Computational Electronics

Mobility Modeling

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- This model is due to Shockley.
- Assumption: The electric field variation in the direction parallel to the SC/oxide interface is much smaller than the electric field variation in the direction perpendicular to the interface.

\[
\frac{dF_x}{dx} \ll \frac{dF_y}{dy}
\]
• Recall the expressions for the threshold voltage for real MOS capacitor:

\[
V_T = 2\phi_F + \frac{1}{C_{ox}} \sqrt{2qN_A k_s e_0 (2\phi_F)} + V_{FB}
\]

Flat–band voltage:
\[
V_{FB} = \frac{1}{q} \phi_{MS} + \frac{Q_{it}}{C_{ox}} + \frac{Q_f}{C_{ox}} + \gamma_{ot} \frac{Q_{ot}}{C_{ox}} + \gamma_m \frac{Q_m}{C_{ox}}
\]

• Beyond the point that determines the onset of strong inversion (\(\phi_s = 2\phi_F\)), any excess charge on the gate balanced with excess charge in the semiconductor, is given by:

\[
Q_G = -(Q_B + Q_N) = C_{tot} (V_G - V_T) \rightarrow Q_N \approx -C_{ox} (V_G - V_T) - Q_B
\]

\[
Q_B = Q_B(\phi_s) - Q_B(2\phi_F)
\]

• Based on how we consider \(Q_B\) we have:
  
  (A) Square-law theory: \(Q_B = 0\)
  
  (B) Bulk-charge theory: \(Q_B \neq 0\)

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**Square Law Theory**

• The charge on the gate is completely balanced by \(Q_N(x)\), i.e:

\[
Q_N(x) = -C_{tot} [V_G - V_T - V(x)]
\]

\[
V_s = 0
\]

\[
E_{FS} \quad E_{FD} = E_{FS} - V_D
\]

\[
x = 0 \quad V(x)
\]

Total current density in the channel:

\[
J_n = qn\mu_n F(x) + qD_n \frac{dn}{dx} \approx -qn\mu_n \frac{dV}{dx} \quad \text{negligible}
\]

Note: Total current density approximately equal to the electron current density (unipolar device).
Integrating the current density, we obtain drain current $I_D$:

$$I_D = -W \int_0^y \int_0^x \left[ -q n(x, y) \mu_n(x, y) \frac{dV}{dx} \right] dx dy$$

$$= W \frac{dV}{dx} \left[ q n(x, y) \mu_n(x, y) dy - Q_N(y) \mu_{eff} \right]$$

$$\approx -Q_N(x) \mu_{eff} W \frac{dV}{dx}$$

$$\approx C_{ox} W \mu_{eff} \left[ V_G - V_T - V(x) \right] \frac{dV}{dx}$$

Effective electron mobility, in which interface-roughness is taken into account.

The role of interface-roughness on the low-field electron mobility:

- **Coulomb**
  - $N_s = 7 \times 10^{17} \text{cm}^{-3}$

- **Phonon**

- **Interface-roughness**
  - $\sigma N_s^{-1/3}$

- **Bulk samples**
  - $10^{13} \leq N_s \leq 10^{17}$

- **Si inversion layers**
  - $10^{15} \leq N_s \leq 10^{17}$

- **Inversion charge density $N_s$ [cm$^{-2}$]**
  - $10^{12} \leq N_s \leq 10^{17}$

- **Mobility [cm$^2$/V-s]**
  - $100 \leq \mu_0 \leq 1500$

- **Doping [cm$^{-3}$]**
  - $10^{13} \leq N_s \leq 10^{17}$

- **Experimental data**
  - **uniform**
  - **step-like (low-high)**
  - **retrograde (Gaussian)**

- **Bulk samples**
  - $N_s = (aN_s + bN_{depl})^{-1}$

- **Si inversion layers**
  - $N_s^{-1/3}$

- **Mobility [cm$^2$/V-s]**
  - $100 \leq \mu_0 \leq 1500$
Summarized below are the various scattering mechanisms that affect the magnitude of the electron or hole mobility:

**Scattering Mechanisms**

- **Defect Scattering**
  - Crystal Defects
  - Impurity
  - Alloy
  - Neutral
  - Ionized

- **Carrier-Carrier Scattering**

- **Lattice Scattering**
  - Intravalley
  - Intervalley
  - Acoustic
  - Optical
  - Nonpolar
  - Polar
  - Deformation potential
  - Piezoelectric

**Mobility Measurement**

\[
R_H = \frac{r_h p - r_e b^2 n}{e(p + bn)^2}
\]

\[
\mu_H = r \mu_{\text{eff}} \quad \text{Hall Mobility}
\]

\[
b = \mu_e/\mu_h
\]
Effective mobility

\[ \mu_{\text{eff}} \approx \frac{Lg_D}{ZC_{ox}(V_{GS} - V_T)} \]

Field-effect mobility

\[ \mu_{FE} = \frac{Lg_m}{ZC_{ox}V_{DS}} \quad g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} = \text{const}} \]

Saturation Mobility

\[ \mu_{\text{sat}} = \frac{2Lm^2}{BZC_{ox}} \]

Mobility modeling is normally divided into:

- **Low-field mobility models** (bulk materials and inversion layers)
  - Characterization of \( \mu_0 \) as a function of doping and lattice scattering
  - Characterization of \( v_{\text{sat}} \) as a function of lattice temperature
  - Describing the transition between the low-field and the saturation velocity region

- **High-field mobility models**

  **Bulk mobility**:
  1. Characterization of \( \mu_0 \) as a function of doping and lattice scattering
  2. Characterization of \( v_{\text{sat}} \) as a function of lattice temperature
  3. Describing the transition between the low-field and the saturation velocity region

  **Inversion layers**:
  1. Characterization of surface-roughness scattering
  2. Description of the carrier-carrier scattering
  3. Quantum-mechanical size-quantization effect
(A) Low-field models for bulk materials

**Phonon scattering:**
- Simple power-law dependence of the temperature
- Sah et al. model:
  - acoustic + optical and intervalley phonons combined via Mathiessen’s rule

**Ionized impurity scattering:**
- Conwell-Weiskopf model
- Brooks-Herring model

**Combined phonon and ionized impurity scattering:**
- Dorkel and Leturg model:
  - temperature-dependent phonon scattering + ionized impurity scattering + carrier-carrier interactions
- Caughey and Thomas model:
  - temperature independent phonon scattering + ionized impurity scattering

**Carrier-carrier scattering**
- modified Dorkel and Leturg model

**Neutral impurity scattering:**
- Li and Thurber model:
  - mobility component due to neutral impurity scattering is combined with the mobility due to lattice, ionized impurity and carrier-carrier scattering via the Mathiessen’s rule
(B) Field-dependent mobility

The field-dependent mobility model provides smooth transition between low-field and high-field behavior.

\[
\mu(E) = \frac{\mu_0}{1 + \left( \frac{\mu_0 E}{v_{\text{sat}}} \right)^\beta} \quad \beta = 1 \text{ for electrons} \quad \beta = 2 \text{ for holes}
\]

\( v_{\text{sat}} \) is modeled as a temperature-dependent quantity:

\[
v_{\text{sat}}(T) = \frac{2.4 \times 10^7}{1 + 0.8 \exp \left( \frac{T_L}{600} \right)} \text{ cm/s}
\]

(C) Inversion layer mobility models

- CVT model:
  combines acoustic phonon, non-polar optical phonon and surface-roughness scattering (as an inverse square dependence of the perpendicular electric field) via Mathiessen's rule

- Yamaguchi model:
  - low-field part combines lattice, ionized impurity and surface-roughness scattering
  - there is also a parametric dependence on the in-plane field (high-field component)

- Shirahata model:
  - uses Klaassen's low-field mobility model
  - takes into account screening effects into the inversion layer
  - has improved perpendicular field dependence for thin gate oxides

- Tasch model:
  the best model for modeling the mobility in MOS inversion layers; uses universal mobility behavior