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ECE695: Reliability Physics of Nano-Transistors

Lecture 3: Reliability as a Threshold Problem

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The story so far

1. In Lectures 1-3, we are discussing the general issues of reliability physics.

2. Electronics is evolving rapidly, with many new reliability and variability concerns.

3. Historically, reliability has been discussed in terms of empirical, statistical, and physical models.

4. Examples of empirical models include reliability Triangle or Apgar tests, etc. Statistical model uses combinatorial approaches.

Outline of lecture 3

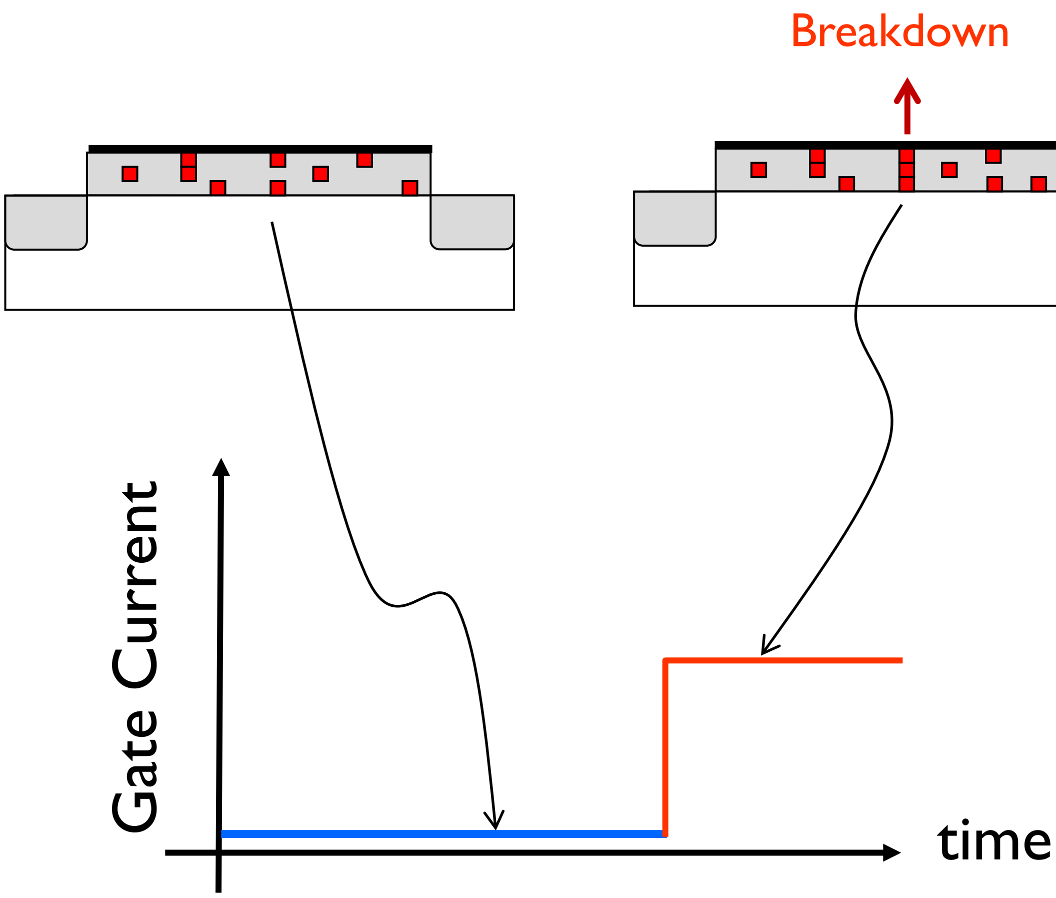
1. Reliability as a Threshold Problem: Empirical vs. Physical Models

2. 'Blind Fish in a Waterfall' as a prototype for Accelerated Testing/Statistical distribution

3. Four elements of Physical Reliability

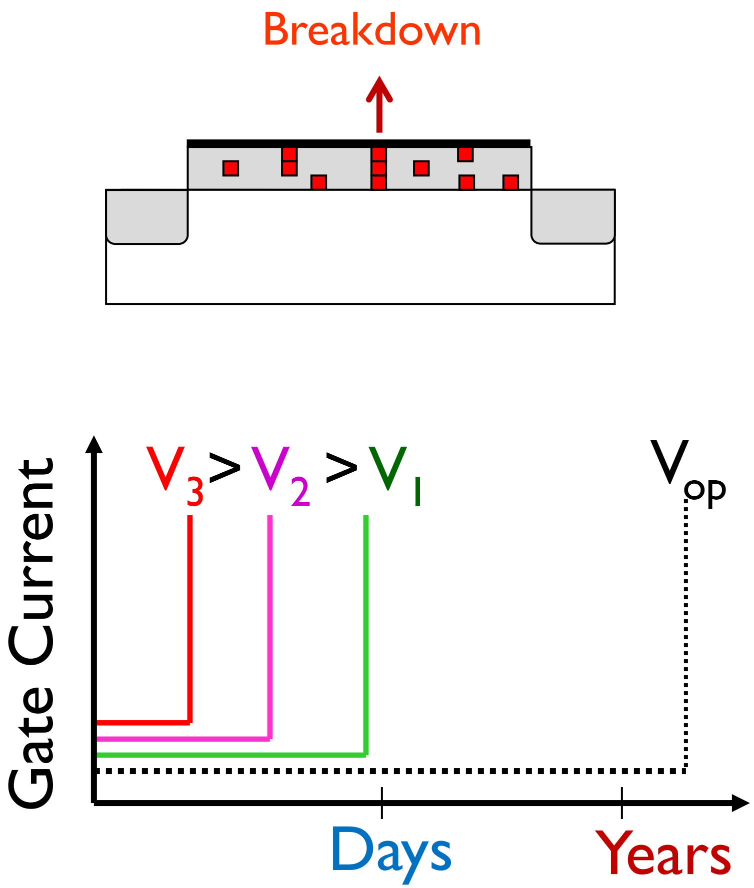
4. Conclusions

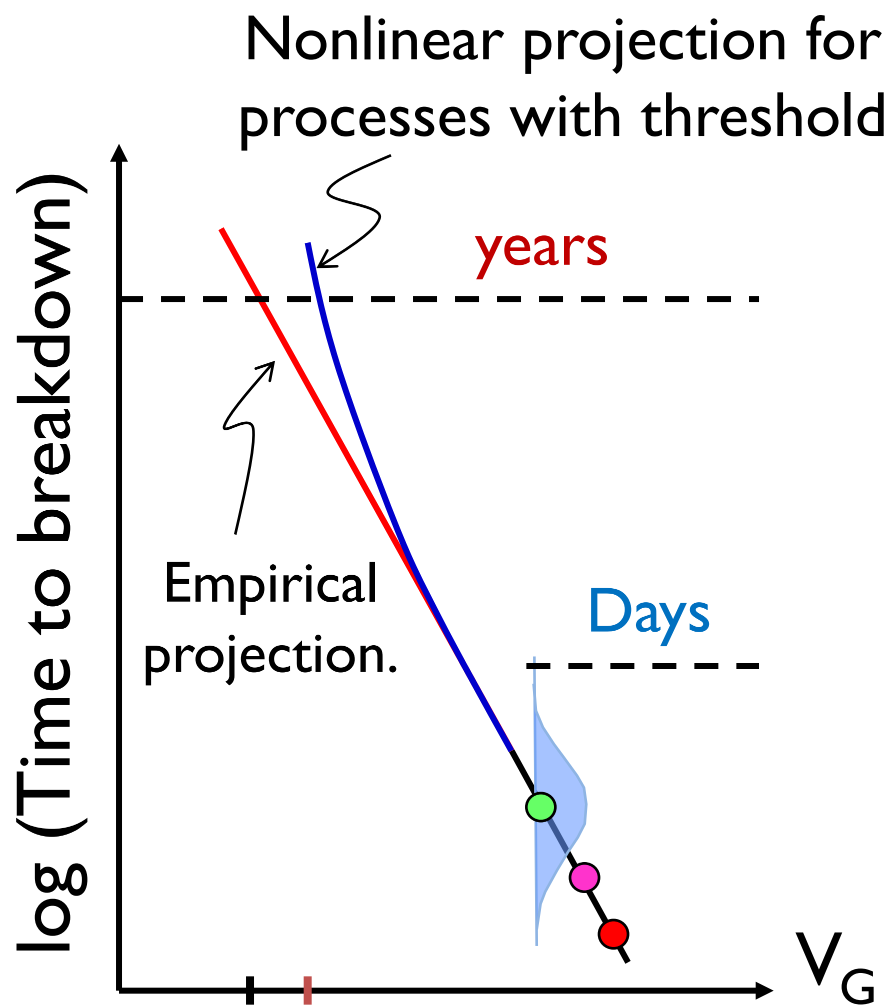
Oxide degradation/breakdown/statistics



Process: Defect generation, *Threshold*: Breakdown

Theory of accelerated & statistical testing





Empirical projection could be overly conservative …

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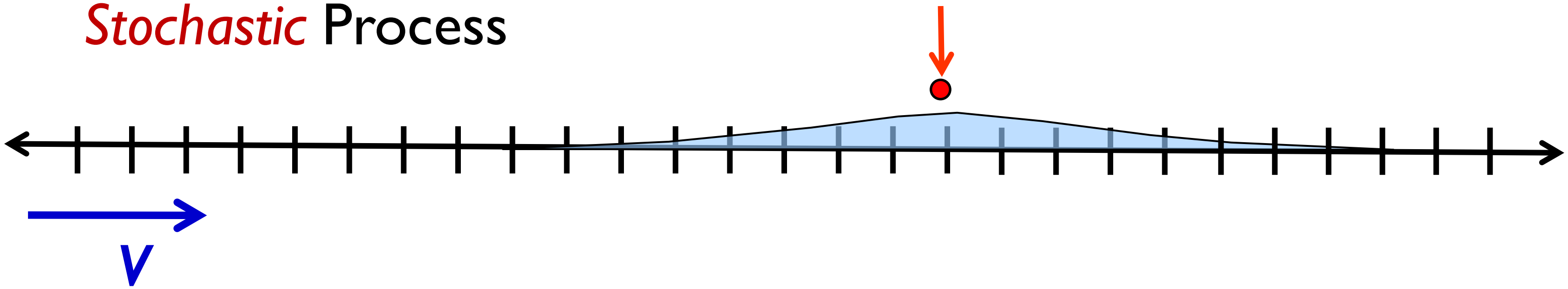
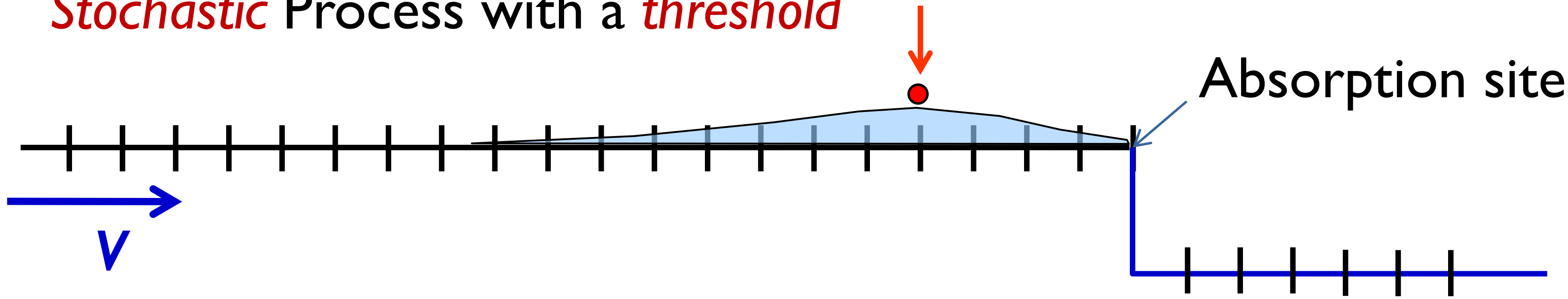
Nonlinear projection: An illustrative example

Stochastic Process

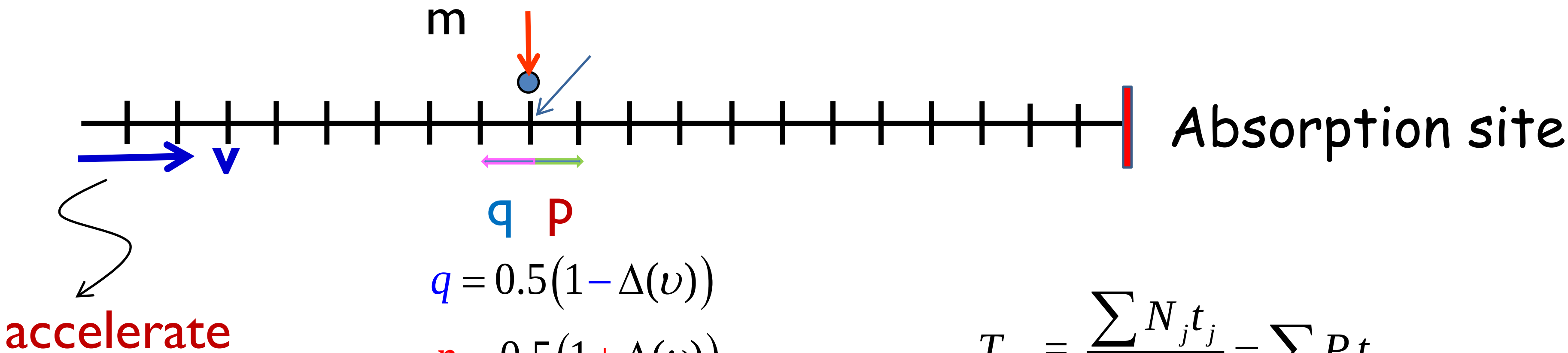


Stochastic Process with a *threshold*



What is the safe velocity **v**, so that after **x sec** of diffusion no more than **y percent of particles** are lost?

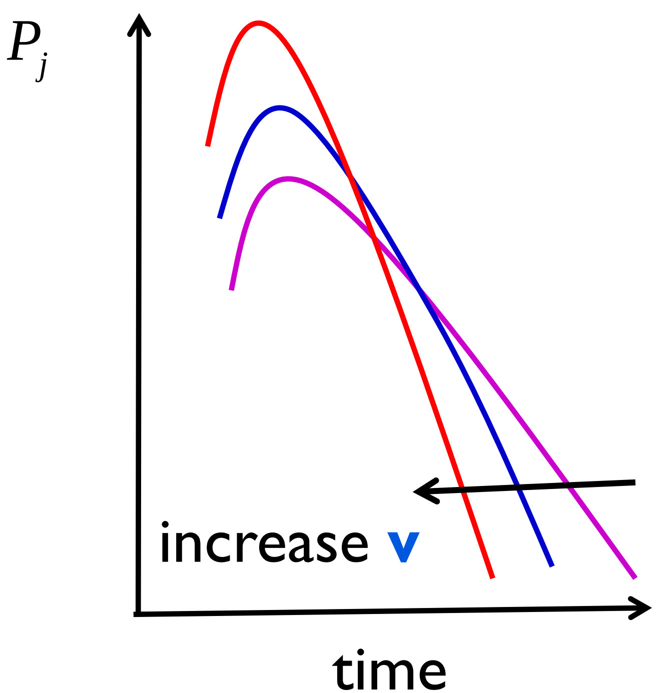
Accelerated testing: Empirical approach

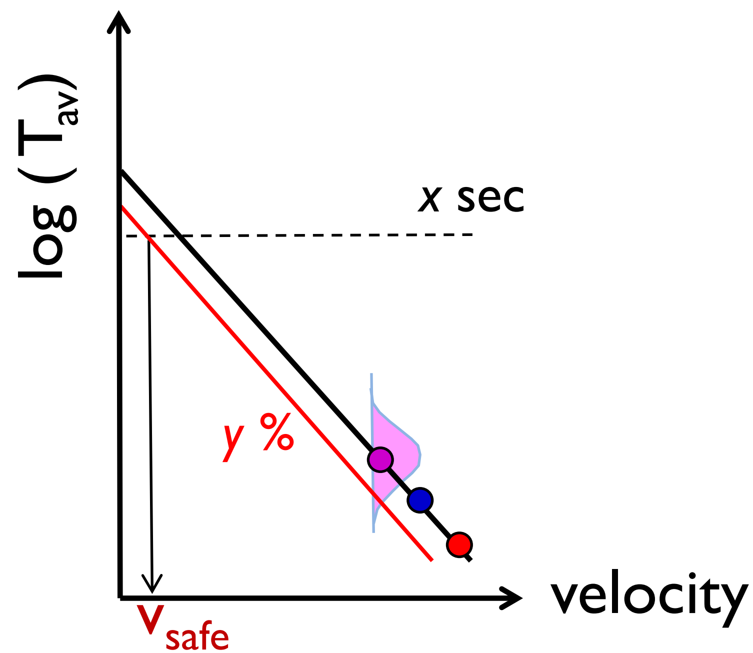


$$q = 0.5\left(1 - \Delta(\upsilon)\right)$$

$$p = 0.5\left(1 + \Delta(\upsilon)\right)$$

$$T_{a\upsilon} = \frac{\sum N_j t_j}{N} = \sum P_j t_j$$

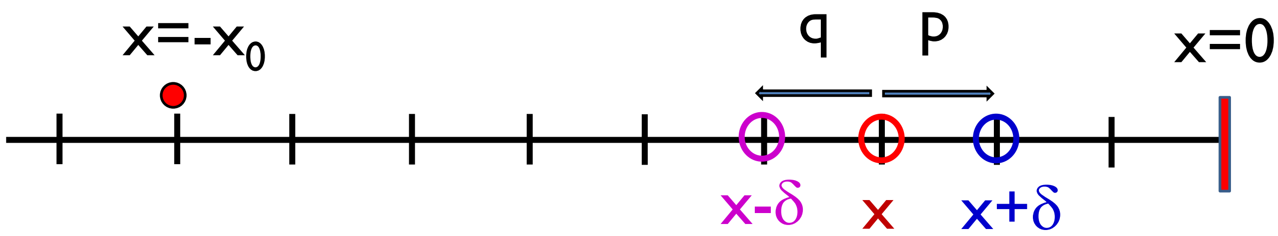




Correspondence

	Dielectric Breakdown	Blind Fish in a Waterfall
Process	Defect generation	Drift-diffusion
Characteristics	Oxide thickness	Point of injection
Accelerator	Voltage/temperature	Flow velocity
Threshold	Breakdown by percolation	Lost at the waterfall
Result	Mean time to failure	Mean time to waterfall

Physical reliability: Mean arrival time



T(x) ....average time to waterfall, starting from position x.

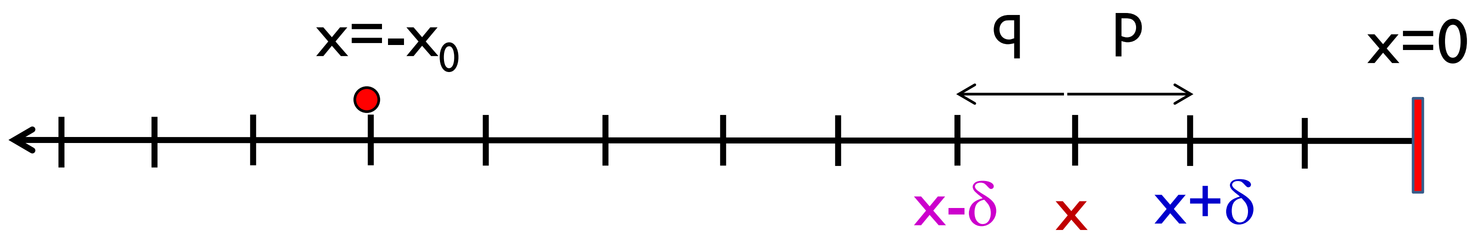
$$T(x) = \tau + \left[p \times T(x+\delta)\right] + \left[q \times T(x-\delta)\right]$$

$$\frac{T(x) - \frac{1}{2}\left(T(x+\delta) + T(x-\delta)\right)}{\delta^2} - \frac{\Delta}{2}\frac{\left(T(x+\delta) - T(x-\delta)\right)}{\delta^2} - \frac{\tau}{\delta^2} = 0$$

$$\boxed{\frac{d^2T}{dx^2} + \frac{2}{v}\frac{dT}{dx} + \frac{2}{D} = 0}$$

$$v \equiv \delta / 2\Delta \qquad D \equiv \delta^2 / \tau$$

Average arrival time distribution



$$\frac{d^2T}{dx^2}+\frac{2}{v}\frac{dT}{dx}+\frac{2}{D}=0$$

$$dT/dx\big|_{x=-L}=0 \qquad T(x=0)=0$$

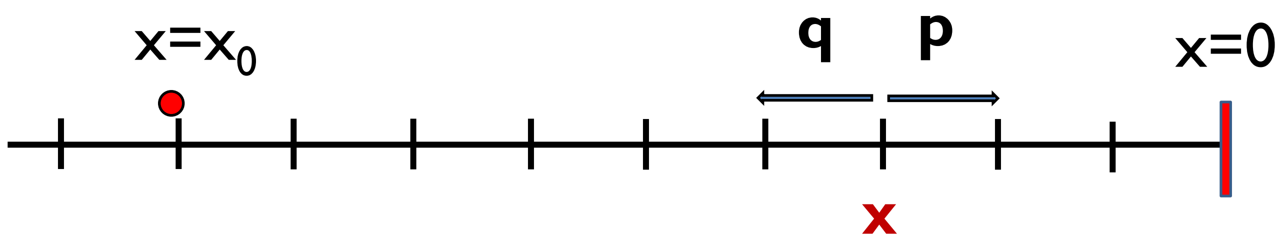
$$T(x)=C_1+C_2x+C_3e^{-2x/v} \qquad C_3=0 \qquad C_1=0$$

$$=C_2x \longrightarrow \frac{2}{v}C_2+\frac{2}{D}=0 \qquad \therefore C_2=-\frac{v}{D}$$

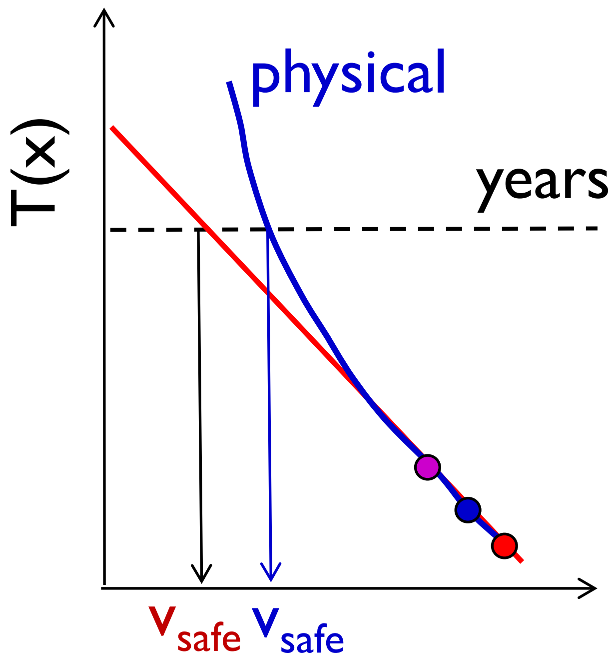
$$T(x)=\frac{v}{D}|x|=\frac{\tau}{2\delta}\frac{|x|}{\Delta}\equiv\frac{|x|}{\upsilon}$$

Average lifetime diverges at small velocity,

Physical vs. empirical projection



$$T(x_0) = \frac{x_0}{\upsilon}$$



Empirical meas. & comp. simulation would not do

Outline

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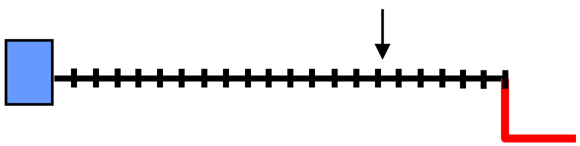
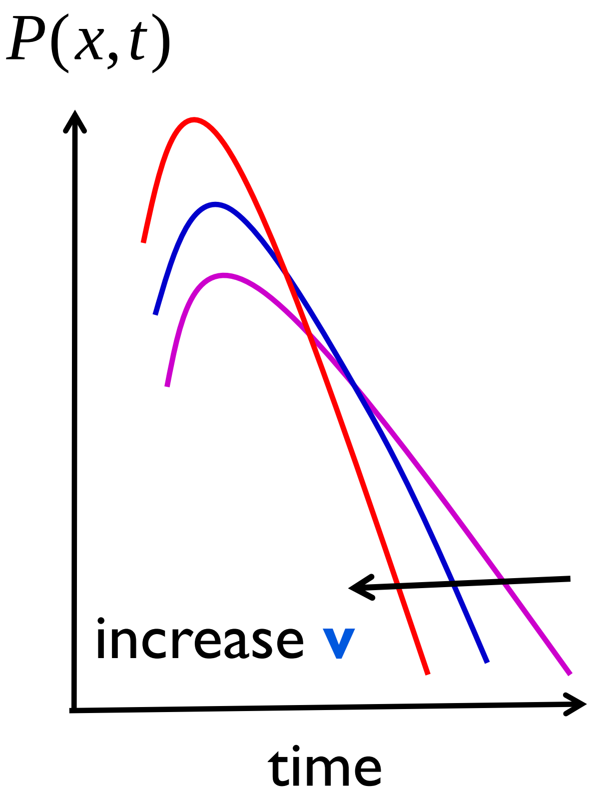
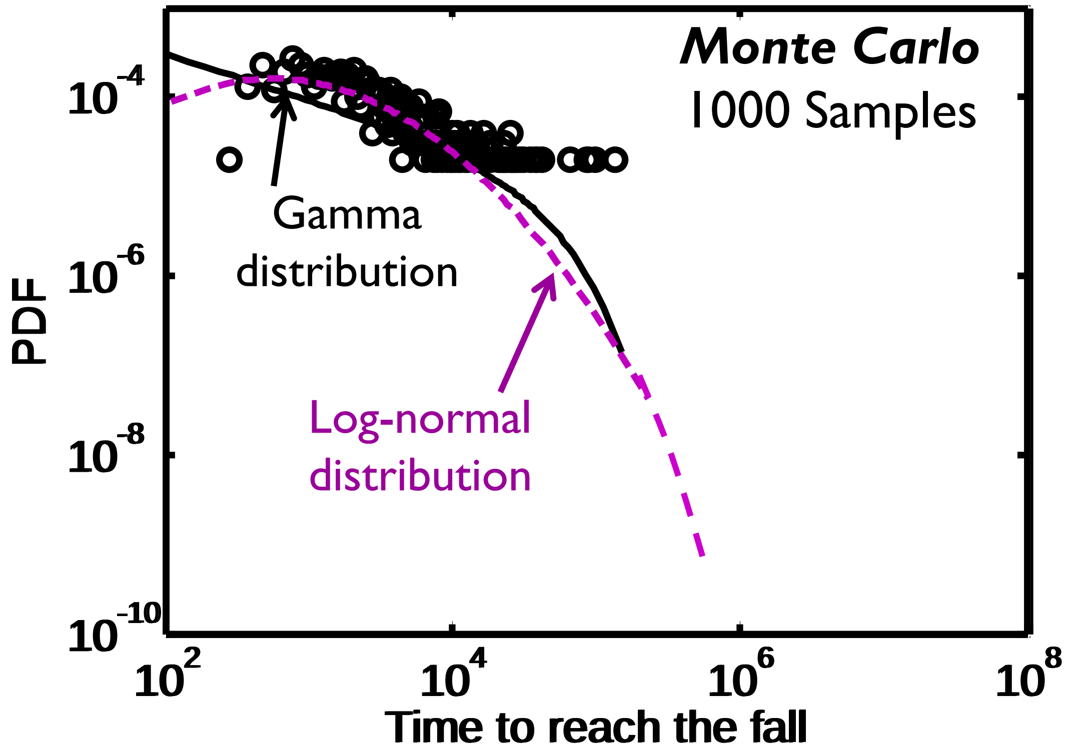
The trouble with empirical distribution



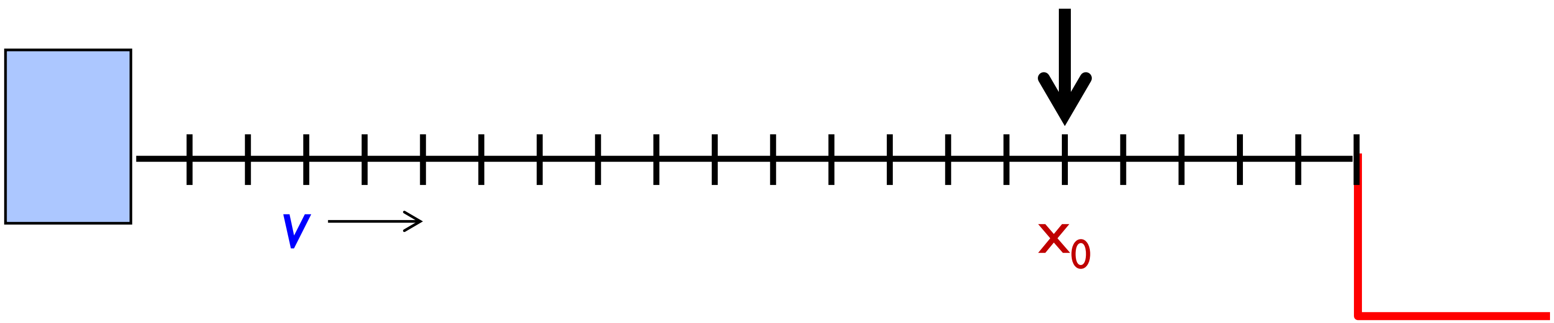
$$f_G(t) = \frac{t^{k-1}e^{-t/\theta}}{\Gamma(k)\theta^k} \qquad T_{avg} = k\theta$$





Derivation Of “Fishy” (or BFRW) Distribution

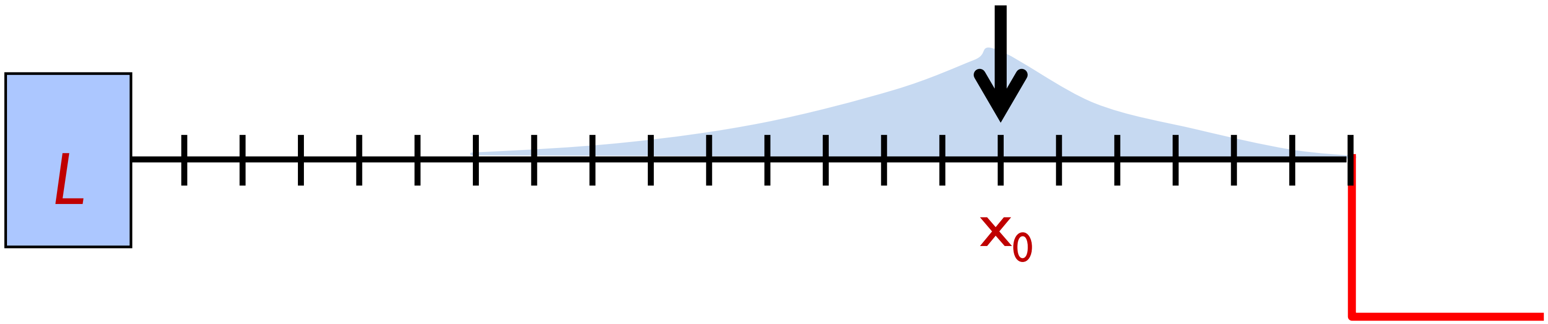
$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2} - \upsilon\frac{\partial P}{\partial x} \qquad \begin{aligned} &P(x, t=0) = \delta(x - x_0) \\ &P(x=0, t) = 0 \end{aligned}$$



Solution by the method of images, c(0,t)=0

$$P(x,t) = \frac{1}{\sqrt{4\pi Dt}}\left[e^{-\frac{(x - x_0 - \upsilon t)^2}{4Dt}} - e^{-\frac{\upsilon x_0}{D}}\, e^{-\frac{(x + x_0 - \upsilon t)^2}{4Dt}} \right]$$

Derivation Of “Fishy” (or BFRW) Distribution



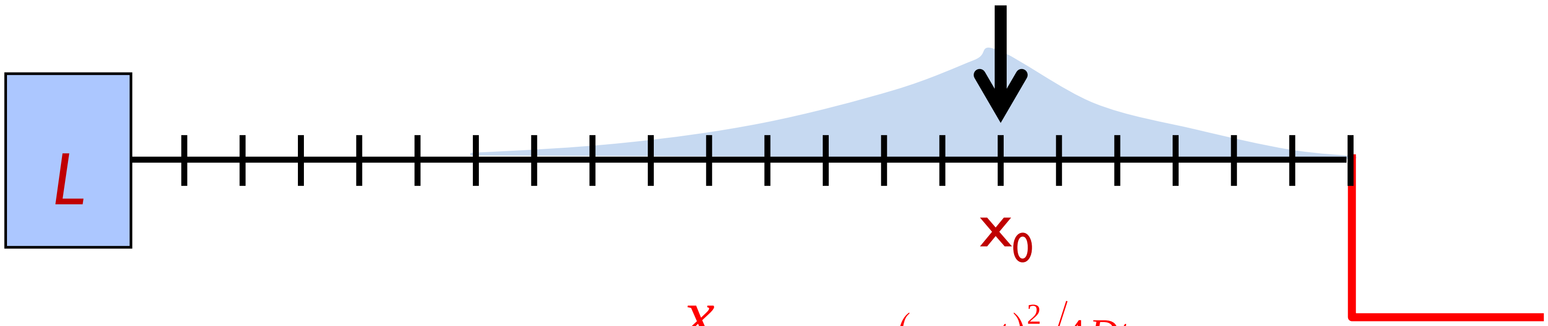
$$P(x,t) = \frac{1}{\sqrt{4\pi Dt}}\left[e^{-\frac{(x-x_0-\upsilon t)^2}{4Dt}} - e^{-\frac{\upsilon x_0}{D}} e^{-\frac{(x+x_0-\upsilon t)^2}{4Dt}} \right]$$

Conservation of particles …

$$\int_0^t f(\tau)d\tau + \int_0^{L\to\infty} P(x,t)dx = 1$$

$$f(t) = \frac{x_0}{\sqrt{4\pi Dt^3}} e^{-(x_0+\upsilon t)^2/4Dt}$$

Moments of BFRW Distribution



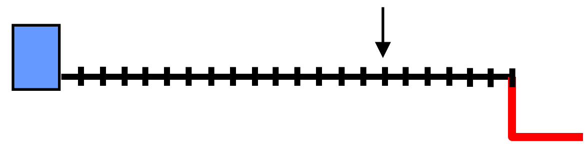
$$f(t) = \frac{x_0}{\sqrt{4\pi D t^3}} e^{-(x_0 - \upsilon t)^2 / 4Dt}$$

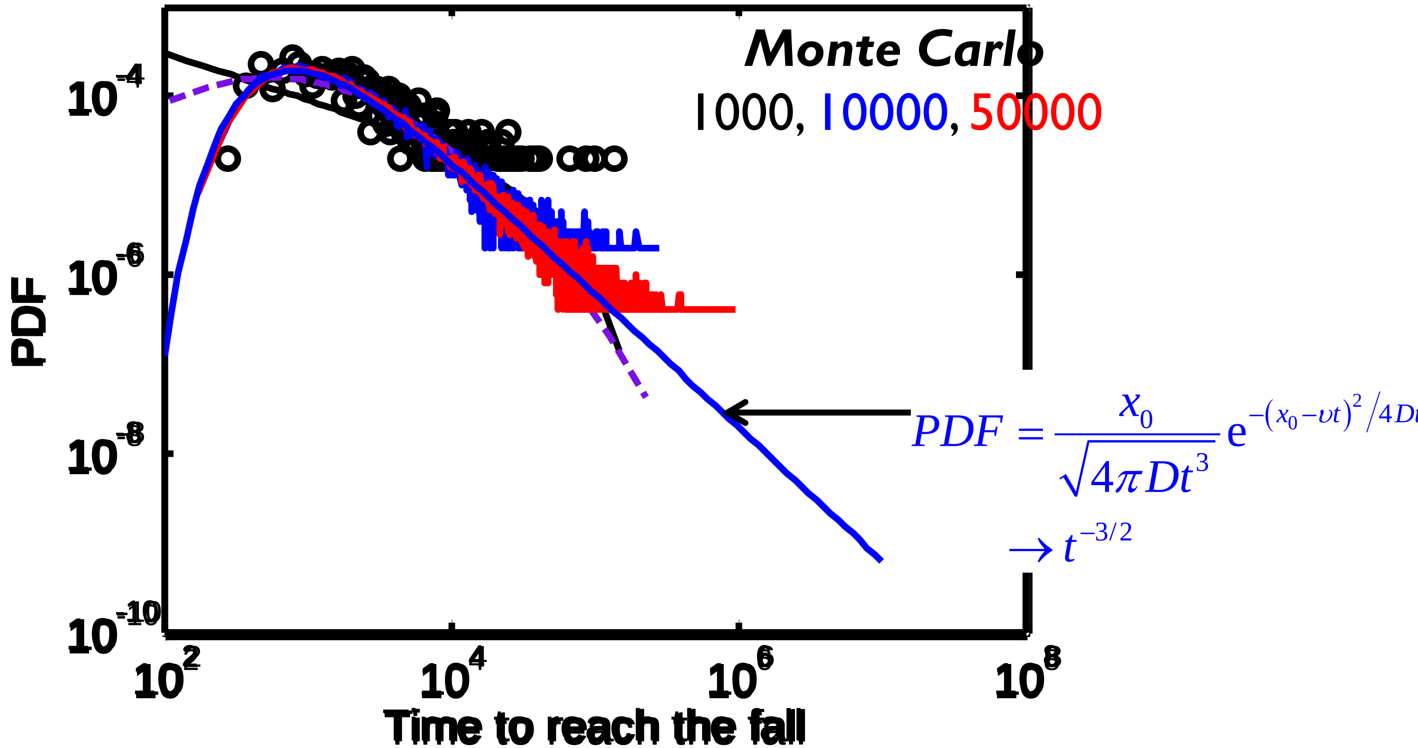
$$\langle t^n \rangle = \int_0^\infty t^n \times f(t) dt = \int_0^\infty \frac{x_0 \times t^n}{\sqrt{4\pi D t^3}} e^{-\frac{(x_0 - \upsilon t)^2}{4Dt}} dt \rightarrow \text{finite}$$

$$T \equiv \langle t^1 \rangle = \int_0^\infty t \times f(t) dt = \int_0^\infty \frac{x_0 \times t}{\sqrt{4\pi D t^3}} e^{-\frac{(x_0 - \upsilon t)^2}{4Dt}} dt = \frac{x_0}{\upsilon}$$

Shape (or distribution) depend on velocity!

Physical vs. empirical distribution





Statistical distribution is physical, empirical approximation often not adequate

Long (or fat) tail of a BFRW distribution

$$T(x_0) = \int_0^\infty t\, f(t)\,dt$$

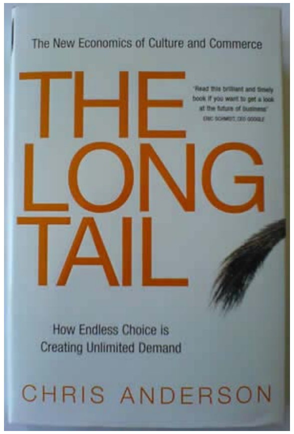
$$= \int_0^\infty \frac{x_0 t}{\sqrt{4\pi D t^3}} \mathrm{e}^{-x_0{}^2/4Dt}\, dt$$

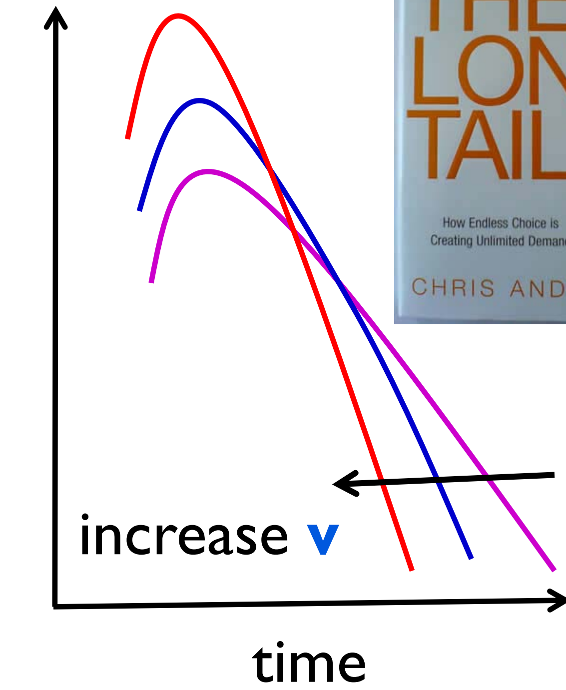
$$\propto \int_0^\infty t \times t^{-\frac{3}{2}} dt \rightarrow \infty$$

Distribution does not obey central-limit theorem!

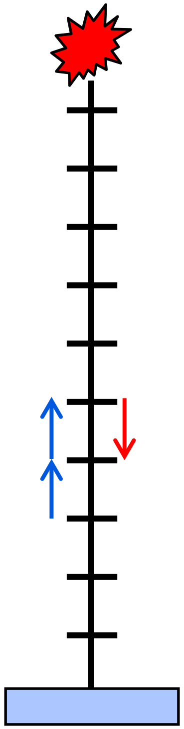
Although there is no average,
a huge fraction of field-return will
occur in short period of time.

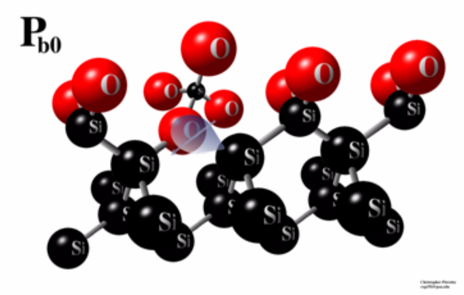
Drug release
Amazon model
Neuron firing





BFRW Distributions in other systems





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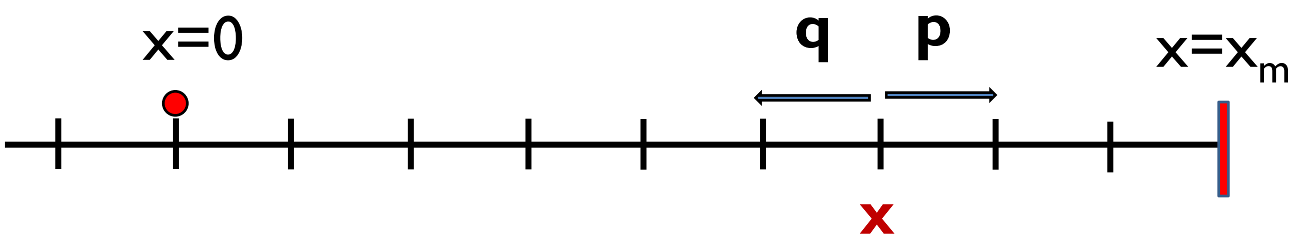
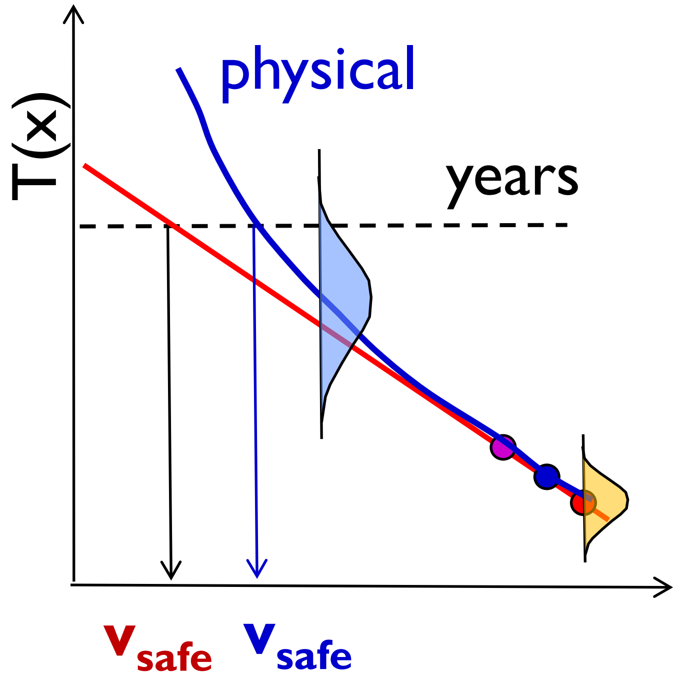
Four Elements of Physical Reliability





1. Theory of Stress Acceleration

$$T(\upsilon, x_0) = \frac{x_0}{\upsilon}$$

2. Theory of Stochastic Distribution

$$f(t;\upsilon,x_0) = \frac{x_0}{\sqrt{4\pi D t^3}} e^{-(x_0-\upsilon t)^2/4Dt}$$

3. Characterization D, x_0

4. Analysis of Statistical data

Conclusions

- Highlighted the difference between empirical vs. physical models and demonstrated how a threshold makes an acceleration model inherently nonlinear.

- Statistical distribution is physical. And physics based distributions differ significantly from empirical presumption about such distributions (Gaussian). Central limit theorems need not apply.

- Many problems in reliability physics has close analog in engineering, physics, biology, and finance. For an electrical analog to BFRW problem, see Shockley-Haynes experiment.

- Reliability problems are too complex to be exclusively predicted from first principles. Characterization experiments determine the parameters of the model.