Lecture 5: Transport: ballistic, diffusive, non-local, and quantum

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level 0 “Virtual Source model”

There are only 5 device-specific input parameters to this model:

\[ C_{ox}, V_T, \nu_{SAT}, \mu_{eff}, L \]
level 0.5 “Virtual Source model”

1) \( I_D/W = Q_n(V_G)\langle \nu(V_D) \rangle \)

2) \( V_{GS} < V_T : Q_n(\psi_S) = -(m-1)C_{ox}\left(\frac{k_BT_L}{q}\right)e^g(V_{GS}-V_T)/mk_BT_L \)
   \( V_{GS} > V_T : Q_n = -C_{inv}(V_{GS} - V_T) \)

3) \( \langle \nu(V_D) \rangle = F_{SAT}(V_D)\nu_{SAT} \)

4) \( F_{SAT}(V_D) = \frac{V_D/V_{DSAT}}{\left[1 + \left(\frac{V_D}{V_{DSAT}}\right)^\beta\right]^{1/\beta}} \)

5) \( V_{DSAT} = \frac{\nu_{SAT}L}{\mu_{eff}} \)

Device-specific input parameters to this model:

\( C_{ox}, C_{inv}, V_T, \nu_{SAT}, \mu_{eff}, L \)
Level 1 Virtual Source Model

With a few extensions

(e.g. continuous, empirical expression for $Q_n$ above and below threshold, series resistance, etc.)

we arrive at the Level 1 VS model, which does a remarkably good job of describing modern transistors.

but...

Our derivation has been based on concepts like mobility and high-field saturation velocity:

\[ \nu_{SAT}, \mu_{eff} \]

that are only valid for long channel devices.

To understand what these parameters mean in a nanoscale MOSFET, we need to understand a bit about carrier transport.
carrier transport

- diffusive transport
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- Landauer approach to transport
equilibrium

\[ \langle v_x \rangle = 0 \]

\[ \langle KE \rangle = k_B T_L \]

\[ \langle KE \rangle = \frac{1}{2} m_n^* \langle v^2 \rangle \]

\[ \sqrt{\langle v^2 \rangle} = v_{rms} = \sqrt{\frac{2k_B T}{m_n^*}} \]

\[ v_{rms} \approx 10^7 \text{ cm/s} \]

uniform n-type layer
1) Random walk with a small bias from left to right.

2) Assume that electrons “drift” to the right at an average velocity, $v_d$

3) what is $I$?

The average distance between scattering events is called the “mean-free-path”.

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uniform n-type layer

$L$

$W$

$I$

$V$
current flow

\[
I = \frac{Q}{t_t}
\]

\[
Q = -qn_sWL = Q_nWL
\]

\[
t_t = \frac{L}{\nu_d}
\]

\[
I = WQ_n\nu_d
\]
velocity and electric field

\[
\frac{dp}{dt} = F_e \\
F_e = -qE \\
\Delta p = -qE \tau = m^*_n \Delta \nu \\
\Delta \nu = -\frac{q \tau}{m^*_n} E \\
\nu_{dn} = -\left(\frac{q \langle \tau \rangle}{m^*_n} \right) E = -\mu_n E \\
\text{“mobility”}
\]
velocity and electric field

\[ \nu_{dn} = -\mu_n E \]

\[ \mu_n = \left( \frac{q\langle \tau \rangle}{m^*_n} \right) \text{ cm}^2/\text{V-s} \]

low \( V_{DS} \) in a MOSFET \( \rightarrow \)

“low-field” or “near-equilibrium” or “linear” transport
diffusion

\[ \frac{I}{W} = J = (-q) \left[ -D_n \frac{dn_s}{dx} \right] \]

“Fick’s Law”

\[ D_n \left( \text{cm}^2/\text{s} \right) \]

“diffusion coefficient”

\[ \frac{D_n}{\mu_n} = \frac{k_B T_L}{q} \]

“Einstein relation”
$J_n = n_S q \mu_n \overline{E} + q D_n \frac{dn_S}{dx}$

$\frac{D_n}{\mu_n} = \frac{k_B T_L}{q}$

“drift-diffusion equation”
carrier transport

- diffusive transport ✓
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
- Landauer approach to transport
high-field transport

\[ |\nu_{dn}| = \mu_n |\mathcal{E}| \approx 10^7 \text{ cm/s} \]

\[ \approx 10 \text{kV/cm} \]

high \( V_{DS} \) in a MOSFET

\[ \nu_{dn} = -\mu_n \mathcal{E} \]

\[ \mu_n = -\left( \frac{q \langle \tau \rangle}{m^*_n} \right) \]

\[ \frac{1}{\tau(E)} \propto D(E) \]

\[ \mathcal{E} \uparrow \langle E \rangle \uparrow \langle \tau \rangle \downarrow \mu_n \downarrow \]

\[ \mu_n(\mathcal{E}) \]
electric fields in nanoscale MOSFETs

\[ \frac{V_{DS}}{L} \approx \frac{1.0 \text{ V}}{32 \text{ nm}} \approx 3 \times 10^5 \text{ V/cm} \]

\( \nu = \nu_{sat} \)

\( \nu = \mu E \)

\( \nu_{dn} \) (cm/s)

\[ 10^7 \]

\[ 10^4 \]

\[ 10^5 \]

\( E \) (V/cm)
non-local transport

ballistic transport

The sample length, $L$, is much shorter than the MFP for scattering.
ballistic transport in a MOSFET

$L \ll \lambda$

\[ \langle KE \rangle = \frac{1}{2} m^* \nu^2 \]

\[ E(x) \]

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diffusive transport in a MOSFET

$L \gg \lambda$

$E$

$E_C(x)$

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quasi-ballistic, non-local transport in a MOSFET

quantum transport

$L = 10 \text{ nm}$

nanoMOS (www.nanoHUB.org)
carrier transport

- diffusive transport ✔
- high-field (hot carrier) transport ✔
- non-local transport ✔
- ballistic transport ✔
- quantum transport ✔
- Landauer approach to transport
current in a nano-device

\[ f_1(E) = \frac{1}{1 + e^{(E-E_{F1})/k_BT_L}} \]

\[ f_2(E) = \frac{1}{1 + e^{(E-E_{F2})/k_BT_L}} \]

\[ I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE \]
transmission and modes (channels)

\[ I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE \]

\[ T(E) = \frac{\lambda(E)}{\lambda(E) + L} \]

“transmission”

\[ M(E) = \frac{h}{4} \langle v_x^+ (E) \rangle D(E) \]

- \( L \ll \lambda(E) \) \( T(E) = 1 \) ballistic
- \( L \gg \lambda(E) \) \( T(E) = \frac{\lambda(E)}{L} \) diffusive
- \( L \sim \lambda(E) \) quasi-ballistic

number of channels for current flow at energy, \( E \).
near-equilibrium current

\[ I = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2)\,dE \]

\[ (f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) qV \]

\[ I = \left[ \frac{2q^2}{h} \int T(E)M(E)\left( -\frac{\partial f_0}{\partial E} \right)\,dE \right] V = GV \]

\[ \left( -\frac{\partial f_0}{\partial E} \right) \approx \delta(E_F) \]

\[ f_0 = \frac{1}{1 + e^{(E - E_F)/k_BT_L}} \]

\[ f(E) \]

\[ G = \frac{2q^2}{h} T(E_F)M(E_F) \]
quantized conductance


\[ G = \frac{2q^2}{h} T(E_F) M(E_F) \]

1) conductance is quantized
2) upper limit to conductance

\[ G \neq \sigma_s \left( \frac{W}{L} \right) \]
carrier transport

- diffusive transport
- high-field (hot carrier) transport
- non-local transport
- ballistic transport
- quantum transport
  - *Landauer approach to transport*
Landauer approach to transport

\[
\begin{align*}
M(E), T(E) & = \frac{1}{1 + e^{(E - E_{F1})/k_B T_L}} \\

f_1(E) &= \frac{1}{1 + e^{(E - E_{F1})/k_B T_L}} \\

f_2(E) &= \frac{1}{1 + e^{(E - E_{F2})/k_B T_L}}
\end{align*}
\]

\[
I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE
\]

\[
I = GV
\]

\[
G = \frac{2q^2}{h} \int T(E) M(E) \left(- \frac{\partial f_0}{\partial E} \right) dE
\]

high drain bias

low drain bias
For a more extensive discussion of carrier transport in semiconductors, see:

https://nanohub.org/resources/11872

Near-equilibrium ballistic and diffusive transport, Lectures 1-29.
High-field transport, Lecture 36.
Non-local transport, Lecture 37
Ballistic transport in devices, Lectures 39 and 40.
Introduction to quantum transport, Lectures 39 and 40.
Landauer approach to transport, Lectures 4-8, and 12.