Lecture 5: Transport:

ballistic, diffusive, non-local, and quantum

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level 0 “Virtual Source model”

1) $I_D/W = Q_n(V_G)\langle\upsilon(V_D)\rangle$

2)
$$V_{GS} \le V_T: \quad Q_n(V_{GS}) = 0$$
$$V_{GS} > V_T: \quad Q_n(V_{GS}) = C_{ox}(V_{GS} - V_T)$$

3) $\langle\upsilon(V_D)\rangle = F_{SAT}(V_D)\upsilon_{SAT}$

4) $F_{SAT}(V_D) = \dfrac{V_D/V_{DSAT}}{\left[1+(V_D/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{SAT}L}{\mu_{eff}}$

There are only 5 device-specific input parameters to this model:

$$C_{ox}, V_{T,} \upsilon_{SAT}, \mu_{eff}, L$$

level 0.5 “Virtual Source model”

1) $I_D/W = Q_n(V_G)\langle \upsilon(V_D)\rangle$

2) $V_{GS} < V_T:\quad Q_n(\psi_S) = -(m-1)C_{ox}\left(\frac{k_B T_L}{q}\right) e^{q(V_{GS}-V_T)/mk_B T_L}$

$V_{GS} > V_T:\quad Q_n = -C_{inv}(V_{GS} - V_T)$

3) $\langle \upsilon(V_D)\rangle = F_{SAT}(V_D)\upsilon_{SAT}$

4) $F_{SAT}(V_D) = \dfrac{V_D/V_{DSAT}}{\left[1+(V_D/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{SAT} L}{\mu_{eff}}$

$C_{ox}, C_{inv}, V_{T,} \upsilon_{SAT}, \mu_{eff}, L$

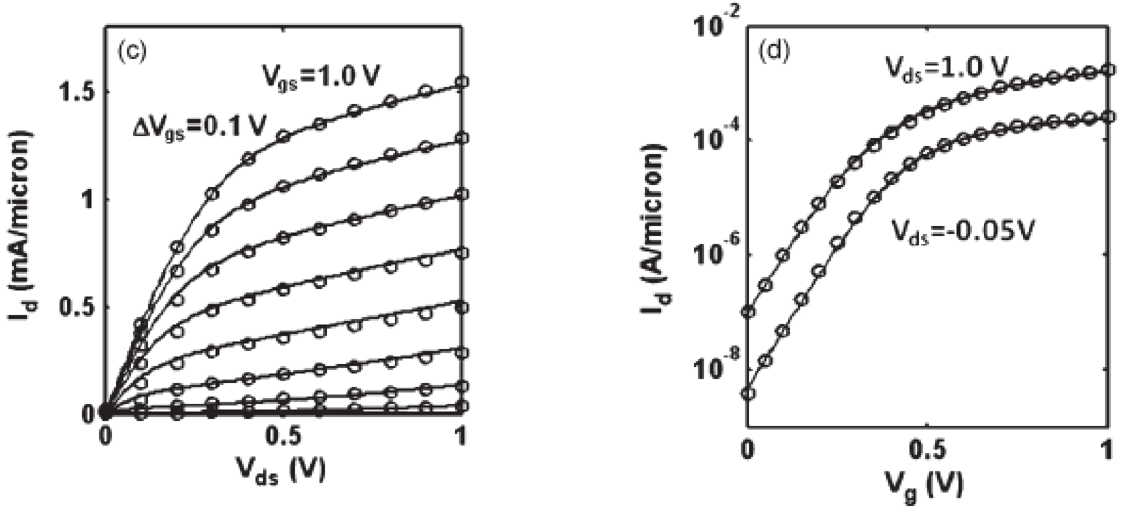
Device-specific input parameters to this model:

Level 1 Virtual Source Model

With a few extensions

(e.g. continuous, empirical expression for Q_n above and below threshold, series resistance, etc.)

we arrive at the Level 1 VS model, which does a remarkably good job of describing modern transistors.



32 nm high-k technology

Ali Khakifirooz, Osama M. Nayfeh, and Dimitri Antoniadis, “Simple Semiempirical Short-Channel MOSFET Current–Voltage Model Continuous Across All Regions of Operation and Employing Only Physical Parameters,” *IEEE Trans. Electron Dev*., **56**, 1674-1680, 2009.

but…

Our derivation has been based on concepts like mobility and high-field saturation velocity:

$$\upsilon_{SAT}, \mu_{eff}$$

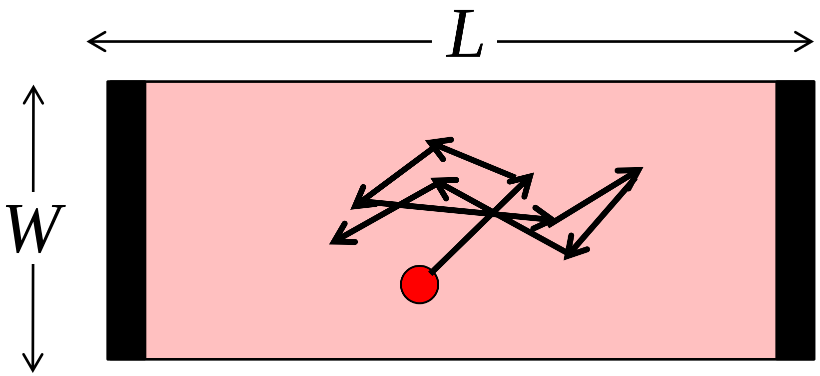
that are only valid for long channel devices.

To understand what these parameters mean in a nanoscale MOSFET, we need to understand a bit about carrier transport.

carrier transport

- diffusive transport
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- Landauer approach to transport

equilibrium



uniform n-type layer

$$\langle \upsilon_x \rangle = 0$$

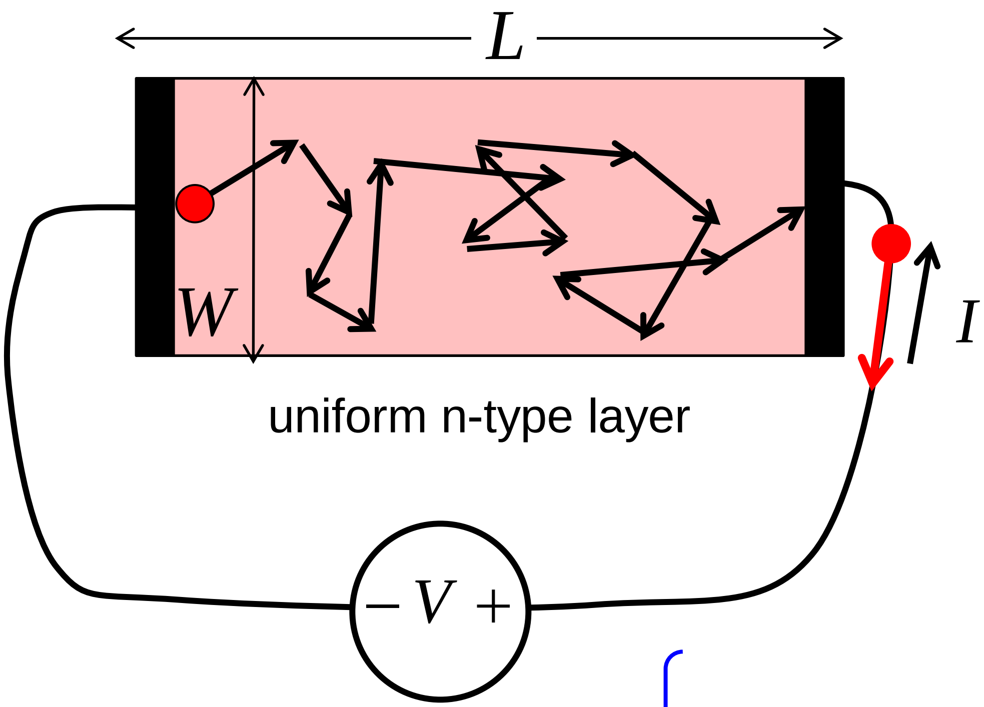
$$\langle KE \rangle = k_B T_L$$

$$\langle KE \rangle = \frac{1}{2} m_n^* \langle \upsilon^2 \rangle$$

$$\sqrt{\langle \upsilon^2 \rangle} = \upsilon_{rms} = \sqrt{\frac{2k_B T}{m_n^*}}$$

$$\upsilon_{rms} \approx 10^7 \text{ cm/s}$$

current flow



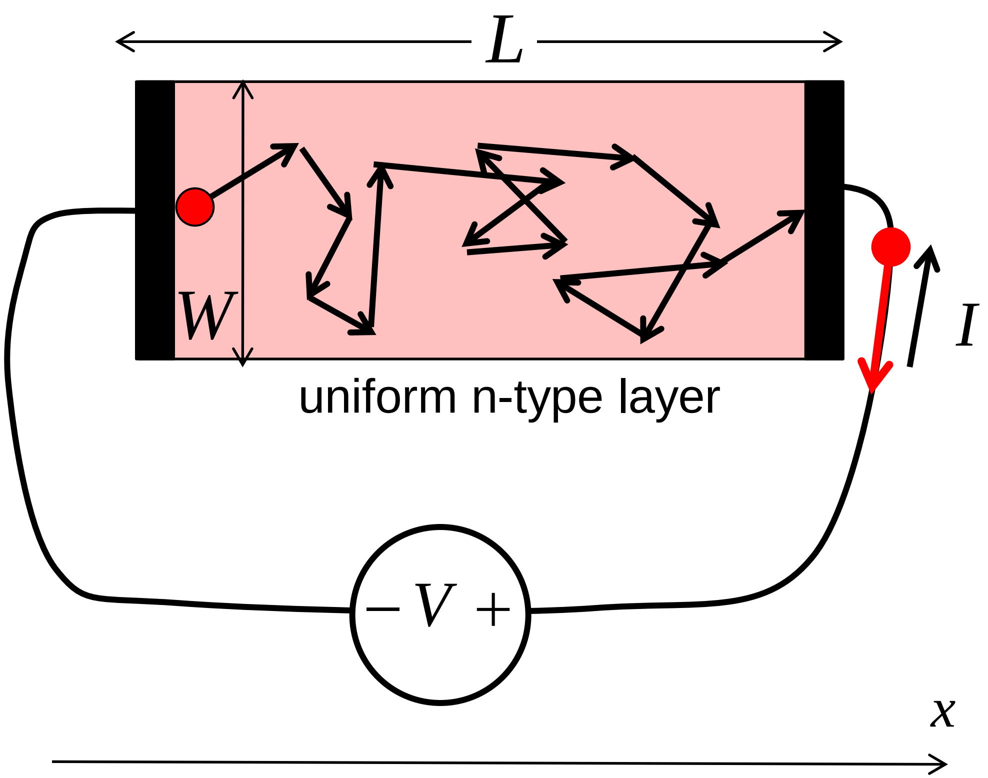
1) Random walk with a small bias from left to right.

2) Assume that electrons "drift" to the right at an average velocity, υ_d

3) what is *I* ?

The average distance between scattering events is called the "mean-free-path".

current flow



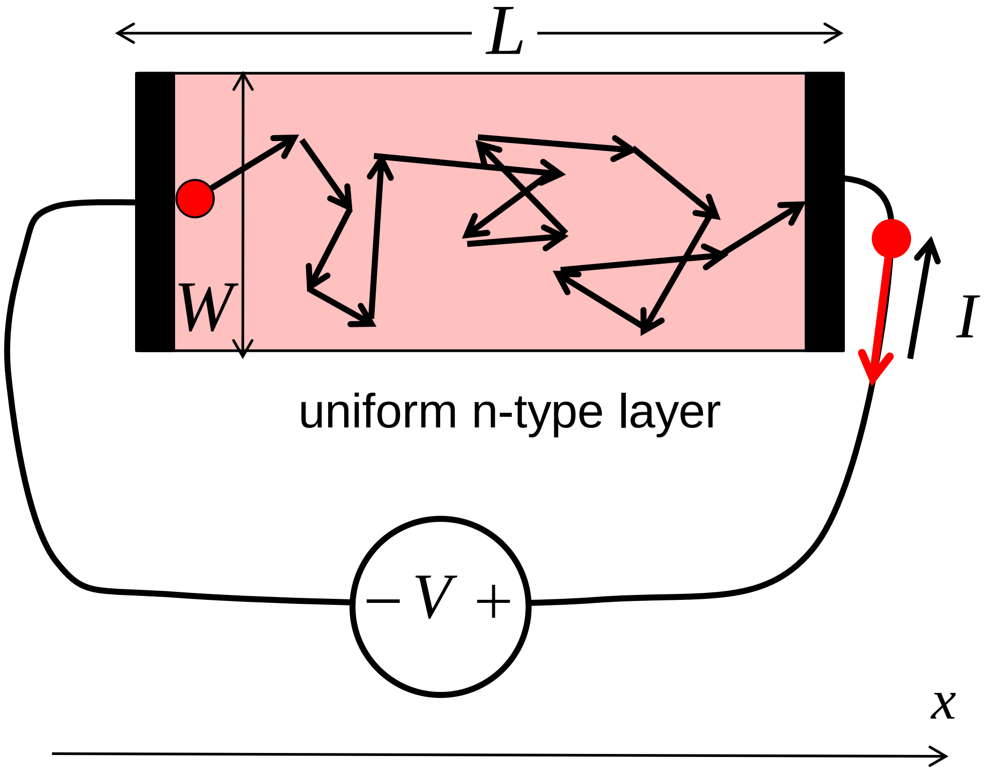
$$I = Q/t_t$$

$$Q = -qn_S WL = Q_n WL$$

$$t_t = L/\upsilon_d$$

$$I = WQ_n \upsilon_d$$

velocity and electric field



$$\frac{dp}{dt} = F_e$$

$$F_e = -q\mathcal{E}$$

$$\Delta p = -q\mathcal{E}\tau = m_n^*\Delta\upsilon$$

$$\Delta\upsilon = -\frac{q\tau}{m_n^*}\mathcal{E}$$

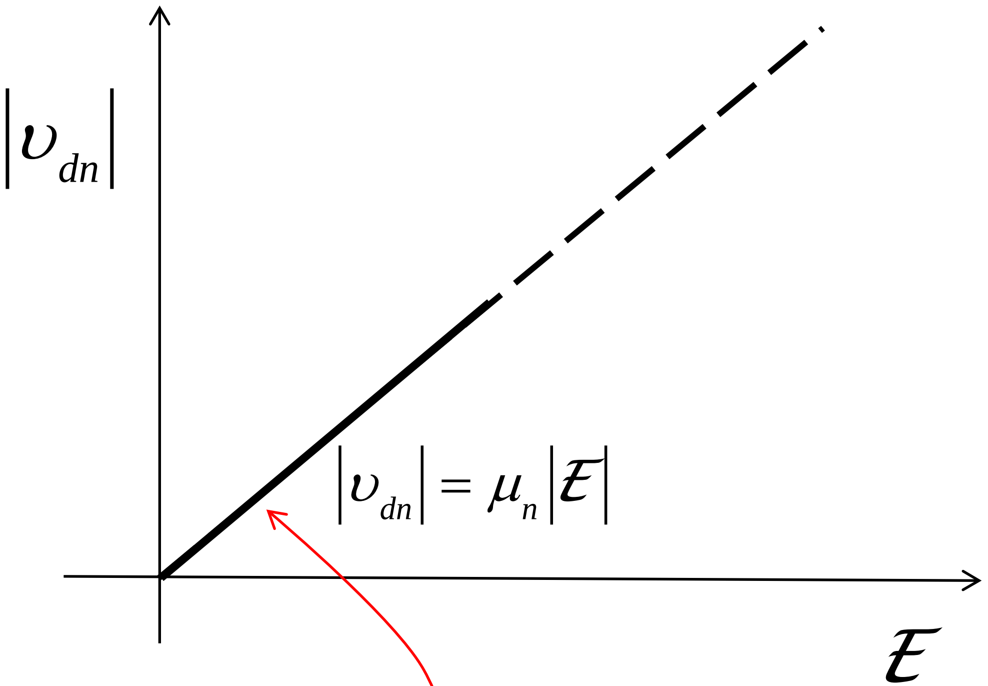
$$\upsilon_{dn} = -\left(\frac{q\langle\tau\rangle}{m_n^*}\right)\mathcal{E} = -\mu_n\mathcal{E}$$

"mobility"

velocity and electric field

$$\upsilon_{dn} = -\mu_n \mathcal{E}$$

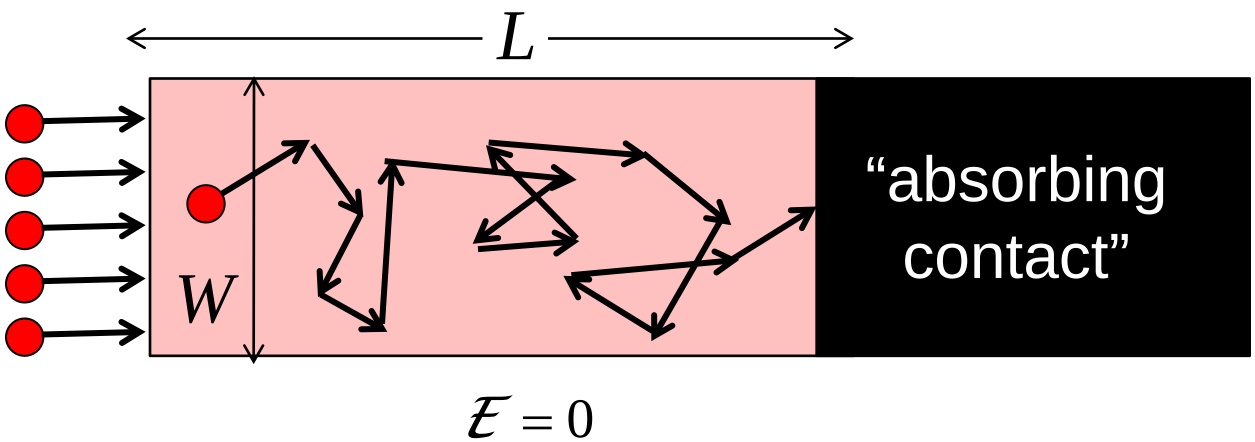
$$\mu_n = \left(\frac{q\langle\tau\rangle}{m_n^*} \right) \text{cm}^2/\text{V-s}$$



low V_{DS} in a MOSFET →

"low-field" or "near-equilibrium" or "linear" transport

diffusion



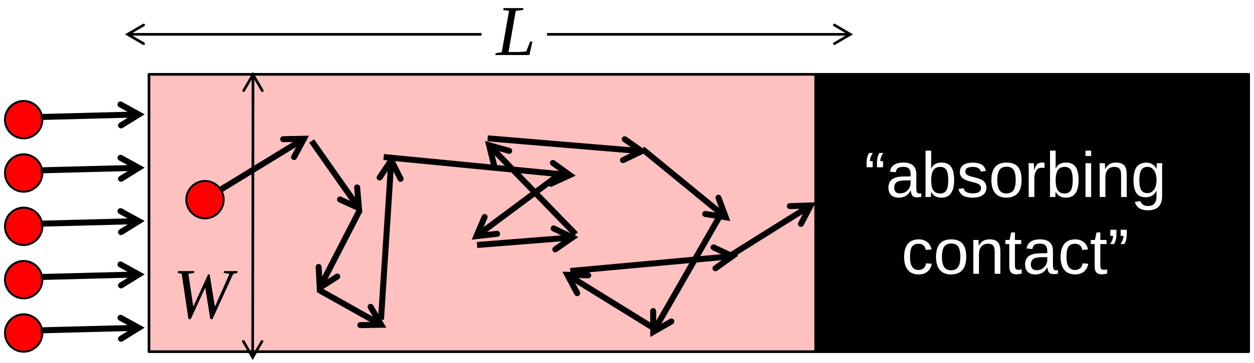
$$I/W = J = (-q)\left[-D_n \frac{dn_S}{dx}\right]$$ "Fick's Law"

$$\frac{D_n}{\mu_n} = \frac{k_B T_L}{q}$$

"Einstein relation"

$D_n \left(\mathrm{cm}^2/\mathrm{s}\right)$ "diffusion coefficient"

drift + diffusion



$$\mathcal{E} \neq 0$$

$$J_n = n_S q \mu_n \mathcal{E} + q D_n \frac{dn_S}{dx}$$

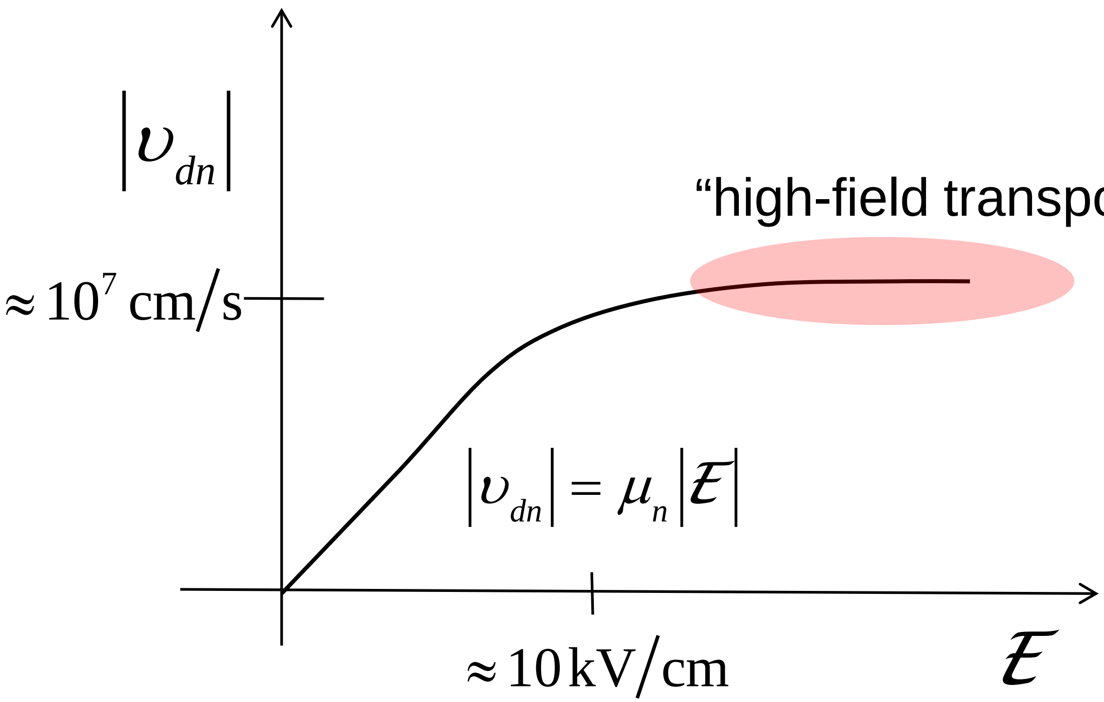
"drift-diffusion equation"

$$\frac{D_n}{\mu_n} = \frac{k_B T_L}{q}$$

carrier transport

- diffusive transport ✓
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- Landauer approach to transport

high-field transport



high V_{DS} in a MOSFET

$$\upsilon_{dn} = -\mu_n \mathcal{E}$$

$$\mu_n = -\left(\frac{q\langle \tau \rangle}{m_n^*} \right)$$

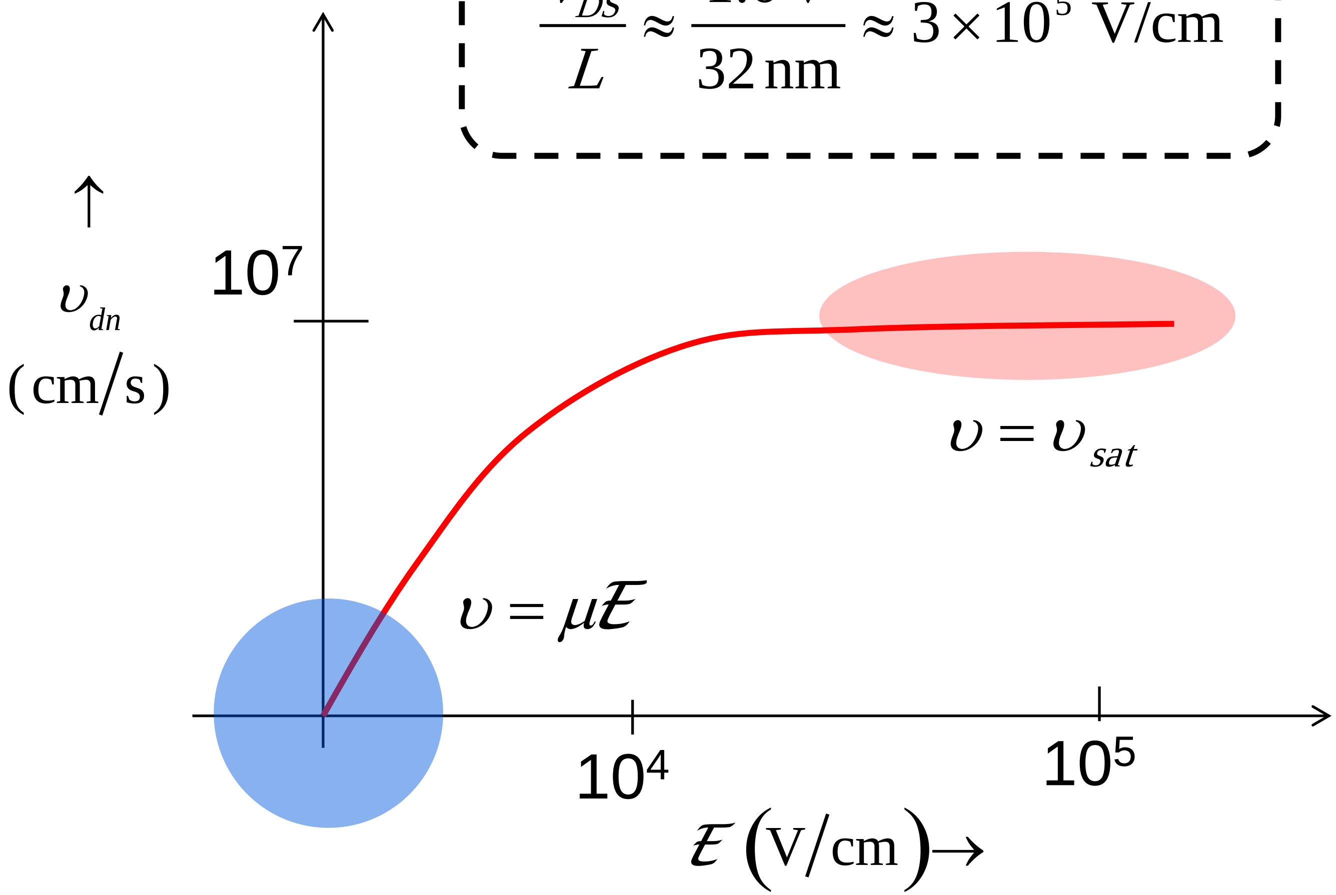
$$\frac{1}{\tau(E)} \propto D(E)$$

$$\mathcal{E}\uparrow \quad \langle E \rangle \uparrow \quad \langle \tau \rangle \downarrow \quad \mu_n \downarrow$$

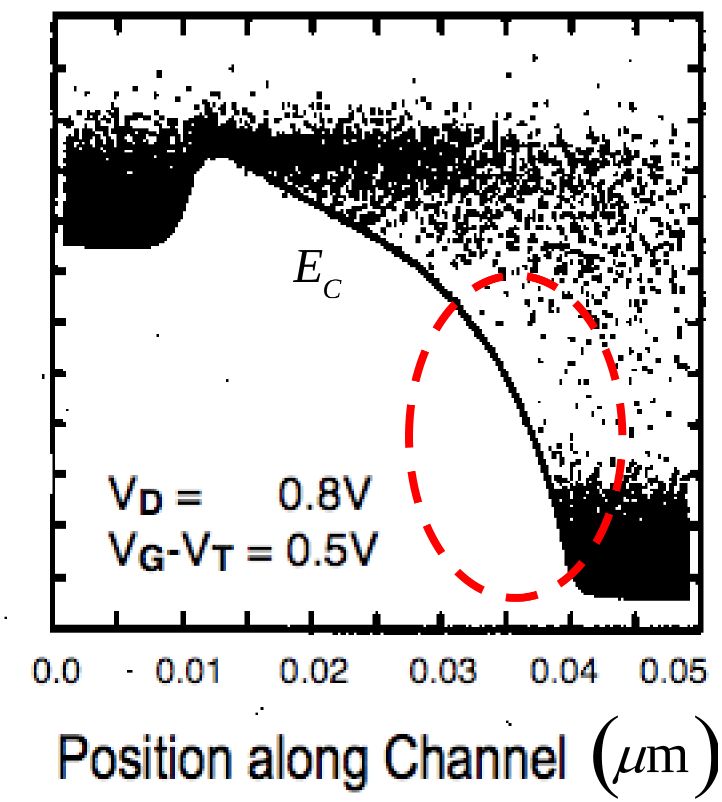
$$\mu_n(\mathcal{E})$$

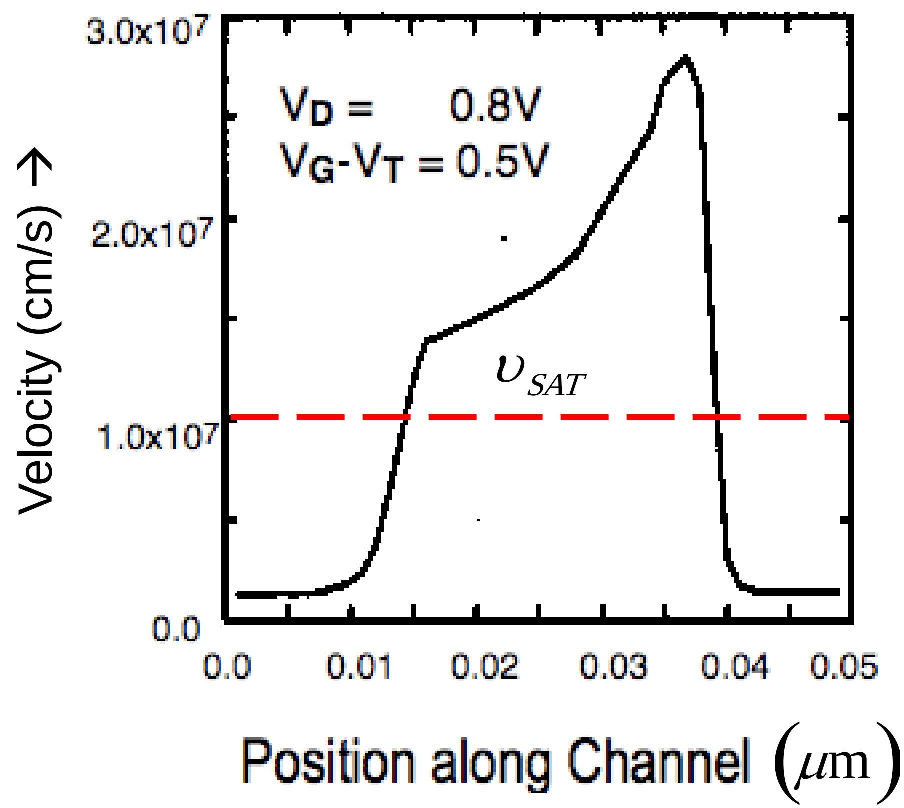
electric fields in nanoscale MOSFETs

$$\frac{V_{DS}}{L} \approx \frac{1.0\,\mathrm{V}}{32\,\mathrm{nm}} \approx 3 \times 10^{5}\ \mathrm{V/cm}$$



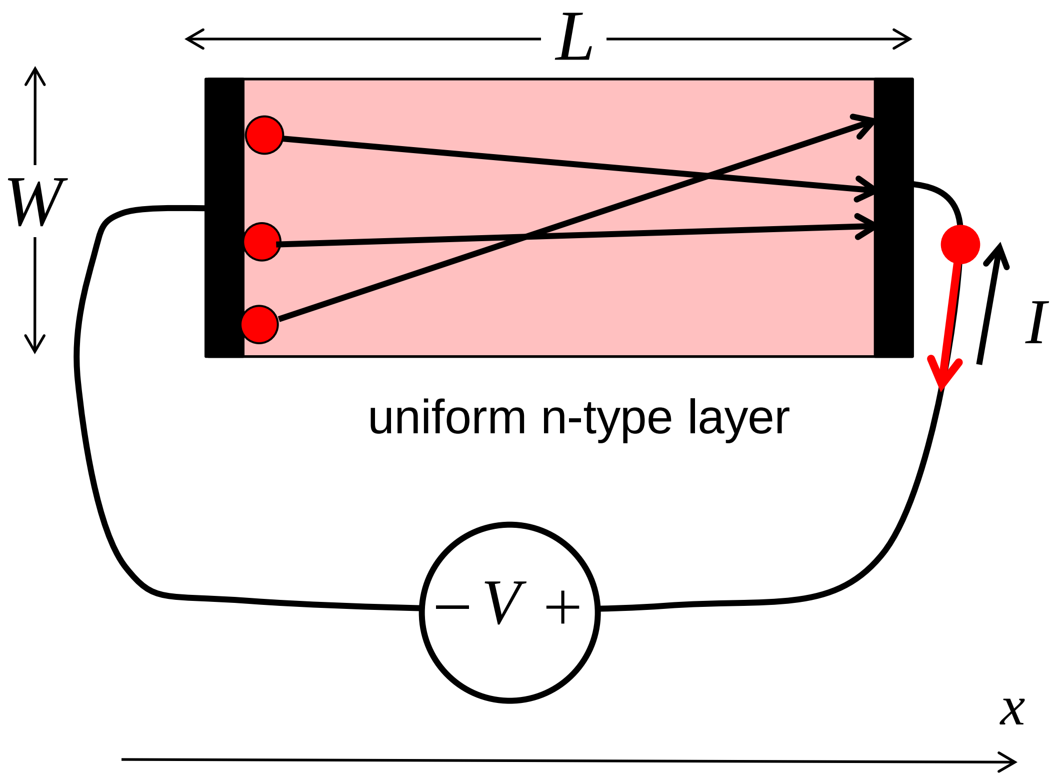
non-local transport





D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

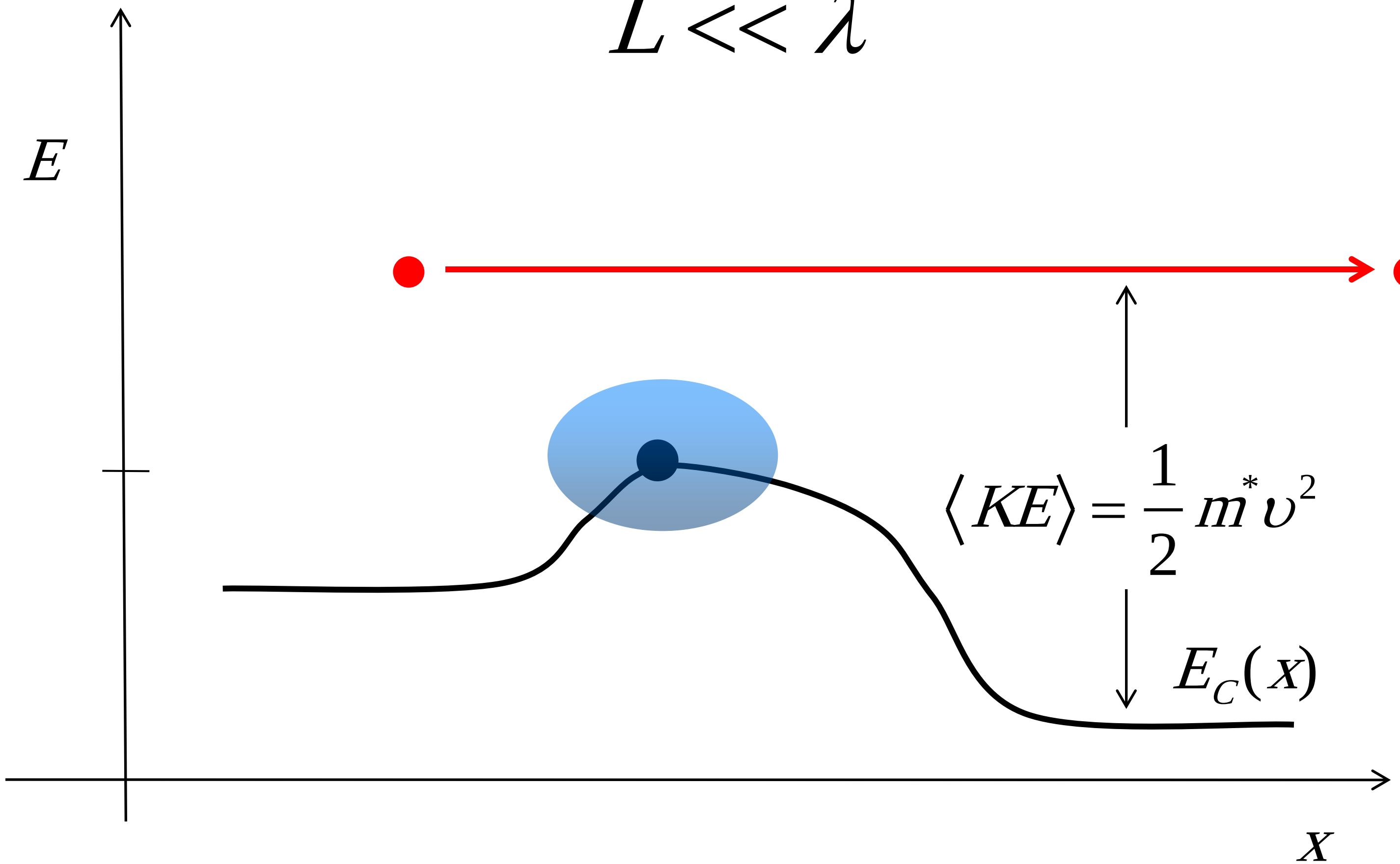
ballistic transport



The sample length, *L*, is much shorter than the MFP for scattering.

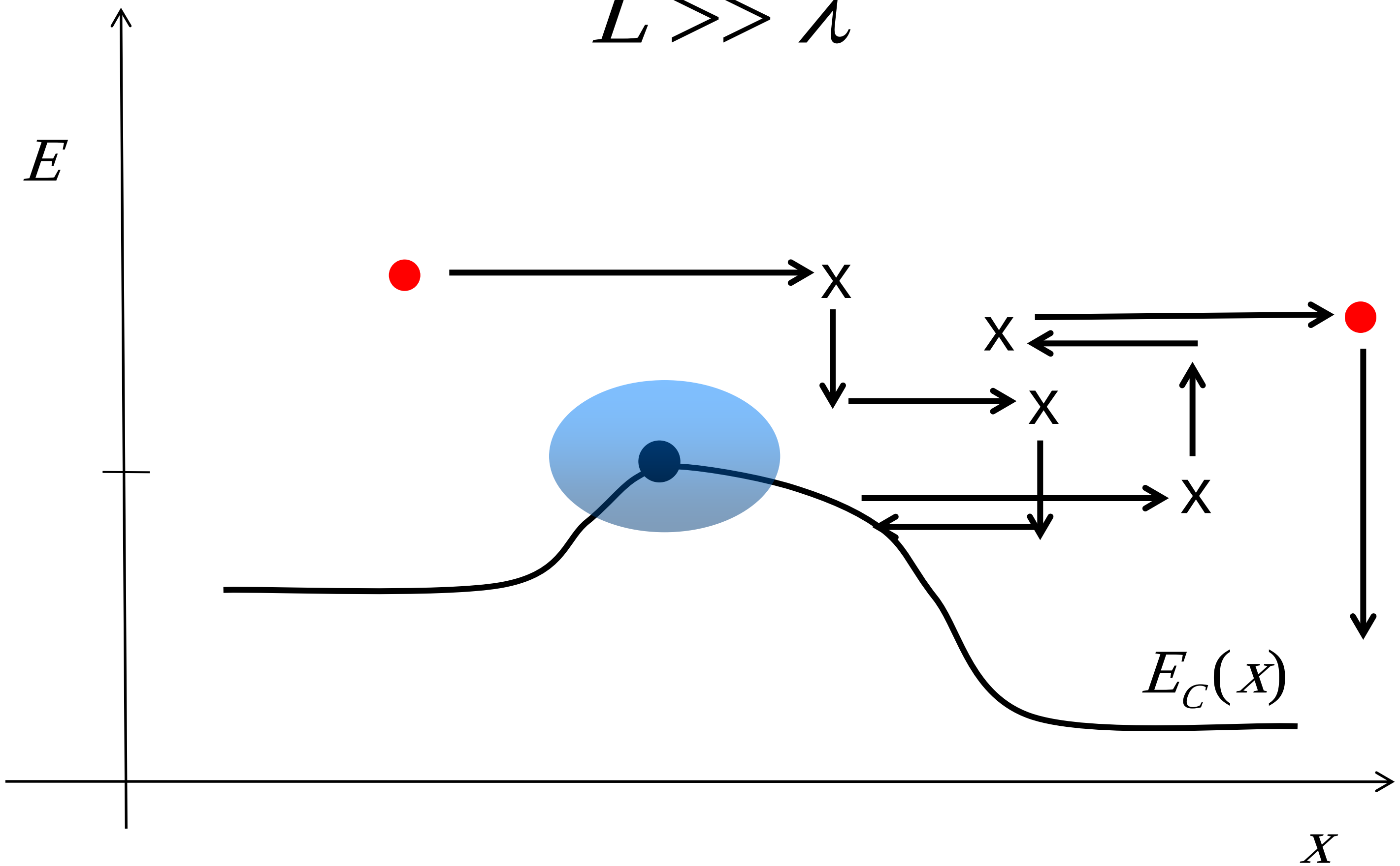
ballistic transport in a MOSFET

$$L << \lambda$$

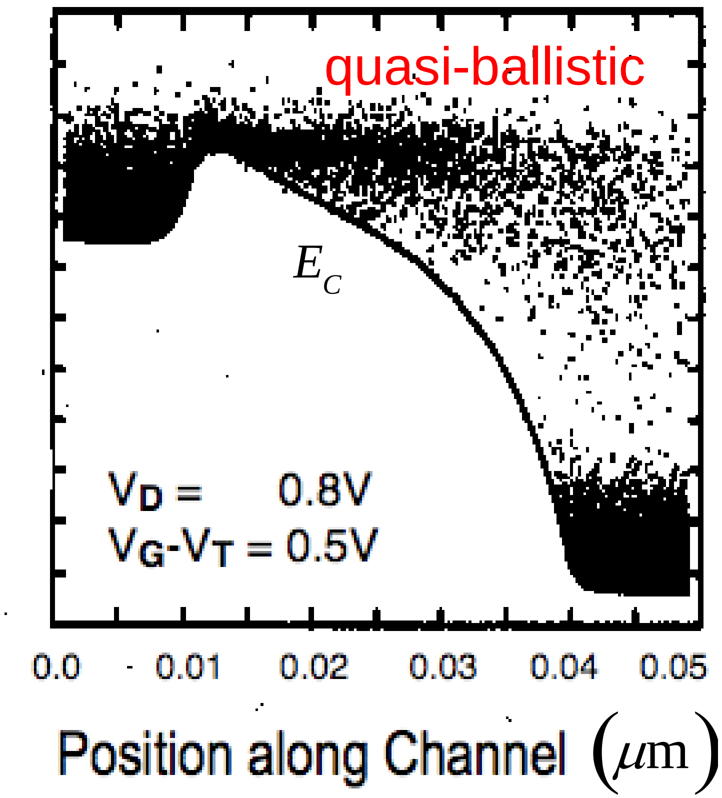


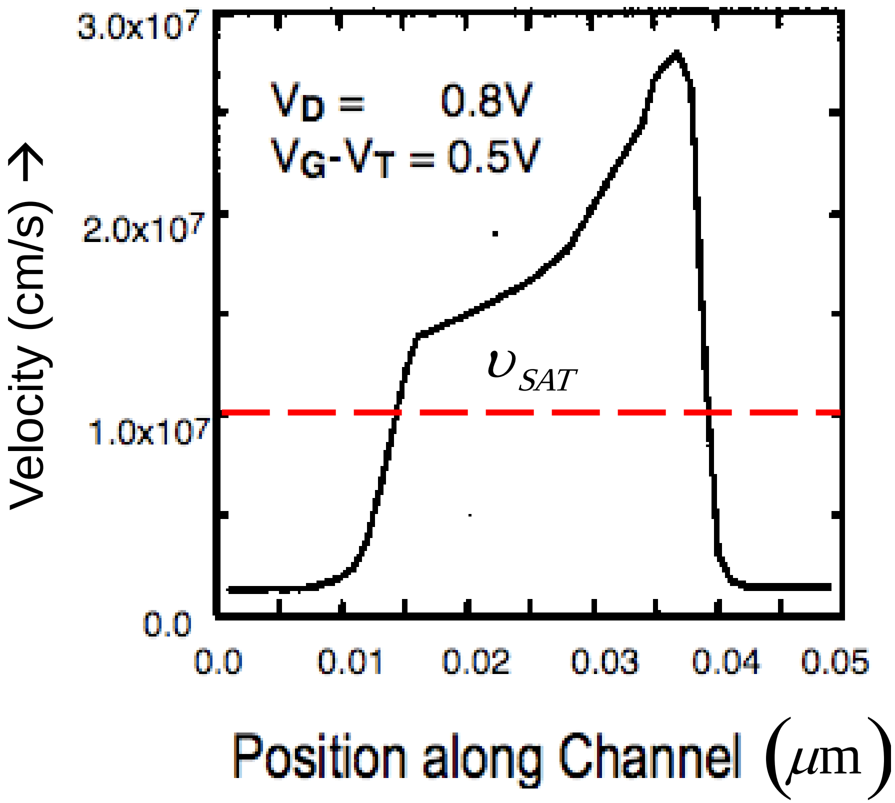
diffusive transport in a MOSFET

$$L >> \lambda$$



quasi-ballistic, non-local transport in a MOSFET

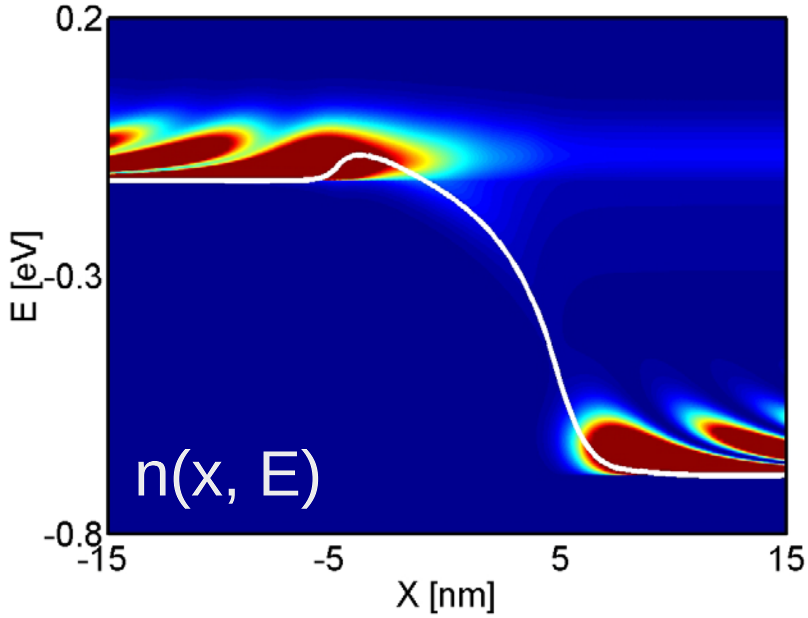




D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

quantum transport

L = 10 nm

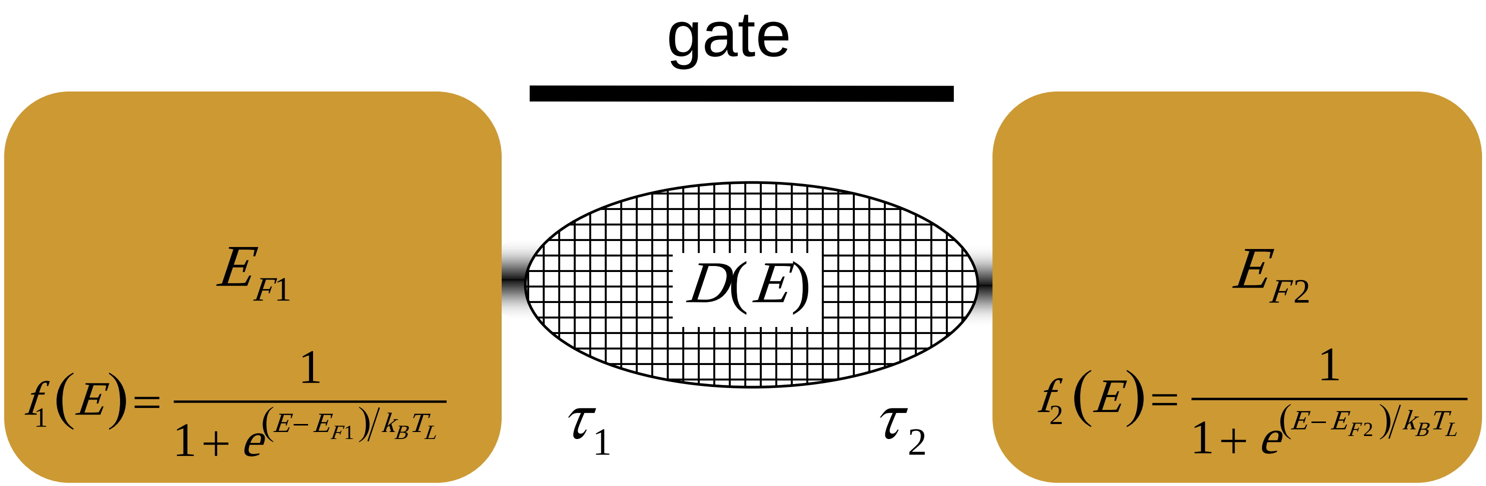


nanoMOS (www.nanoHUB.org)

carrier transport

- diffusive transport ✓
- high-field (hot carrier) transport ✓
- non-local transport ✓
- ballistic transport ✓
- quantum transport ✓
- Landauer approach to transport

current in a nano-device



$$I = \frac{2q}{h} \int T(E) M(E) \left(f_1 - f_2 \right) dE$$

transmission and modes (channels)

$$I = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

"transmission"

$L << \lambda(E)$	$T(E) = 1$	ballistic
$L >> \lambda(E)$	$T(E) = \dfrac{\lambda(E)}{L}$	diffusive
$L \sim \lambda(E)$		quasi-ballistic

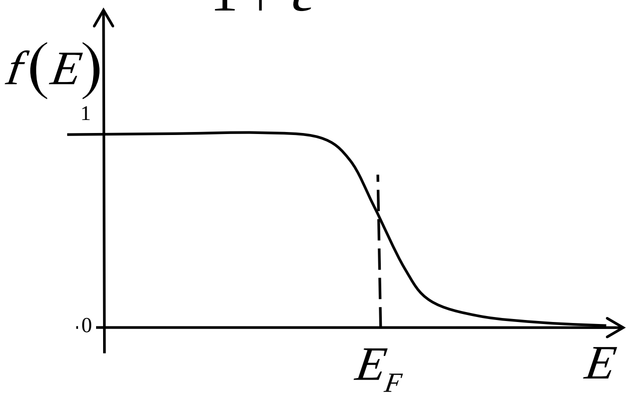
$M(E)$ number of channels for current flow at energy, *E*.

$$M(E) = \frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D(E)$$

near-equilibrium current

$$I = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

$$f_0 = \frac{1}{1+e^{(E-E_F)/k_BT_L}}$$



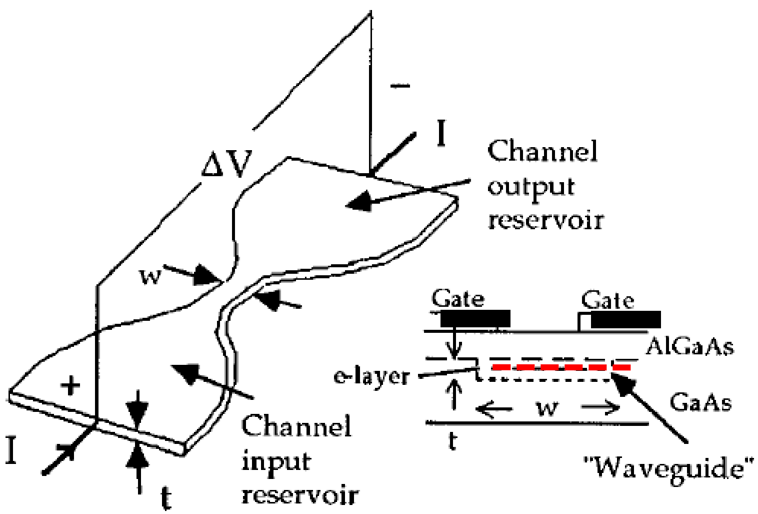
$$\left(f_1 - f_2\right) \approx \left(-\frac{\partial f_0}{\partial E}\right) qV$$

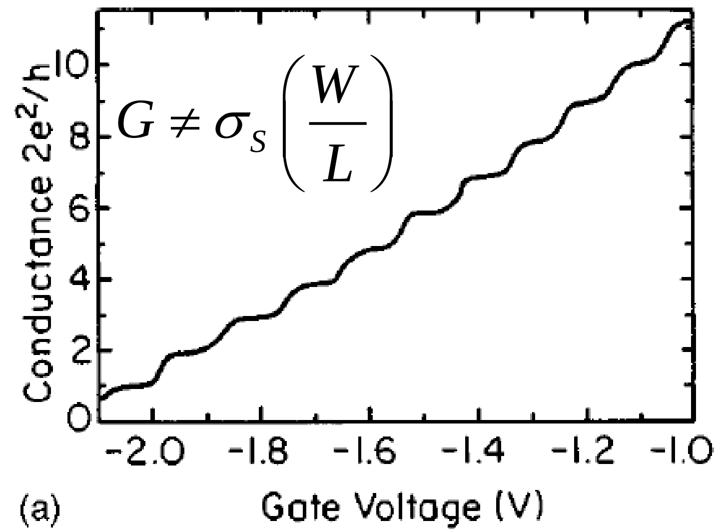
$$I = \left[\frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right]V = GV$$

$$\left(-\frac{\partial f_0}{\partial E}\right) \approx \delta\left(E_F\right)$$

$$G = \frac{2q^2}{h}T\left(E_F\right)M\left(E_F\right)$$

quantized conductance





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B. J. van Wees, et al. *Phys. Rev. Lett.* **60**, 848–851,1988.

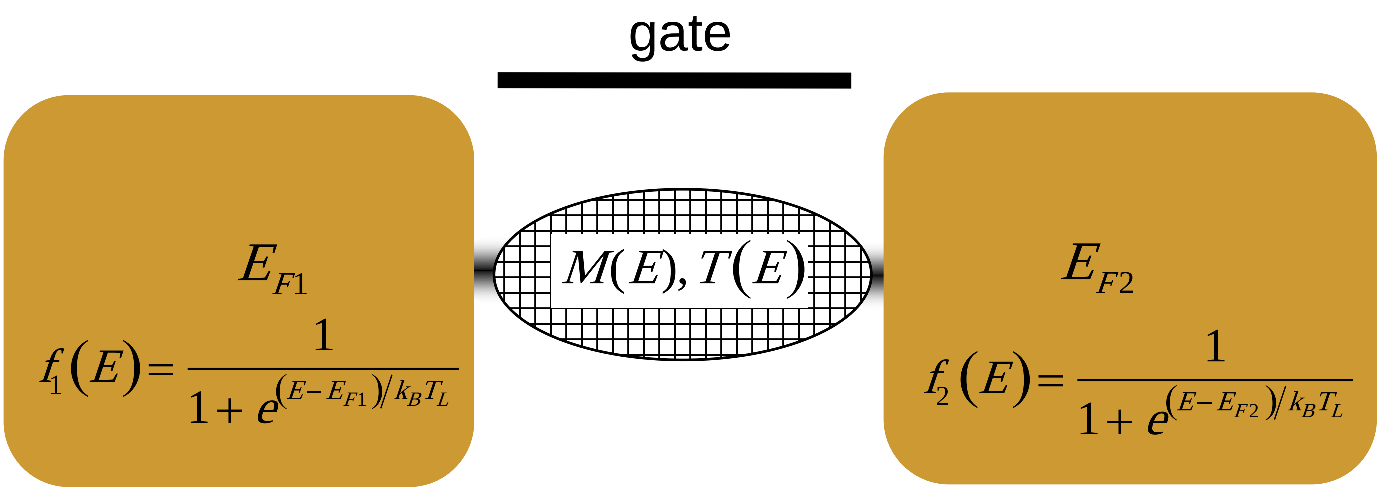
$$G = \frac{2q^2}{h} T(E_F) M(E_F)$$

1) conductance is quantized
2) upper limit to conductance

carrier transport

- diffusive transport
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- ***Landauer approach to transport***

Landauer approach to transport



$$I = \frac{2q}{h} \int T(E) M(E) \left(f_1 - f_2 \right) dE$$ high drain bias

$$I = GV$$

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$ low drain bias

references

For a more extensive discussion of carrier transport in semiconductors, see:

M.S. Lundstrom, ECE-656: “Electronic Transport in Semiconductors,” Fall, 2011.
https://nanohub.org/resources/11872

Near-equilibrium ballistic and diffusive transport, Lectures 1-29.

High-field transport, Lecture 36.

Non-local transport, Lecture 37

Ballistic transport in devices, Lectures 39 and 40.

Introduction to quantum transport, Lectures 39 and 40.

Landauer approach to transport, Lectures 4-8, and 12.