Computational Electronics

Mobility Modeling

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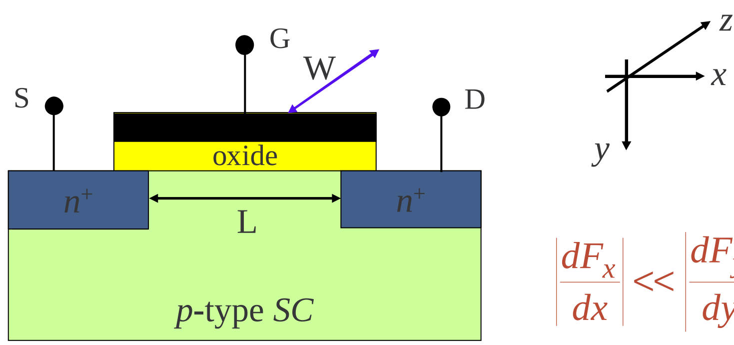
Gradual Channel Approximation

- This model is due to Shockley.
- Assumption: The electric field variation in the direction parallel to the SC/oxide interface is much smaller than the electric field variation in the direction perpendicular to the interface.



$$\left|\frac{dF_x}{dx}\right| << \left|\frac{dF_y}{dy}\right|$$

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- Recall the expressions for the threshold voltage for real MOS capacitor:

$$\text{Gate voltage}: \quad V_T = 2\varphi_F + \frac{1}{C_{ox}}\sqrt{2qN_A k_s \varepsilon_0 (2\varphi_F)} + V_{FB}$$

$$\text{Flat-band voltage}: \quad V_{FB} = \frac{1}{q}\phi_{MS} + \frac{Q_{it}}{C_{ox}} + \frac{Q_f}{C_{ox}} + \gamma_{ot}\frac{Q_{ot}}{C_{ox}} + \gamma_m \frac{Q_m}{C_{ox}}$$

- Beyond the point that determines the onset of strong inversi-on ($\varphi_s=2\varphi_F$), any excess charge on the gate balanced with excess charge in the semiconductor, is given by:

$$Q_G = -(Q_B + Q_N) = C_{tot}(V_G - V_T) \rightarrow Q_N \approx -C_{ox}(V_G - V_T) - Q_B$$
$$Q_B = Q_B(\varphi_s) - Q_B(2\varphi_F)$$

- Based on how we consider Q_B, we have:
  - (A) Square-law theory: $Q_B = 0$
  - (B) Bulk-charge theory: $Q_B \neq 0$

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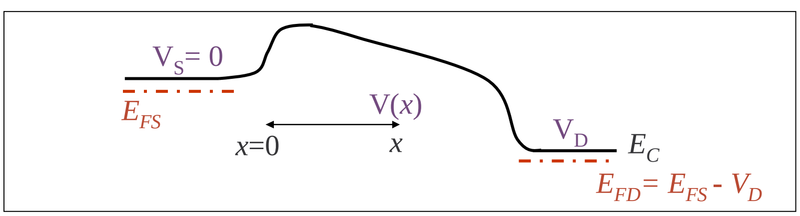
Square Law Theory

- The charge on the gate is completely balanced by $Q_N(x)$, i.e:

$$Q_N(x) \approx -C_{tot}[V_G - V_T - V(x)]$$



- Total current density in the channel:

$$J_n = qn\mu_n F(x) + \underbrace{qD_n \frac{dn}{dx}}_{\text{negligible}} \approx -qn\mu_n \frac{dV}{dx}$$

Note: Total current density approximately equal to the electron current density (unipolar device).

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- Integrating the current density, we obtain drain current I_D:

$$I_D = -\int_0^W dz \int_0^{y_c(x)} dy\left[-qn(x,y)\mu_n(x,y)\frac{dV}{dx}\right]$$

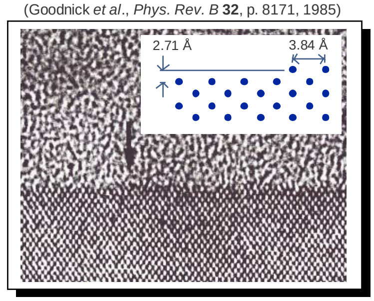
$$= W\frac{dV}{dx}\underbrace{\int_0^{y_c(x)} qn(x,y)\mu_n(x,y)dy}_{-Q_N(x)\mu_{eff}}$$

$$\approx -Q_N(x)\mu_{eff}W\frac{dV}{dx}$$

$$\approx C_{ox}W\mu_{eff}\left[V_G - V_T - V(x)\right]\frac{dV}{dx}$$

Effective electron mobility, in which interface-roughness is taken into account.

High-resolution transmission electron micrograph of the interface between Si and SiO_2
(Goodnick *et al*., *Phys. Rev. B* **32**, p. 8171, 1985)



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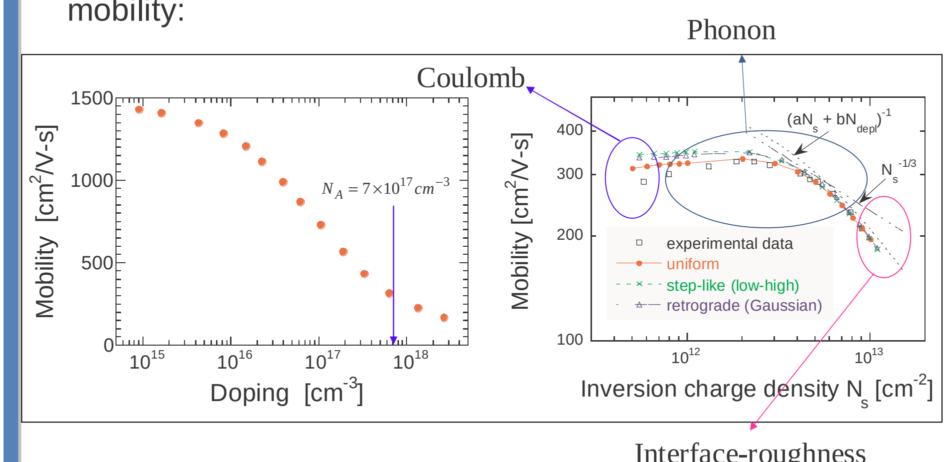
- The role of interface-roughness on the low-field electron mobility:

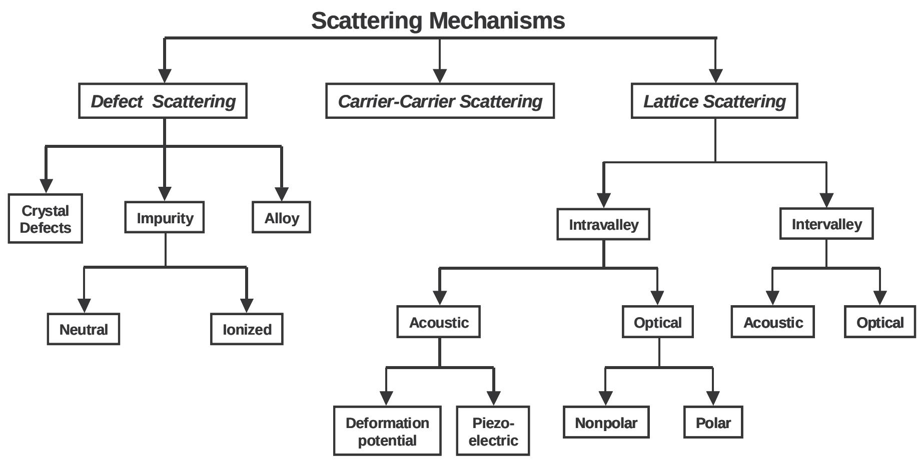


Bulk samples

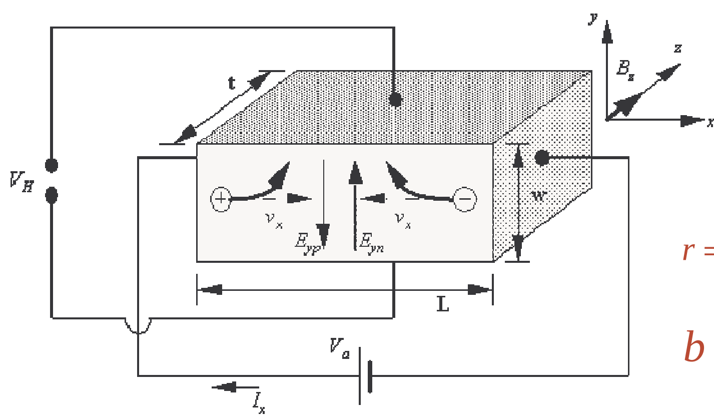
Si inversion layers

Mobility Scattering Mechanisms

Summarized below are the various scattering mechanisms that



Mobility Measurement



$$r = \frac{\langle \tau^2 \rangle}{\langle \tau \rangle^2}$$

$$b = \mu_e / \mu_h$$

$$R_H = \frac{r_h p - r_e b^2 n}{e(p + bn)^2} \qquad \mu_H = r\mu_{eff}$$ Hall Mobility

MOSFET Mobility Measurement

Effective mobility

$$\mu_{eff} \approx \frac{Lg_D}{ZC_{ox}(V_{GS} - V_T)} \qquad g_D = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS}=const.}$$

Field-effect mobility

$$\mu_{FE} = \frac{Lg_m}{ZC_{ox}V_{DS}} \qquad g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS}=const.}$$

Saturation Mobility

$$\mu_{sat} = \frac{2Lm^2}{BZC_{ox}}$$

Mobility Modeling

Mobility modeling is normally divided into:

- Low-field mobility models (bulk materials and inversion layers)
- High-field mobility models

Bulk mobility:

1. Characterization of μ_0 as a function of doping and lattice scattering
2. Characterization of v_{sat} as a function of lattice temperature
3. Describing the transition between the low-field and the saturation velocity region

Inversion layers:

1. Characterization of surface-roughness scattering
2. Description of the carrier-carrier scattering
3. Quantum-mechanical size-quantization effect

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(A) Low-field models for bulk materials

Phonon scattering:

- Simple power-law dependence of the temperature
- Sah et al. model:
  acoustic + optical and intervalley phonons combined via Mathiessen's rule

Ionized impurity scattering:

- Conwell-Weiskopf model
- Brooks-Herring model

Combined phonon and ionized impurity scattering:

- Dorkel and Leturg model:
  temperature-dependent phonon scattering + ionized impurity scattering + carrier-carrier interactions
- Caughey and Thomas model:
  temperature independent phonon scattering + ionized impurity scattering

Mobility Modeling, …

- Sharfetter-Gummel model:
  phonon scattering + ionized impurity scattering (parameterized expression – does not use the Mathiessen's rule)
- Arora model:
  similar to Caughey and Thomas, but with temperature dependent phonon scattering

Carrier-carrier scattering

- modified Dorkel and Leturg model

Neutral impurity scattering:

- Li and Thurber model:
  mobility component due to neutral impurity scattering is combined with the mobility due to lattice, ionized impurity and carrier-carrier scattering via the Mathiessen's rule

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(B) Field-dependent mobility

The field-dependent mobility model provides smooth transition between low-field and high-field behavior

$$\mu(E) = \frac{\mu_0}{\left[1 + \left(\frac{\mu_0 E}{v_{sat}}\right)^{\beta}\right]^{1/\beta}}$$

β = 1 for electrons
β = 2 for holes

v_{sat} is modeled as a temperature-dependent quantity:

$$v_{sat}(T) = \frac{2.4\times 10^7}{1 + 0.8\exp\left(\frac{T_L}{600}\right)} \text{ cm/s}$$

Mobility Modeling, …

(C) Inversion layer mobility models

- CVT model:
  combines acoustic phonon, non-polar optical phonon and surface-roughness scattering (as an inverse square dependence of the perpendicular electric field) via Mathiessen's rule
- Yamaguchi model:
  - low-field part combines lattice, ionized impurity and surface-roughness scattering
  - there is also a parametric dependence on the in-plane field (high-field component)
- Shirahata model:
  - uses Klaassen's low-field mobility model
  - takes into account screening effects into the inversion layer
  - has improved perpendicular field dependence for thin gate oxides
- Tasch model:
  the best model for modeling the mobility in MOS inversion layers; uses universal mobility behavior