ECE606: Solid State Devices
Lecture 10
Shockley, Reed, Hall and other Recombinations

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1) **Derivation of SRH formula (Shockley, Reed, Hall)**

2) Application of SRH formula for special cases

3) Direct and Auger recombination

4) Conclusion

Ref. ADF, Chapter 5, pp. 141-154
Sub-processes of SRH Recombination

(1)+(3): one electron reduced from Conduction-band & one-hole reduced from valence-band

(2)+(4): one hole created in valence band and one electron created in conduction band
Physical picture

(1)↓ (3)↓
(2)↑ (4)↑

Equivalent picture

(1)↑ (3)↑
(2)↓ (4)↓

(1)+(3): one electron reduced from C-band & one-hole reduced from valence-band

(2)+(4): one hole created in valence band & one electron created in conduction band
Changes in electron and hole Densities

\[
\frac{\partial n}{\partial t}\bigg|_{1,2} = -c_n n p_T + e_n n_T (1 - f_c)
\]

\[
\frac{\partial p}{\partial t}\bigg|_{3,4} = -c_p p n_T + e_p p_T f_v
\]
Assume non-degenerate

$$(1 - f_c) \approx 1$$

$$\frac{\partial n}{\partial t}_{1,2} = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$0 = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$e_n = c_n \frac{n_0 p_{T0}}{n_{T0}} \equiv c_n n_1$$

$$0 = -c_n \left( n_0 p_{T0} - n_{T0} n_1 \right)$$

$$\frac{\partial p}{\partial t}_{3,4} = -c_p p n_T + e_p p_T$$

$$0 = -c_p p_0 n_{T0} + p_{T0} e_p$$

Fundamental concept of detailed balance, connecting 2 counteracting processes.

$$e_p \equiv \frac{c_p p_0 n_{T0}}{p_{T0}} = c_p p_1$$

$$0 = -c_p \left( p_0 n_{T0} - p_{T0} p_1 \right)$$
Expressions for \((n_1)\) and \((p_1)\)

\[
n_1 = \frac{n_0 p_{T0}}{n_{T0}}
\]

\[
p_1 = \frac{p_0 n_{T0}}{p_{T0}}
\]

\[
n_1 p_1 = \frac{n_0 p_{T0}}{n_{T0}} \times \frac{p_0 n_{T0}}{p_{T0}} = n_0 p_0 = n_i^2
\]
Expressions for \((n_1)\) and \((p_1)\)

\[ n_{T0} = N_T \left(1 - f_{00}\right) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}} \]

\[(1 - f_{00}) = \frac{1}{1 + g \exp} \quad f_{00} = 1 - \frac{1}{1 + g \exp} \]

\[ f_{00} = \frac{g \exp}{1 + g \exp} \quad f_{00} / (1 - f_{00}) = \frac{g \exp}{1 + g \exp} / \frac{1}{1 + g \exp} = g \exp \]

\[ n_1 = \frac{n_0 p_{T0}}{n_{T0}} = n_0 \frac{(N_T f_{00})}{N_T (1 - f_{00})} \]

\[ n_1 = n_i e^{\beta(E_F - E_i)} g_D e^{\beta(E_T - E_F)} = n_i g_D e^{\beta(E_T - E_i)} \]

\[ p_{1n_1} = n_i^2 \]

\[ p_1 = \frac{n_i^2}{n_1} = n_i g_D^{-1} e^{\beta(E_i - E_T)} \]
Dynamics of Trap Population

\[
\frac{\partial n_T}{\partial t} = - \left. \frac{\partial n}{\partial t} \right|_{1,2} + \left. \frac{\partial p}{\partial t} \right|_{3,4}
\]

\[
= c_n n p_T - e_n n_T - c_p p n_T + e_p p_T
\]

\[
= c_n \left( n p_T - n_T n_1 \right) - c_p \left( p n_T - p_T p_1 \right)
\]
Steady-state Trap Population

\[
\frac{\partial n_T}{\partial t} = 0 = c_n \left( np_T - n_T n_1 \right) - c_p \left( p n_T - p_T p_1 \right)
\]

\[
0 = c_n \left( n \left( N_T - n_T \right) - n_T n_1 \right) - c_p \left( p_T - n \left( N_T - n_T \right) p_1 \right)
\]

\[
n_T \left( c_n n + c_n n_1 + c_p p + c_p p_1 \right) = c_n n T + c_p p_1 N_T
\]

\[
n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n \left( n + n_1 \right) + c_p \left( p + p_1 \right)}
\]

\[
N_T = p_T + n_T
\]

\[
p_T = N_T - n_T
\]
Net Rate of Recombination-Generation

\[ p_T = N_T - n_T \]

\[ n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n (n + n_1) + c_p (p + p_1)} = \frac{c_n N_T n + c_p N_T p_1}{A} \]

\[ p_1 n_1 = n_i^2 \]

\[ R = -\frac{dp}{dt} = c_p (p_n T - p_T p_1) = c_p (p_n T - N_T p_1 + n_T p_1) \]

\[ = c_p n_T (p + p_1) - c_p N_T p_1 = c_p (p + p_1) \frac{c_n N_T n + c_p N_T p_1}{A} - c_p N_T p_1 \frac{A}{A} \]

\[ = \frac{c_p N_T}{A} \left( p c_n + p_1 c_n n + p c_p p_1 + p_1 c_p p_1 - p_1 c_n n - p_1 c_n n_1 - p_1 c_p p - p_1 c_p p_1 \right) \]

\[ = \frac{c_p N_T c_n}{A} (p n - p_1 n_1) \]

\[ = \frac{np - n_i^2}{\left( \frac{1}{c_p N_T} \right) (n + n_1) + \left( \frac{1}{c_n N_T} \right) (p + p_1)} \]
Net Rate of Recombination-Generation

\[ n_1 = n_i g_D e^{\beta(E_T - E_i)} \]

\[ p_1 = \frac{n_i^2}{n_1} \]

\[ = n_i g_D^{-1} e^{\beta(E_i - E_T)} \]

\[ R = -\frac{dp}{dt} = c_p \left( p n_T - p_T p_1 \right) \]

\[ = \frac{np - n_i^2}{\left( \frac{1}{c_p N_T} \right)(n + n_1) + \left( \frac{1}{c_n N_T} \right)(p + p_1)} \]

Minority carrier recombination lifetimes

\[ \tau_n \]

\[ \tau_p \]
1) Derivation of SRH formula
2) Application of SRH formula for special cases
3) Direct and Auger recombination
4) Conclusion
Case 1: Low-level Injection in p-type

\[ R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \]

\[ = \frac{(n_0 + \Delta n)(p_0 + \Delta n) - n_i^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)} \]

\[ = \frac{\Delta n(n_0 + p_0) + \Delta n^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)} \]

\[ = \frac{\Delta n(p_0)}{\tau_n(p_0)} = \frac{\Delta n}{\tau_n} \quad \Delta n^2 \approx 0 \]

\[ p_0 \gg \Delta n \gg n_0 \]

Lots of holes, few electrons => independent of holes
Case 2: High-level Injection

\[ R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \]

\[ = \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)} \]

\[ = \frac{\Delta n(n_0 + p_0) + \Delta n^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta n + p_1)} \]

\[ = \frac{\Delta n^2}{(\tau_n + \tau_p) \Delta n} = \frac{\Delta n}{(\tau_n + \tau_p)} \]

\[ \Delta n \gg p_0 \gg n_0 \]

Lots of holes, lots of electrons => dependent on both relaxations

e.g. organic solar cells
\[ R_{\text{high}} = \frac{\Delta n}{(\tau_n + \tau_p)} \]
\[ \Delta n \gg p_0 \gg n_0 \]
\[ R_{\text{low}} = \frac{\Delta n}{\tau_p} \]
\[ p_0 \gg \Delta n \gg n_0 \]

which one is larger and why?
Depletion region – in PN diode: \( n=p=0 \)

\[
n \ll n_1 \quad p \ll p_1
\]

\[
R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} = \frac{-n_i^2}{\tau_p(n_1) + \tau_n(p_1)}
\]

NEGATIVE Recombination => Generation

\( n=p=0 \quad \ll n_i \rightarrow \) generation to create n,p

Equilibrium restoration!
1) Derivation of SRH formula
2) Application of SRH formula for special cases
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Band-to-band Recombination

\[ R = B \left( np - n_i^2 \right) \quad \text{B is a material property} \]

Direct recombination at low-level injection

\[ n_0 \ll \left( \Delta n = \Delta p \right) \ll p_0 \]

\[ R = B \left[ (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2 \right] \approx Bp_0 \times \Delta n \]

Direct generation in depletion region

\[ n, p \sim 0 \]

\[ R = B \left( np - n_i^2 \right) \approx -Bn_i^2 \]
Auger Recombination

2 electron & 1 hole

\[ R = c_n \left( n^2 p - n_i^2 n \right) + c_p \left( np^2 - n_i^2 p \right) \]

\[ c_n, c_p \sim 10^{-29} \text{ cm}^6/\text{sec} \]

Auger recombination at low-level injection

\[ n_0 \ll (\Delta n = \Delta p) \ll (p_0 = N_A) \]

\[ R \approx c_p N_A^2 \Delta n = \frac{\Delta n}{\tau_{\text{auger}}} \quad \tau_{\text{auger}} = \frac{1}{c_p N_A^2} \]

Dominant recombination in heavy doped semiconductors
Effective Carrier Lifetime

\[ \Delta n(t) = \Delta n(t = 0) e^{-\frac{t}{\tau_{\text{eff}}}} \]

\[ \tau_{\text{eff}} = \left( c_n N_T + B N_D + c_{n,\text{auger}} N_D^2 \right)^{-1} \]

\[ R = R_{\text{SRH}} + R_{\text{direct}} + R_{\text{Auger}} \]

\[ = \Delta n \left( \frac{1}{\tau_{\text{SRH}}} + \frac{1}{\tau_{\text{direct}}} + \frac{1}{\tau_{\text{Auger}}} \right) \]

\[ = \Delta n \left( c_n N_T + B N_D + c_{n,\text{auger}} N_D^2 \right) \]
Effective Carrier Lifetime with all Processes

\[ \tau_{\text{eff}} \approx c_{n,\text{auger}} N_D^{-2} \]

\[ \tau_{\text{eff}} = \left( c_n N_T + B N_D + c_{n,\text{auger}} N_D^2 \right)^{-1} \]

SRH is an important recombination mechanism in important semiconductors like Si and Ge.

SRH formula is complicated, therefore simplification for special cases are often desired.

Direct band-to-band and Auger recombination can also be described with simple phenomenological formula.

These expressions for recombination events have been widely validated by measurements.
1) Nature of interface states
2) SRH formula adapted to interface states
3) Surface recombination in depletion region
4) Conclusion

REF: ADF, Chapter 5, pp. 154-167
Surface states: Little bit of history

Hoerni’s diagram of Mesa and planar transistors

Defect Induced Leakage Path

Silicon oxide

One of the fundamental advances in semiconductor history
Atomic configuration of Surface States

Single bonds

Poly Si

Si substrate

SiO₂

16 Å
Surface States

The surface states are depicted in the diagram. The states are shown in a 3D coordinate system with axial $k$, spatial $x$, and energy $E$ axes. The surface states are represented by a series of connected nodes in a lattice structure. The energy distribution is shown below the lattice, indicating the energy levels associated with these surface states.
Multiple Levels of Surface States

The diagram illustrates the concept of multiple levels of surface states, which are represented by energy bands in the context of semiconductor physics. The left side of the diagram shows a crystal lattice, while the right side demonstrates the band structure with energy levels (E) and wavevector (k) for electrons.

The energy bands are labeled with red and blue lines, indicating different levels or bands within the material. The horizontal axis represents the position (x), and the vertical axis represents the energy (E). The red lines indicate higher energy states, while the blue lines represent lower energy states. The transitions between these bands are depicted with arrows, showing how electrons can move from one state to another.

The diagram also includes a graph with energy (E) and wavevector (k) axes, showing how the energy levels change as the wavevector varies. This is crucial for understanding the electronic properties of materials at the nanoscale.
Multiple Levels of Surface States

![Diagram showing multiple levels of surface states with a grid and arrows indicating correct and incorrect states.]

*Okay*  *Wrong*
1) Nature of interface states

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For single level bulk traps ....

\[ R_{\text{bulk}} = \frac{np - n^2_i}{c_p N_T} (n + n_1) + \frac{1}{c_n N_T} (p + p_1) = \frac{(np - n^2_i) N_T}{c_p (n + n_1) + \frac{1}{c_n} (p + p_1)} \]

For single level interface trap at \( E \) ...

\[ R(E) = \frac{\left( n_s p_s - n^2_i \right)}{c_{ps}} \frac{D_T(E)}{n_s + n_{1s}} + \frac{1}{c_{ns}} \left( p_s + p_{1s} \right) \]

\[ R = \int_{E_v}^{E_C} R(E) \, dE \]
Case 1: Minority Carrier Recombination

$$R(E) = \left( \frac{n_{s0} + \Delta n_{s0}}{c_{ps}} + \frac{1}{c_{ns}} \left( p_{s0} + \Delta p_{s0} + p_{1s} \right) \right) \frac{1}{c_{ps}} \left( n_{s0} + \Delta n_{s0} + n_{1s} \right)$$

$$= \frac{n_{s0} \Delta p_{s0} D_{IT}(E) dE}{n_{s0} \left[ 1 + \frac{n_{1s}}{c_{ps}} + \frac{p_{1s}}{c_{ns} n_{s0}} \right]}$$

$$= \frac{c_{ps} \Delta p_{s0} D_{IT}(E) dE}{1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns} n_{s0}} \frac{p_{1s}}{n_{s0}}}$$

Donor doped

$$n_{s0} + \Delta n_{s0}$$

$$p_{s0} + \Delta p_{s0}$$
Consider the Denominator ...

\[ D = 1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}} \frac{P_{1s}}{n_{s0}} = 1 + \frac{n_{1s}}{N_D} + \frac{c_{ps}}{c_{ns}} \frac{P_{1s}}{N_D} \]

\[ = 1 + \frac{n_i e^{(E-E_i)\beta}}{n_i e^{(E_F-E_i)\beta}} + \frac{c_{ps}}{c_{ns}} \frac{n_i e^{-(E-E_i)\beta}}{n_i e^{(E_F-E_i)\beta}} \]

\[ = 1 + e^{(E-E_F)\beta} + \frac{c_{ps}}{c_{ns}} e^{(E_F-E)\beta} \]

\[ = 1 + e^x + ae^{-x} \quad x \equiv \beta(E-E_F) \]
Consider the Denominator ...

\[
D = 1 + \frac{n_i e^{(E-E_i)\beta}}{N_D} + \frac{c_{ps}}{c_{ns}} \frac{n_i e^{-(E-E_i)\beta}}{N_D} \\
= 1 + e^{(E-E_F)\beta} + \frac{c_{ps}}{c_{ns}} e^{(E_F-E)\beta}
\]

At \( E = E_i \Rightarrow D = 1 + \frac{n_i}{N_D} + \frac{c_{ps}}{c_{ns}} \frac{n_i}{N_D} \approx 1
\]

At \( E = E_F > E_i, x = 0 \) \( D = 1 + 1 + \frac{c_{ps}}{c_{ns}} \times small \approx 2 \)

At \( E = E_F' < E_i, \) \( D = 1 + small + 1 = 2 \)
Approximate the Denominator ...

\[ D = 1 + \frac{n_i e^{(E-E_i)\beta}}{N_D} + \frac{c_{ps} n_i e^{-(E-E_i)\beta}}{c_{ns} N_D} \]

\[ D \approx \begin{cases} 1 & \text{for } E_F \leq E \leq E'_F \\ \infty & \text{otherwise} \end{cases} \]
\[ R = \int_{E_V}^{E_C} R(E) = \int_{E_V}^{E_C} c_{ps} \Delta p_{s0} D_{IT}(E) dE \]

\[ \approx \int_{E_F'}^{E_F} c_{ps} \Delta p_{s0} D(E) dE \]
Surface Recombination Velocity

\[ R \approx \int_{E_F}^{E_F} c_{ps} \Delta p_{s0} D_{IT}(E) \, dE \]

\[ = c_{ps} D_{IT} \left( E_F - E'_F \right) \Delta p_{s0} \]

\[ = s_g \Delta p_{s0} \]

Surface recombination velocity
1) Nature of interface states
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