$$\mathcal{E}_s = \frac{qN_A}{(m-1)C_{ox}}$$



subthreshold charge vs. gate voltage

$$Q_n(\psi_s) = -qn_B e^{q\psi_s/k_B T_L} \left(\frac{k_B T_L / q}{\mathcal{E}_s} \right)$$

$$\psi_s = \frac{V_{GS}}{m} \quad m = 1 + C_D / C_{ox}$$

$$\mathcal{E}_s = \frac{qN_A}{(m-1)C_{ox}} \quad n_B = n_i^2 / N_A$$

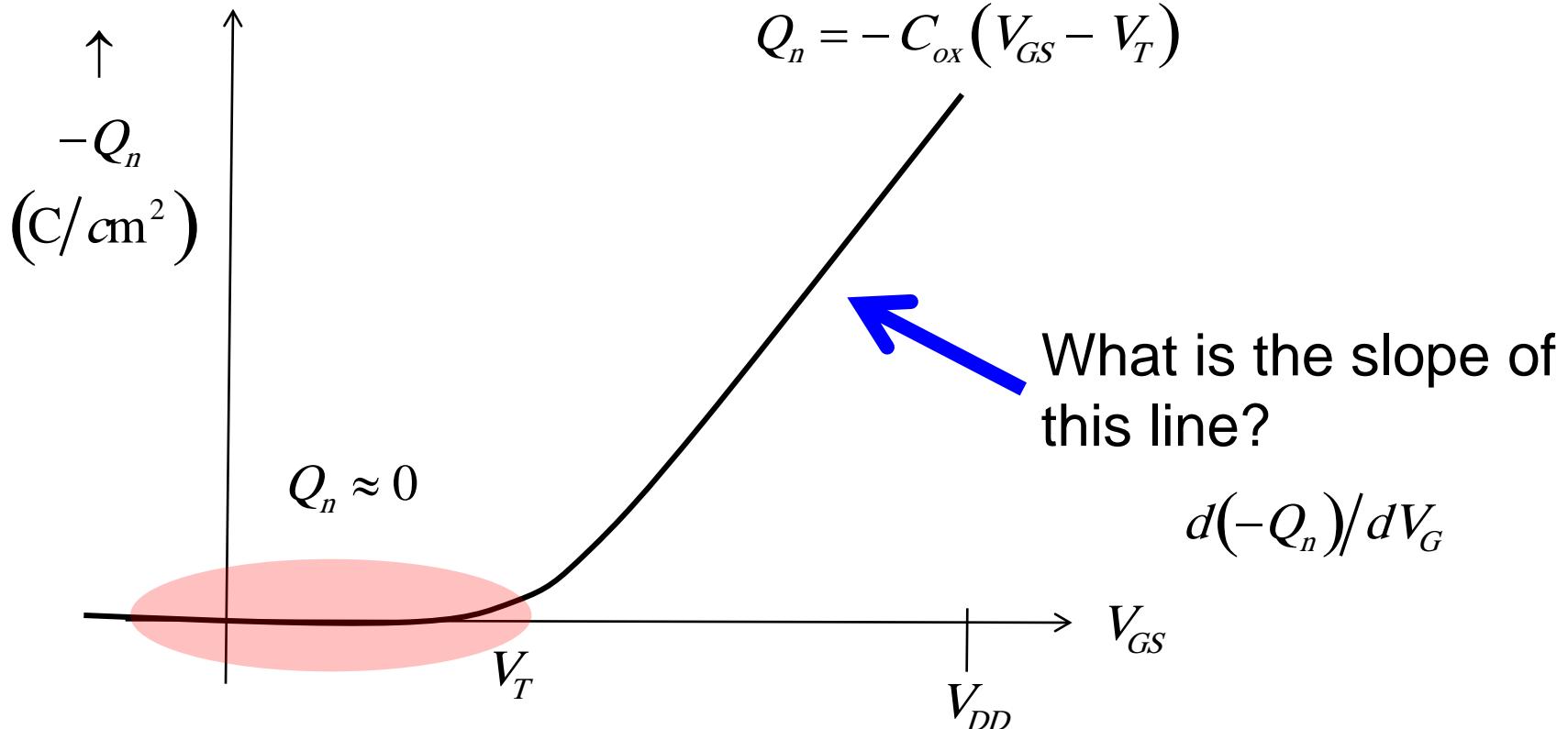
$$\psi_B = \frac{k_B T_L}{q} \ln \left(\frac{N_A}{n_i} \right)$$

$$\left(\frac{n_i}{N_A} \right) = e^{-q\psi_B/k_B T_L}$$

$$2\psi_B = \frac{V_T}{m}$$

$$Q_n(\psi_s) = -(m-1)C_{ox} \left(\frac{n_i}{N_A} \right)^2 e^{qV_{GS}/mk_B T_L} \left(\frac{k_B T_L}{q} \right)$$

subthreshold charge vs. gate voltage



$$Q_n(\psi_S) = -(m-1)C_{ox}\left(\frac{k_B T_L}{q}\right) e^{q(V_{GS} - V_T)/mk_B T_L}$$

above threshold charge vs. gate voltage

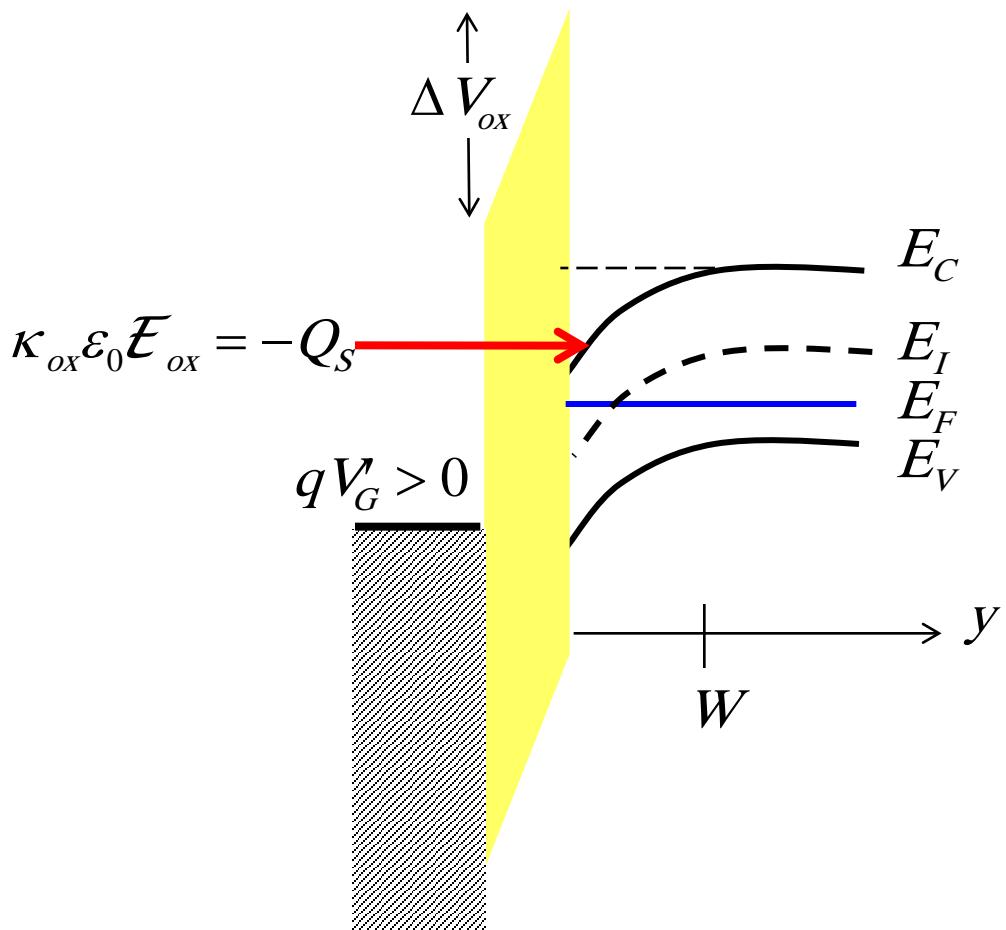
$$V_G = V_{FB} + \psi_S + \Delta V_{ox}$$

$$\Delta V_{ox} = \mathcal{E}_{ox} t_{ox}$$

$$V_G = V_{FB} + \psi_S - \frac{Q_s(\psi_S)}{C_{ox}}$$

$$Q_s = -Q_M$$

$$C_G \equiv \frac{dQ_M}{dV_G} \quad \frac{1}{C_G} = \frac{dV_G}{d(-Q_s)}$$



$$qV_G = \Delta V_{ox} + \psi_S$$

above threshold charge vs. gate voltage

$$V_G = V_{FB} + \psi_s - \frac{Q_s(\psi_s)}{C_{ox}}$$

$$\frac{1}{C_G} = \frac{dV_G}{d(-Q_s)}$$

$$\frac{1}{C_G} = \frac{d\psi_s}{d(-Q_s)} + \frac{1}{C_{ox}}$$

$$\frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_{ox}}$$

well above threshold....

$$Q_s \approx Q_n$$

$$C_S = C_S(\text{inv})$$

$$\frac{1}{C_{inv}} = \frac{1}{C_S(\text{inv})} + \frac{1}{C_{ox}}$$

$$C_{inv} = \frac{C_{ox} C_S(\text{inv})}{C_{ox} + C_S(\text{inv})} < C_{ox}$$

re-cap

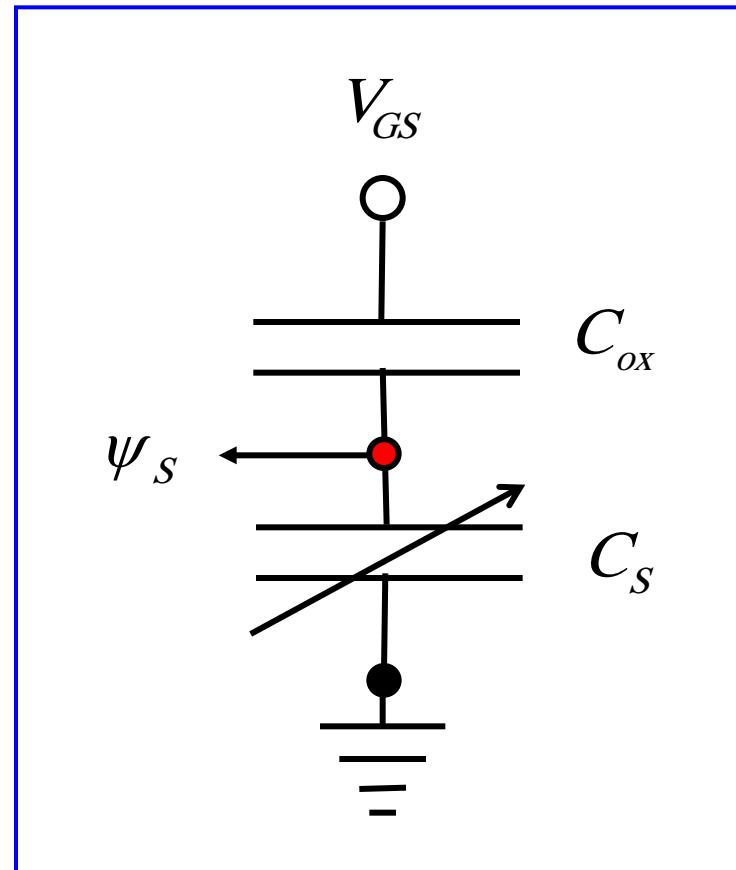
$$\psi_s = V_{GS} \frac{C_{ox}}{C_{ox} + C_s}$$

below threshold:

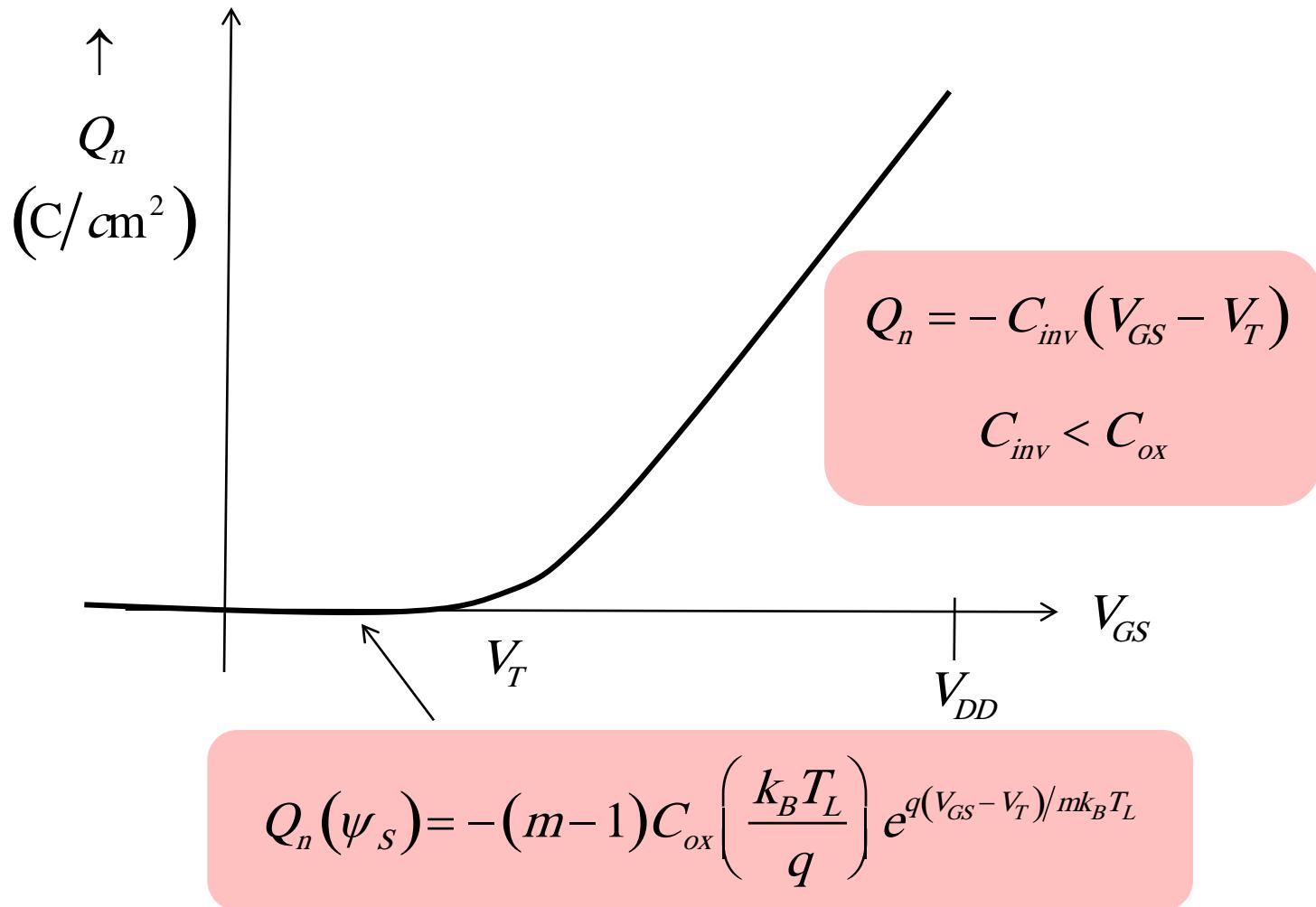
$$C_s = C_D = \frac{\kappa_s \epsilon_0}{W_D}$$

above threshold:

$$C_s = C_s(\text{inv}) = \frac{\kappa_s \epsilon_0}{t_{inv}}$$



channel charge vs. gate voltage



channel charge vs. gate voltage

$V_G \ll V_T$:

$$Q_n(V_G) = -(m-1)C_{ox} \left(\frac{k_B T_L}{q} \right) e^{q(V_{GS} - V_T)/mk_B T_L}$$

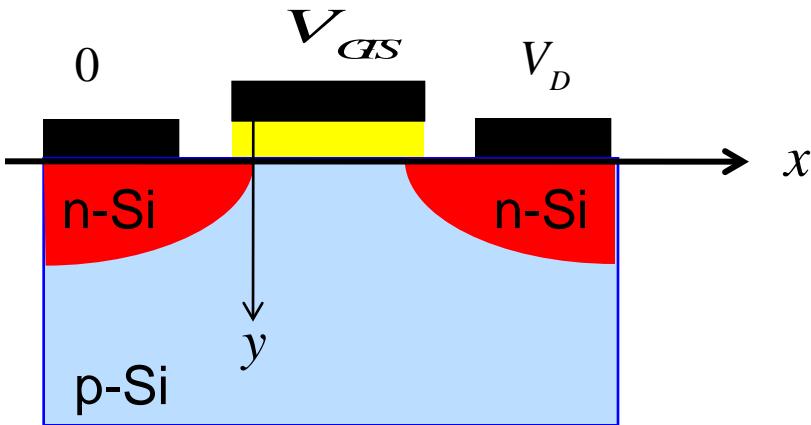
$V_G \gg V_T$:

$$Q_n = -C_{inv}(V_{GS} - V_T)$$

Is there a single expression that works both below and above threshold?

Yes – a numerical one – the “surface potential model”
Yes – an empirical one – with some fitting parameters

2D electrostatics

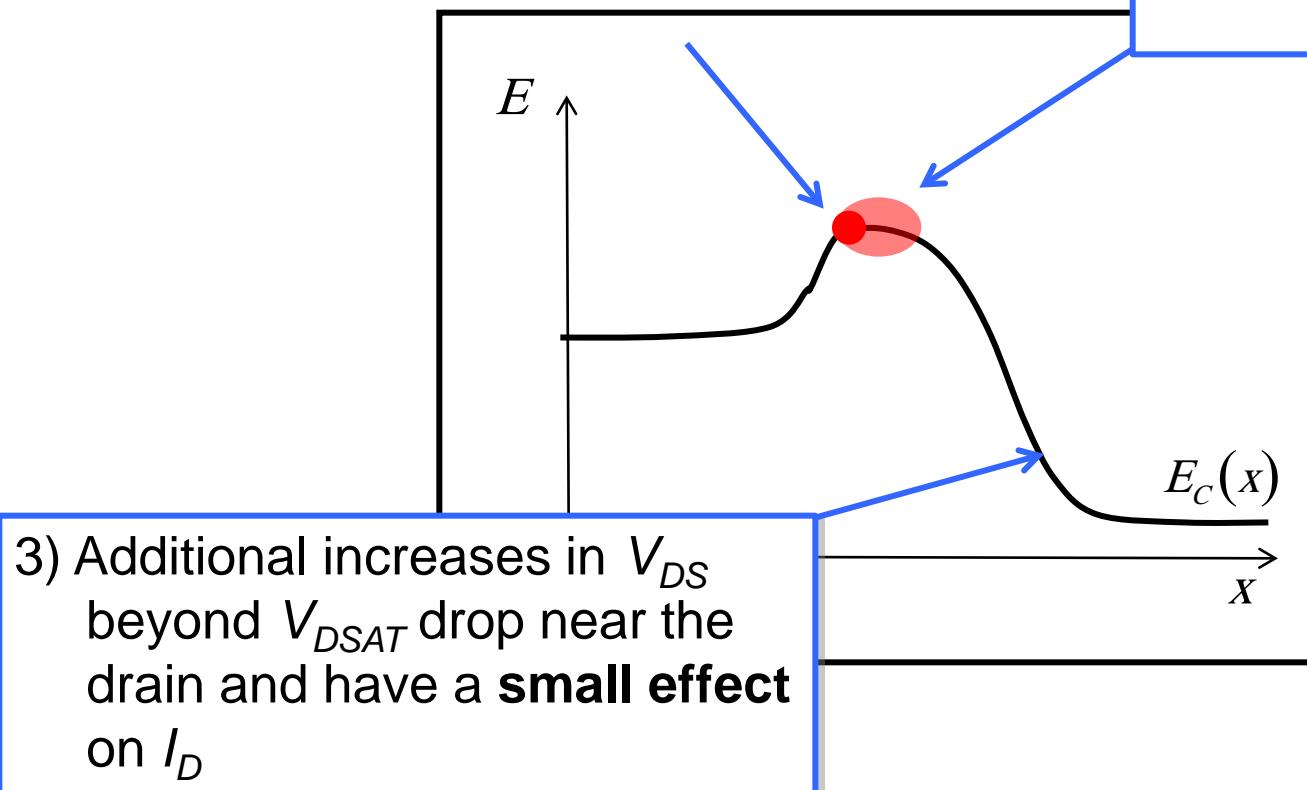


The operation of a MOSFET is determined by the 2D energy bands, which vary in space according to the 2D electrostatic potential: $\psi(x, y)$

“well-tempered MOSFET”

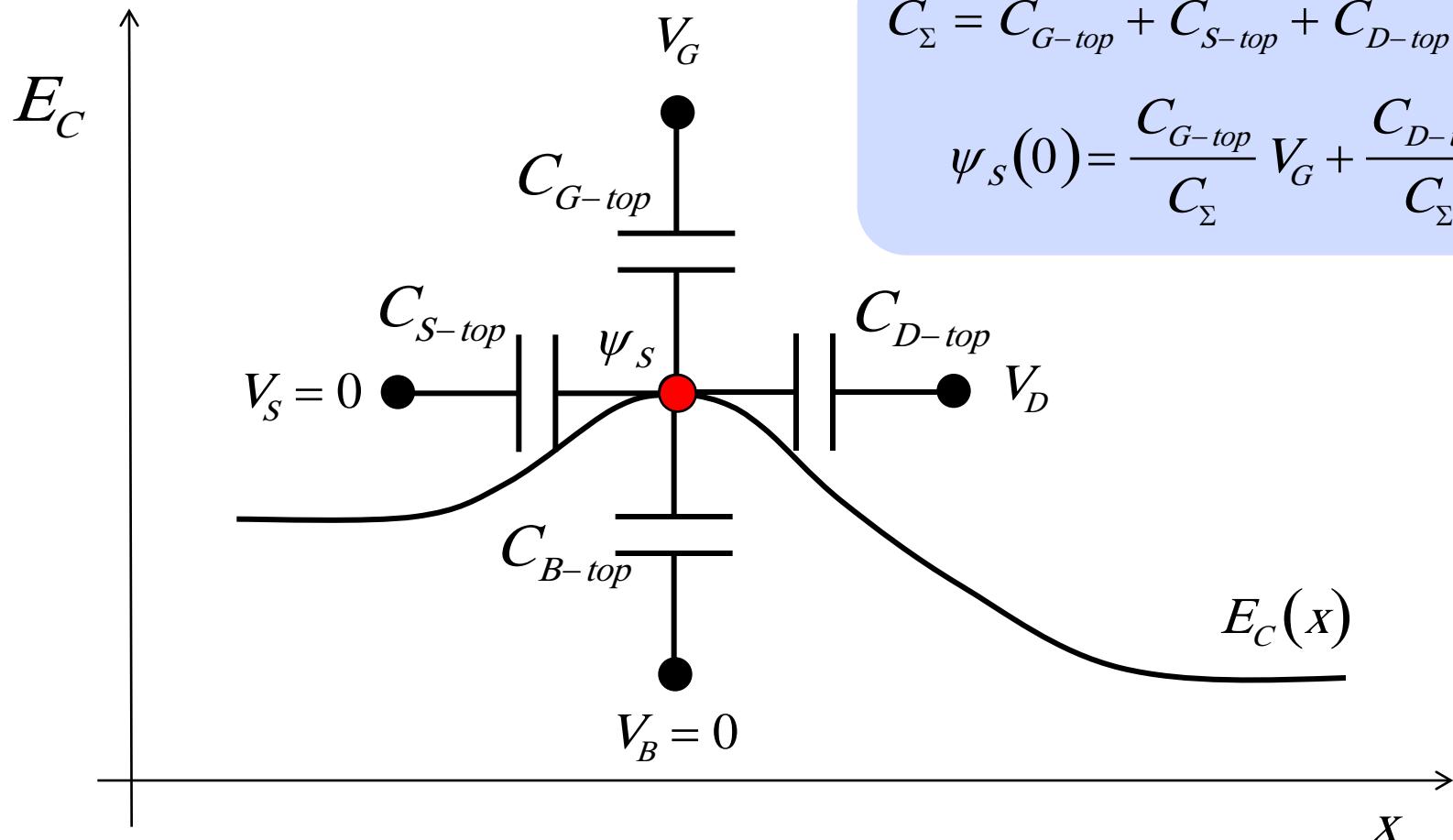
1) $Q_n(0) \approx -C_{inv}(V_G - V_T)$

2) region under strong control of gate



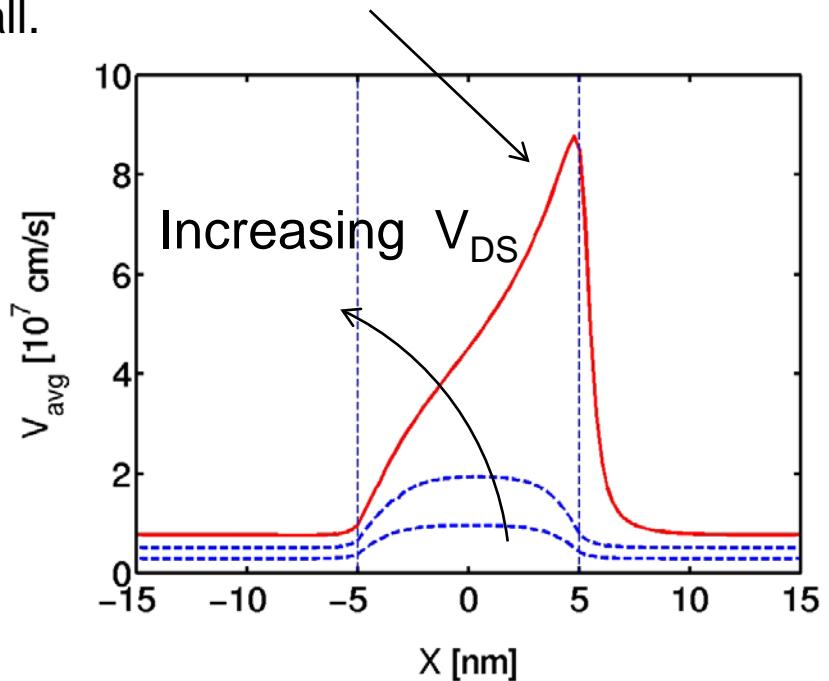
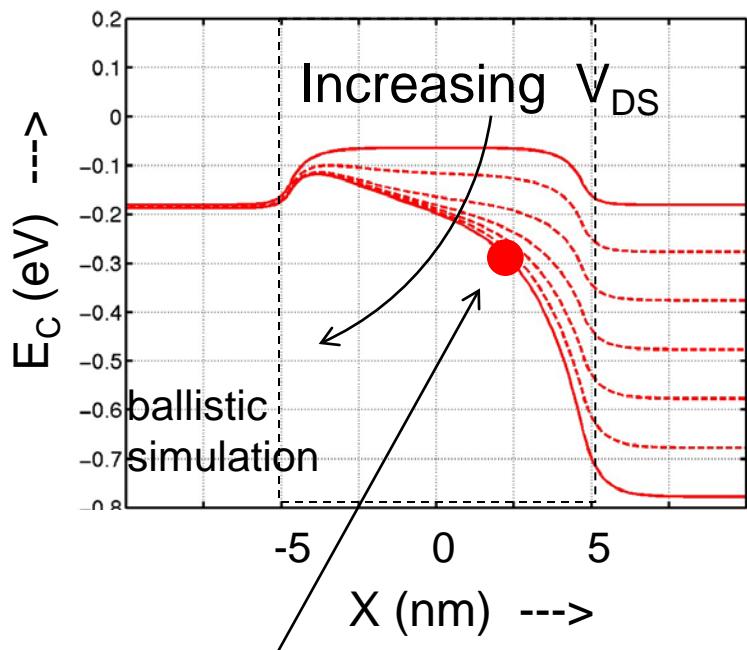
3) Additional increases in V_{DS} beyond V_{DSAT} drop near the drain and have a **small effect** on I_D

2D MOS electrostatics



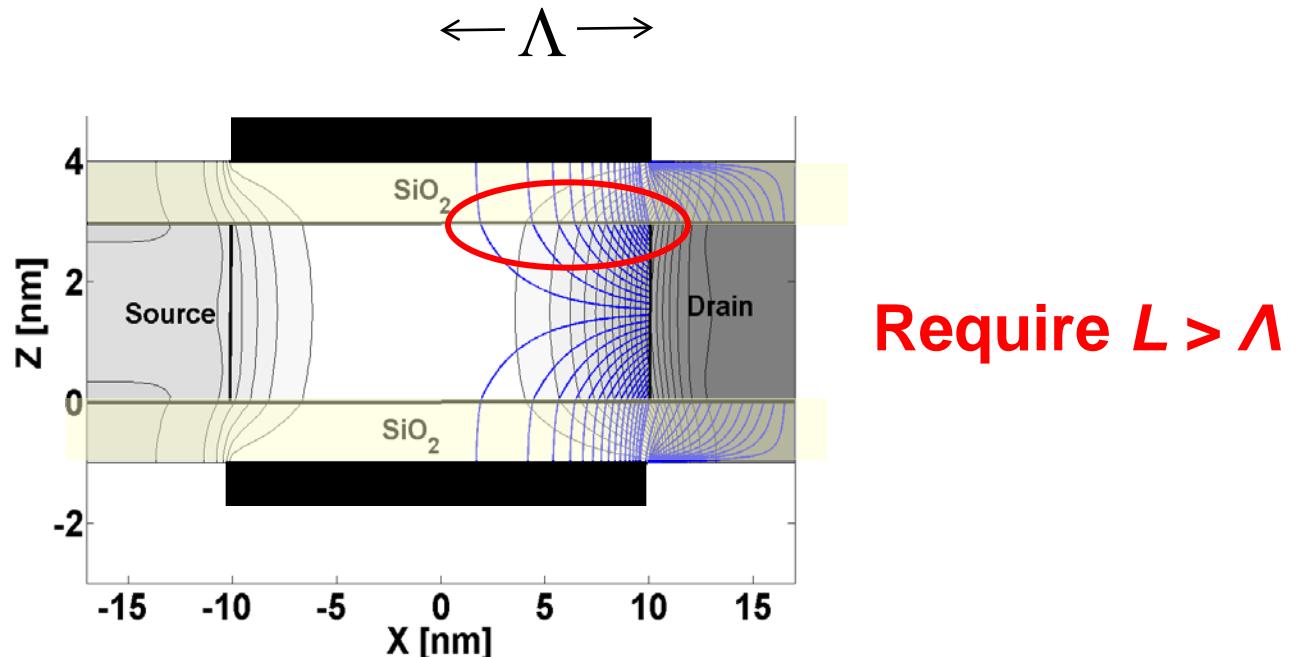
nanoMOS simulations

The electron velocity is very high in the pinch-off region. High velocity implies low inversion layer density (because I_D is constant). In the textbook model, we say $Q_i \approx 0$, but it is not really zero - just very small.



pinch-off point: where the electric field along the channel becomes very large. Note that electrons are simply swept across the high-field (pinched-off) portion at very high velocity.

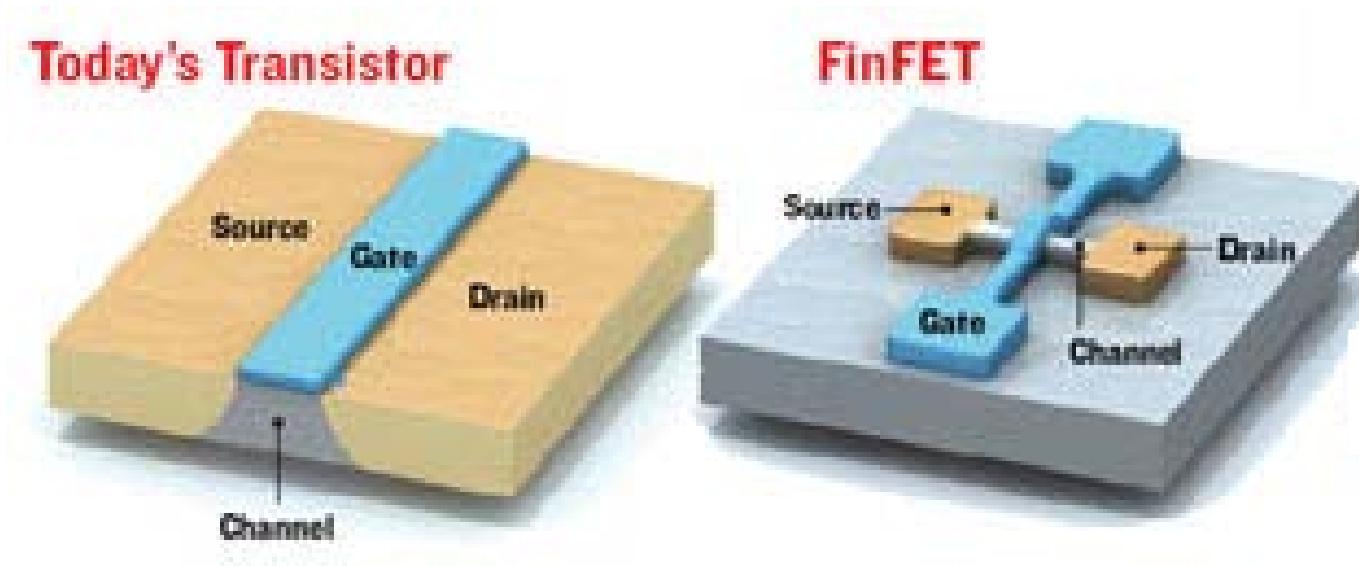
geometric scaling length, Λ



$$T_{OX} = 1 \text{ nm}$$

Off-state: $V_G = 0V$, $V_D = 1V$, $I_{off} = 0.1\mu\text{A}/\mu\text{m}$ (by H. Pal, Purdue)

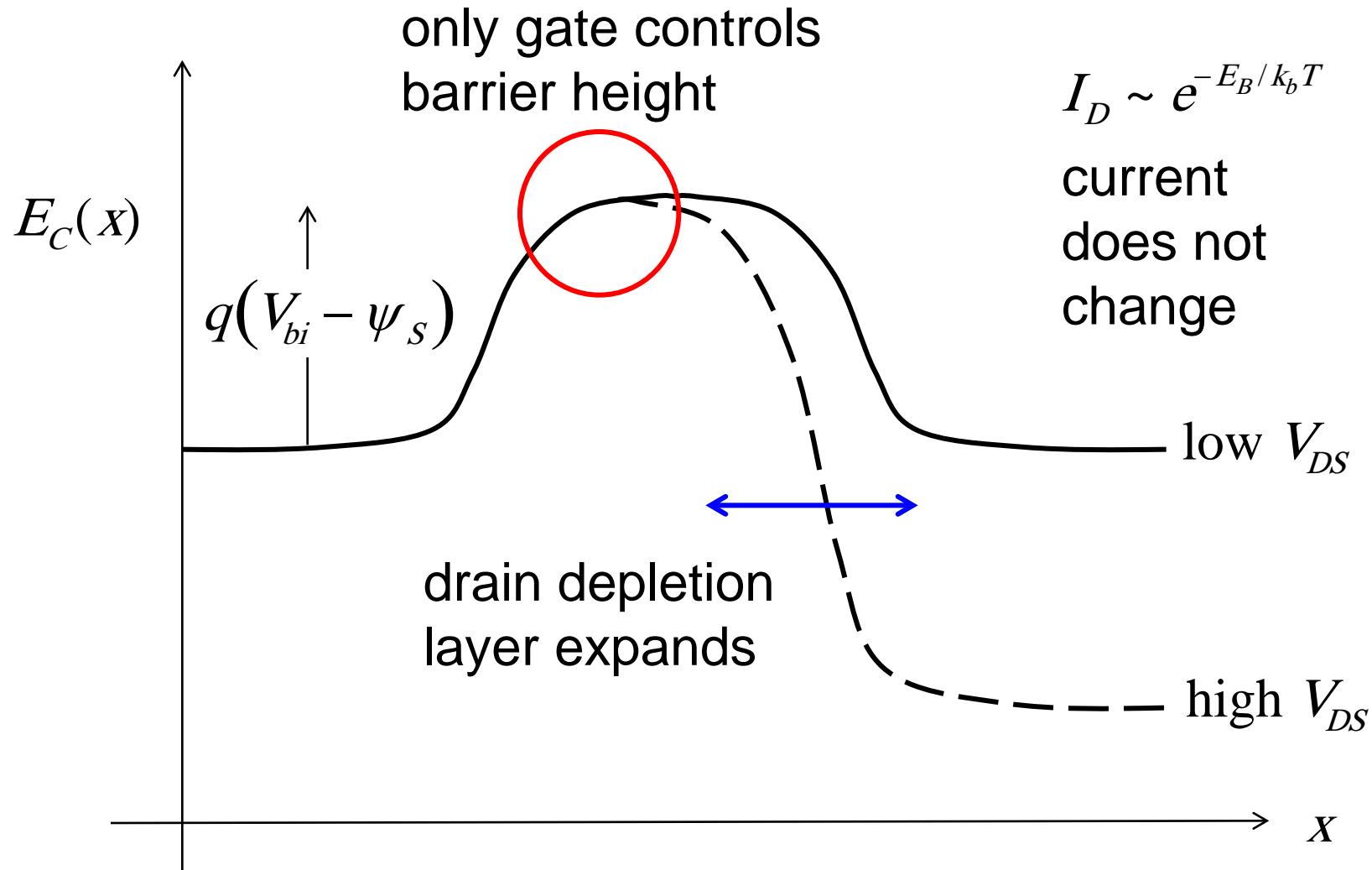
non-planar MOSFETs



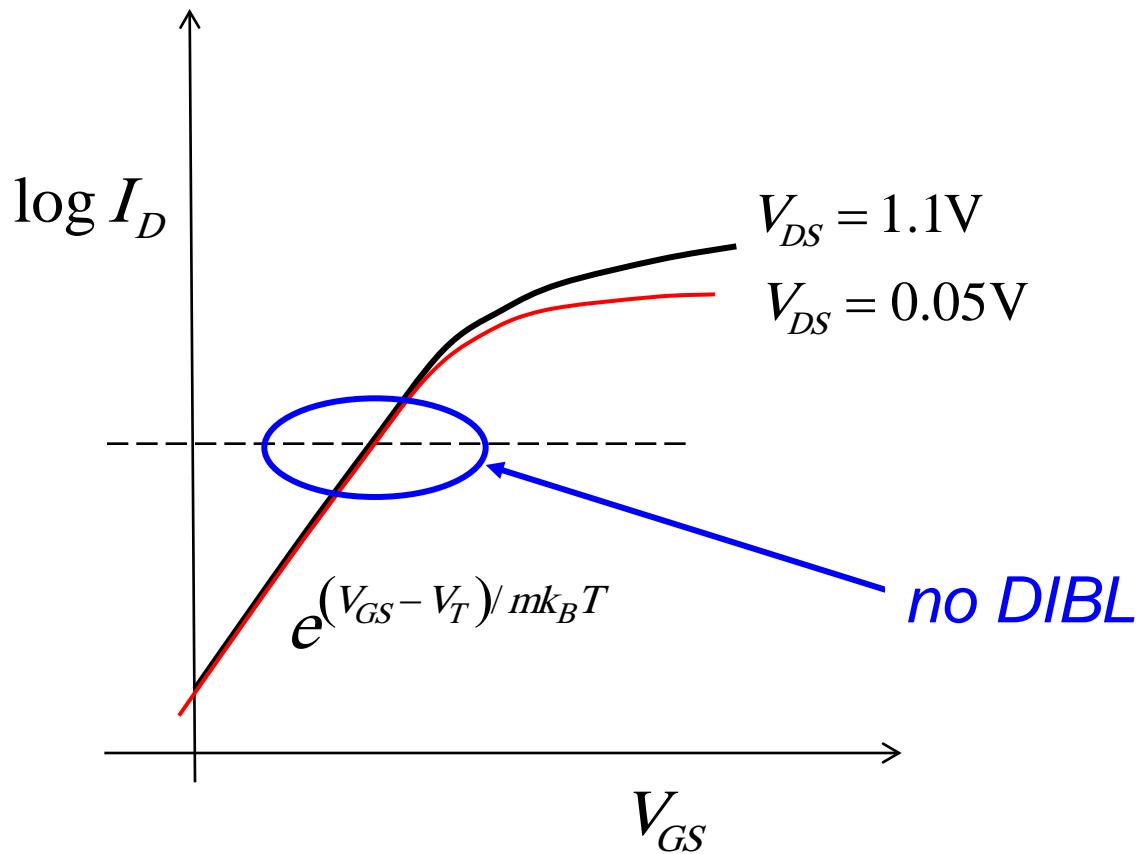
“Transistors go Vertical,” *IEEE Spectrum*, Nov. 2007.

See also: “Integrated Nanoelectronics of the Future,” Robert Chau, Brian Doyle, Suman Datta, Jack Kavalieros, and Kevin Zhang, *Nature Materials*, Vol. 6, 2007

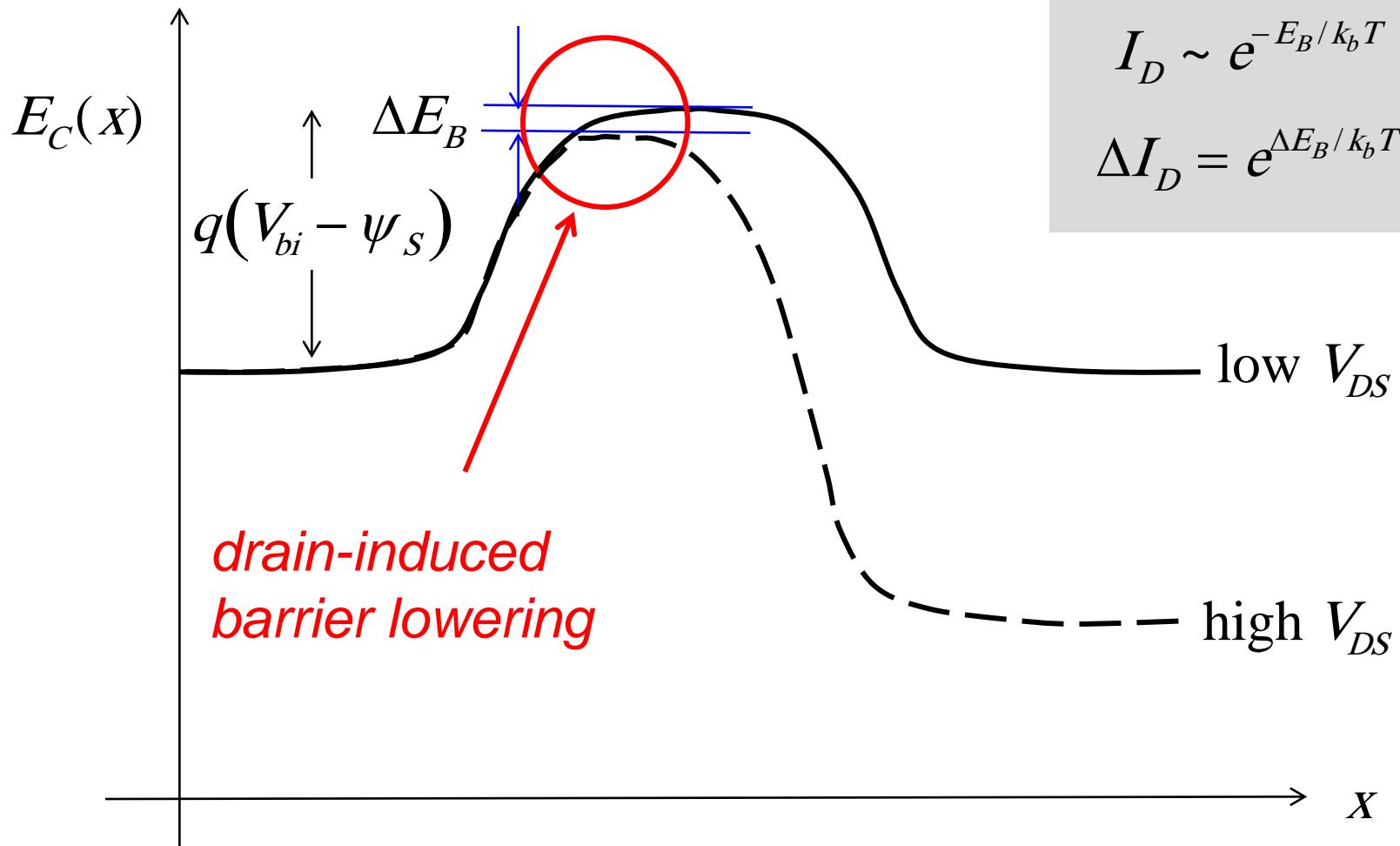
barrier lowering



no barrier lowering \rightarrow no DIBL

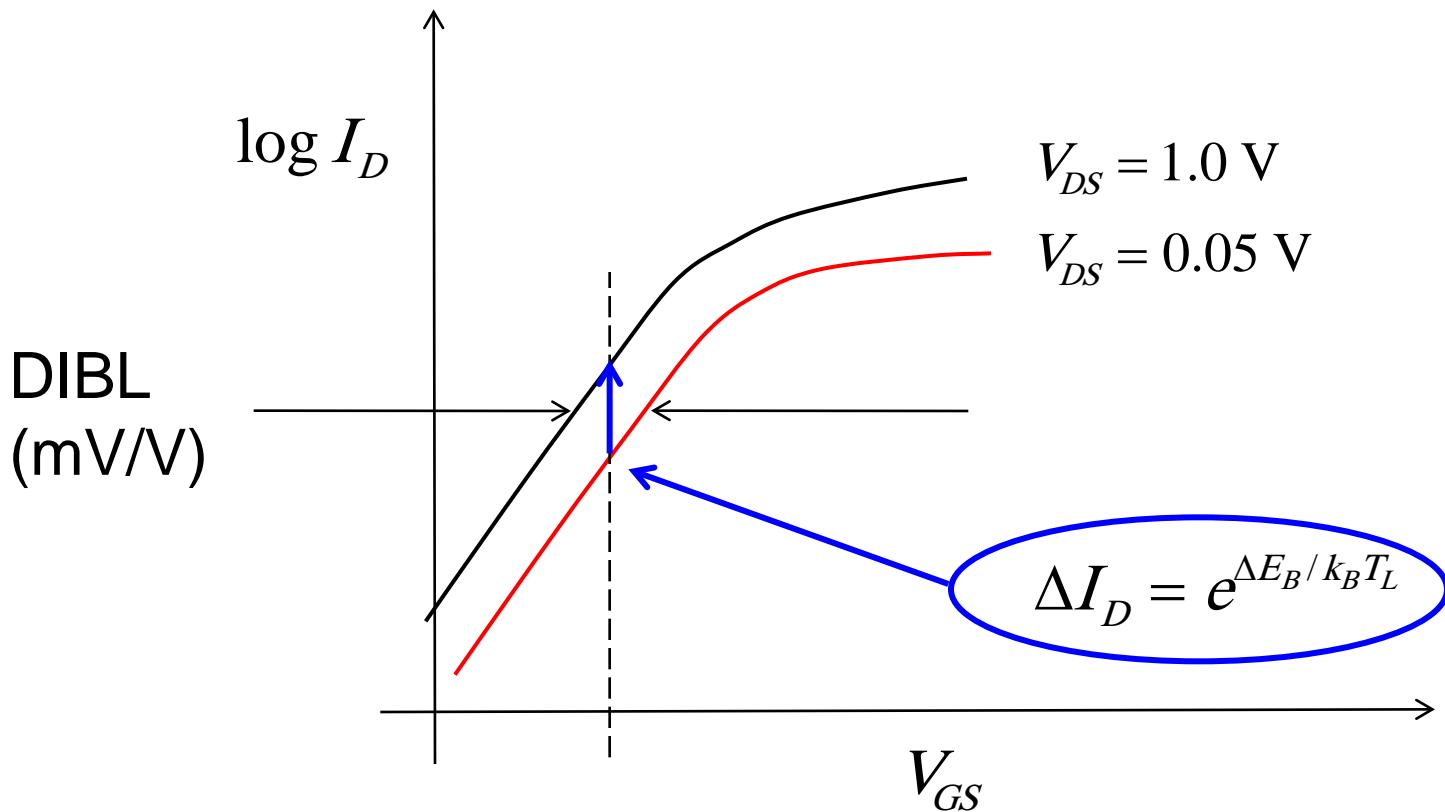


barrier lowering



$$I_D \sim e^{-E_B/k_b T}$$
$$\Delta I_D = e^{\Delta E_B/k_b T}$$

barrier lowering increases current



summary

- 1) 1D MOS electrostatics (subthreshold)
- 2) 1D MOS electrostatics (above threshold)
- 3) 2D MOS electrostatics, “geometric screening”
- 4) barrier lowering, DIBL

references

For a discussion of 1D MOS electrostatics, see:

M.S. Lundstrom, ECE-612: "Nanoscale Transistors," Lectures 1-4, Fall, 2008. <https://nanohub.org/resources/5328>

For a discussion of 2D MOS electrostatics, see:

M.S. Lundstrom, ECE-612: "Nanoscale Transistors," Lecture 12, Fall, 2008. <https://nanohub.org/resources/5328>