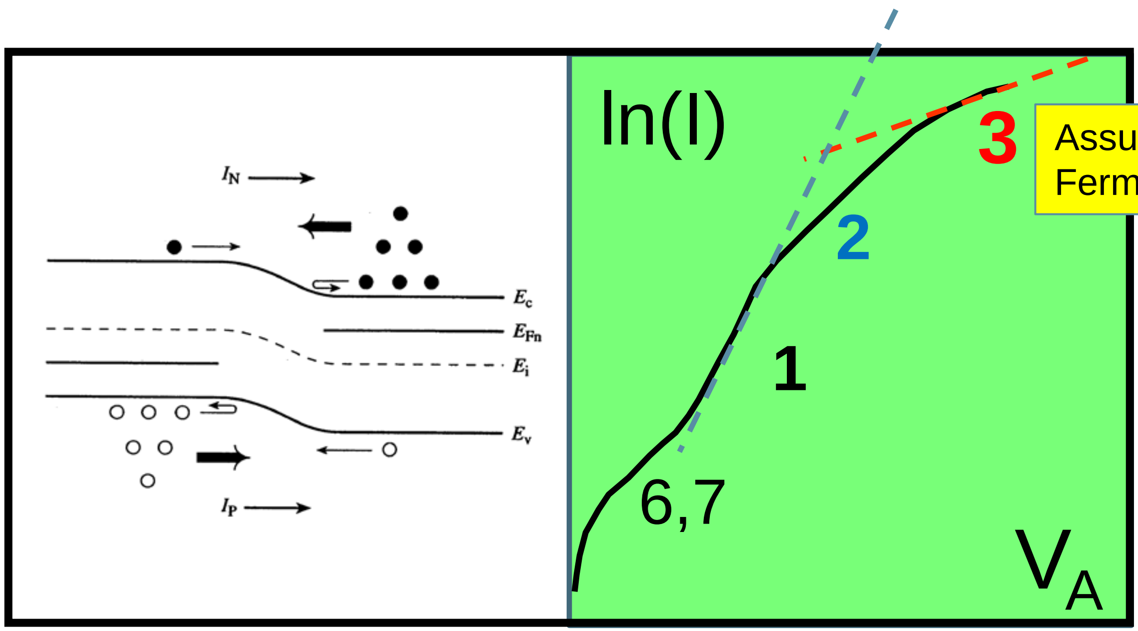
ECE606: Solid State Devices
Lecture 15
p-n diode characteristics

Gerhard Klimeck
gekco@purdue.edu

1) **Solution in the nonlinear regime**
2) I-V in the ambipolar regime
3) Tunneling and I-V characteristics
4) Non-ideal effects: Impact ionization
5) Non-ideal effects: Junction recombination
6) Conclusion

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{(qV_A-\Delta F_n-\Delta F_p)\beta} - 1\right) = I_0\left(e^{q(V_A-aJ_n-bJ_p)\beta} - 1\right)$$

Today's lecture: Nonlinear Regime (2,3)



Assumption of **flat** Quasi-Fermi levels invalid here

Flat Quasi-Fermi Level up to Junction ?

$$\mathbf{J}_N = qn\mu_N \mathcal{E} + qD_N \frac{dn}{dx}$$

$$J_n = n\mu_n \frac{dF_n}{dx} \Rightarrow \Delta F_n = \frac{J_n W_n}{\mu_n N_D} = \tilde{R} J_n$$

Drop of Quasi-Fermi level across the junction **proportional** to current!

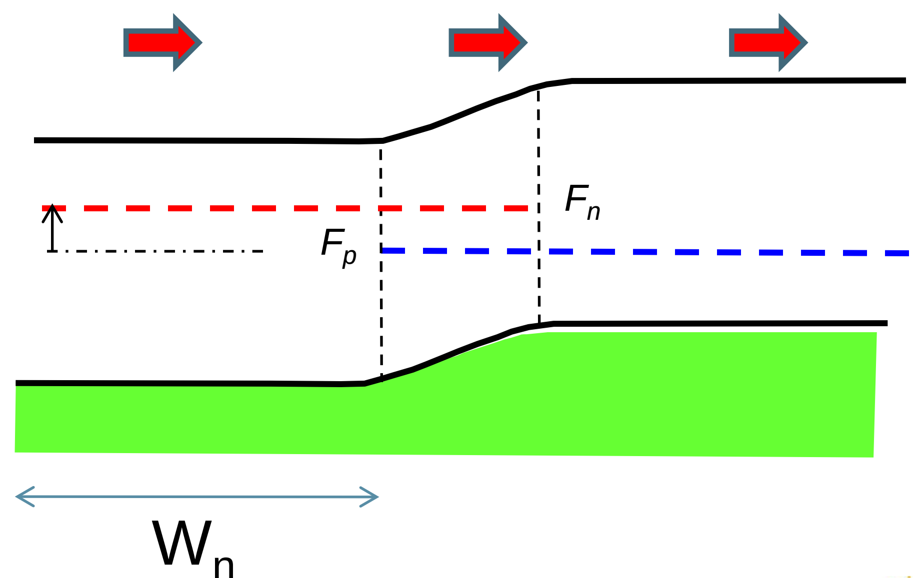
Rewrite ***n*** into non-equilibrium form, re-arrange $\mathbf{J}_n$ equation

$$n = n_i e^{\beta(F_n - E_i)} \qquad qD_N \frac{dn}{dx} = qD_N \beta \left[\frac{dF_n}{dx} - \mathcal{E}\right]\left[n_i e^{\beta(F_n - E_i)}\right]$$

New diffusion component: Plug this into original $\mathbf{J}_n$ equation

$$qD_N \frac{dn}{dx} = qD_N n\beta \left[\frac{dF_n}{dx} - \mathcal{E}\right]$$

$$= q\mu_N n \left[\frac{dF_n}{dx} - \mathcal{E}\right] \quad \because \frac{D_N}{\mu_n} = \frac{k_B T}{q}$$



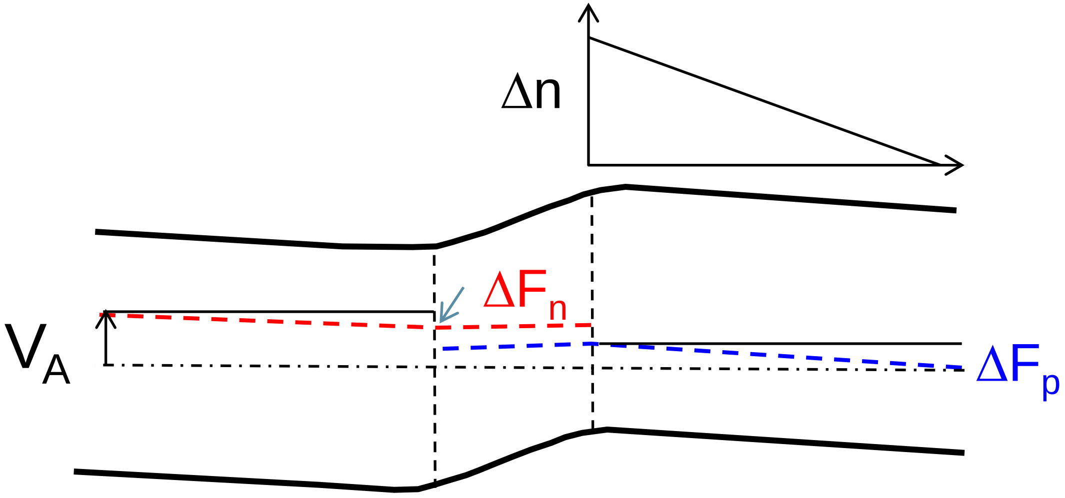
Forward Bias: Nonlinear Regime …

$$n(0^+) = \frac{n_i^2}{N_A} e^{(F_n - F_p)\beta}\Big|_{junction} = \frac{n_i^2}{N_A} e^{\left(qV_A - \Delta F_n - \Delta F_p\right)\beta} \Rightarrow \Delta n(0^+) = \frac{n_i^2}{N_A}\left(e^{\left(qV_A - \Delta F_n - \Delta F_p\right)\beta} - 1\right)$$

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{\left(qV_A - \Delta F_n - \Delta F_p\right)\beta} - 1\right)$$

$$\Delta F_n = \frac{J_n W_n}{\mu_n N_D}$$

$$\Delta F_p = \frac{J_p W_n}{\mu_n N_D}$$



Still diffusion dominated transport? Since Quasi-Fermi levels are not flat in nonlinear regime (drift), this approximation becomes worse.

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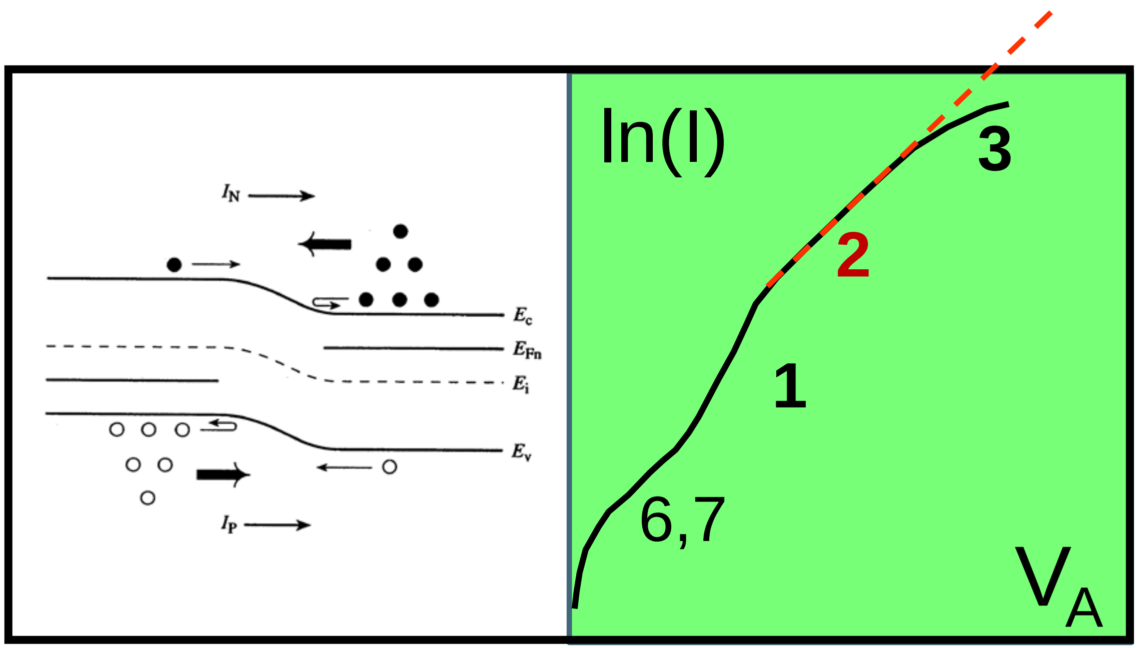
Region (2): Ambipolar Transport

$$J_T \approx -q\left[\frac{D_n}{W_p}+\frac{D_p}{W_n}\right]n_i e^{\left(qV_A-\Delta F_n-\Delta F_p\right)\beta/2}$$

$$\ln(J_T)\approx\frac{qV_A}{2k_BT}$$

Today's lecture: **Ambipolar** Transport regime (2)

Question: Where does the 2 come from?



Nonlinear Regime: Ambipolar Transport

$$np = n_i^2 e^{(F_n - F_p)\beta}$$

Here not negligibly small.
Ambipolar transport !

$$\left(\frac{n_i^2}{N_A} + \Delta n\right)(N_A + \Delta p) = n_i^2 \left(e^{q\left(V_A - \Delta F_n - \Delta F_p\right)\beta} - 1\right)$$

Excess carrier concentrations >> N_A Thus…

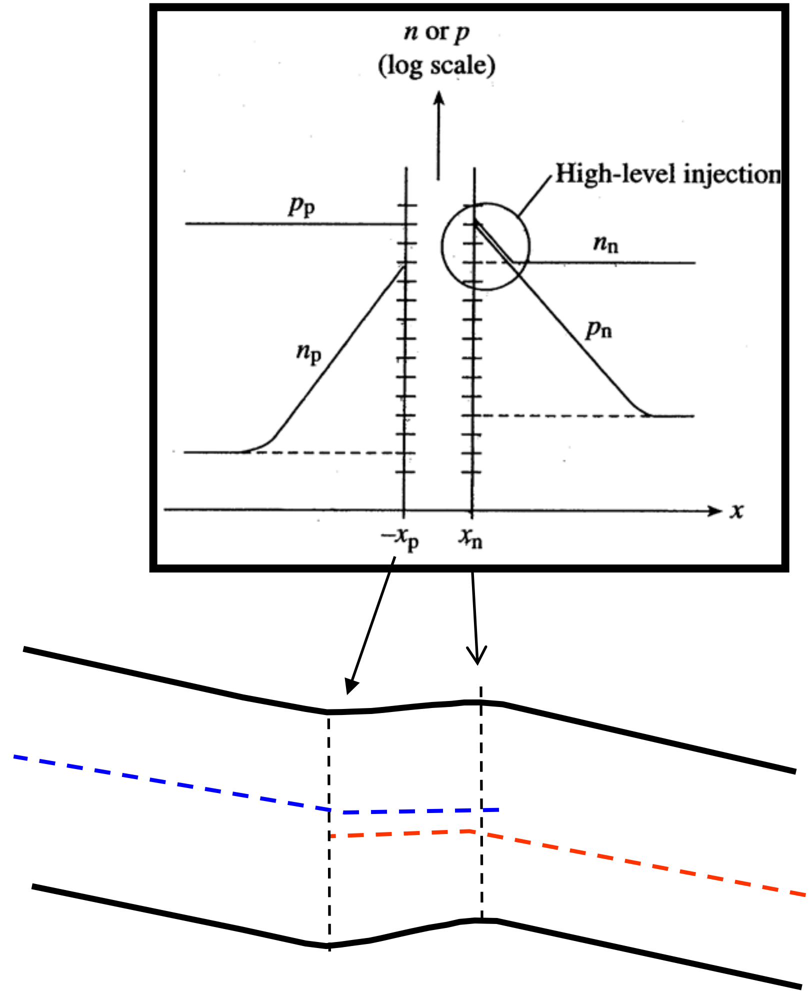
$$\Delta n \approx \Delta p = n_i \sqrt{\left(e^{q\left(V_A - \Delta F_n - \Delta F_p\right)\beta} - 1\right)}$$

$$\approx n_i\, e^{q\left(V_A - \Delta F_n - \Delta F_p\right)\beta/2}$$

Currents

$$J_n = -qD_n \frac{\Delta n}{W_p} = \frac{qD_n n_i}{W_p} e^{\left(qV_A - \Delta F_n - \Delta F_p\right)\beta/2}$$

$$J_p = -qD_p \frac{\Delta n}{W_n} = \frac{qD_p n_i}{W_n} e^{\left(qV_A - \Delta F_n - \Delta F_p\right)\beta/2}$$



Note: junction never disappears,
even for large forward bias!

Electric field is not really negligible, but we do it anyhow….

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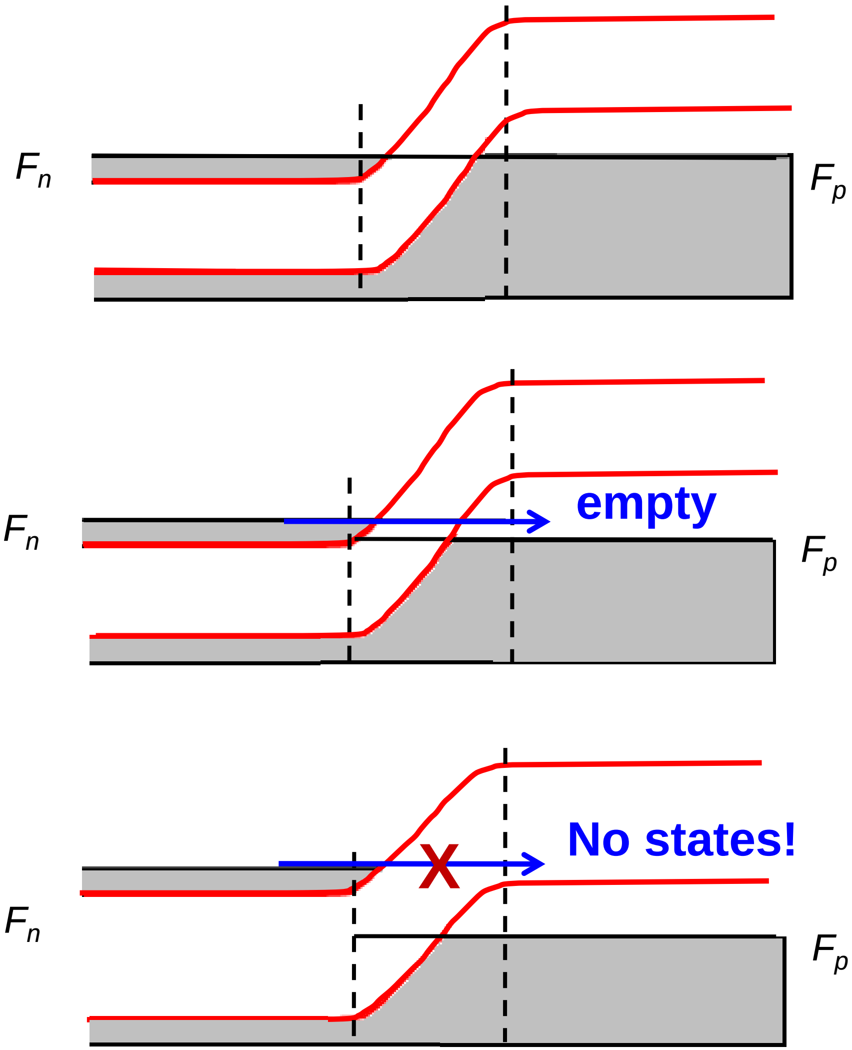
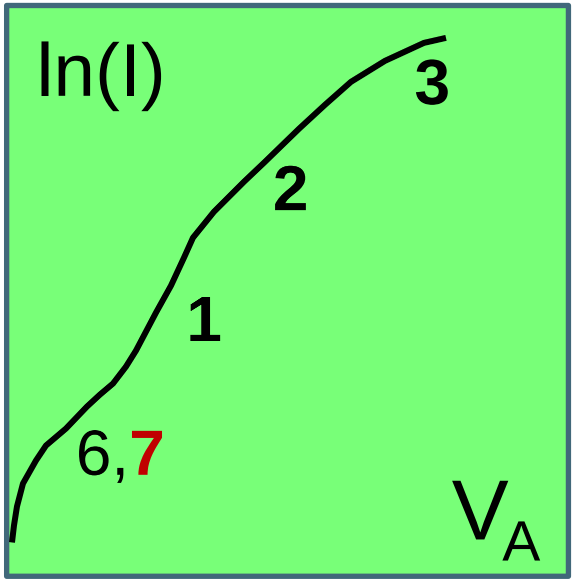
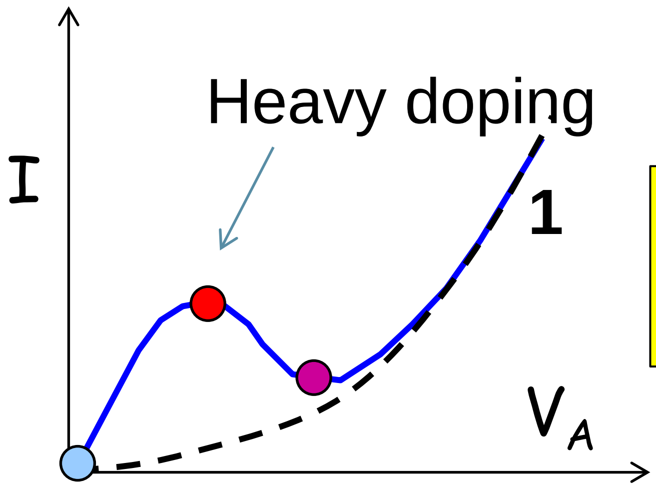
Forward Bias Nonlinearity (7): Esaki Diode

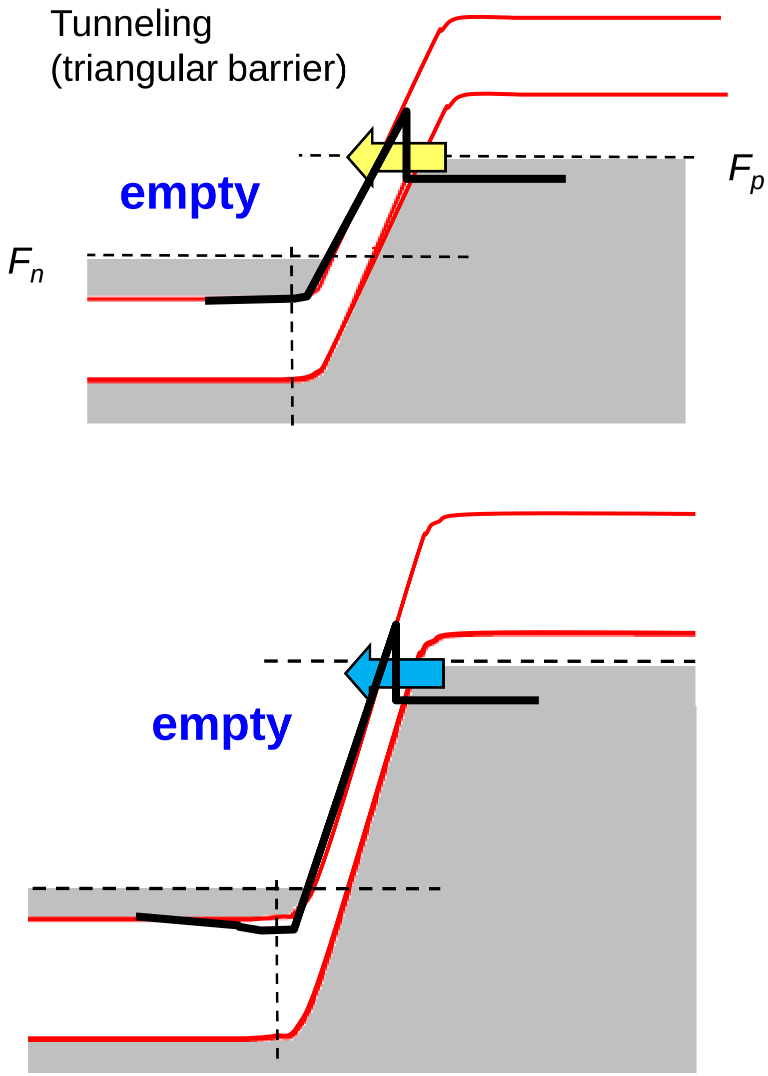
Esaki-Diode: **Heavily** doped diode

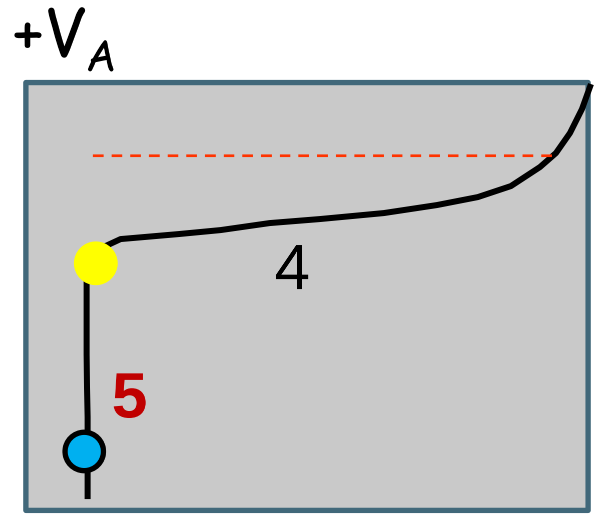






Tunneling in diodes.
Nobel Prize (Esaki)





Zener tunneling occurs in every diode. (reverse bias)

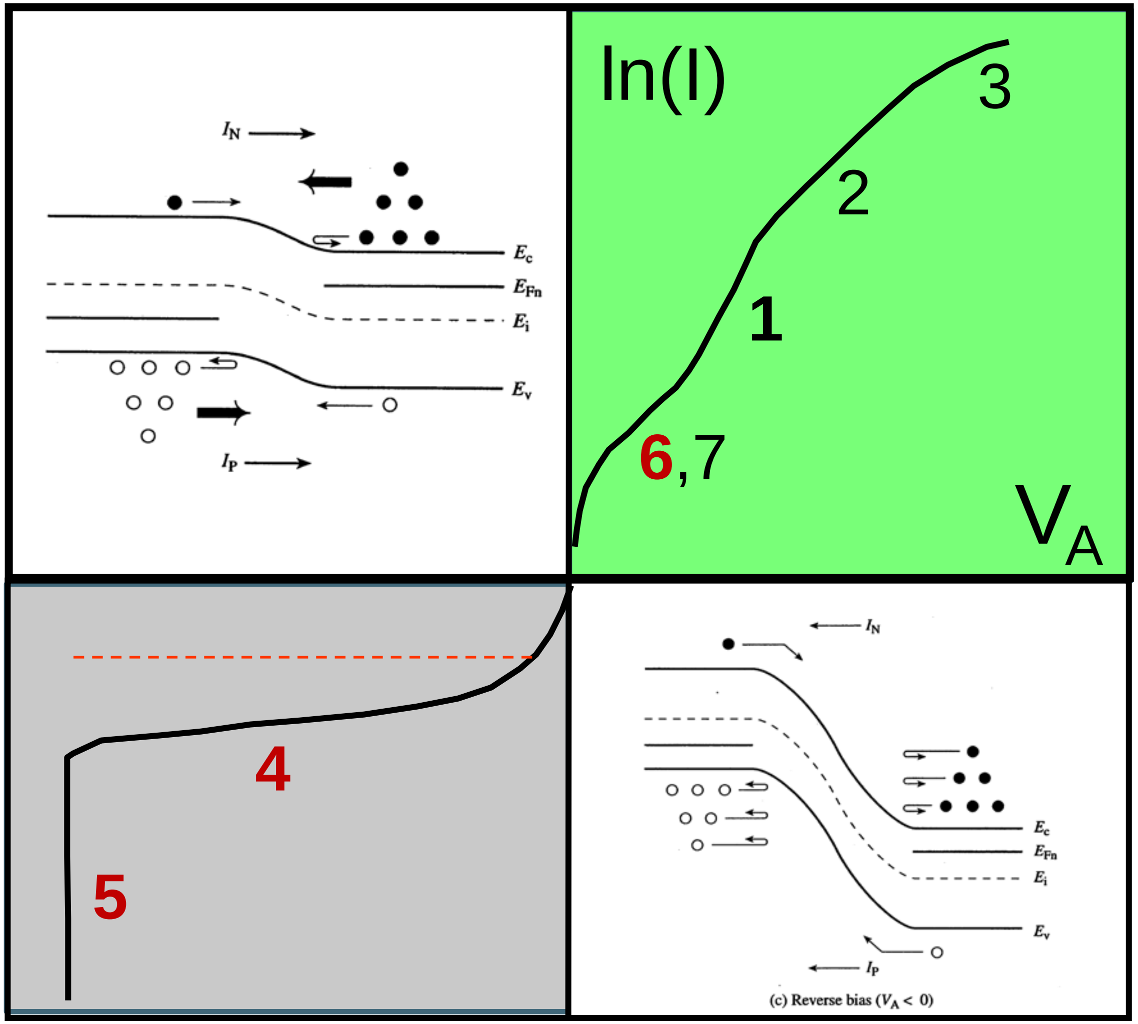
Remember: Tunneling through a triangular barrier

$$I = qpT\upsilon$$

$$T = \frac{4}{4\cosh^2 \alpha d + \left(\frac{\alpha}{k} - \frac{k}{\alpha}\right)\sinh^2 \alpha d}$$

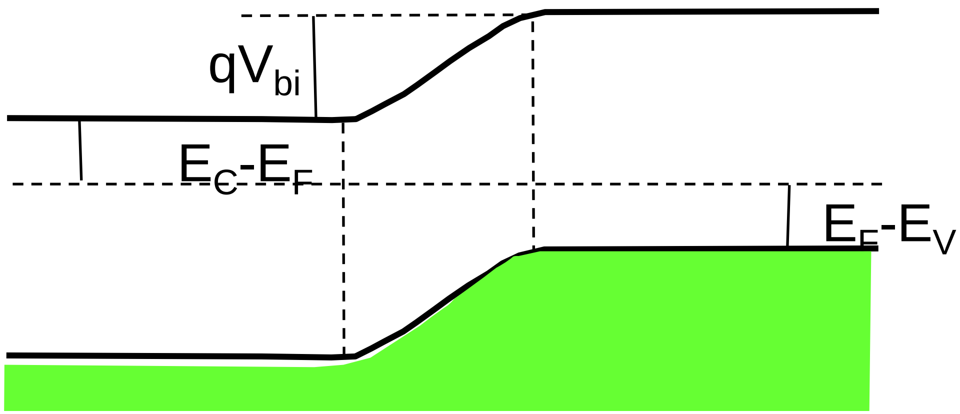
(p.49 ADF)

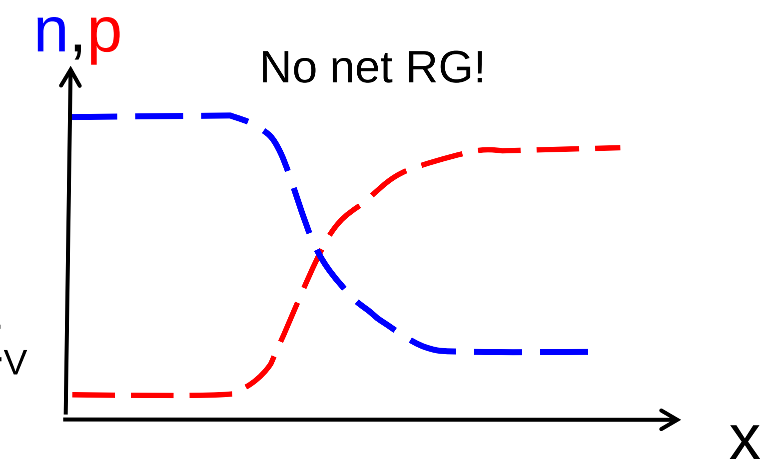
Various Regions of I-V Characteristics



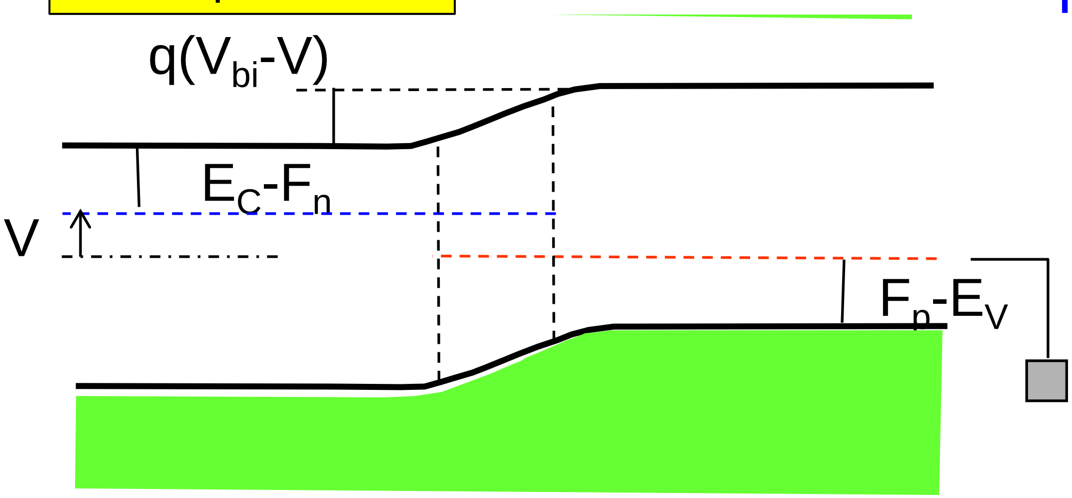
1. *Diffusion limited*
2. *Ambipolar transport*
3. *High injection*
4. ***R-G in depletion***
5. ***Breakdown***
6. ***Trap-assisted R-G***
7. *Esaki Tunneling*

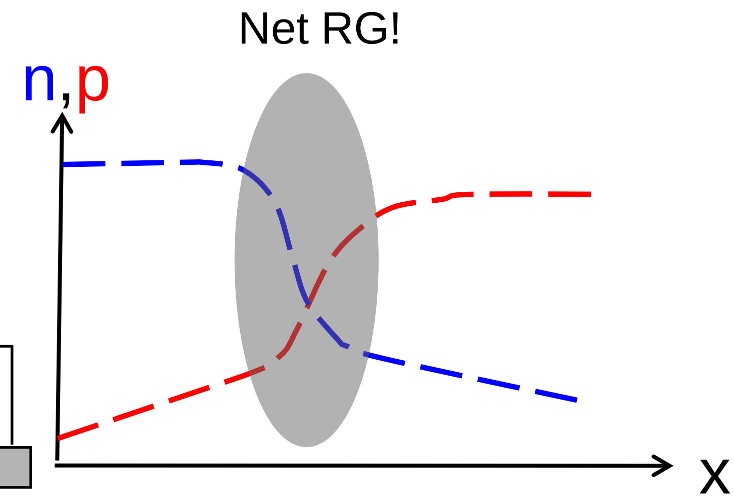
Equilibrium





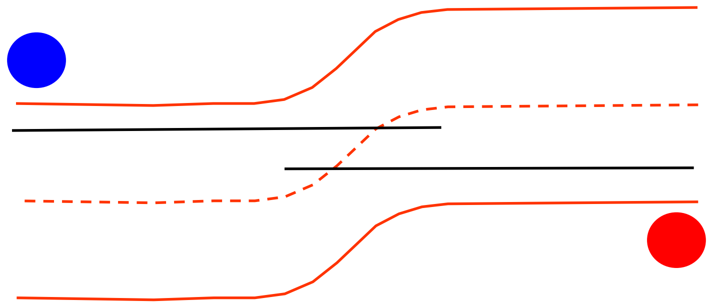
Non-Equilibrium





What is the recombination current?

$$I_R = -qA\int_0^W \frac{\partial n}{\partial t}dx$$



Shockley-Reed Hall

$$\frac{\partial n}{\partial t} = -\frac{[n(x)p(x) - n_i^{\,2}]}{\tau_p[n(x)+n_1] + \tau_n[p(x)+p_1]}$$

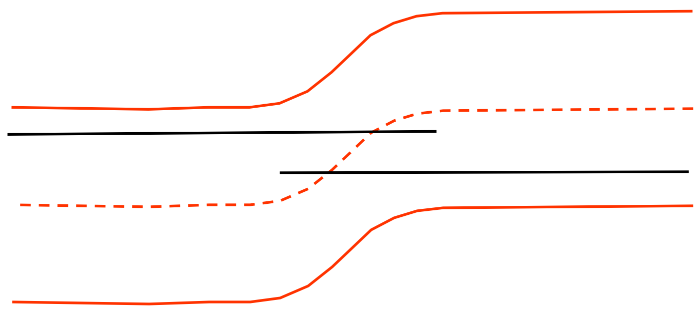
Follows from assuming midgap traps

Assume $\tau_n = \tau_p$ $\quad E_i = E_T$ $\quad n_1 = p_1 = n_i$

$$\frac{\partial n}{\partial t} = -\frac{n_i^{\,2}(e^{qV_A/kT} - 1)}{\tau[n(x)+p(x)+2n_i]}$$

Note: Do you remember this HW ?

Mass action in non-equilibrium

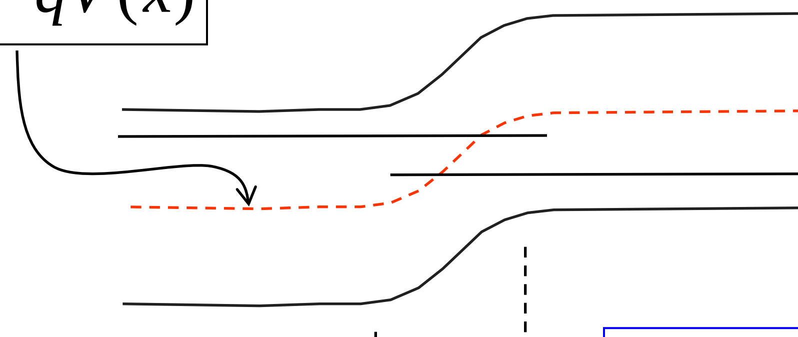


$$n(x)\,p(x) = n_i^{\,2} e^{(F_N - F_P)/kT}$$
$$= n_i^{\,2} e^{qV_A/kT}$$

For non-equilibrium at **low** current values.

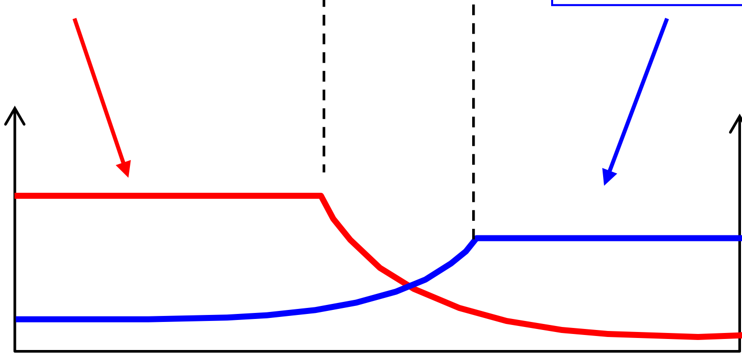
Electron/Hole Concentrations at Junction

$$E_i(x) = E_{iL} - qV(x)$$

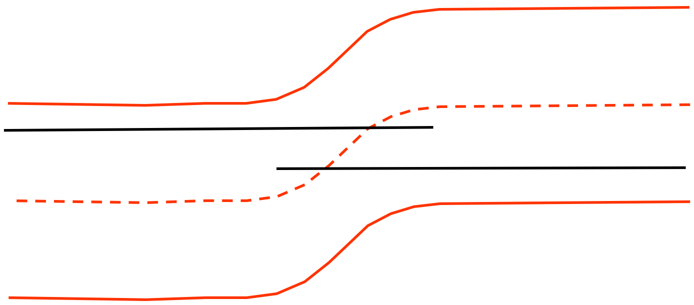


$$n(x) = n_i e^{(F_N - E_i(x))/kT}$$
$$= n_i e^{[F_N - E_{iL} + qV(x)]/kT}$$

$$p(x) = \frac{n_i^{\,2} e^{qV_A/kT}}{n_i e^{[F_N - E_{iL} + qV(x)]/kT}}$$
$$= n_i e^{-[F_N - E_{iL} + qV(x)]/kT + qV_A/kT}$$



position



$$U_{FN} = \frac{F_N - E_{iL}}{kT} \qquad U_A = \frac{V_A}{kT/q}$$

$$\frac{\partial n}{\partial t} = -\frac{n_i(e^{U_A} - 1)}{\tau[e^{U_{FN}+U} + e^{-U_{FN}-U+U_A}]}$$

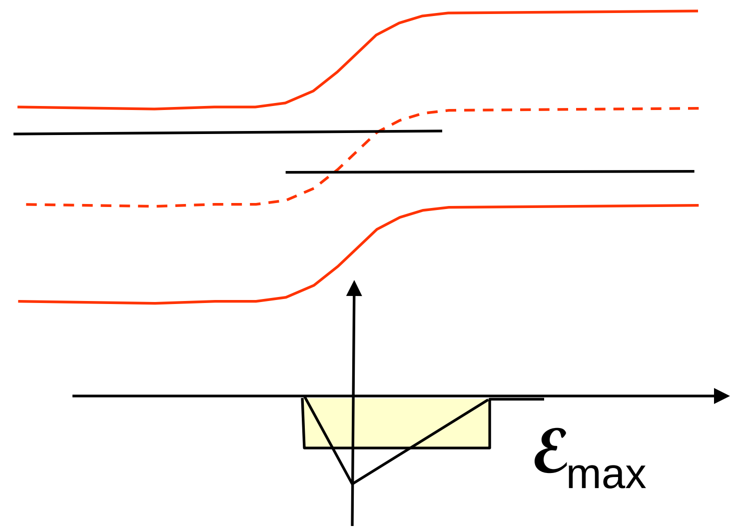
$$I_R = -qA\left(\frac{n_i}{\tau}\right) \times sinh\left(\frac{U_A}{2}\right) \times \int_0^W \frac{dx}{cosh[U_{FN} + U - U_A/2]}$$

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau}\frac{e^{U_A/2}(e^{U_A/2} - e^{-U_A/2})}{e^{U_A/2}[e^{U_{FN}+U-U_A/2} + e^{-U_{FN}-U+U_A/2}]}$$

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau}\frac{sinh(U_A/2)}{cosh[U_{FN} + U - U_A/2]}$$

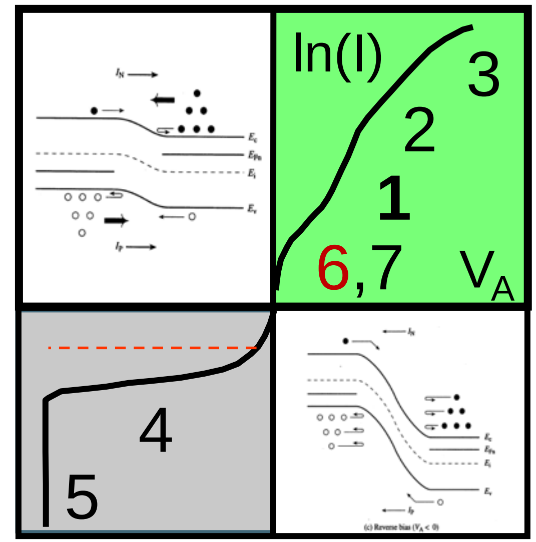
Junction Recombination in Forward Bias

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau}\frac{\sinh(U_A/2)}{\cosh[U_{FN}+U-U_A/2]}$$



$$\Rightarrow I_R \approx -qA(\frac{n_i}{\tau})\sinh(\frac{U_A}{2})\int_0^W \frac{dx}{e^{(U_{FN}+U-U_A/2)}}$$

$$\Rightarrow I_R \approx -qA(\frac{n_i}{\tau})\times \sinh(\frac{U_A}{2})\int_0^W \frac{dx}{e^{-(\mathcal{E}_{max}x)/(kT/2q)}}$$

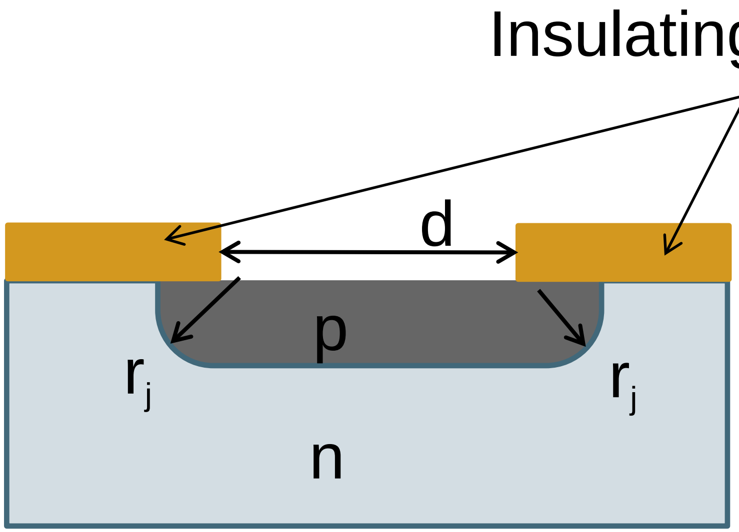


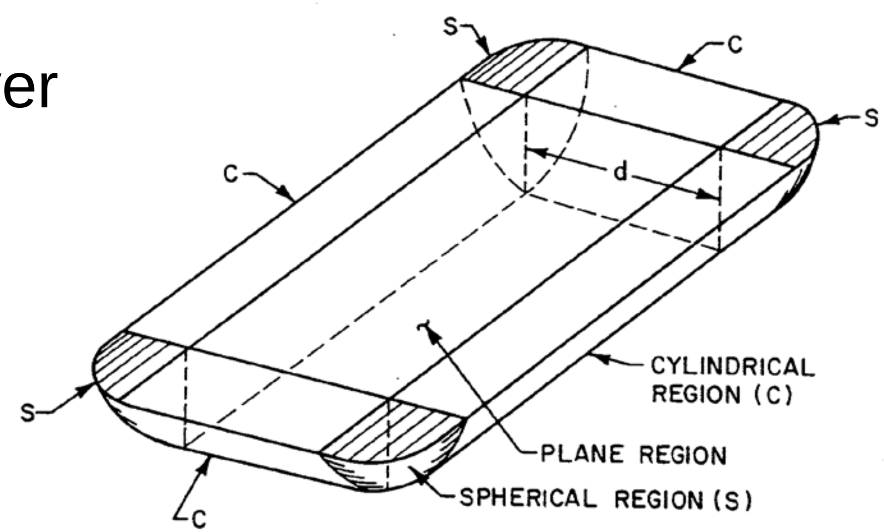
$$\Rightarrow I_{Dep} = -qA\left[\frac{kT}{2q\mathcal{E}_{max}}\right]\left[\frac{n_i}{\tau}e^{qV_A/2kT}\right]$$

Effective width

Excess Carrier at mid-junction

Junction Leakage in Practice



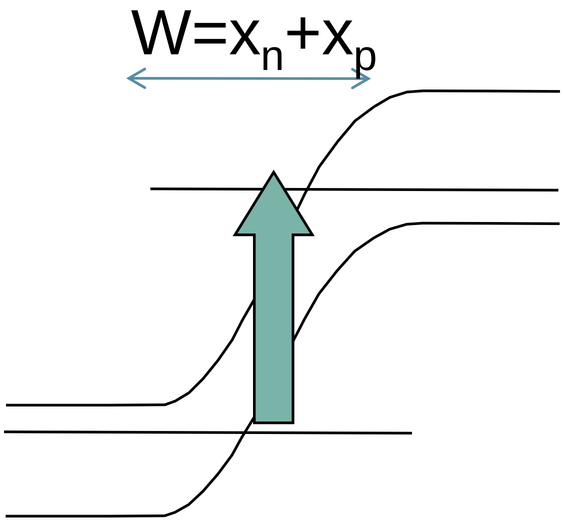


Junction Design Considerations
Electric field stronger at corners, sharp edges. → increased recombination!

Junction Recombination in Reverse Bias

$$\frac{\partial n}{\partial t} = -\frac{n_i}{2\tau}$$

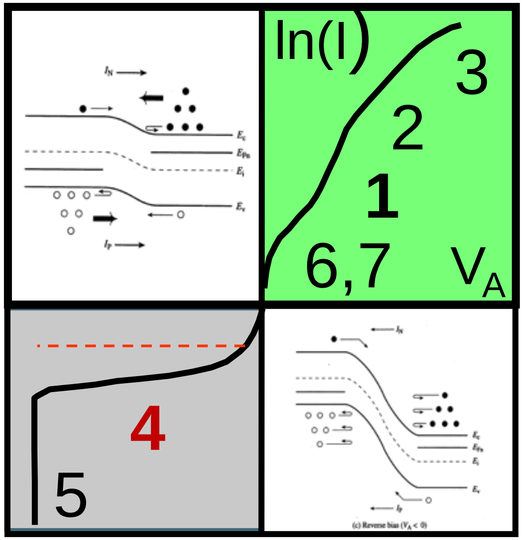
(Recombination in depletion region)



Integrate…

$$I_R \approx -qA\int_0^W \left(\frac{n_i}{2\tau}\right) dx$$

$$= -qA\frac{n_i W}{2\tau} \propto \sqrt{V_{bi} - V_A}$$



Outline

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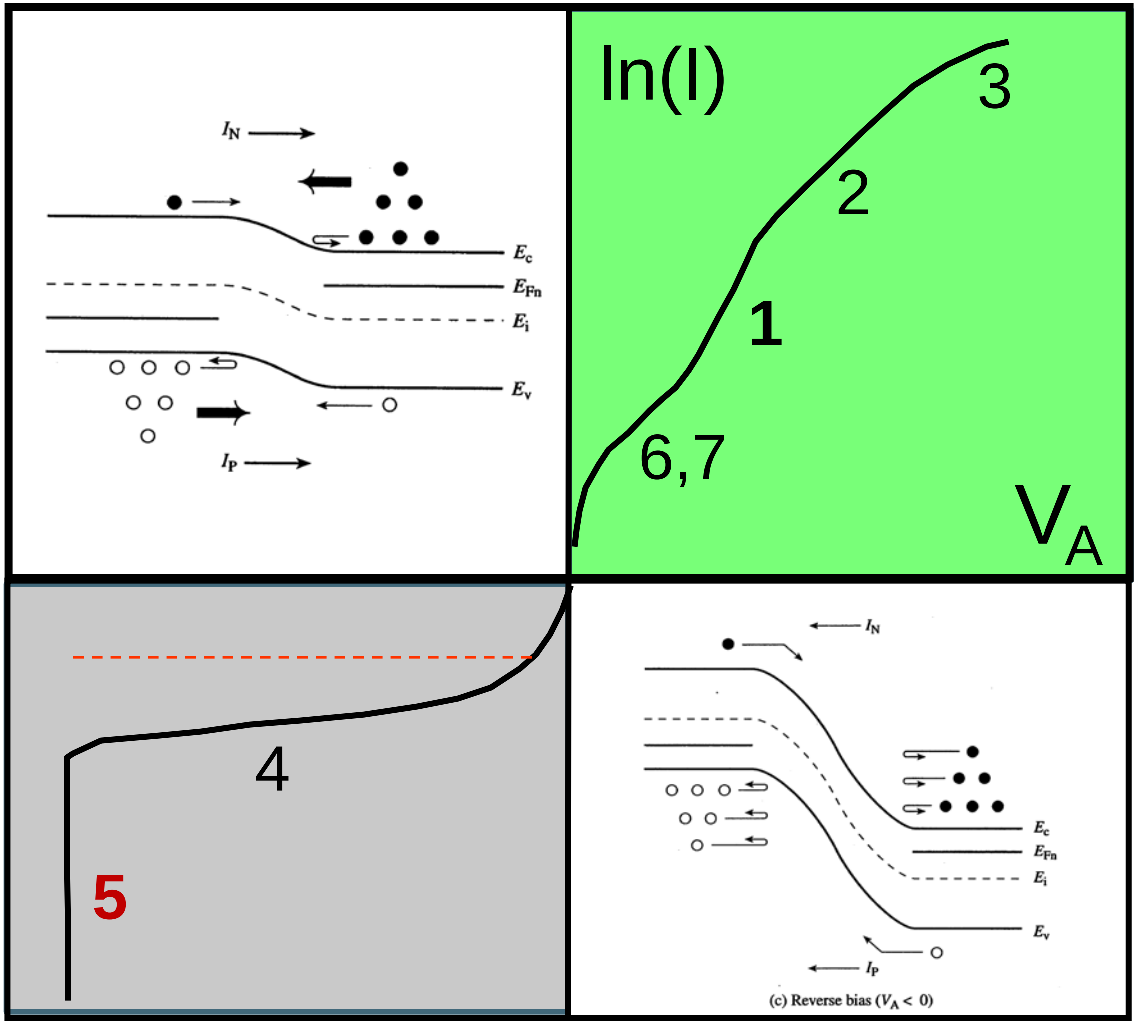
Avalanche Breakdown



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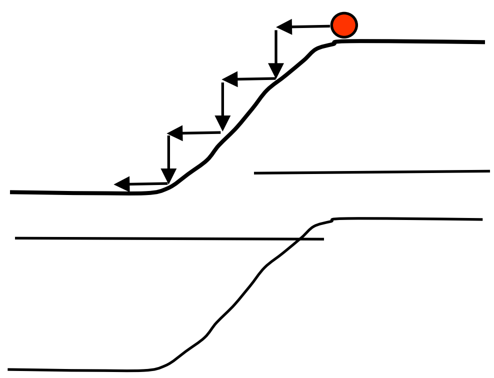
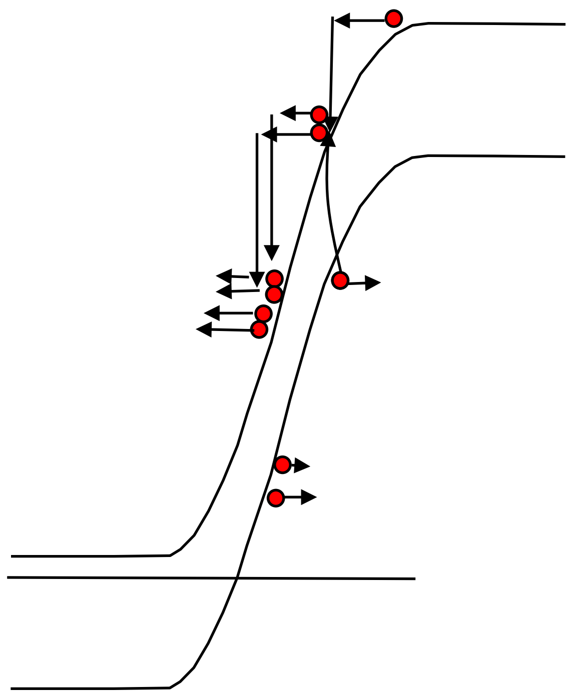
Nonlinearity due to Impact-Ionization

Reverse Bias



High Reverse Bias



Exponential current growth
(Impact Ionization or Inverse Auger process)

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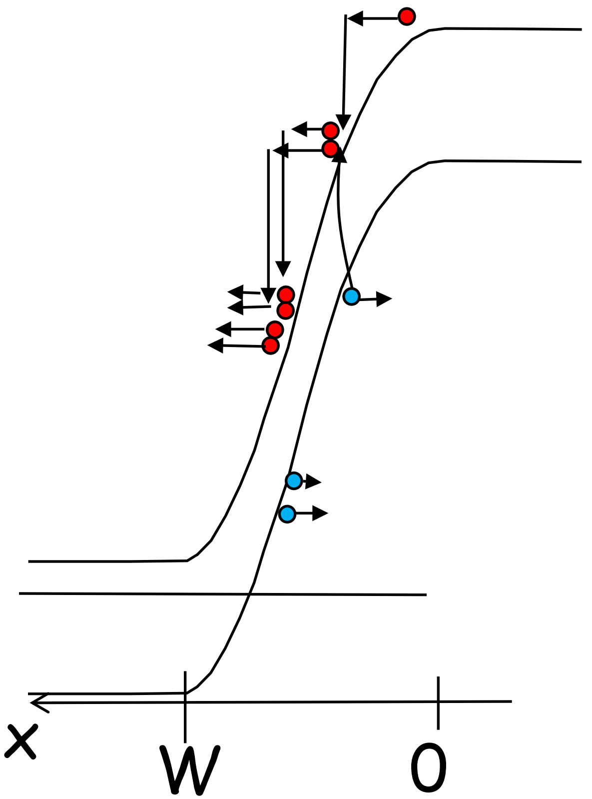
Impact-ionization and Flux Conservation



$$\boxed{I_n(x+dx) = I_n(x) + \alpha_n I_n(x)dx + \alpha_p I_p(x)dx}$$

Impact Ionization probabilities

$$\frac{I_n(x+d\ )x - I_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x)$$

$$\Rightarrow \frac{dI_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x)$$

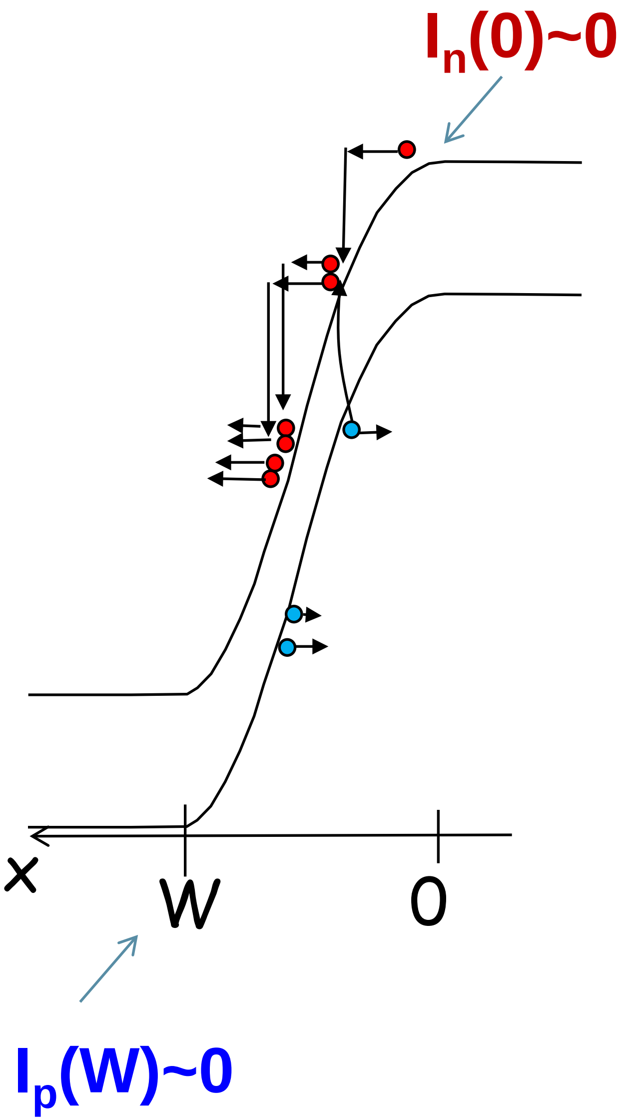
Steady state: Define
$I_T = I_N + I_P$ (total current)

$$\frac{dI_n(x)}{dx} = \alpha_p \left[I_T - I_n(x) \right] + \alpha_n I_n(x)$$

$$\frac{dI_n(x)}{dx} - \left(\alpha_n - \alpha_p\right) I_n(x) = \alpha_p I_T$$

Differential equation

Impact-ionization



Solution form of differential equation

$$\frac{I_n(W)}{I_T} = \frac{\int_0^W \alpha_p e^{-\int_0^x (\alpha_n-\alpha_p)dx'} dx + \frac{I_n(0)}{I_T}}{1+\int_0^W (\alpha_p-\alpha_n) e^{-\int_0^x (\alpha_n-\alpha_p)dx'} dx}$$

Reverse diffusion current

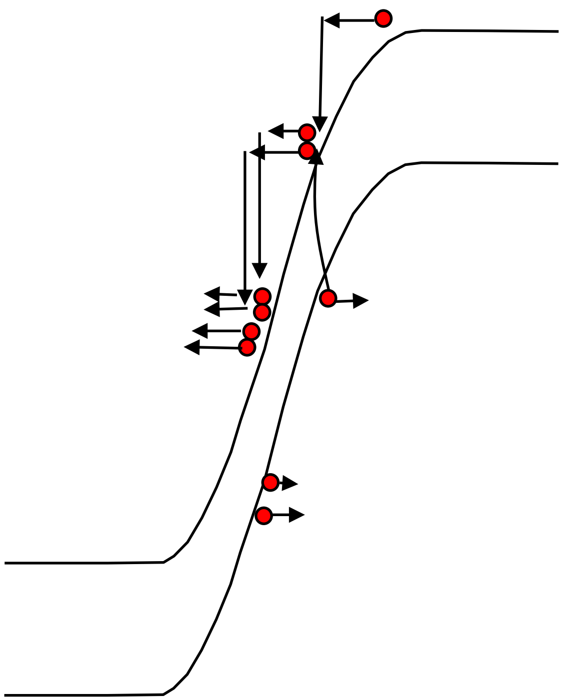
$$I_p(W) + I_n(W) = I_T \Rightarrow I_n(W) \approx I_T$$

$$\frac{I_n(0)}{I_T} \equiv \frac{1}{M_p}$$ Multiplication Factor

At x=W, I_N has grown exponentially, and I_P is now negligible.

$$\left(1-\frac{1}{M_p}\right) \approx 1 = \int_0^W \alpha_p e^{-\int_0^x (\alpha_p-\alpha_n)dx'} dx$$

Impact-ionization



Simplify further...

$$\int_0^W \alpha_p e^{-\int_0^x (\alpha_p - \alpha_n) dx'} dx \approx 1$$

$$\alpha_p = \alpha_n \Rightarrow \alpha_p W = 1$$

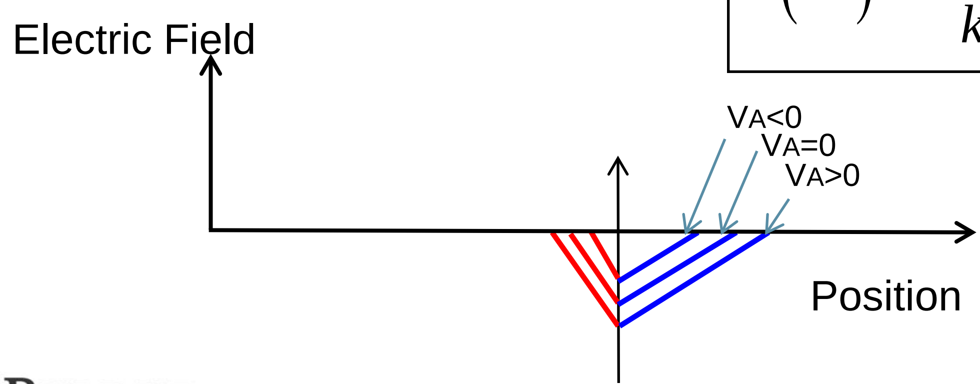
Assume: Significant impact ionization

$$\alpha_p = A_0 e^{-B/\mathcal{E}}$$

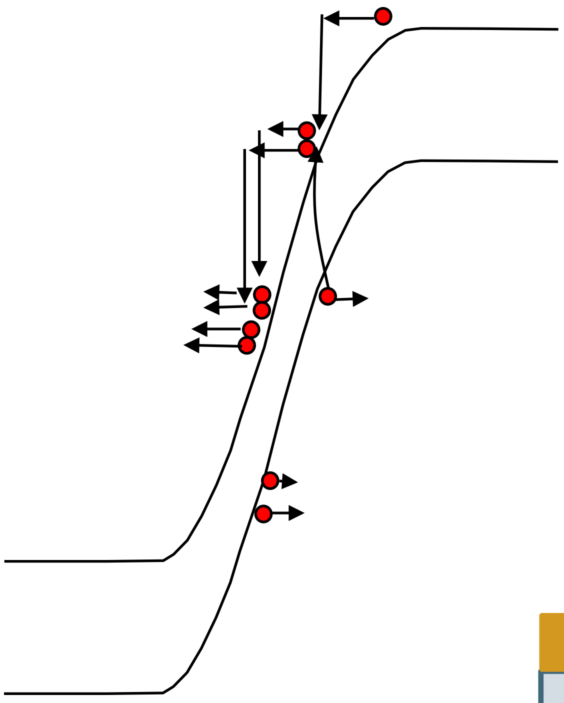
from experiment and theory

Breakdown-Field

$$\mathcal{E}(0^-) = \frac{q N_D x_n}{k_s \varepsilon_0} = \left[\frac{2q}{k_s \varepsilon_0} \frac{N_D N_A}{N_D + N_A}(V_{bi} - V_A)\right]^{1/2}$$

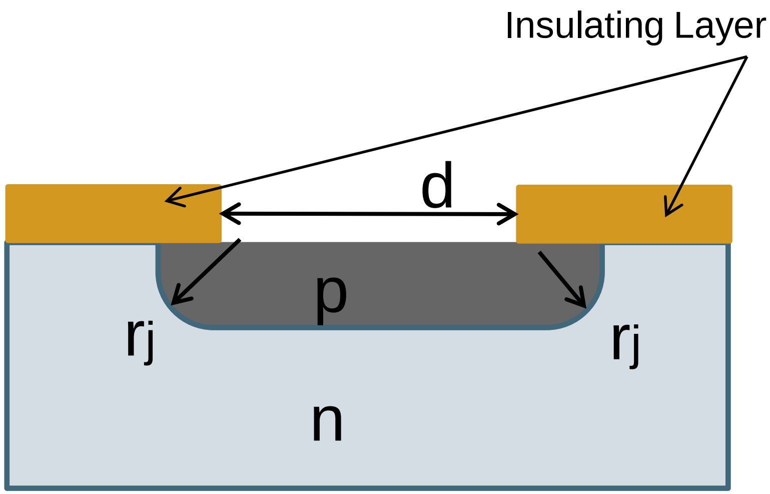


Impact-ionization: In Practice



Photon Detector

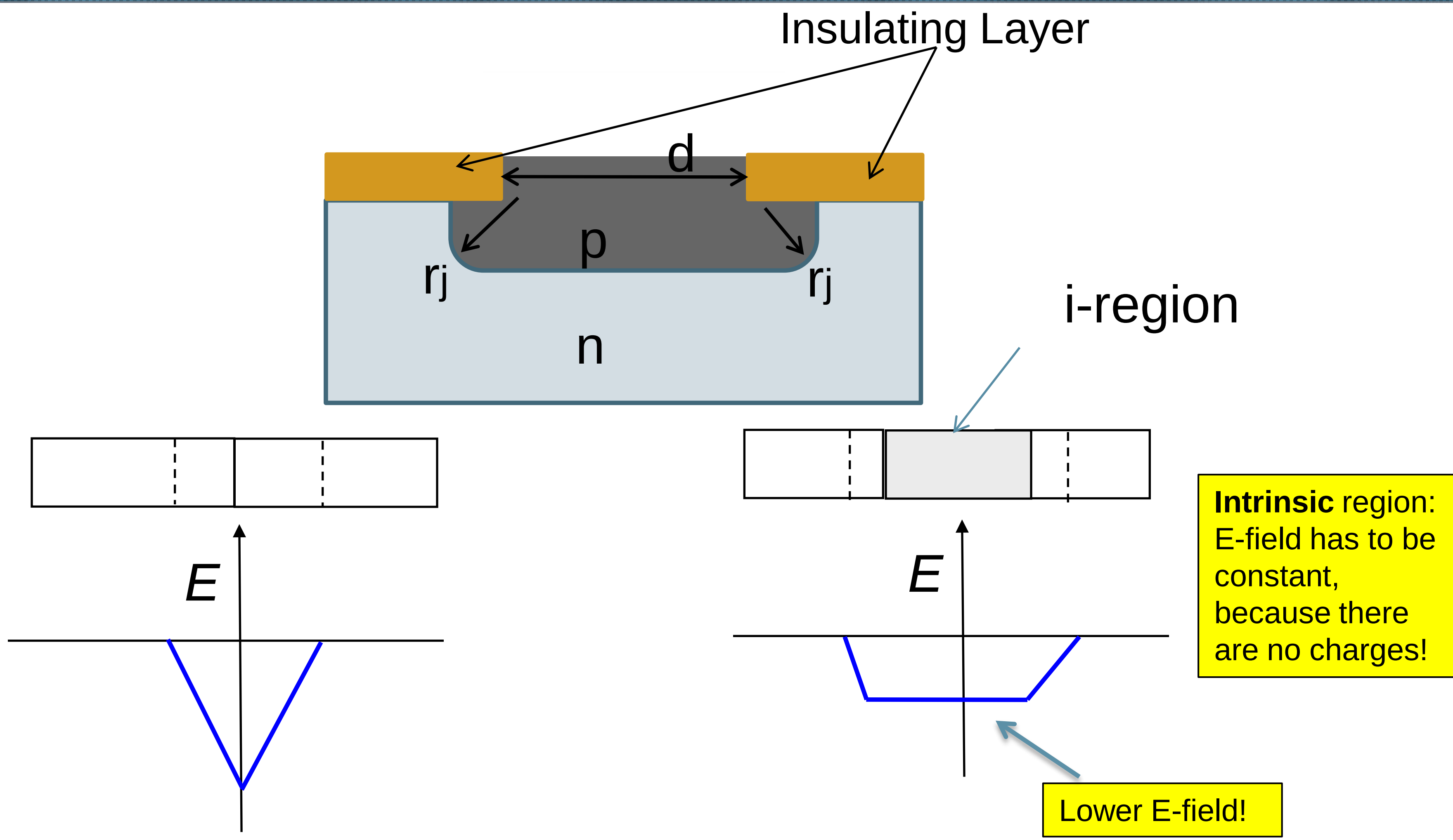
Good ….



High E-fields at junction corners → Breakdown

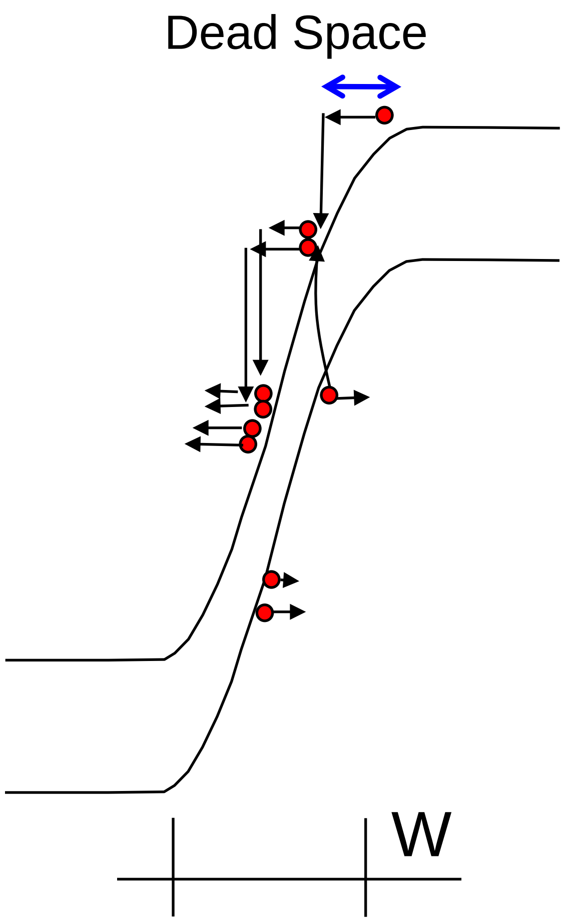
Bad….

Junction Engineering



Reduced field for p-i-n junction, because V_{bi} (area under the curve) must be the same.

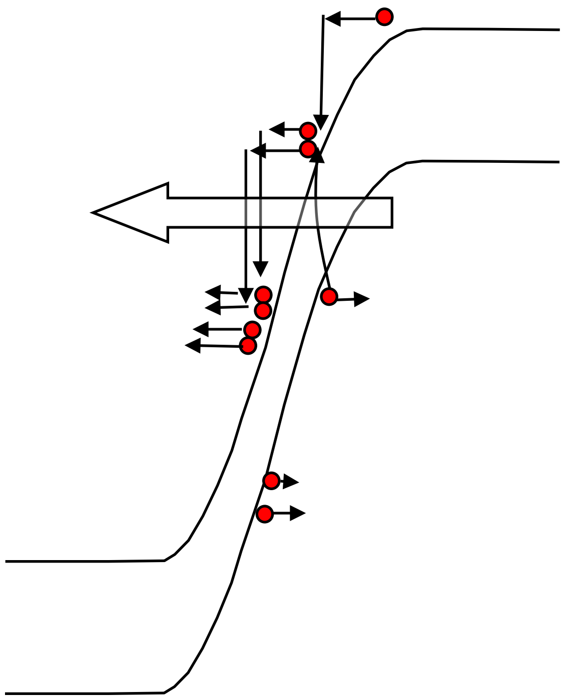
Modern Considerations: Dead Space

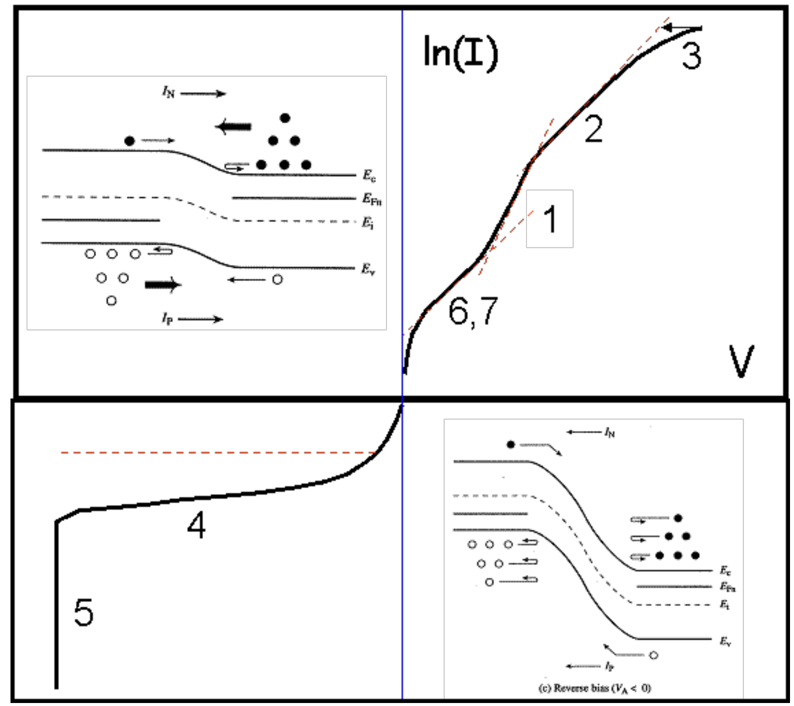


Dead Space:
Space you need before an electron can impact ionize.

For very small (ballistic) junctions, electrons can cross the junction without inducing impact Ionization.
(Dead space too small)

Zener Breakdown vs. Impact Ionization





How do you differentiate between Zener tunneling and impact-ionization?

Conclusion

1) Junction recombination is often used as a diagnostic tool for process maturity. Defects in junction arises from misplaced donor impurities, not necessary from deep-trap impurities.

2) Impact ionization plays an important role in wide variety of devices (e.g. avalanche photo-diodes).

3) In the next class, we will discuss AC response of p-n junction diodes.