Lecture 9: Scattering and Transmission

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ballistic MOSFET (MB)

\[ I_D = W Q_n \left( V_{GS}, V_{DS} \right) \nu_T \]

\[ I_D = W Q_n \left( V_{GS}, V_{DS} \right) F_{SAT} \left( V_D \right) \nu_T \]

\[ I_D = W Q_n \left( V_{GS}, V_{DS} \right) \frac{\nu_T}{2 k_B T_L / q} V_{DS} \]
ballistic vs. real MOSFETs

$L_G = 40 \text{ nm}$

- Si MOSFETs deliver > one-half of the ballistic on-current. (Similar for the past 15 years.)
- MOSFETs operate closer to the ballistic limit under high $V_{DS}$.
- But III-V FETs operate close to the ballistic limit.


review: ballistic transport in a MOSFET

$L << \lambda$

\[ KE = \frac{1}{2} m^* \nu^2 \]
review: diffusive transport in a MOSFET

$L \gg \lambda$

$E$

$E_C(0)$

$E_C(x)$

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non-local, quasi-ballistic transport

Nanoscale MOSFETs are neither fully ballistic nor fully diffusive; they operate in a “quasi-ballistic” regime.

How do we understand how carrier scattering affects the performance of a nanoscale MOSFET?
transmission

\[ f_1(E) = \frac{1}{1 + \exp\left(\frac{E - E_{F1}}{k_B T_L}\right)} \]

\[ f_2(E) = \frac{1}{1 + \exp\left(\frac{E - E_{F2}}{k_B T_L}\right)} \]

\[ I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE \]

ballistic transport: \( T(E) = 1 \)
current transmission in a MOSFET

elastic scattering....

\[ R_{11}(E) I(E) = [1 - T_{12}(E)] I(E) \]

\[ I(E) \]

\[ T_{12}(E) I(E) \]

\[ E \]

\[ E_C(0) \]

\[ E_C(x) \]

\[ X \]
current transmission in a MOSFET

elastic scattering…. 

\[ T_{21}(E) I(E) \]

\[ R_{22}(E) I(E) = \left[ 1 - T_{21}(E) \right] I(E) \]

\[ E \]

\[ E_C(0) \]

\[ E_c(x) \]

\[ X \]
transmission in the presence of elastic scattering

\[ T_{12}(E) = T_{21}(E) = T(E) \]
inelastic scattering

\[ T_{12}(E) \neq T_{21}(E) \]

MFP and transmission

\[ \Lambda(E) = \nu(E) \tau(E) \]

\[ T(\Lambda(E)) \]
characteristic times

1) single particle lifetime, $\tau$:

$\tau(\vec{p})$

2) momentum relaxation time, $\tau_m$:

$\tau_m(\vec{p})$

3) energy relaxation time, $\tau_E$:

$\tau_E(\vec{p})$

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transition rate and characteristic times

Transition rate from $p$ to $p'$ (probability per second)

$$S(\vec{p}, \vec{p}')$$

$$\frac{1}{\tau(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}')$$

$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta p_z}{p_z}$$

$$\frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta E}{E_0}$$

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Fermi’s Golden Rule

\[ S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar} |H_{p'p}|^2 \delta(E' - E - \Delta E) \]

\[ H_{p',\vec{p}} = \int_{-\infty}^{+\infty} \psi_f^* U_S(\vec{r}) \psi_i d\vec{r} \]

\[ E' = E + \Delta E \quad \Delta E = 0 \text{ for a static } U_S \]

\[ \Delta E = \pm \hbar \omega \text{ for an oscillating } U_S \]
The number of ways that an incident electron at energy, \( E \), can scatter is expected to be proportional to the density of final states that conserve energy and momentum.

1) elastic scattering

\[
\frac{1}{\tau(E)} \propto D_f(E)
\]

2) phonon absorption

\[
\frac{1}{\tau(E)} \propto D_f(E + \hbar \omega)
\]

3) phonon emission

\[
\frac{1}{\tau(E)} \propto D_f(E - \hbar \omega)
\]
MFP and transmission

\[ \Lambda(E) = \nu(E)\tau(E) \]

\[ T(\Lambda(E)) \]
transmission across a field-free slab

Consider a flux of carriers injected from the left into a field-free slab of length, $L$. The flux that emerges at $x = L$ is $T$ times the incident flux, where $0 < T < 1$. The flux that emerges from $x = 0$ is $R$ times the incident flux, where $T + R = 1$, assuming no carrier recombination-generation.

How is $T$ related to the mean-free-path for backscattering within the slab?
In general, there could be injection from both the left and the right contacts.

For elastic scattering: \( T_{12}(E) = T_{21}(E) = T(E) \)

Near equilibrium: \( T_{12}(E) \approx T_{21}(E) \approx T(E) \) (no built-in fields)
1) Inject from left only.

2) Ignore “vertical transport” (elastic scattering or near-equilibrium), so \( T_{12}(E) = T_{21}(E) = T(E) \).

Then relate \( T \) to the mean-free-path for backscattering within the slab. (No assumption about whether the slab length, \( L \), is long or short compared to the mfp, but we do assume that the mean-free-path is not position-dependent.)
transmission

\[ I^+ (x = 0) \]

\[ R I^+ (x = 0) \]

\[ \text{mfp} = \lambda \quad \mathcal{E} = 0 \]

\[ I^+ (x) \quad \Gamma^+ (x) \]

\[ I^- (x) \quad \Gamma^- (x) \]

\[ I^+ (x = L) = T I^+ (x = 0) \]

absorbing boundary

\[ \frac{d I^+ (x)}{dx} = - \frac{I^+ (x)}{\lambda} + \frac{\Gamma (x)}{\lambda} \]

\[ I = I^+ (x) - \Gamma (x) \quad \text{(constant)} \]

\[ \Gamma (x) = I^+ (x) - I \]

\[ \frac{d I^+ (x)}{dx} = - \frac{I}{\lambda} \]
transmission (ii)

\[ I^+(x = 0) \quad \text{mfp} = \lambda \quad \mathcal{E} = 0 \quad T I^+(0) \]

\[ RI^+(0) \]

\[ I^+(x) \quad I^-(x) \]

absorbing boundary

\[ 0 \quad L \quad x \]

\[ d I^+(x) \quad \frac{d I^+(x)}{dx} = - \frac{I}{\lambda} \]

\[ \int_{r^+(0)}^{r^+(x)} dI^+ = - \frac{I}{\lambda} \int_0^x dx' \]

\[ I^+(x) = I^+(0) - I \frac{x}{\lambda} \]
transmission (iii)

\[ I_1 = I^+ (x = 0) \]

\[ R I_1 \]

\[ mfp = \lambda \quad E = 0 \]

**absorbing boundary**

\[ I^+ (x) \]

\[ I^- (x) \]

\[ I^+ (0) \]

\[ I^- (0) \]

\[ I^+ (L) \]

\[ I^- (L) \]

\[ I^+ (x) = I^+ (0) - \left( I^+ (x) - I^- (x) \right) \frac{x}{\lambda} \]

\[ I^+ (L) = I^+ (0) - \left( I^+ (L) - I^- (L) \right) \frac{L}{\lambda} \]

\[ I^- (L) = 0 \]

\[ I^+ (L) = I^+ (0) - I^+ (L) \frac{L}{\lambda} \]
transmission (iv)

\[ I^+(x = 0) \]

\[ mfp = \lambda \quad \mathcal{E} = 0 \]

\[ I^+(x) \quad I^-(x) \]

\[ I^+(L) = I^+(0) - I^+(L) \frac{L}{\lambda} \]

\[ I^+(L) = \frac{I^+(0)}{1 + L/\lambda} \]

\[ \frac{I^+(L)}{I^+(0)} = T = \frac{\lambda}{\lambda + L} \]

\[ I^+(x = L) = T I^+(0) \]

absorbing boundary

\[ T(E) = \frac{\lambda(E)}{\lambda(E) + L} \quad T(E) + R(E) = 1 \]

\[ T \to 0 \quad L >> \lambda \]

\[ T \to 1 \quad L << \lambda \]
MFP and transmission

\[ \Lambda(E) = \nu(E) \tau(E) \]

\[ T(\Lambda(E)) \]

\[ \Lambda(E) \text{ vs. } \lambda(E)? \]
If we assume that the scattering is \textit{isotropic} (equal probability of scattering forward or back) then average time between backscattering events is \( 2 \lambda \). 

\[
\lambda(E) = 2 \nu(E) \tau_m(E) = 2 \Lambda(E)
\]
If we assume that the scattering is *isotropic*:

\[
\lambda(E) = \frac{\pi}{2} \nu(E) \tau_m(E)
\]
transmission across a slab with an electric field

\[ T(E) = \frac{\lambda(E)}{\lambda(E) + L} \]

This turns out to be a difficult problem.

How can we understand the essential physics?
transport “downhill”

\[ \ell \ll L \]

\[ T \approx 1: \]

High field regions are good carrier collectors.

physics of elastic back-scattering

\[ E_C(x) \]

\[ q\Delta V(x_1) \]

\[ \frac{1}{2} m v_x^2 > q\Delta V(x_1) \]

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field-free region followed by high-field region

Transmission is controlled by the low-field region.
wrap-up

1) Transmission is related to the MFP for backscattering

\[ T = \frac{\lambda}{L + \lambda} \]

2) Ballistic transport: \( L \ll \lambda \quad T \rightarrow 1 \)

3) Diffusive transport: \( L \gg \lambda \quad T \rightarrow \frac{\lambda}{L} \ll 1 \)

4) High-field regions are good collectors (\( T \sim 1 \))
For a more in-depth treatment of carrier scattering, see:
