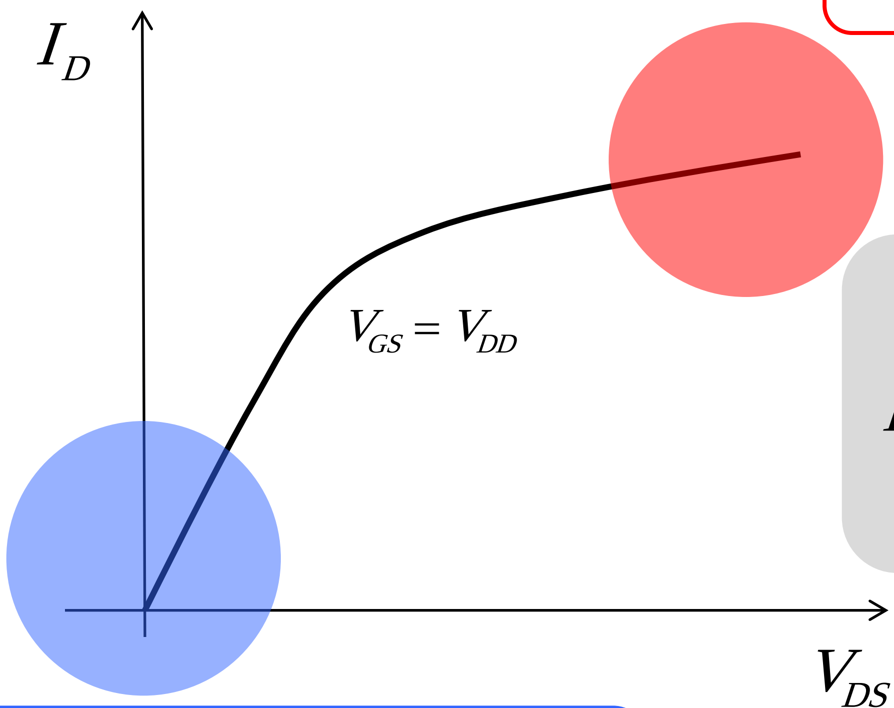
Lecture 9: Scattering and Transmission

Mark Lundstrom

Electrical and Computer Engineering
Network for Computational Nanotechnology
and
Birck Nanotechnology Center
Purdue University
West Lafayette, Indiana USA

ballistic MOSFET (MB)

$$I_D = W Q_n\left(V_{GS}, V_{DS}\right)\upsilon_T$$

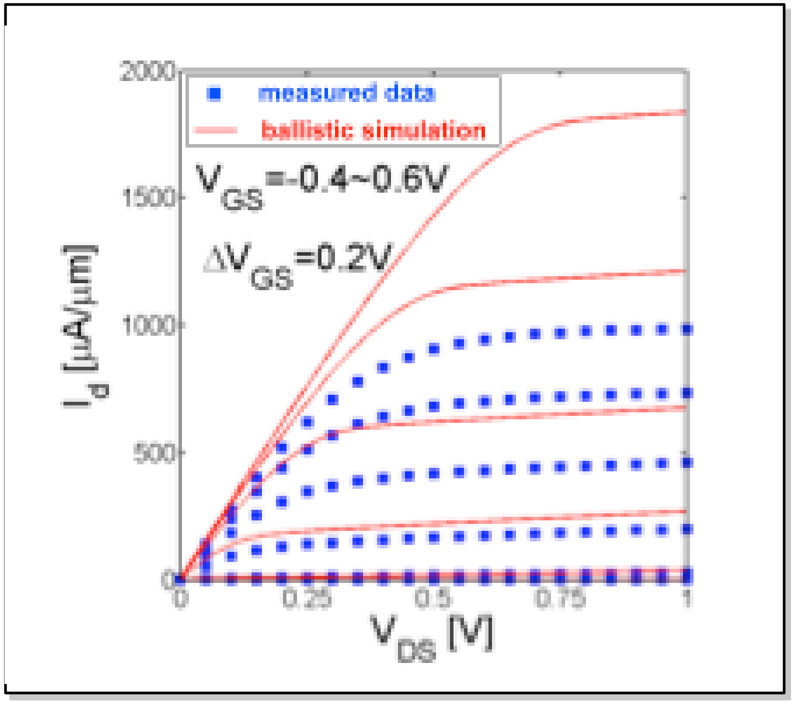


$$I_D = W Q_n\left(V_{GS}, V_{DS}\right) F_{SAT}\left(V_D\right)\upsilon_T$$

$$I_D = W Q_n\left(V_{GS}, V_{DS}\right)\frac{\upsilon_T}{2k_B T_L/q} V_{DS}$$

ballistic vs. real MOSFETs

$$L_G = 40 \text{ nm}$$

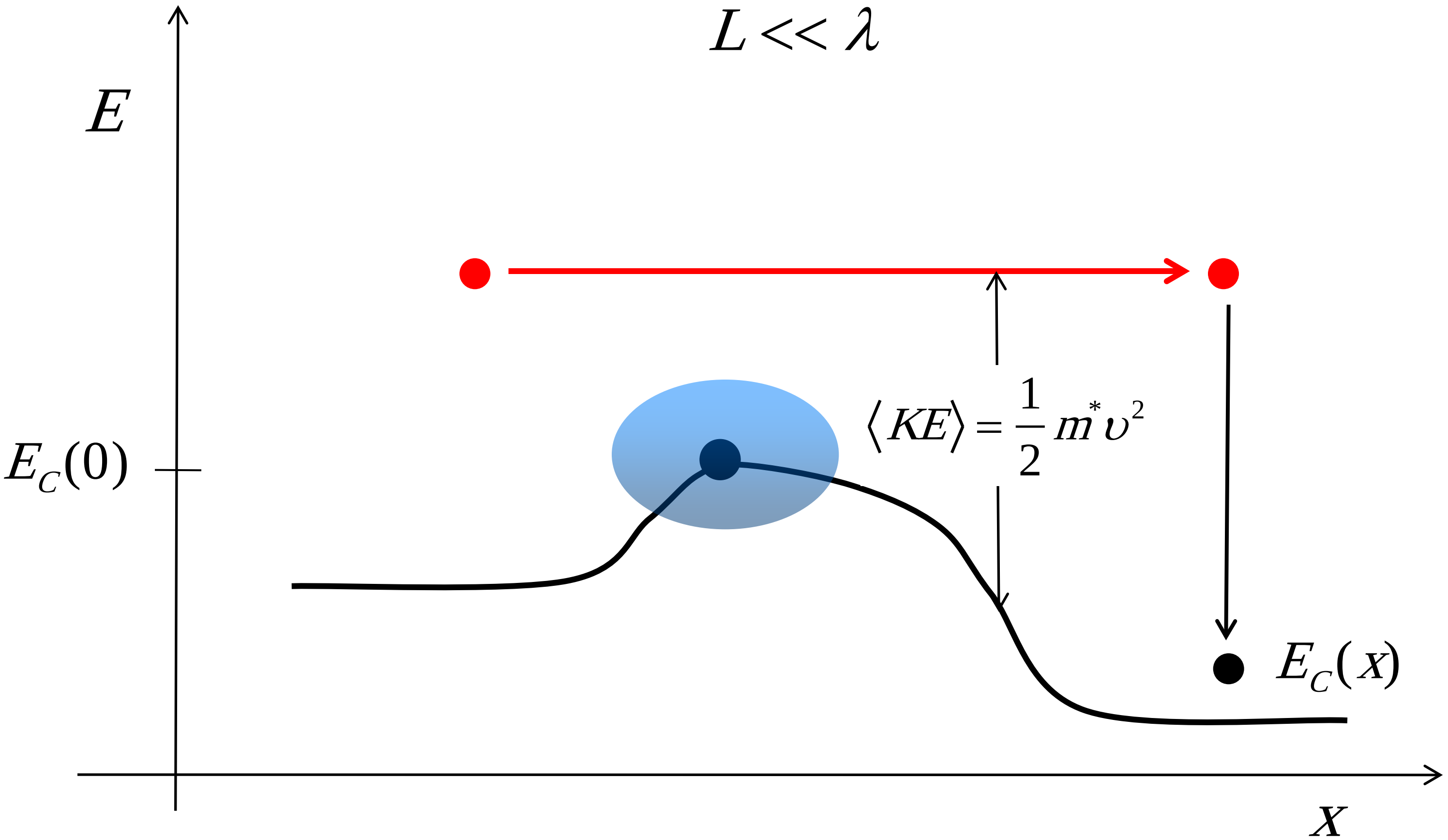


- Si MOSFETs deliver > one-half of the ballistic on-current. (Similar for the past 15 years.)
- MOSFETs operate closer to the ballistic limit under high V_{DS}.
- But III-V FETs operate close to the ballistic limit.

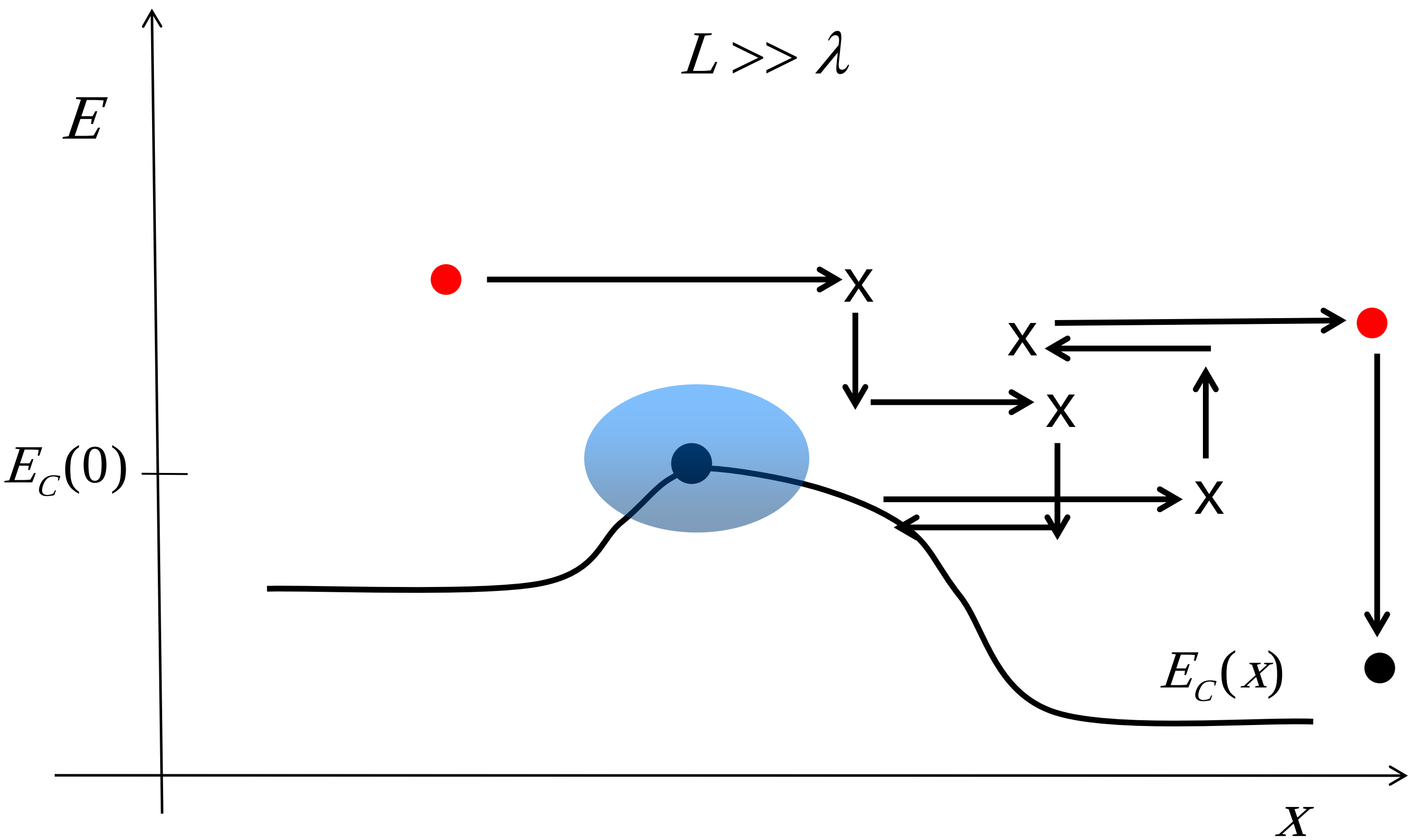
A. Majumdar, Z. B. Ren, S. J. Koester, and W. Haensch, "Undoped-Body Extremely Thin SOI MOSFETs With Back Gates," *IEEE Transactions on Electron Devices*, **56**, pp. 2270-2276, 2009.

Device characterization and simulation: Himadri Pal and Yang Liu, Purdue, 2010.

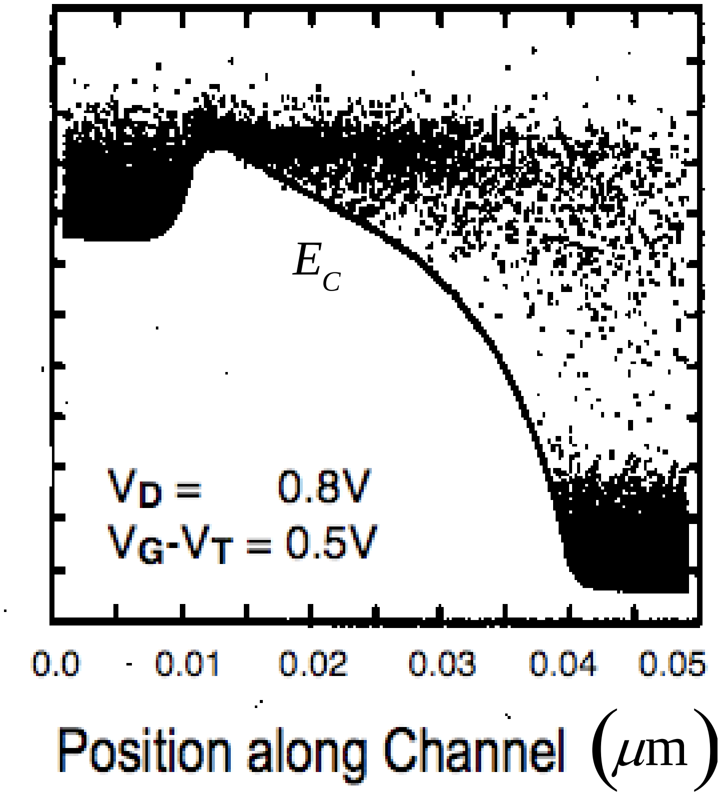
review: ballistic transport in a MOSFET



review: diffusive transport in a MOSFET



non-local, quasi-ballistic transport



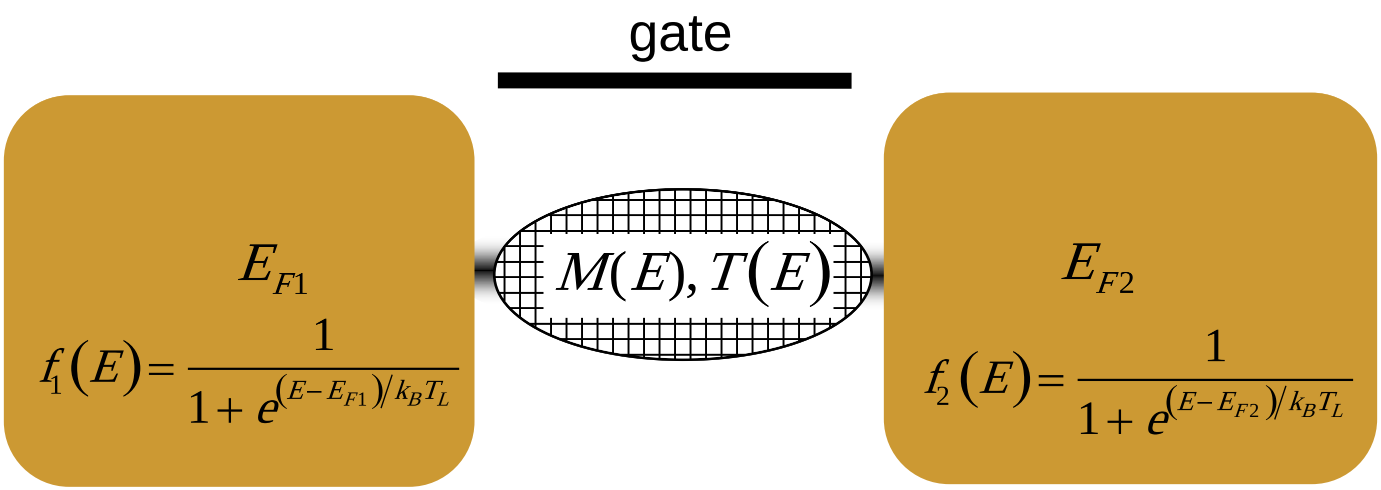
D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

nanoscale MOSFETs

Nanoscale MOSFETs are neither fully ballistic nor fully diffusive; they operate in a “quasi-ballistic” regime.

How do we ***understand*** how carrier scattering affects the performance of a nanoscale MOSFET?

transmission

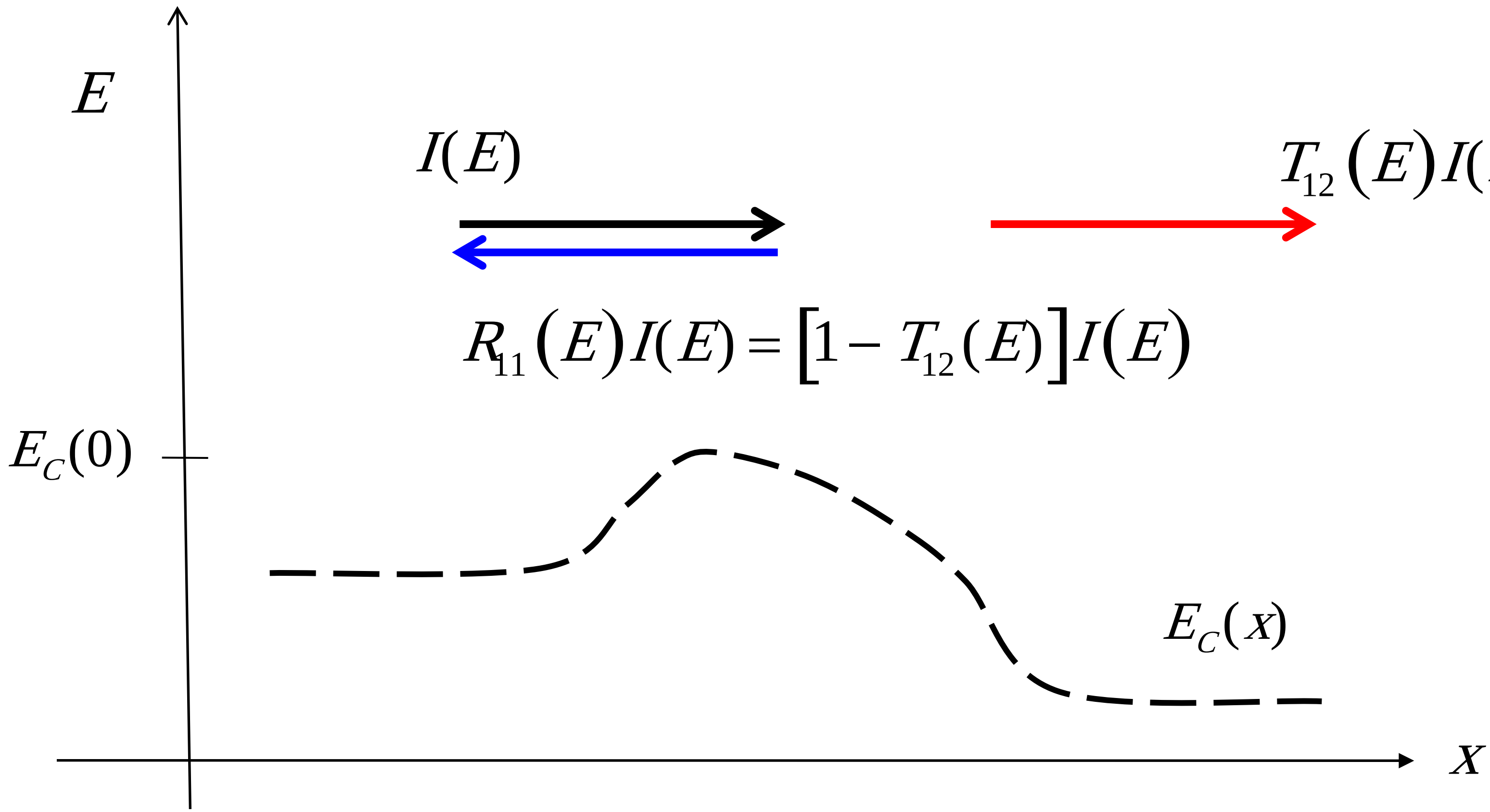


$$I = \frac{2q}{h} \int T(E) M(E) \left(f_1 - f_2 \right) dE$$

ballistic transport: $T(E) = 1$

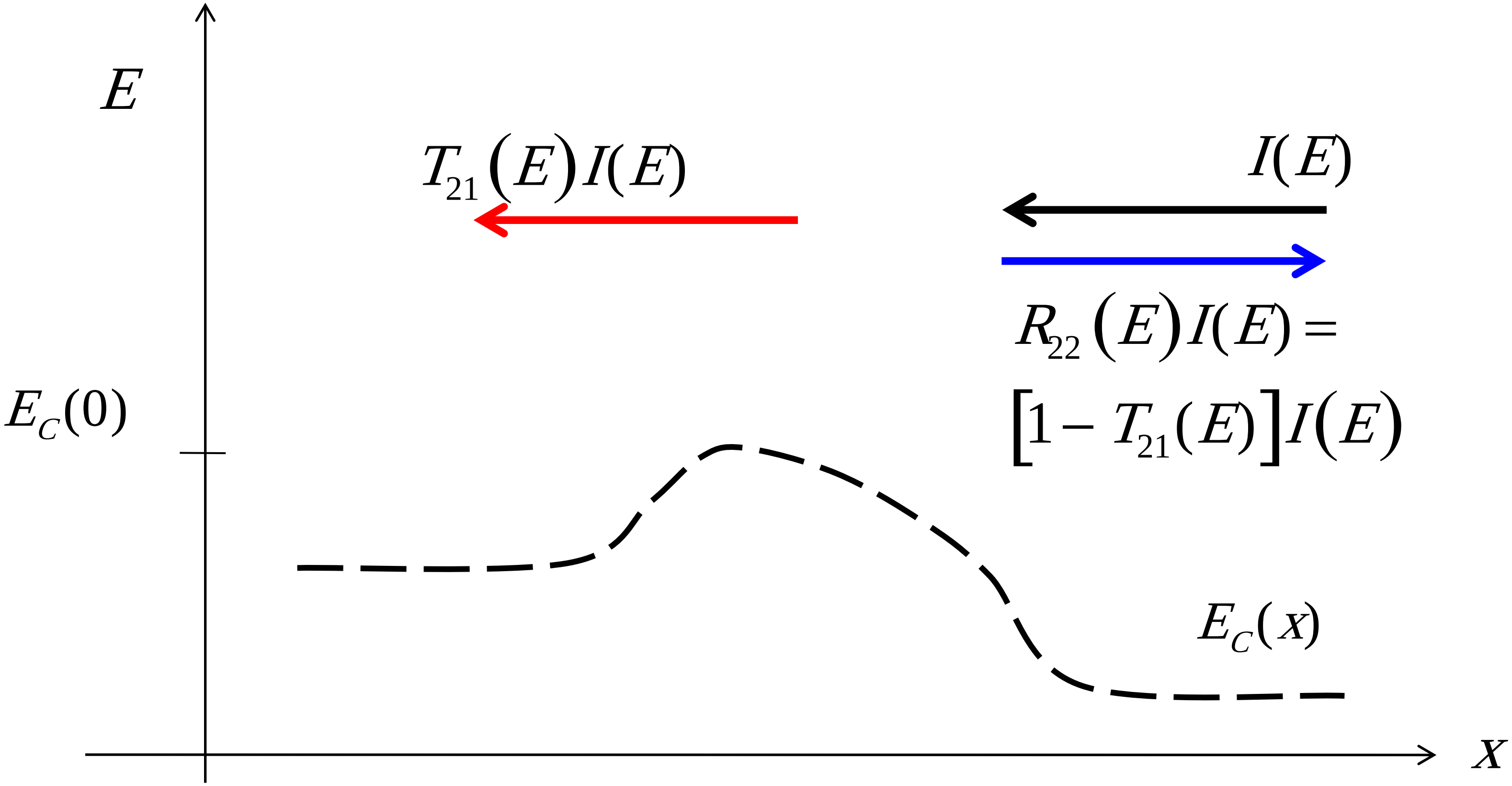
current transmission in a MOSFET

elastic scattering….



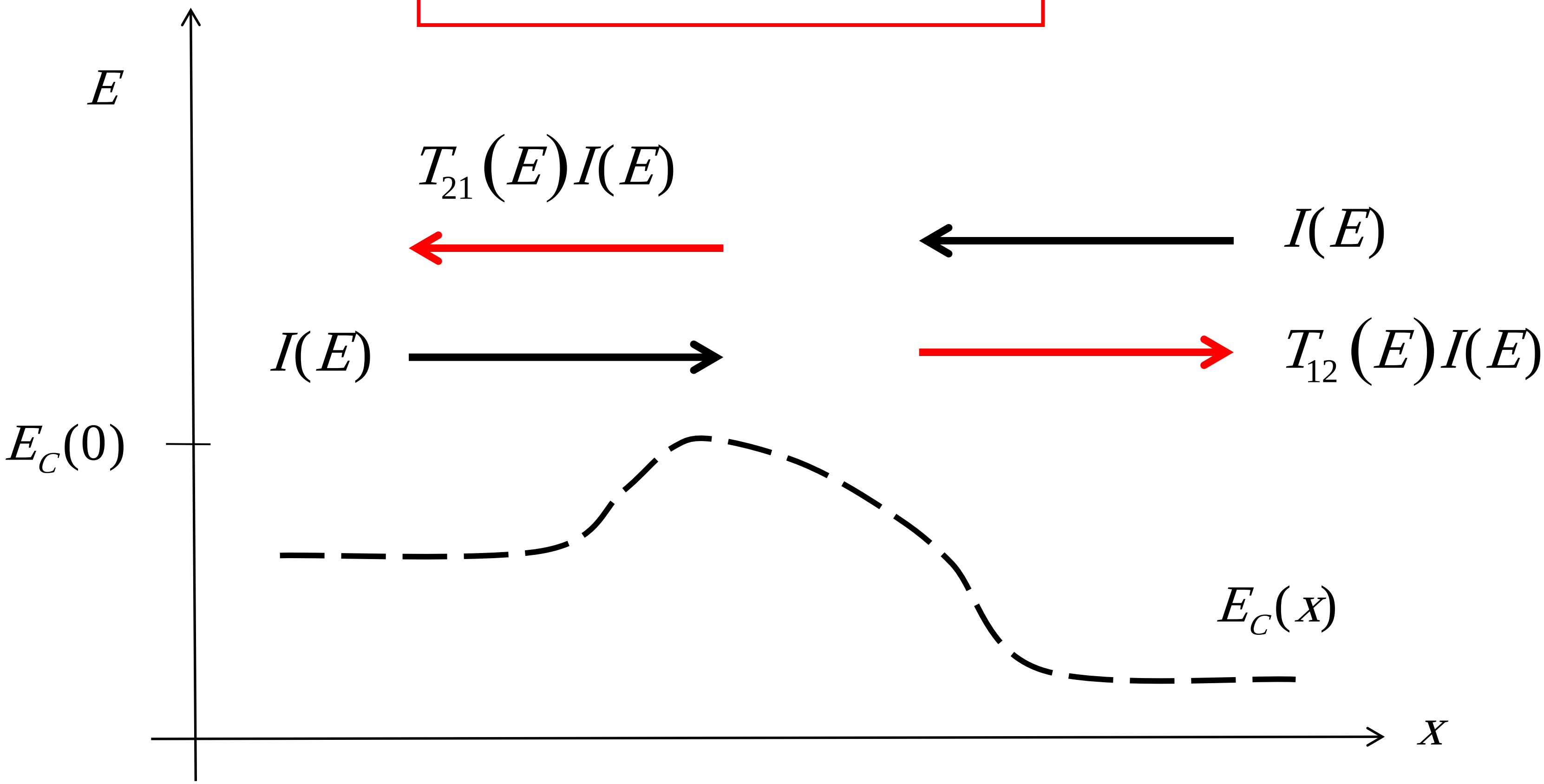
current transmission in a MOSFET

elastic scattering….



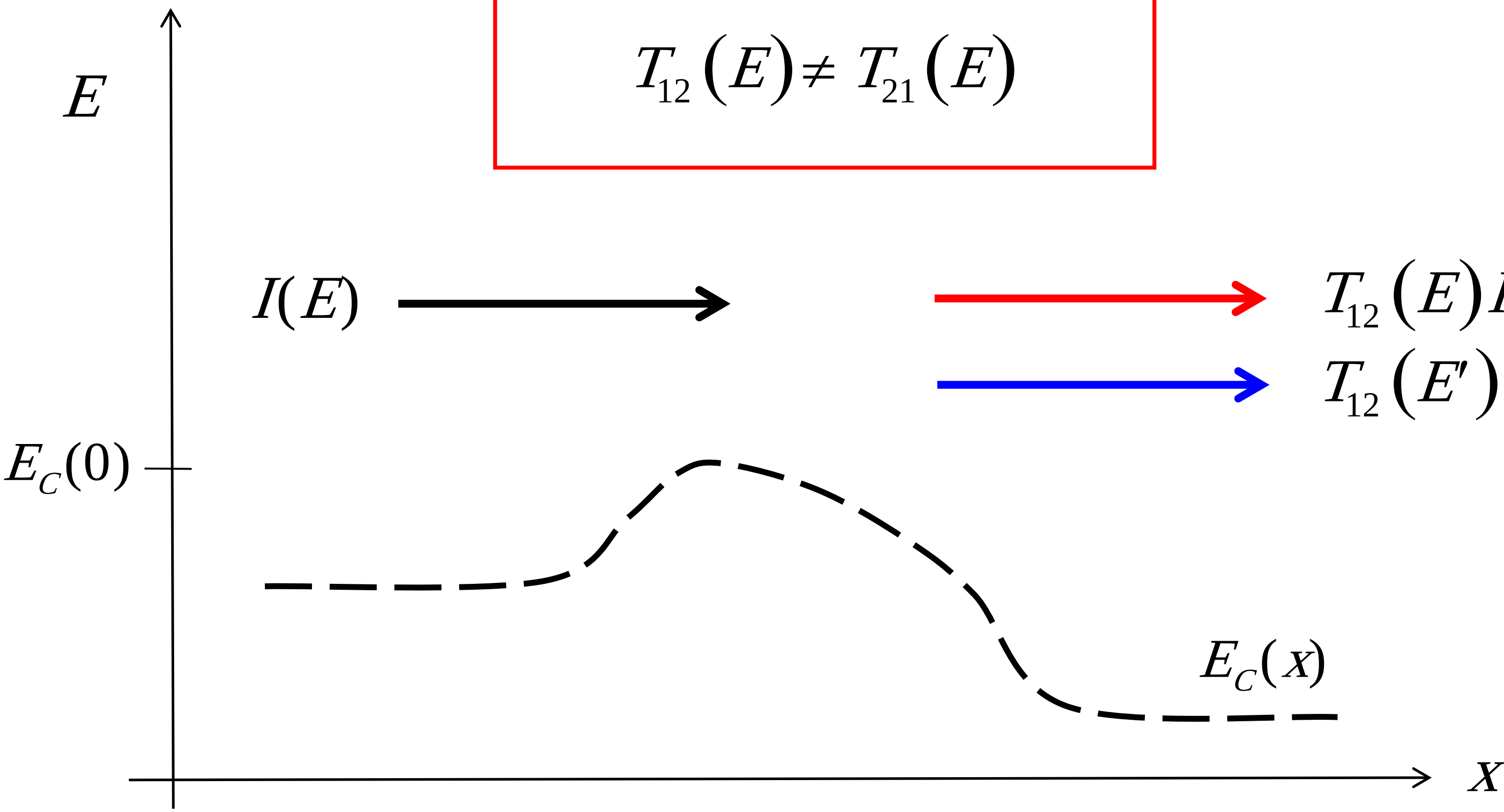
transmission in the presence of **elastic** scattering

$$\boxed{T_{12}(E) = T_{21}(E) = T(E)}$$



inelastic scattering

$$\boxed{T_{12}(E) \neq T_{21}(E)}$$



S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge, 1995.

MFP and transmission

$$\Lambda(E) = \upsilon\left(E\right)\tau\left(E\right)$$

$$T\left(\Lambda(E)\right)$$

characteristic times

1) single particle lifetime, τ :

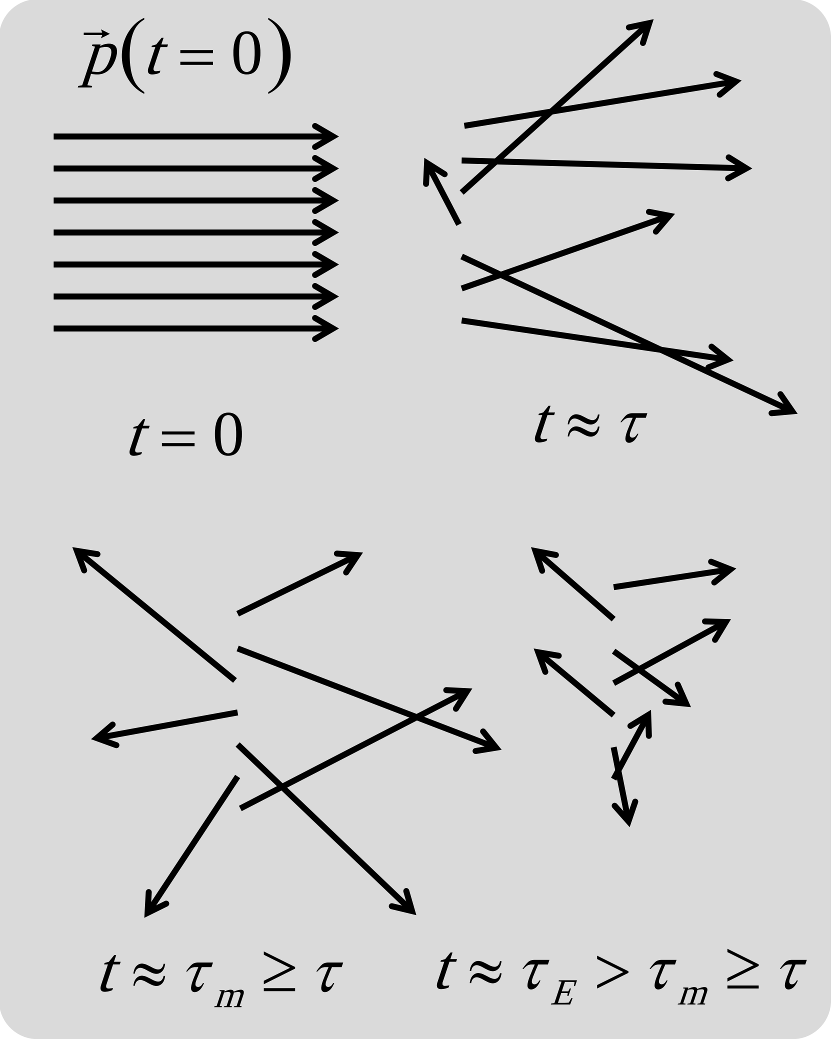
$$\tau(\vec{p})$$

2) momentum relaxation time, τ_m :

$$\tau_m(\vec{p})$$

3) energy relaxation time, τ_E :

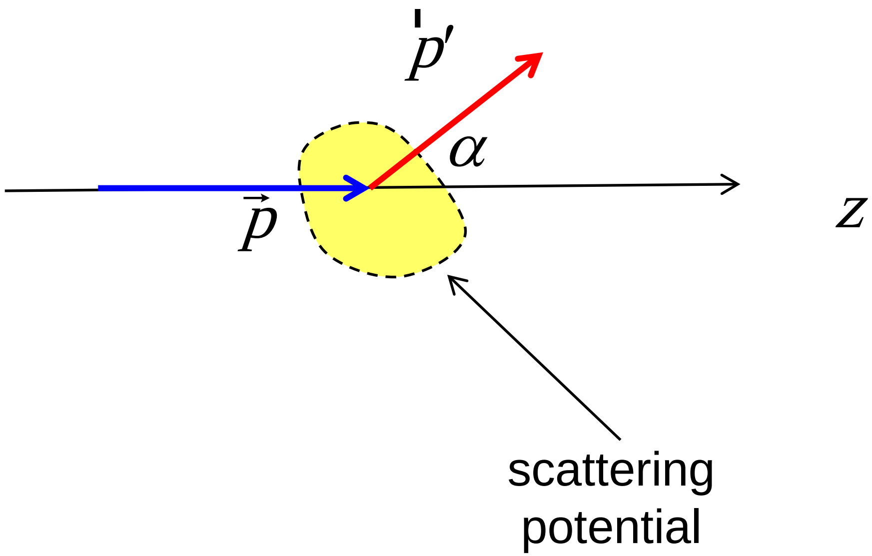
$$\tau_E(\vec{p})$$



transition rate and characteristic times

Transition rate from *p* to *p'*
(probability per second)

$$S\left(\vec{p},\vec{p}\,'\right)$$

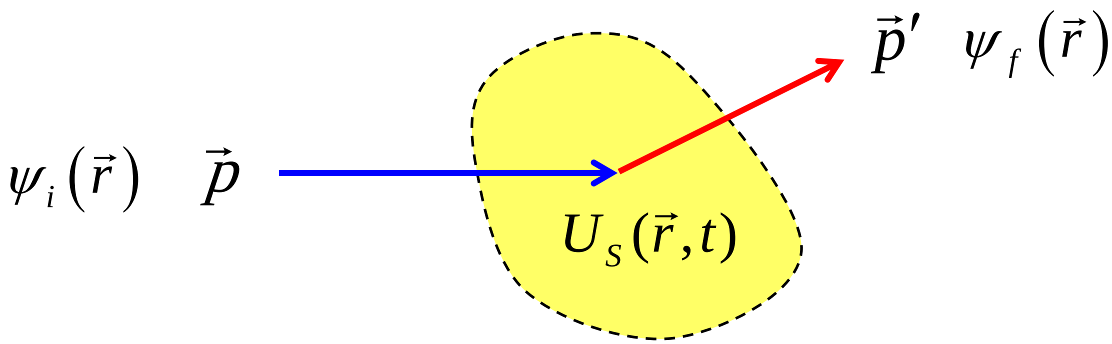


$$\frac{1}{\tau\left(\vec{p}\right)}=\sum_{\vec{p}\,',\uparrow}S\left(\vec{p},\vec{p}\,'\right)$$

$$\frac{1}{\tau_m\left(\vec{p}\right)}=\sum_{\vec{p}\,',\uparrow}S\left(\vec{p},\vec{p}\,'\right)\frac{\Delta p_z}{p_z}$$

$$\frac{1}{\tau_E\left(\vec{p}\right)}=\sum_{\vec{p}\,',\uparrow}S\left(\vec{p},\vec{p}\,'\right)\frac{\Delta E}{E_0}$$

Fermi's Golden Rule

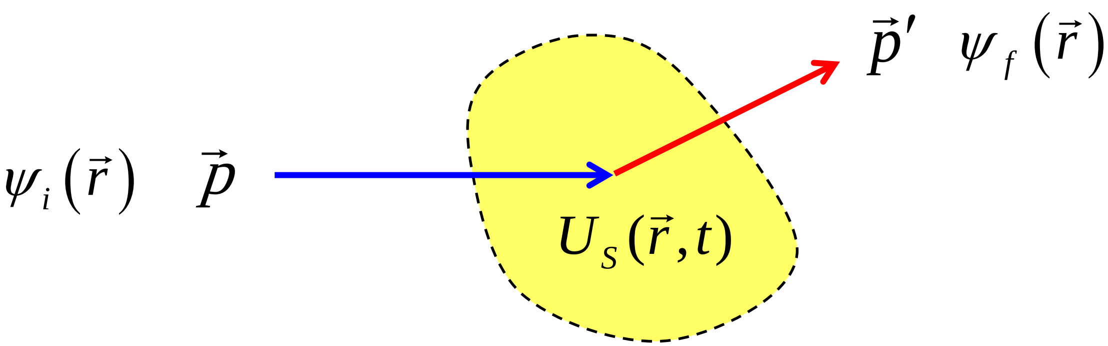


$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{p'p}\right|^2 \delta(E' - E - \Delta E) \qquad H_{\vec{p}\,',\vec{p}} = \int_{-\infty}^{+\infty} \psi_f^* U_S(\vec{r})\psi_i \, d\vec{r}$$

$$E' = E + \Delta E \qquad \Delta E = 0 \text{ for a static } U_S$$

$$\Delta E = \pm\hbar\omega \text{ for an oscillating } U_S$$

scattering and DOS



The number of ways that an incident electron at energy, *E*, can scatter is expected to be proportional to the density of final states that conserve energy and momentum.

1) elastic scattering

$$\frac{1}{\tau(E)} \propto D_f(E)$$

2) phonon absorption

$$\frac{1}{\tau(E)} \propto D_f(E+\hbar\omega)$$

3) phonon emission

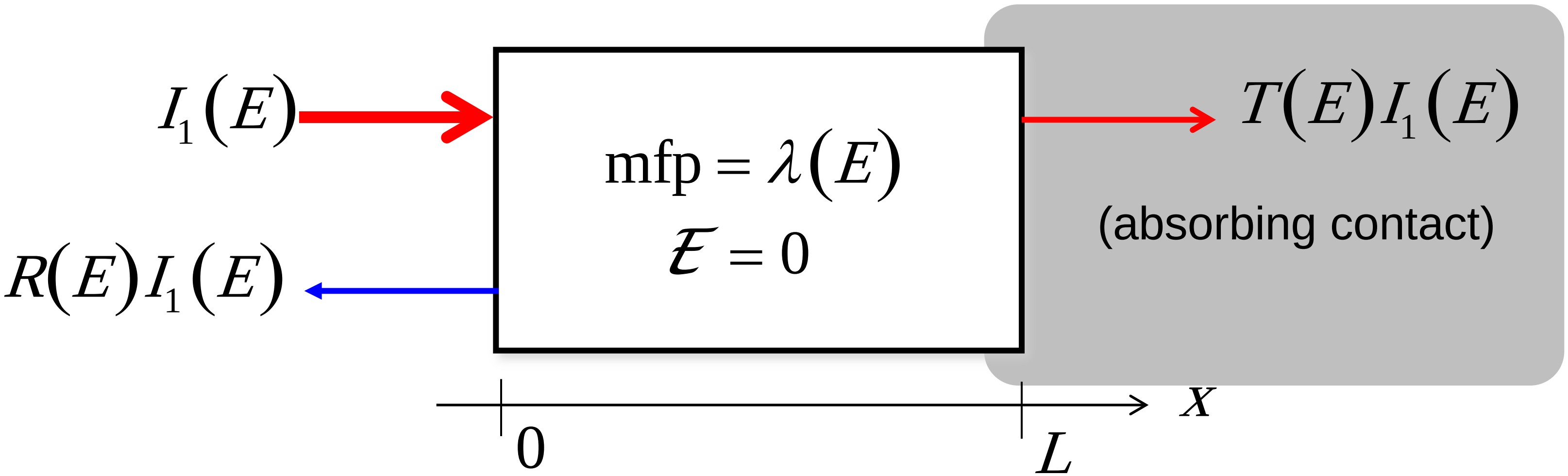
$$\frac{1}{\tau(E)} \propto D_f(E-\hbar\omega)$$

MFP and transmission

$$\Lambda(E) = \upsilon\left(E\right)\tau\left(E\right)$$

$$T\left(\Lambda(E)\right)$$

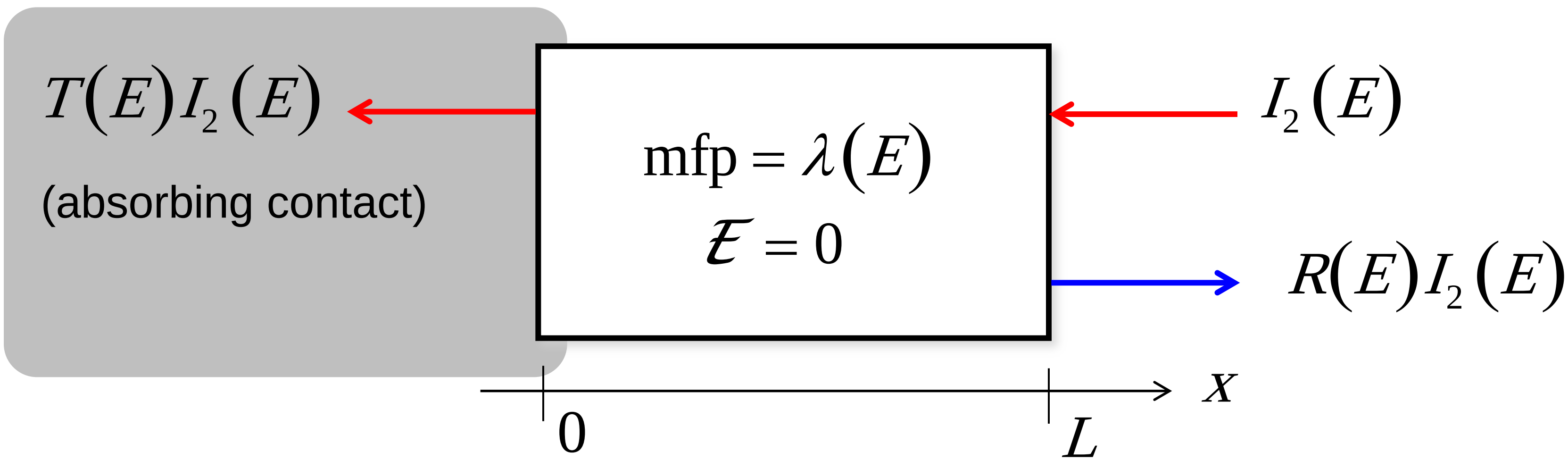
transmission across a field-free slab



Consider a flux of carriers injected from the left into a field-free slab of length, L. The flux that emerges at $x = L$ is T times the incident flux, where $0 < T < 1$. The flux that emerges from $x = 0$ is R times the incident flux, where $T + R = 1$, assuming no carrier recombination-generation.

How is T related to the mean-free-path **for backscattering** within the slab?

injection from the right

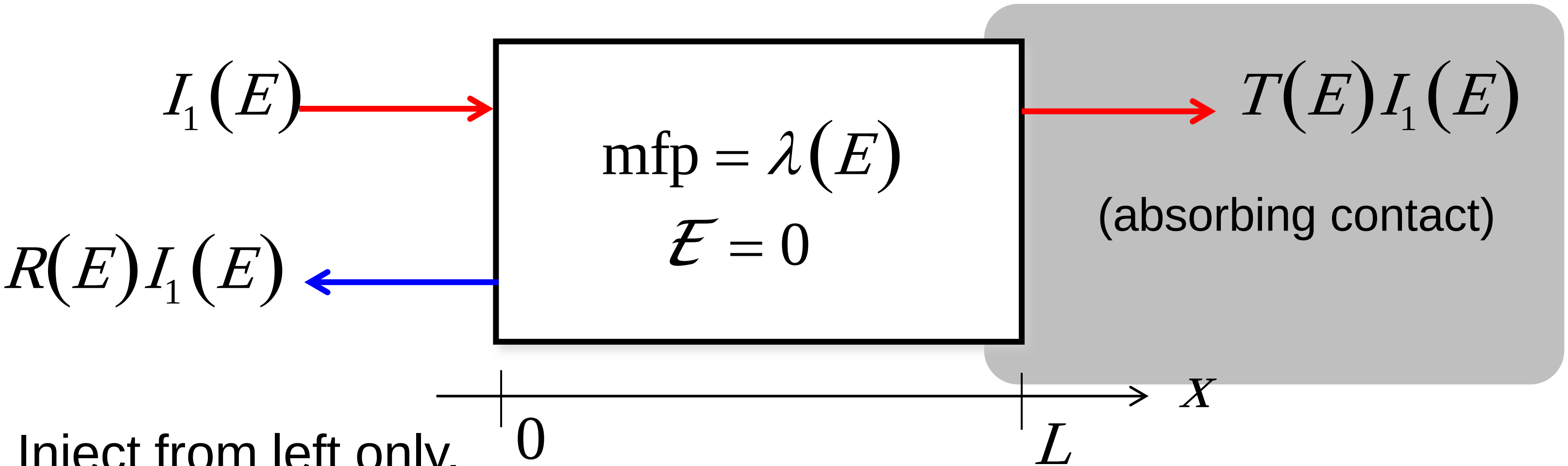


In general, there *could* be injection from both the left and the right contacts.

For elastic scattering: $T_{12}(E) = T_{21}(E) = T(E)$

Near equilibrium: $T_{12}(E) \approx T_{21}(E) \approx T(E)$ (no built-in fields)

problem specification

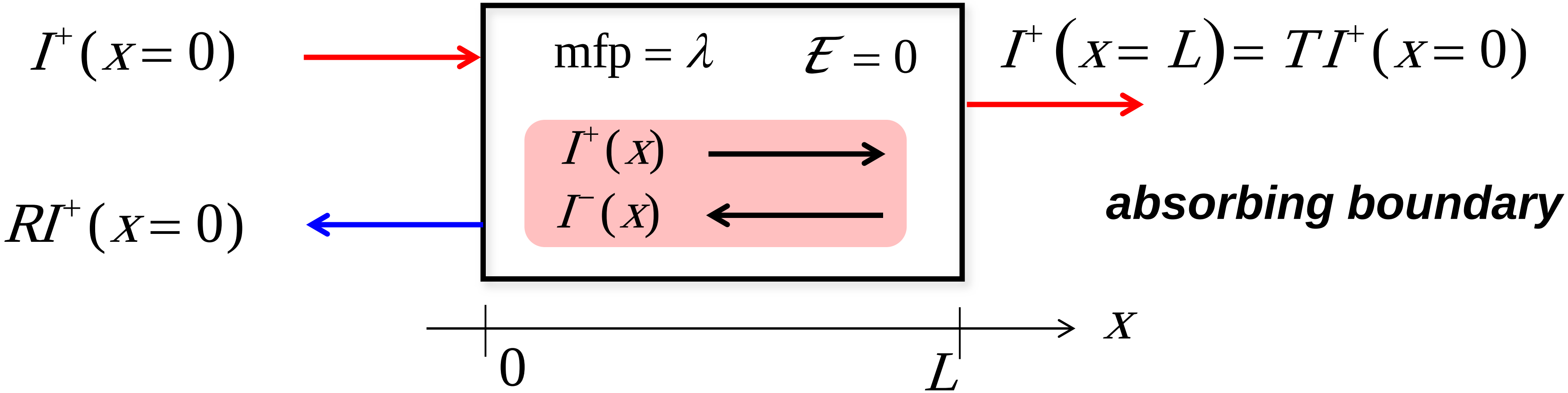


1) Inject from left only.

2) Ignore "vertical transport" (elastic scattering or near-equilibrium), so $T_{12}(E) = T_{21}(E) = T(E)$.

Then relate T to the mean-free-path for backscattering within the slab. (No assumption about whether the slab length, L, is long or short compared to the mfp, but we **do assume** that the mean-free-path is not position-dependent.)

transmission



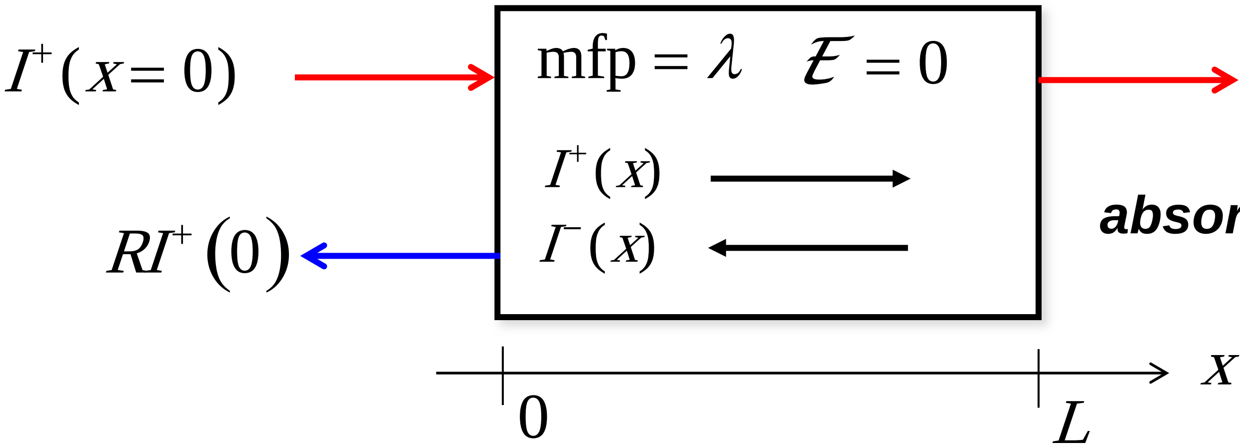
$$\frac{dI^+(x)}{dx} = -\frac{I^+(x)}{\lambda} + \frac{I^-(x)}{\lambda}$$

$$I = I^+(x) - I^-(x) \quad \text{(constant)}$$

$$I^-(x) = I^+(x) - I$$

$$\boxed{\frac{dI^+(x)}{dx} = -\frac{I}{\lambda}}$$

transmission (ii)



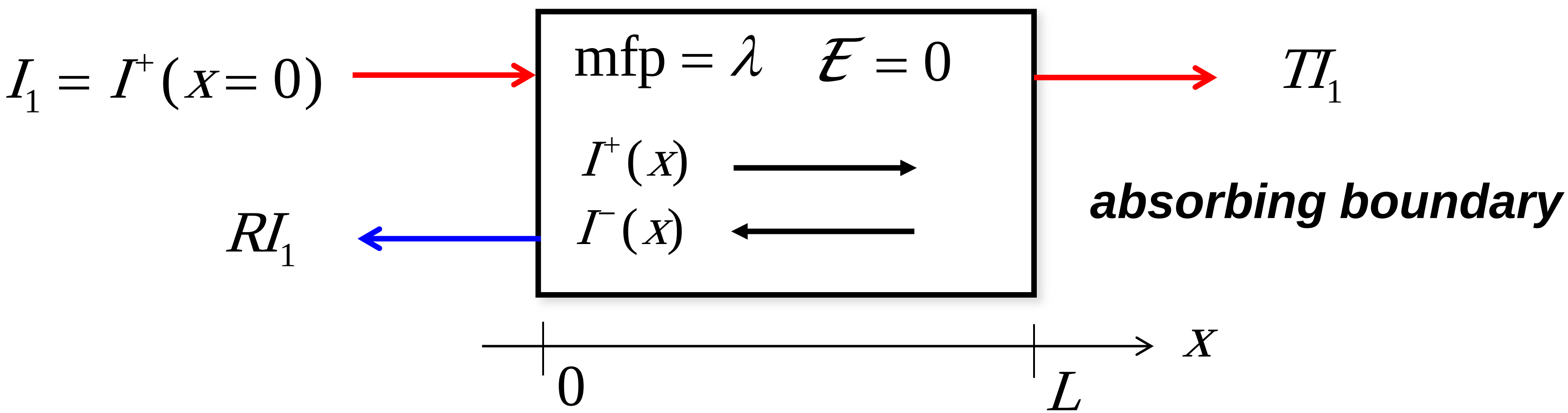
absorbing boundary

$$\boxed{\frac{dI^{+}(x)}{dx}=-\frac{I}{\lambda}}$$

$$\int_{I^{+}(0)}^{I^{+}(x)} dI^{+}=-\frac{I}{\lambda}\int_{0}^{x} dx'$$

$$I^{+}(x)=I^{+}(0)-I\frac{x}{\lambda}$$

transmission (iii)



$$I^+(x) = I^+(0) - \left(I^+(x) - I^-(x)\right)\frac{x}{\lambda}$$

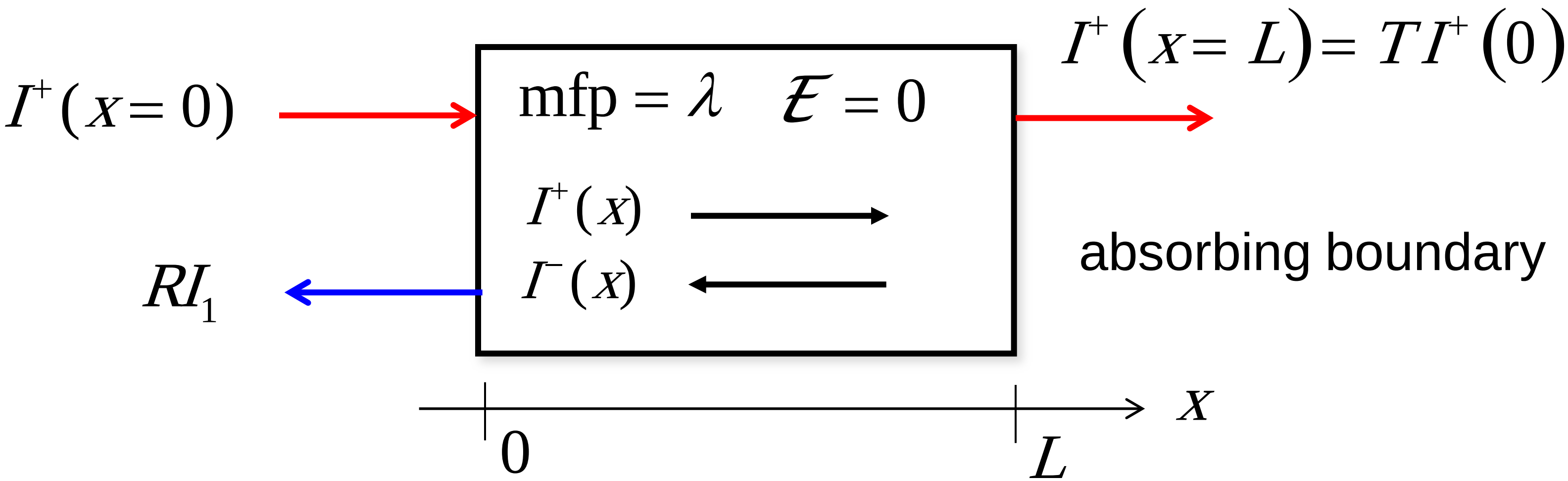
$$\boxed{I^+(x) = I^+(0) - I\frac{x}{\lambda}}$$

$$I^+(L) = I^+(0) - \left(I^+(L) - I^-(L)\right)\frac{L}{\lambda}$$

$$I^-(L) = 0$$

$$I^+(L) = I^+(0) - I^+(L)\frac{L}{\lambda}$$

transmission (iv)



$$\boxed{I^+(L) = I^+(0) - I^+(L)\frac{L}{\lambda}}$$

$$I^+(L) = \frac{I^+(0)}{1 + L/\lambda}$$

$$\frac{I^+(L)}{I^+(0)} = T = \frac{\lambda}{\lambda + L}$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L} \qquad T(E) + R(E) = 1$$

$$T \to 0 \quad L >> \lambda$$

$$T \to 1 \quad L << \lambda$$

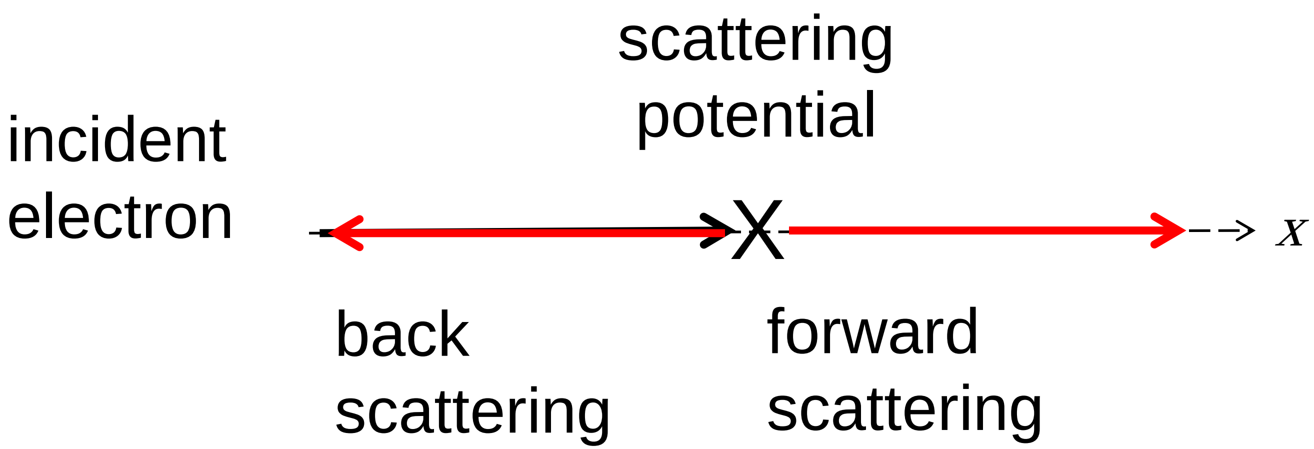
MFP and transmission

$$\Lambda(E) = \upsilon(E)\tau(E)$$

$$T(\Lambda(E))$$

$$\Lambda(E) \text{ vs. } \lambda(E)?$$

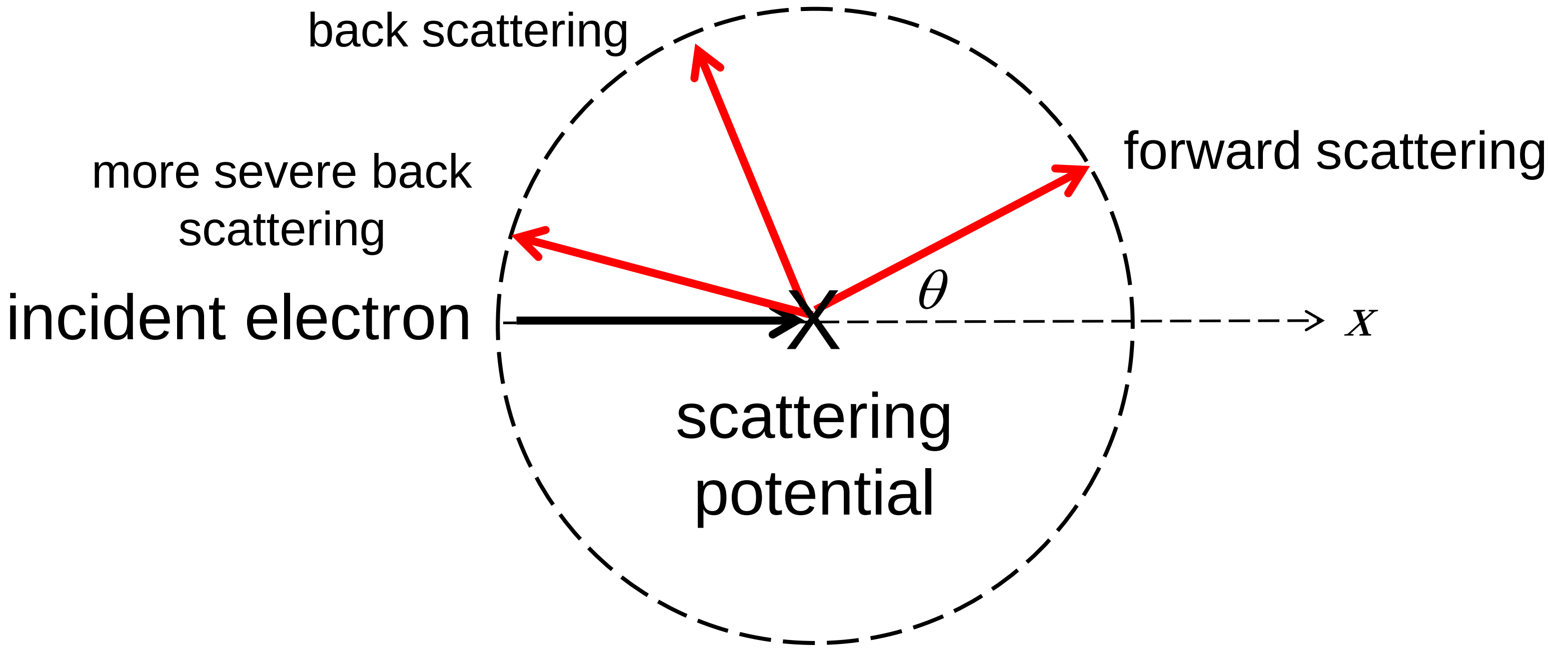
backscattering in 1D



If we assume that the scattering is ***isotropic*** (equal probability of scattering forward or back) then average time between backscattering events is 2|.

$$\lambda(E) = 2\upsilon(E)\tau_m(E) = 2\Lambda(E)$$

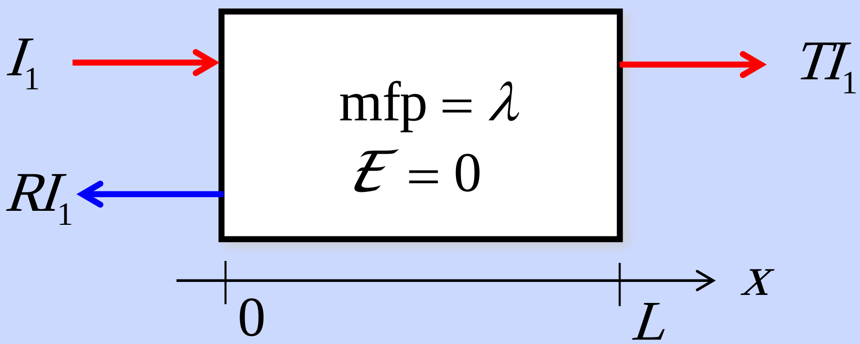
backscattering in 2D



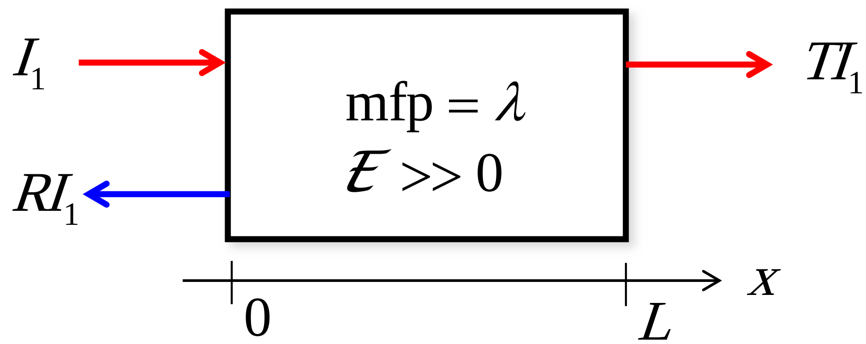
If we assume that the scattering is ***isotropic**:*

$$\lambda(E) = \frac{\pi}{2} \upsilon(E) \tau_m(E)$$

transmission across a slab with an electric field



$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

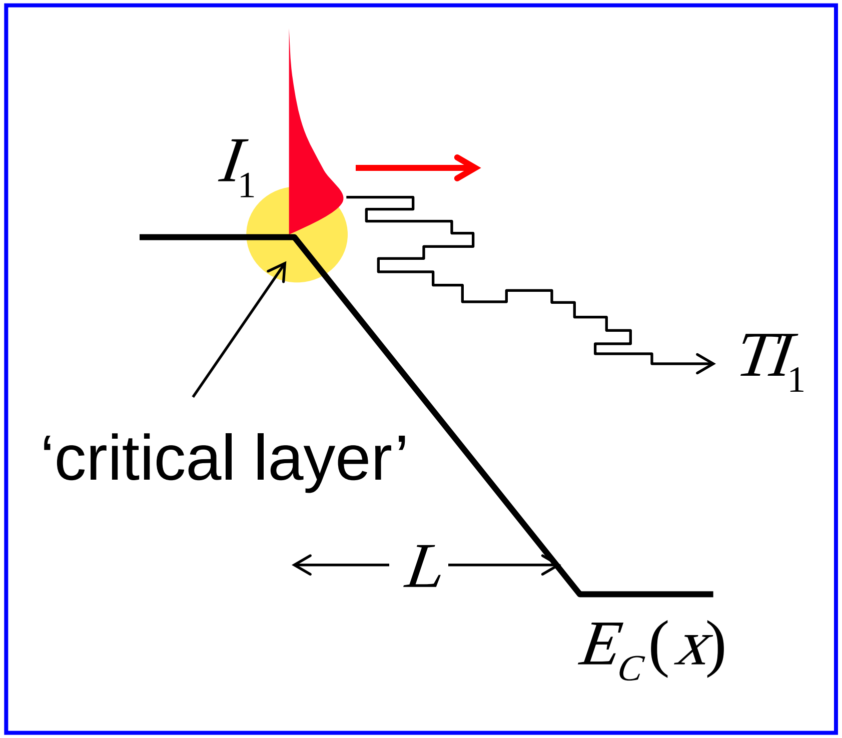


$$T(E) = ?$$

This turns out to be a <u>difficult</u> problem.

How can we understand the essential physics?

transport "downhill"



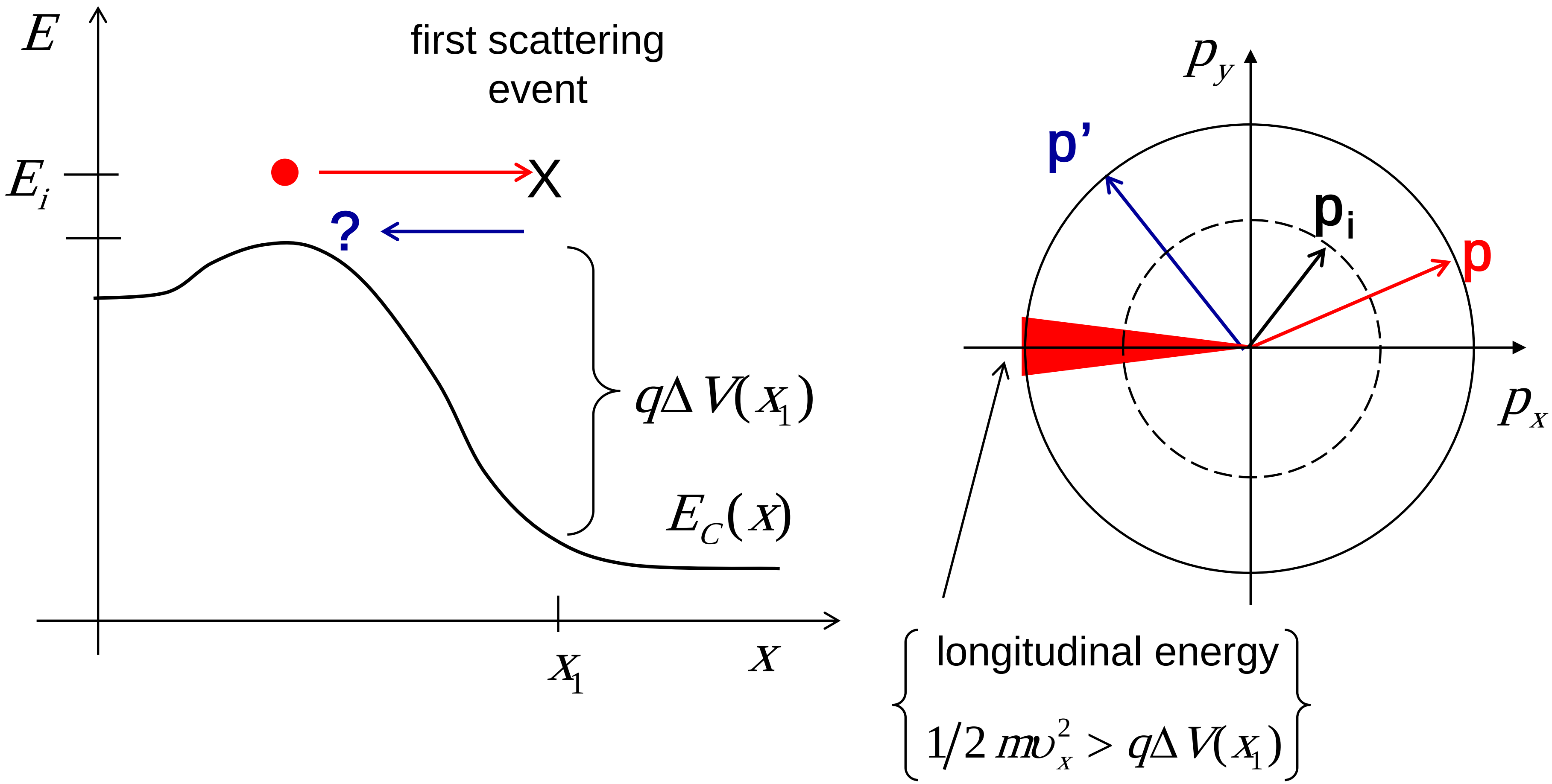
$\ell << L$

$T \approx 1$:

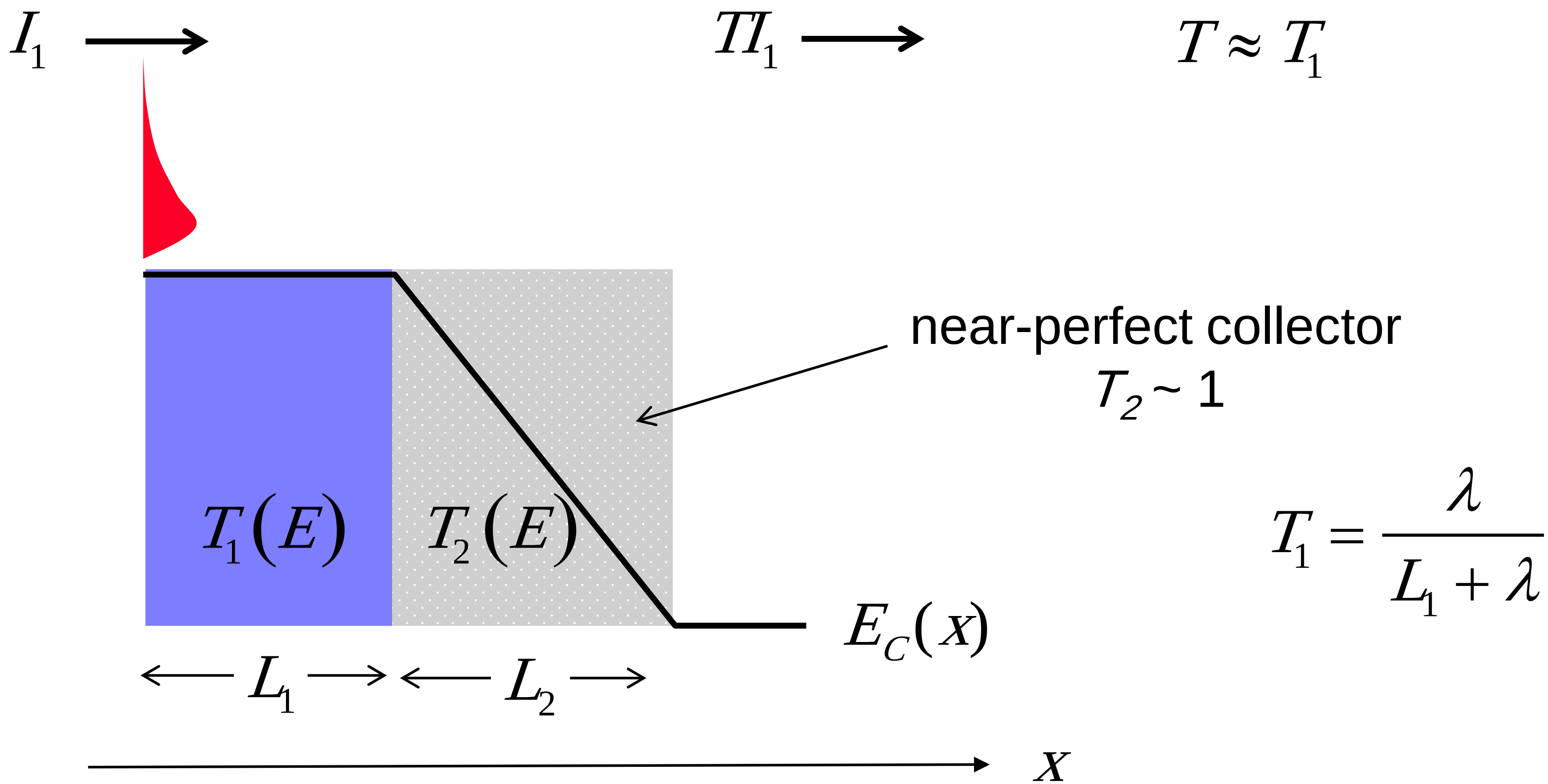
High field regions are good carrier collectors.

Peter J, Price, "Monte Carlo calculation of electron transport in solids," *Semiconductors and Semimetals*, **14**, pp. 249-334, 1979

physics of elastic back-scattering



field-free region followed by high-field region



Transmission is controlled by the low-field region.

wrap-up

1) Transmission is related to the MFP for backscattering

$$T = \frac{\lambda}{L + \lambda}$$

2) Ballistic transport: $L << \lambda \quad T \rightarrow 1$

3) Diffusive transport: $L >> \lambda \quad T \rightarrow \frac{\lambda}{L} << 1$

4) High-field regions are good collectors (T ~ 1)

references

For a more in-depth treatment of carrier scattering, see:

M.S. Lundstrom, ECE-656: “Carrier Transport in Semiconductors,” Lectures 19-28, Fall, 2011. https://nanohub.org/resources/11872.

Mark Lundstrom, *Fundamentals of Carrier Transport*, Chapter 2, Cambridge Univ. Press, 2000.