

Lecture 5: Transport: ballistic, diffusive, non-local, and quantum

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level 0 “Virtual Source model”

$$1) \quad I_D/W = Q_n(V_G) \langle v(V_D) \rangle$$

$$2) \quad \begin{aligned} V_{GS} \leq V_T &: Q_n(V_{GS}) = 0 \\ V_{GS} > V_T &: Q_n(V_{GS}) = C_{ox} (V_{GS} - V_T) \end{aligned}$$

$$3) \quad \langle v(V_D) \rangle = F_{SAT}(V_D) v_{SAT}$$

$$4) \quad F_{SAT}(V_D) = \frac{V_D / V_{DSAT}}{\left[1 + (V_D / V_{DSAT})^\beta \right]^{1/\beta}}$$

$$5) \quad V_{DSAT} = \frac{v_{SAT} L}{\mu_{eff}}$$

There are only 5 device-specific input parameters to this model:

$$C_{ox}, V_T, v_{SAT}, \mu_{eff}, L$$



level 0.5 “Virtual Source model”

$$1) \quad I_D/W = Q_n(V_G) \langle v(V_D) \rangle$$

$$2) \quad V_{GS} < V_T: \quad Q_n(\psi_S) = -(m-1)C_{ox} \left(\frac{k_B T_L}{q} \right) e^{q(V_{GS} - V_T)/mk_B T_L}$$

$$V_{GS} > V_T: \quad Q_n = -C_{inv} (V_{GS} - V_T)$$

$$3) \quad \langle v(V_D) \rangle = F_{SAT}(V_D) v_{SAT}$$

$$4) \quad F_{SAT}(V_D) = \frac{V_D / V_{DSAT}}{\left[1 + (V_D / V_{DSAT})^\beta \right]^{1/\beta}}$$

$$5) \quad V_{DSAT} = \frac{v_{SAT} L}{\mu_{eff}}$$

$C_{ox}, C_{inv}, V_T, v_{SAT}, \mu_{eff}, L$

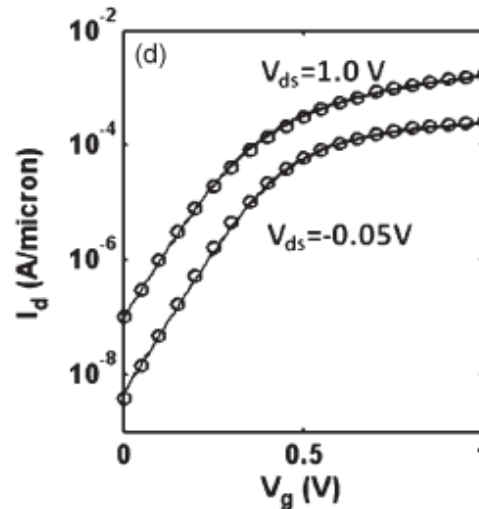
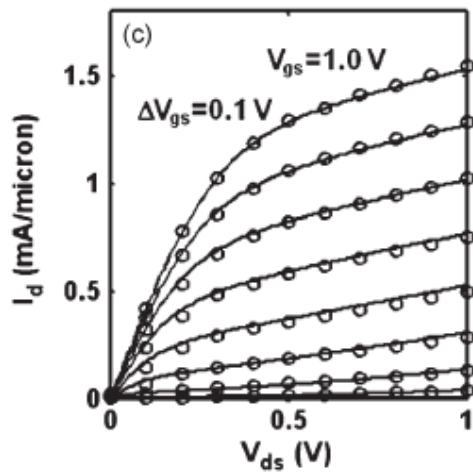
Device-specific input parameters
to this model:

Level 1 Virtual Source Model

With a few extensions

(e.g. continuous, empirical expression for Q_n above and below threshold, series resistance, etc.)

we arrive at the Level 1 VS model, which does a remarkably good job of describing modern transistors.



32 nm high-k technology

but...

Our derivation has been based on concepts like mobility and high-field saturation velocity:

$$v_{SAT}, \mu_{eff}$$

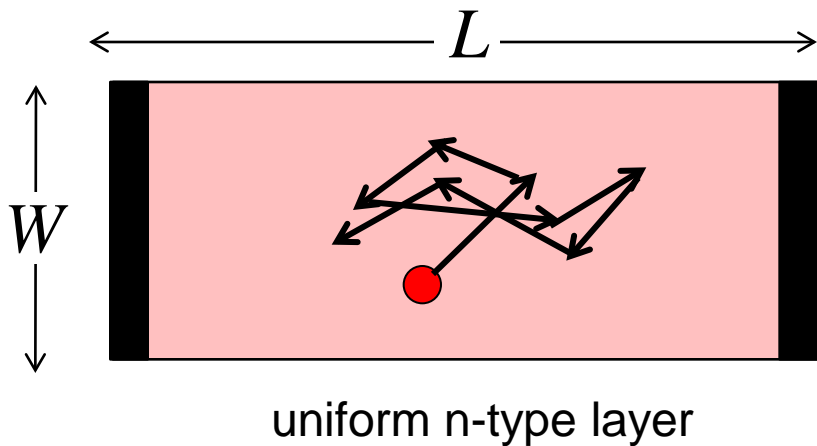
that are only valid for long channel devices.

To understand what these parameters mean in a nanoscale MOSFET, we need to understand a bit about carrier transport.

carrier transport

- diffusive transport
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- Landauer approach to transport

equilibrium



$$\langle v_x \rangle = 0$$

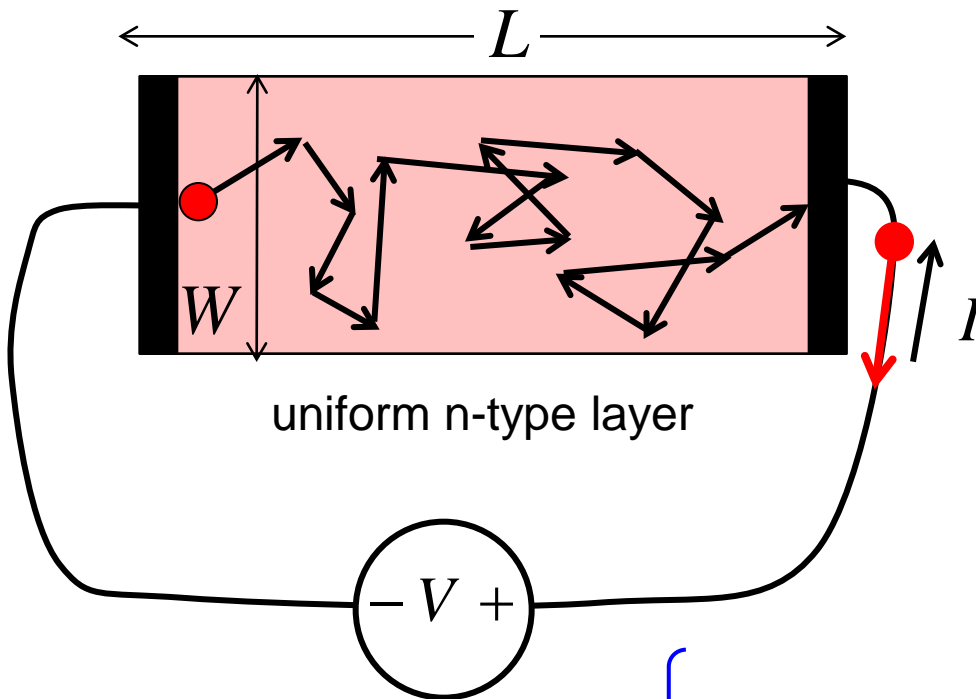
$$\langle KE \rangle = k_B T_L$$

$$\langle KE \rangle = \frac{1}{2} m_n^* \langle v^2 \rangle$$

$$\sqrt{\langle v^2 \rangle} = v_{rms} = \sqrt{\frac{2k_B T}{m_n^*}}$$

$$v_{rms} \approx 10^7 \text{ cm/s}$$

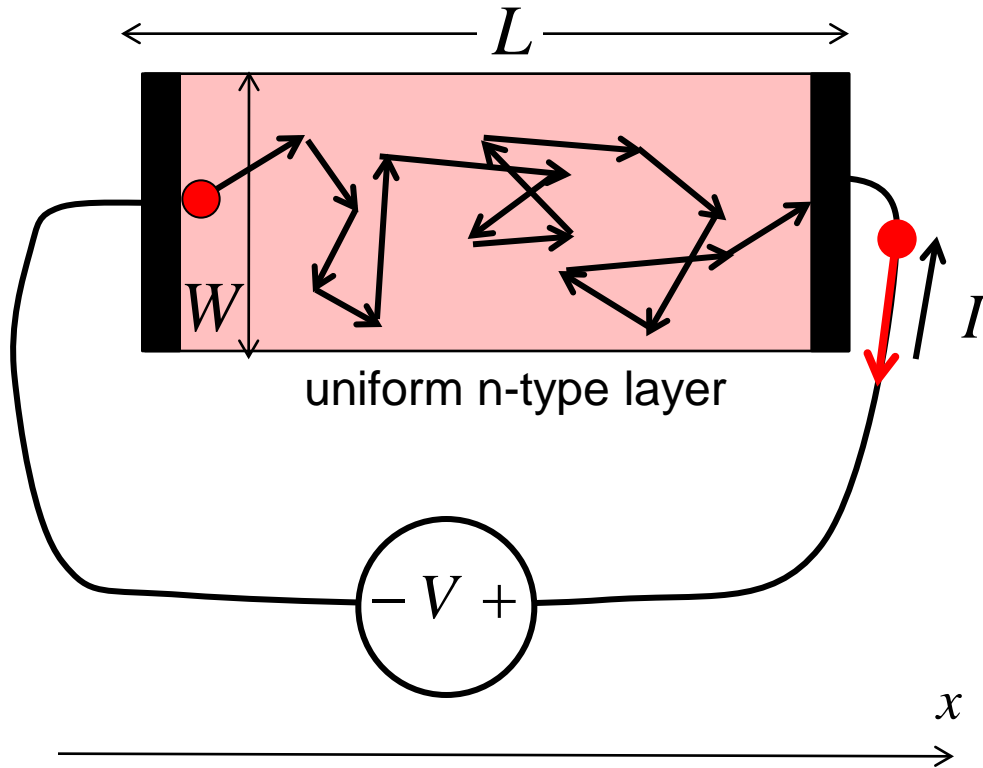
current flow



- 1) Random walk with a small bias from left to right.
- 2) Assume that electrons “drift” to the right at an average velocity, v_d
- 3) what is I ?

{ The average distance between scattering events is called the “mean-free-path”. }

current flow



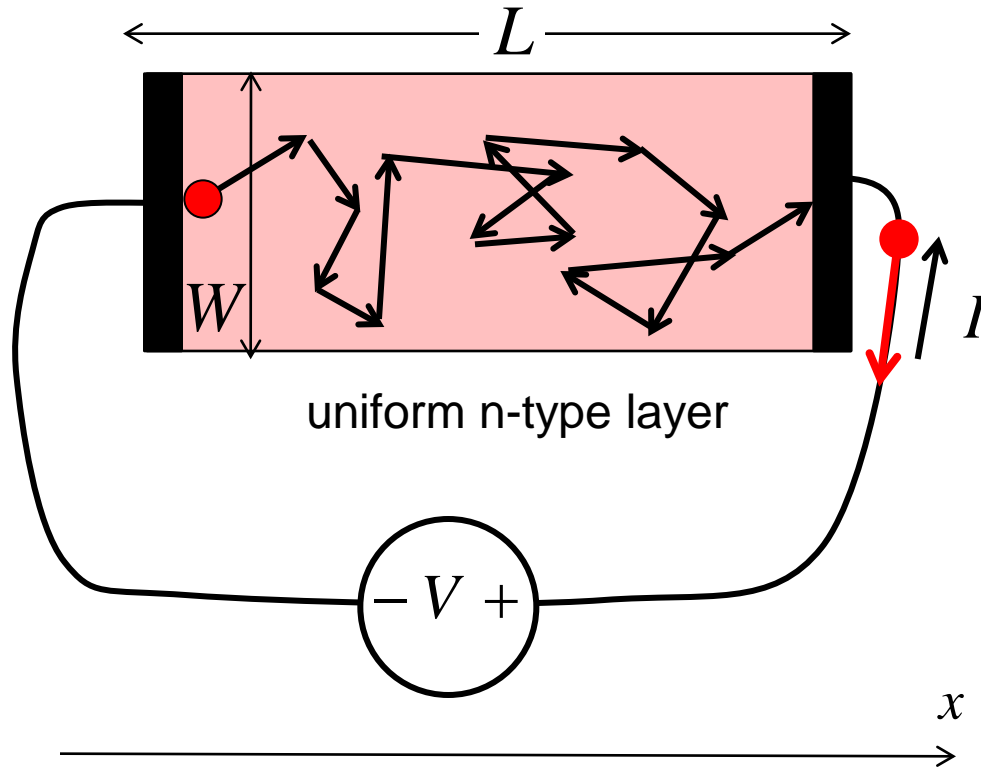
$$I = Q/t_t$$

$$Q = -qn_sWL = Q_nWL$$

$$t_t = L/v_d$$

$$I = WQ_nv_d$$

velocity and electric field



$$\frac{dp}{dt} = F_e$$

$$F_e = -q\mathcal{E}$$

$$\Delta p = -q\mathcal{E}\tau = m_n^* \Delta v$$

$$\Delta v = -\frac{q\tau}{m_n^*} \mathcal{E}$$

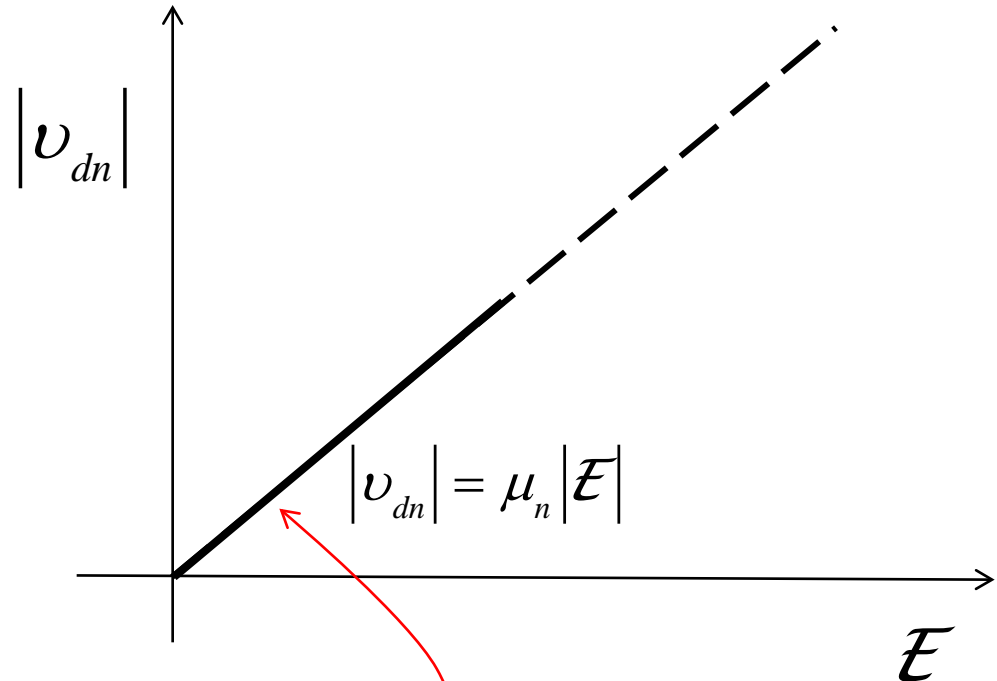
$$v_{dn} = -\left(\frac{q\langle\tau\rangle}{m_n^*}\right) \mathcal{E} = -\mu_n \mathcal{E}$$

"mobility"

velocity and electric field

$$v_{dn} = -\mu_n \mathcal{E}$$

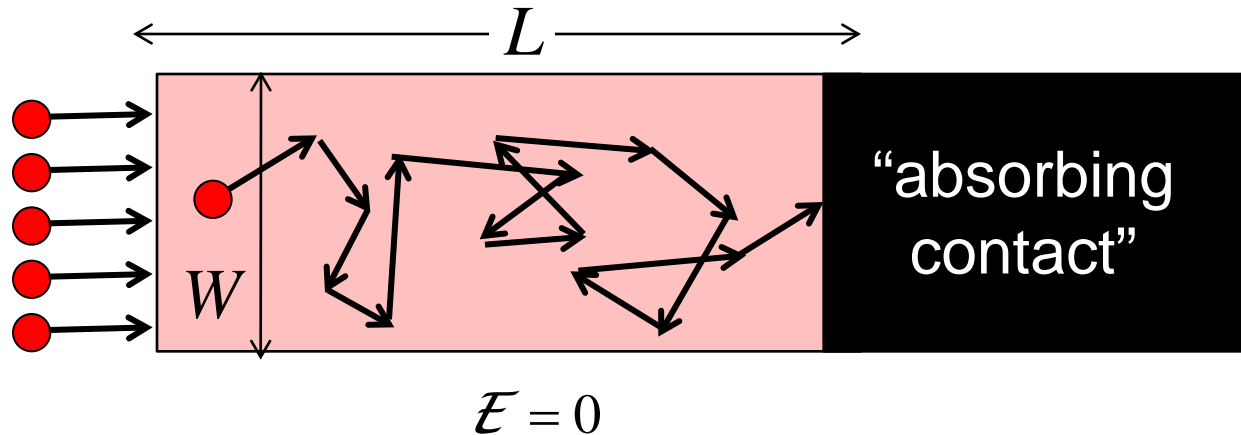
$$\mu_n = \left(\frac{q \langle \tau \rangle}{m_n^*} \right) \text{cm}^2/\text{V-s}$$



low V_{DS} in a MOSFET →

“low-field” or “near-equilibrium”
or “linear” transport

diffusion



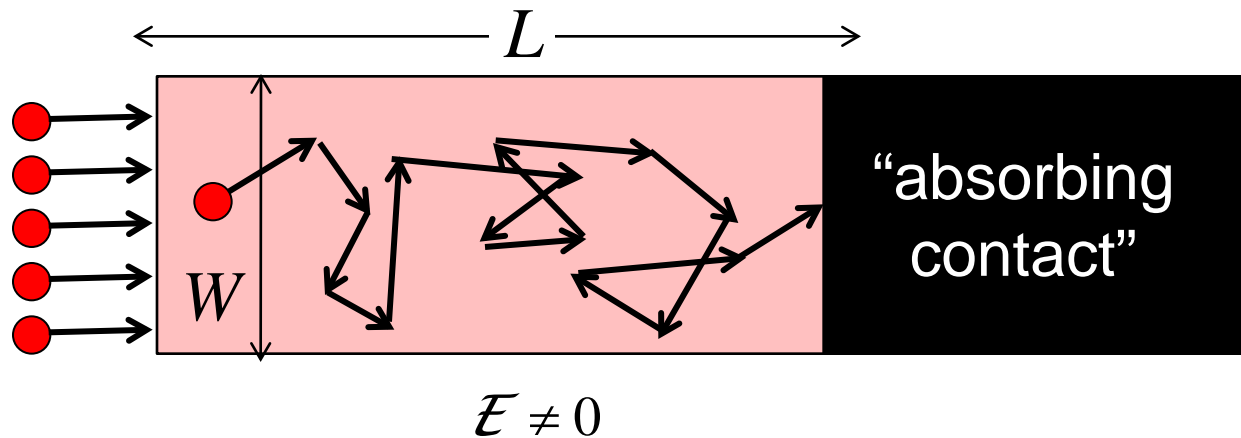
$$I/W = J = (-q) \left[-D_n \frac{dn_s}{dx} \right] \quad \text{“Fick’s Law”}$$

D_n (cm^2/s) “diffusion coefficient”

$$\frac{D_n}{\mu_n} = \frac{k_B T_L}{q}$$

“Einstein relation”

drift + diffusion



$$J_n = n_s q \mu_n \mathcal{E} + q D_n \frac{dn_s}{dx}$$

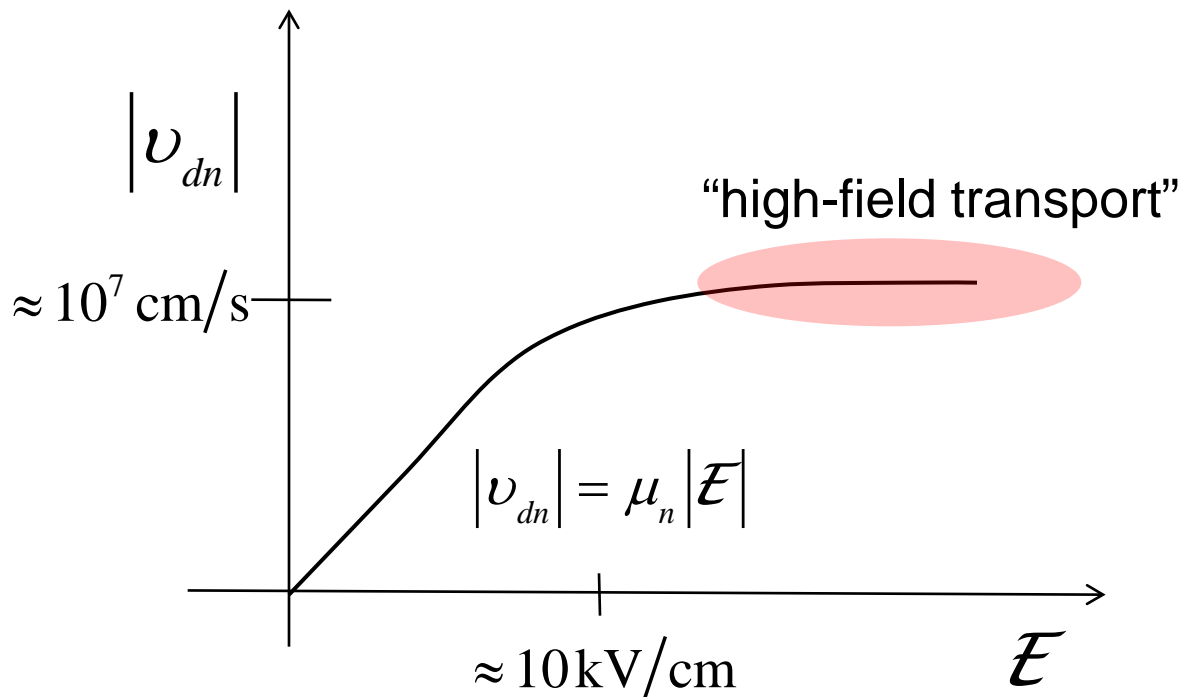
"drift-diffusion equation"

$$\frac{D_n}{\mu_n} = \frac{k_B T_L}{q}$$

carrier transport

- diffusive transport ✓
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- Landauer approach to transport

high-field transport



high V_{DS} in a MOSFET

$$v_{dn} = -\mu_n \mathcal{E}$$

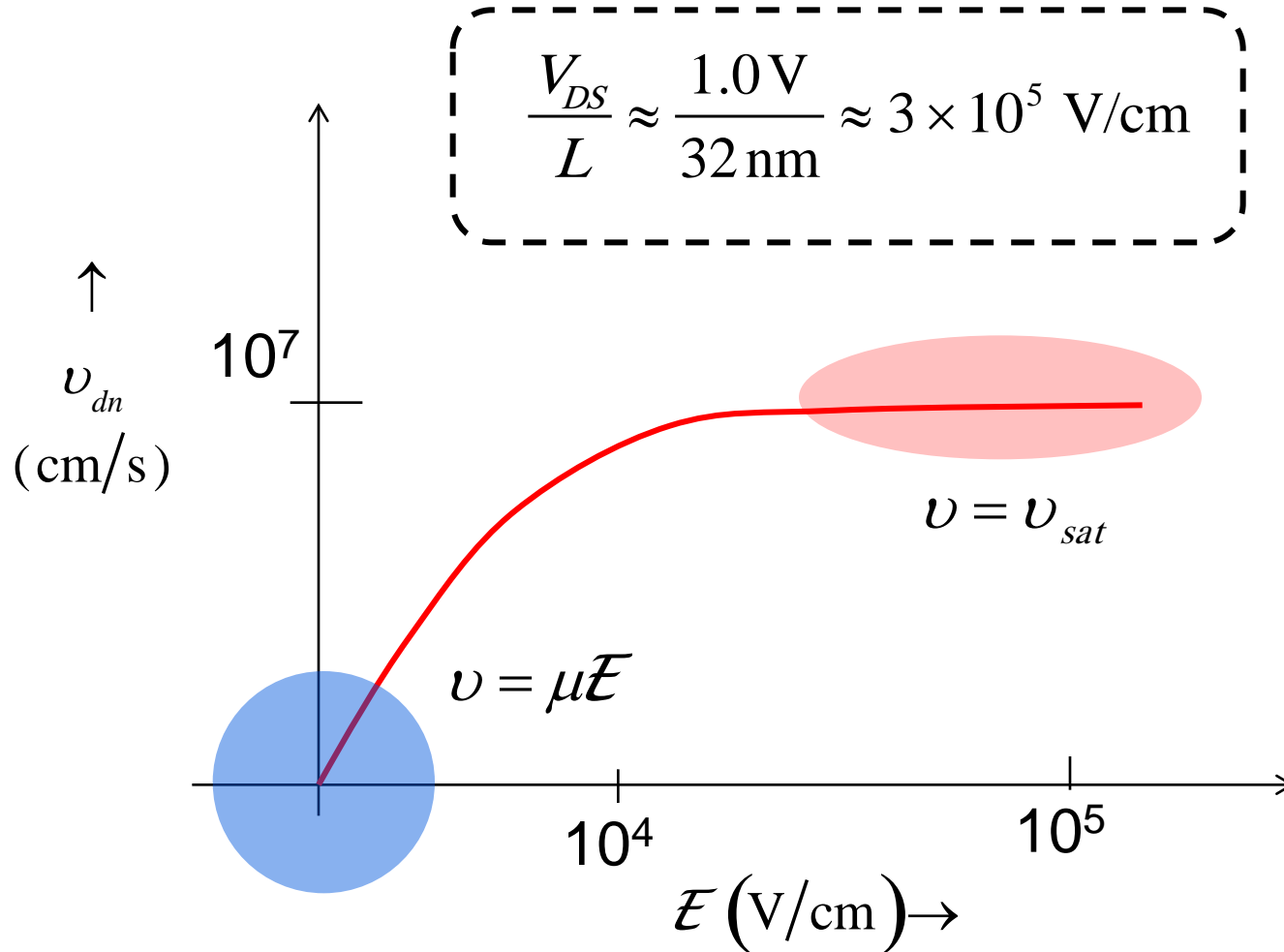
$$\mu_n = -\left(\frac{q \langle \tau \rangle}{m_n^*} \right)$$

$$\frac{1}{\tau(E)} \propto D(E)$$

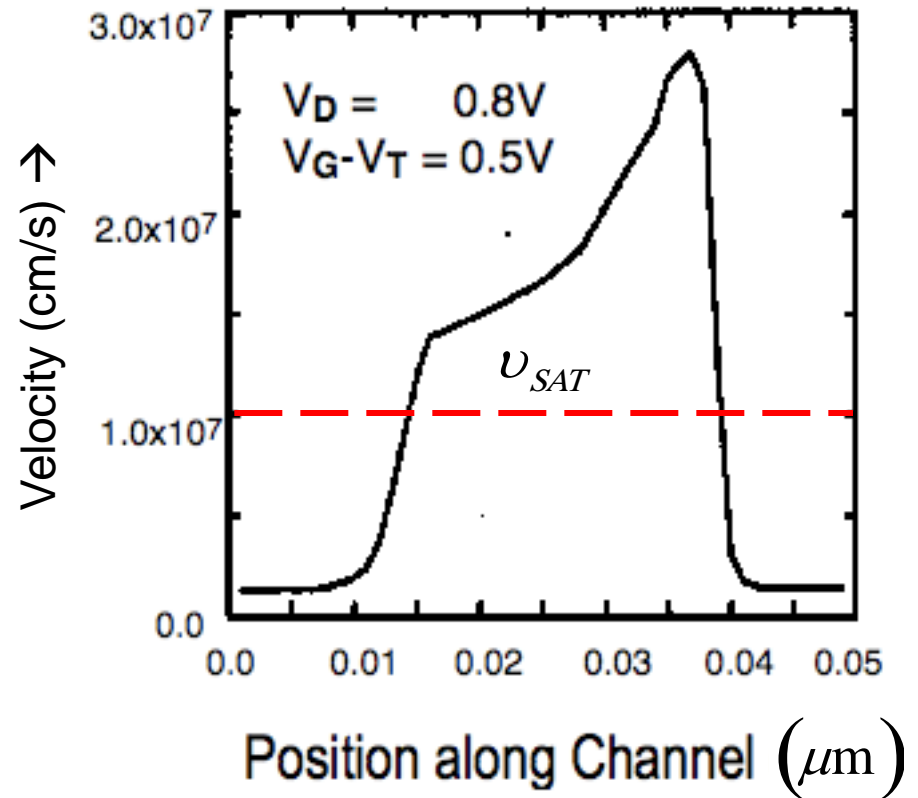
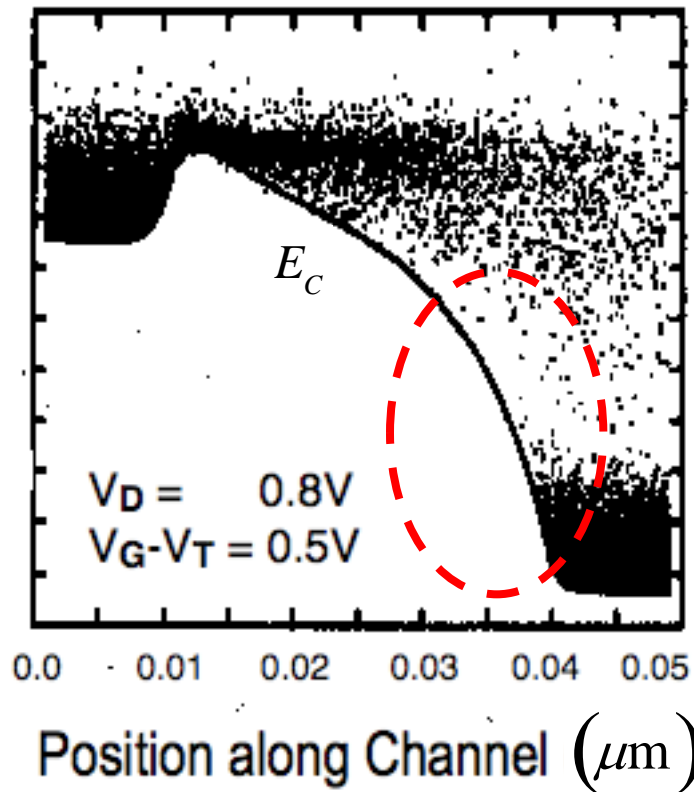
$$\mathcal{E} \uparrow \quad \langle E \rangle \uparrow \quad \langle \tau \rangle \downarrow \quad \mu_n \downarrow$$

$$\mu_n(\mathcal{E})$$

electric fields in nanoscale MOSFETs

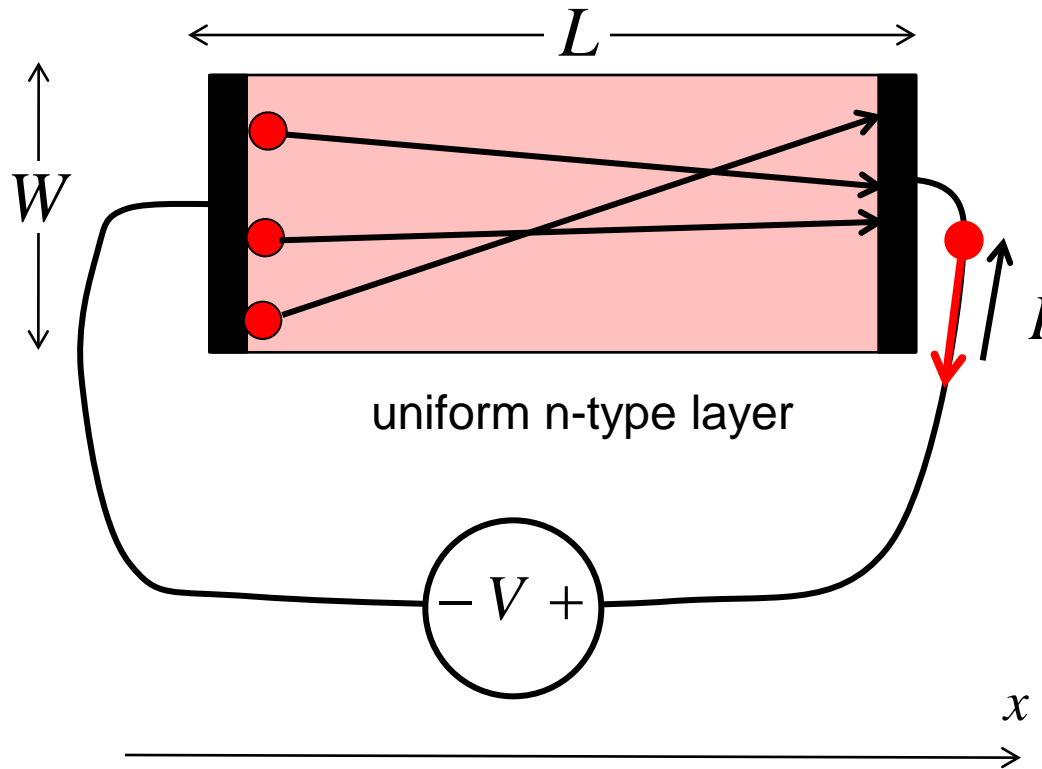


non-local transport



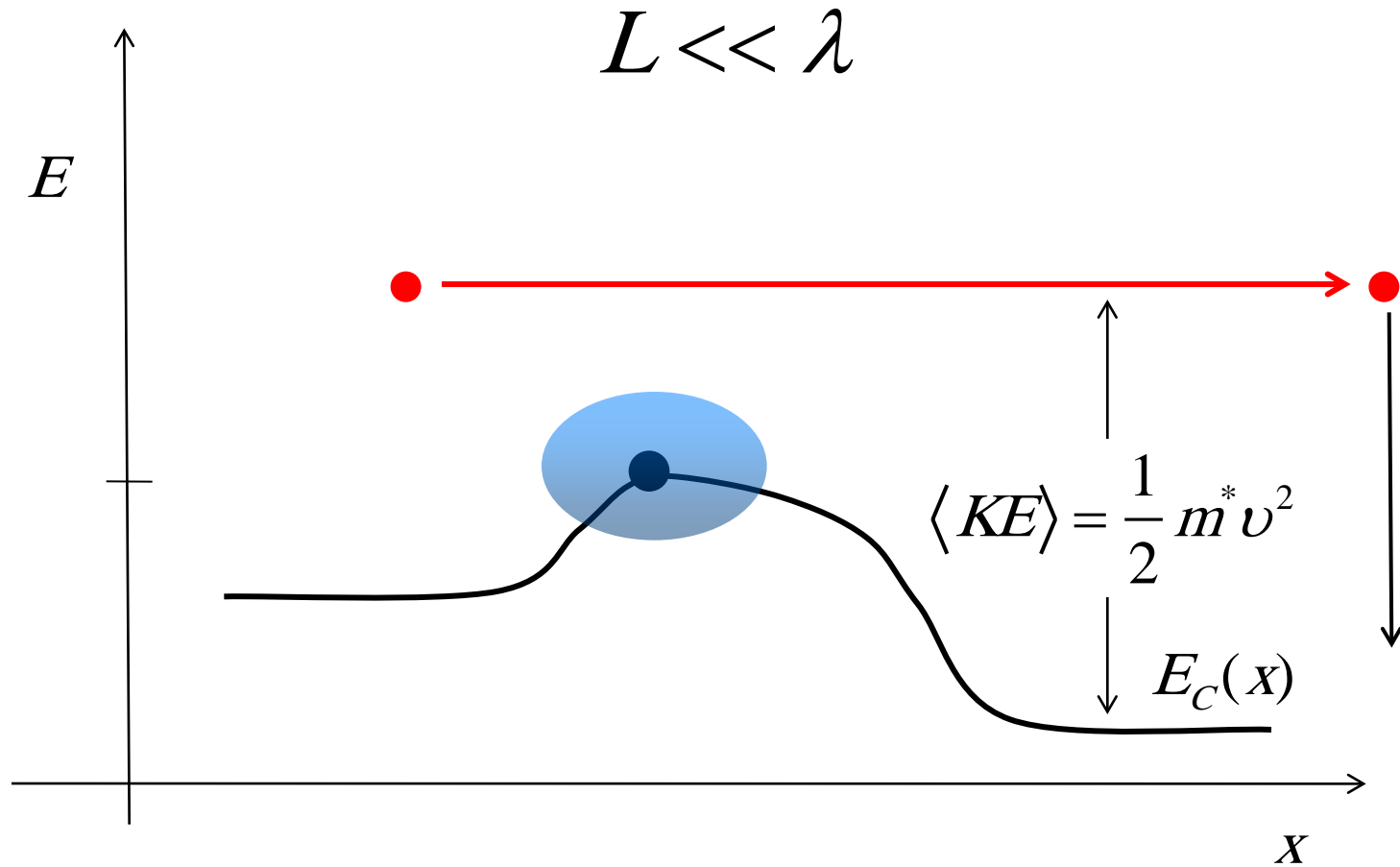
D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

ballistic transport

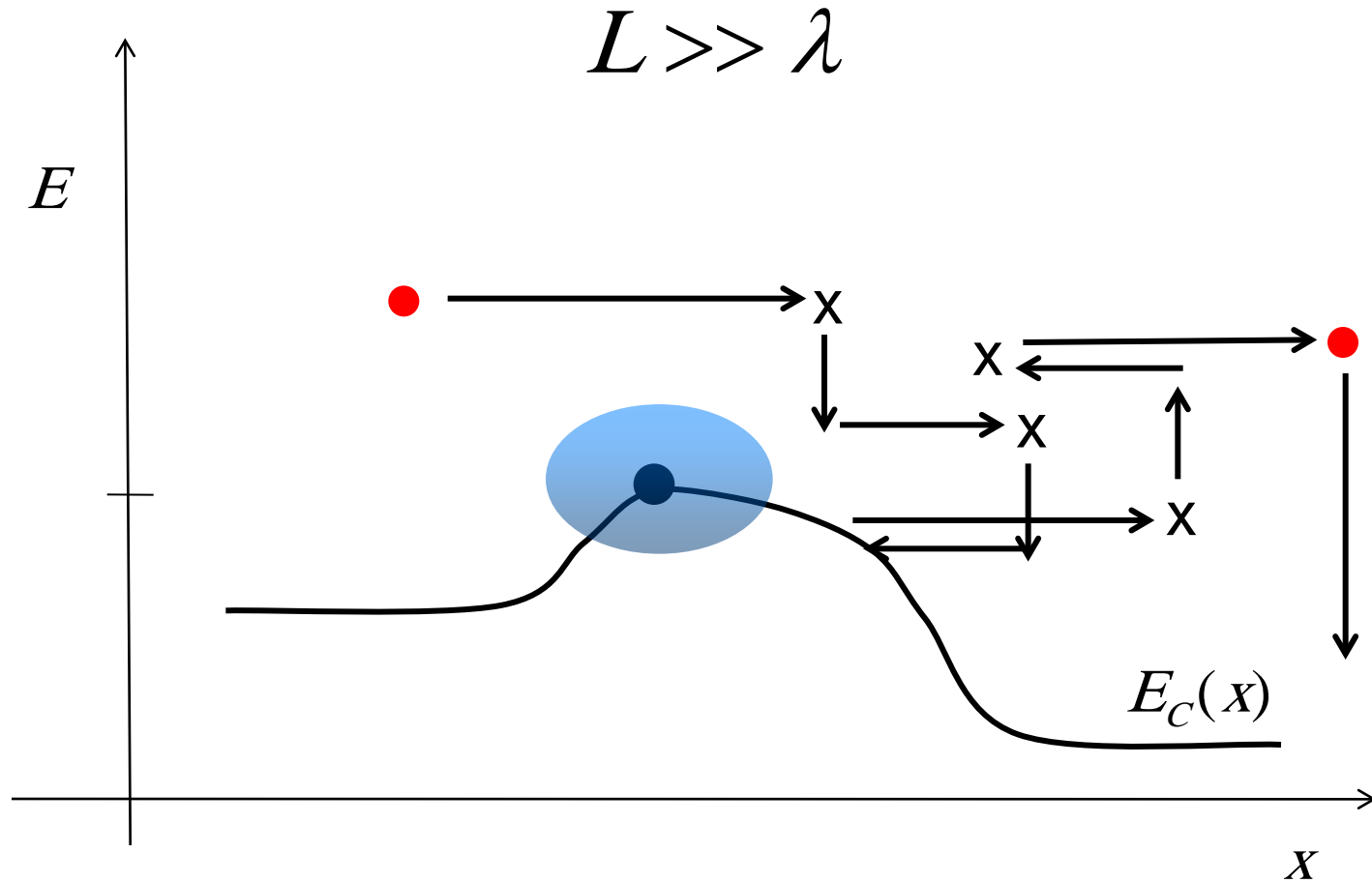


The sample length, L , is much shorter than the MFP for scattering.

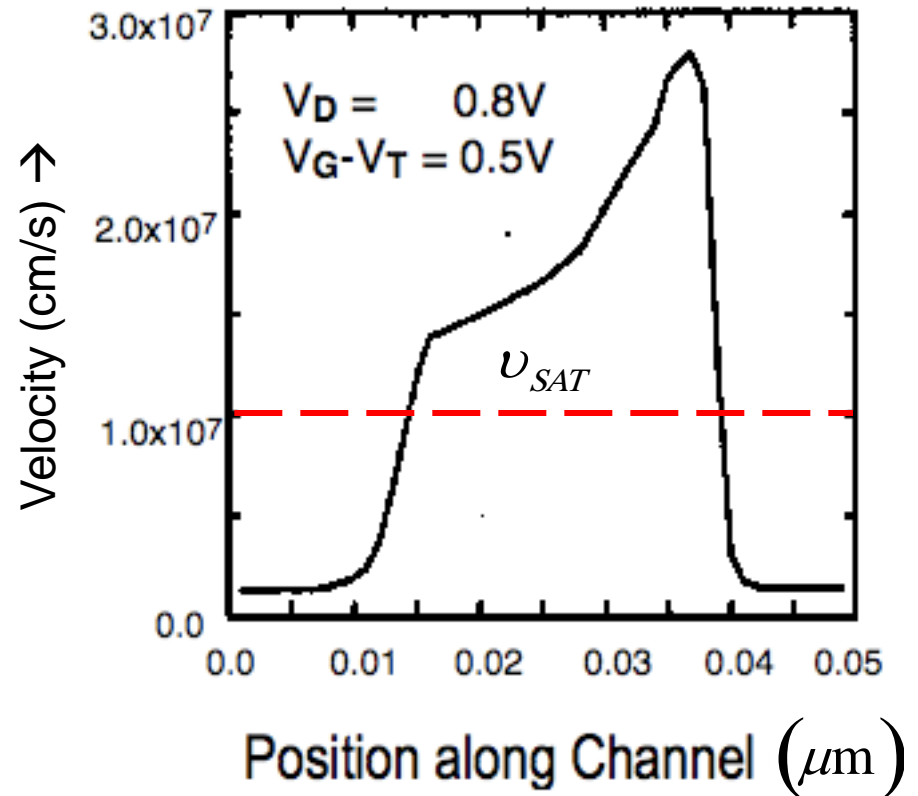
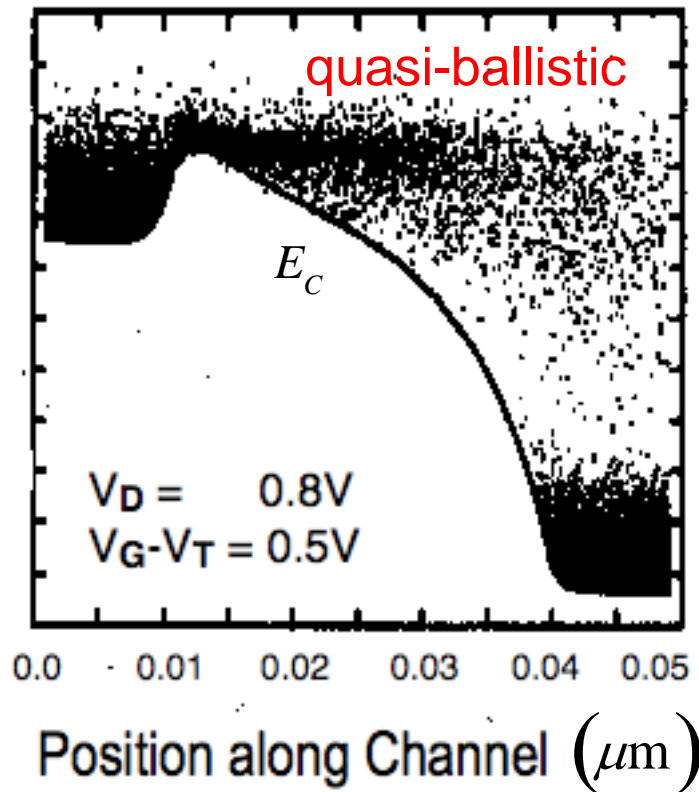
ballistic transport in a MOSFET



diffusive transport in a MOSFET



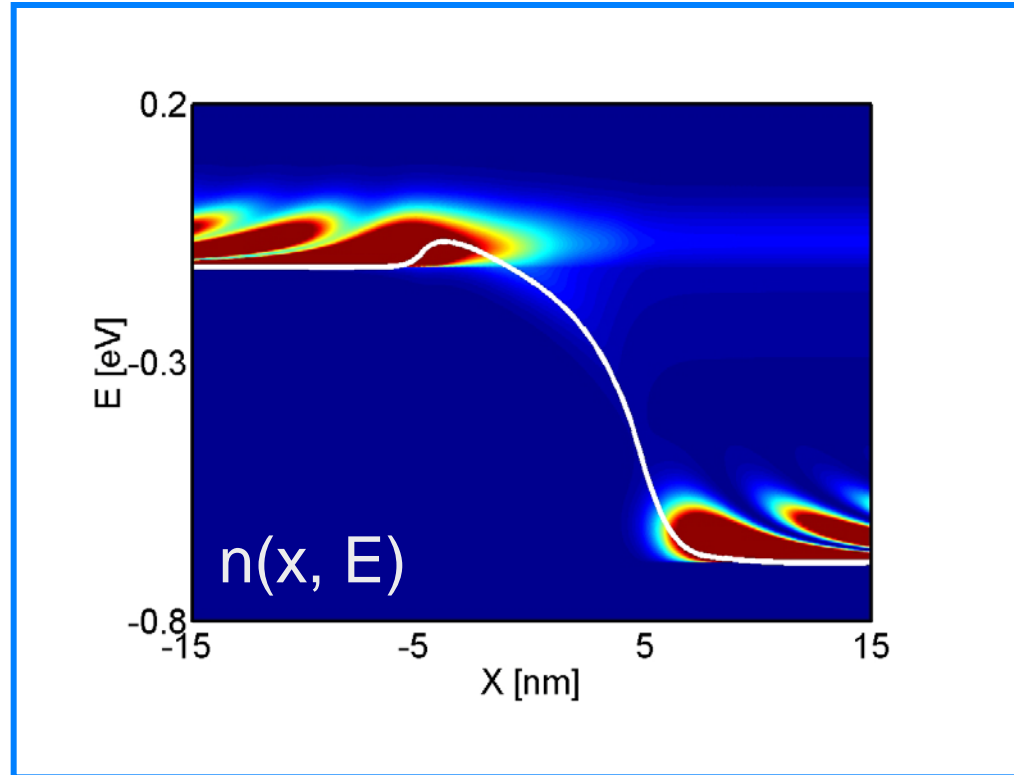
quasi-ballistic, non-local transport in a MOSFET



D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

quantum transport

$L = 10$ nm

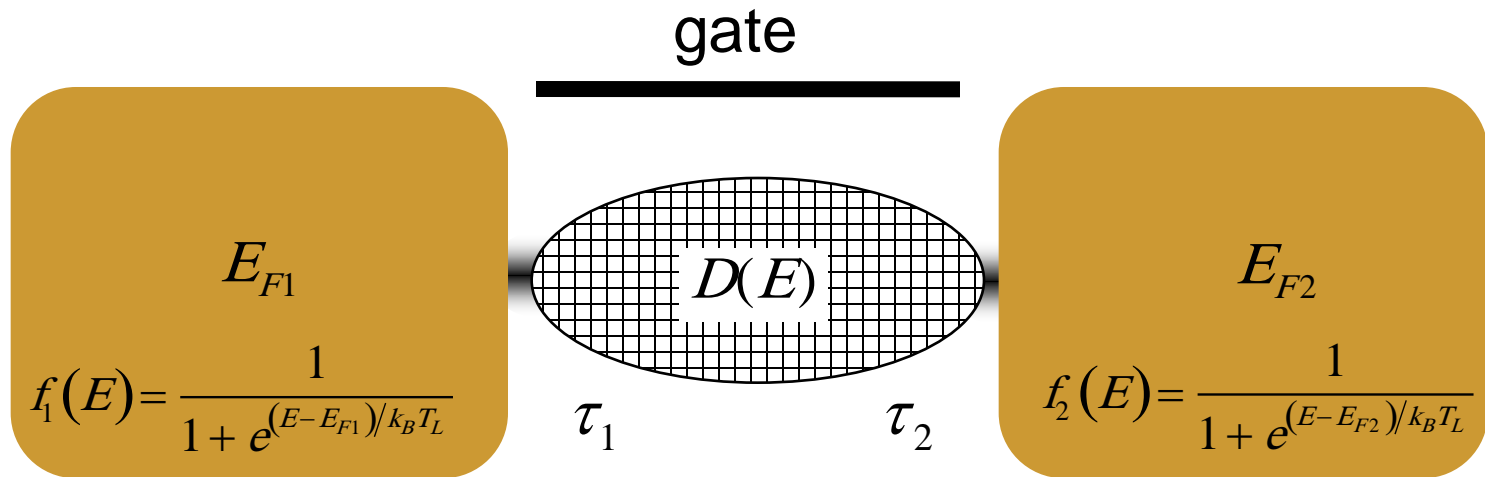


nanoMOS (www.nanoHUB.org)

carrier transport

- diffusive transport ✓
- high-field (hot carrier) transport ✓
- non-local transport ✓
- ballistic transport ✓
- quantum transport ✓
- Landauer approach to transport

current in a nano-device



$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

transmission and modes (channels)

$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

“transmission”

$$L \ll \lambda(E) \quad T(E) = 1 \quad \text{ballistic}$$

$$L \gg \lambda(E) \quad T(E) = \frac{\lambda(E)}{L} \quad \text{diffusive}$$

$$L \sim \lambda(E) \quad \text{quasi-ballistic}$$

$M(E)$ number of channels for current flow at energy, E .

$$M(E) = \frac{h}{4} \langle v_x^+(E) \rangle D(E)$$

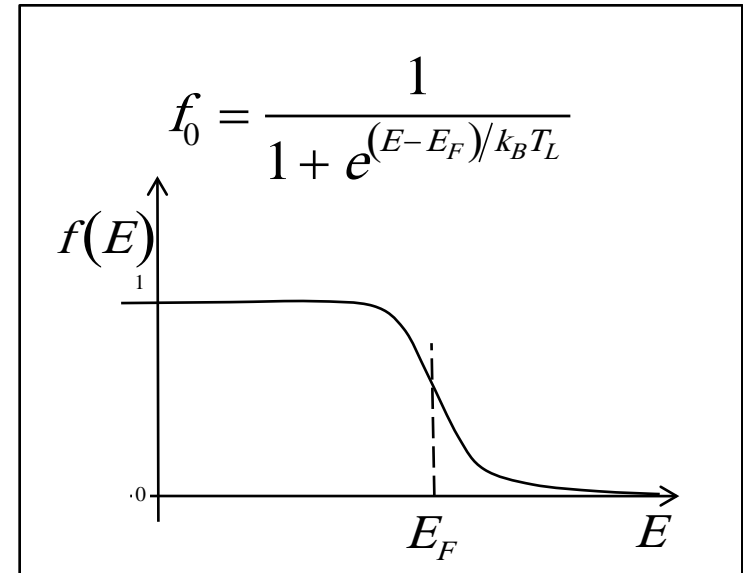
near-equilibrium current

$$I = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2)dE$$

$$(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E} \right) qV$$

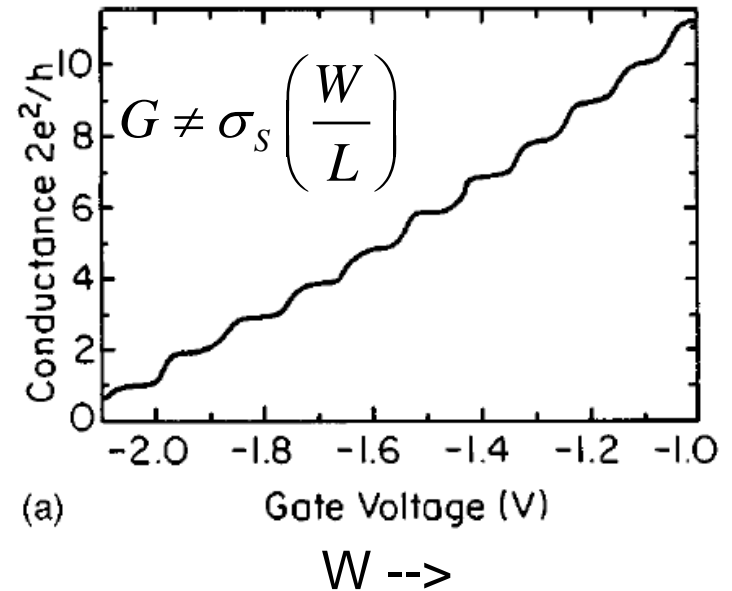
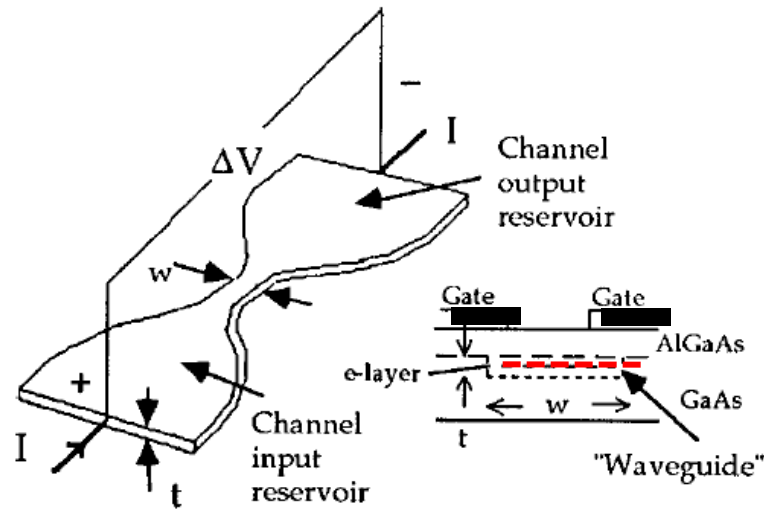
$$I = \left[\frac{2q^2}{h} \int T(E)M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right] V = GV$$

$$\left(-\frac{\partial f_0}{\partial E} \right) \approx \delta(E_F)$$



$$G = \frac{2q^2}{h} T(E_F)M(E_F)$$

quantized conductance



B. J. van Wees, et al. *Phys. Rev. Lett.* **60**, 848–851, 1988.

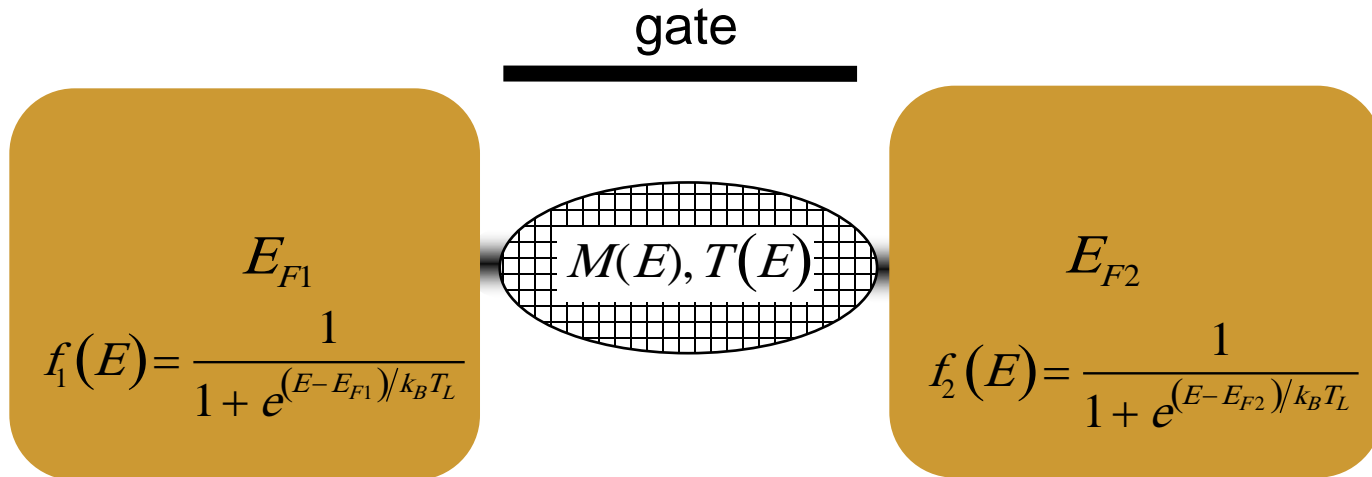
$$G = \frac{2q^2}{h} T(E_F) M(E_F)$$

- 1) conductance is quantized
- 2) upper limit to conductance

carrier transport

- diffusive transport
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- ***Landauer approach to transport***

Landauer approach to transport



$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE \quad \text{high drain bias}$$

$$I = GV$$

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \quad \text{low drain bias}$$

references

For a more extensive discussion of carrier transport in semiconductors, see:

M.S. Lundstrom, ECE-656: “Electronic Transport in Semiconductors,”
Fall, 2011.

<https://nanohub.org/resources/11872>

Near-equilibrium ballistic and diffusive transport, Lectures 1-29.

High-field transport, Lecture 36.

Non-local transport, Lecture 37

Ballistic transport in devices, Lectures 39 and 40.

Introduction to quantum transport, Lectures 39 and 40.

Landauer approach to transport, Lectures 4-8, and 12.