

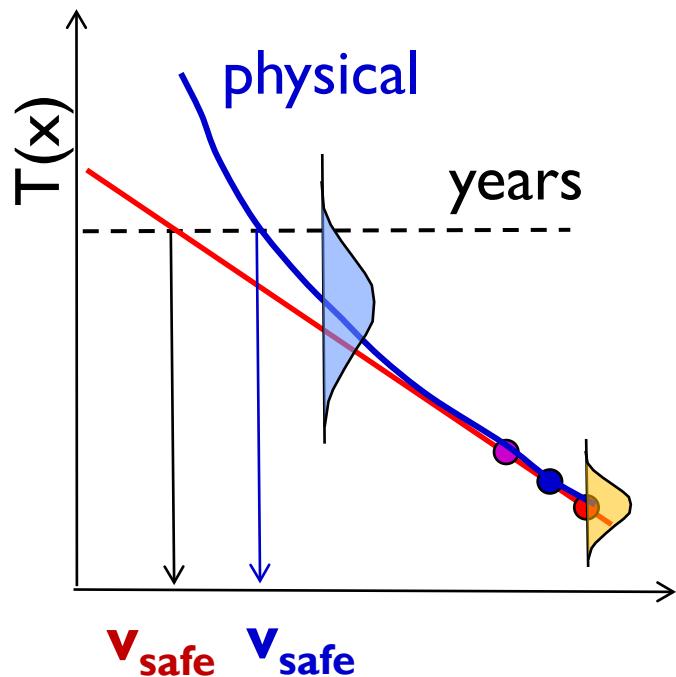
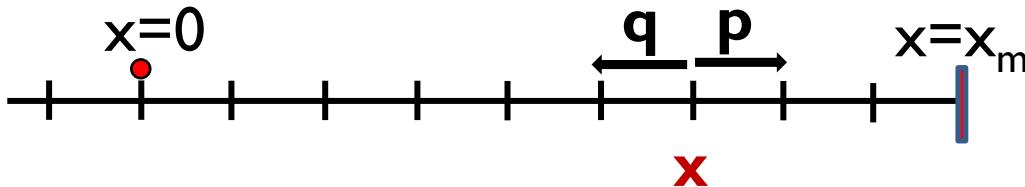
ECE695: Reliability Physics of Nano-Transistors

Lecture 7A: Appendix

- Theory of Stochastic Distribution

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Appendix: Four Elements of Physical Reliability



I. Theory of Stress Acceleration

$$T(v, x_0) = \frac{x_0}{v}$$

2. Theory of Stochastic Distribution

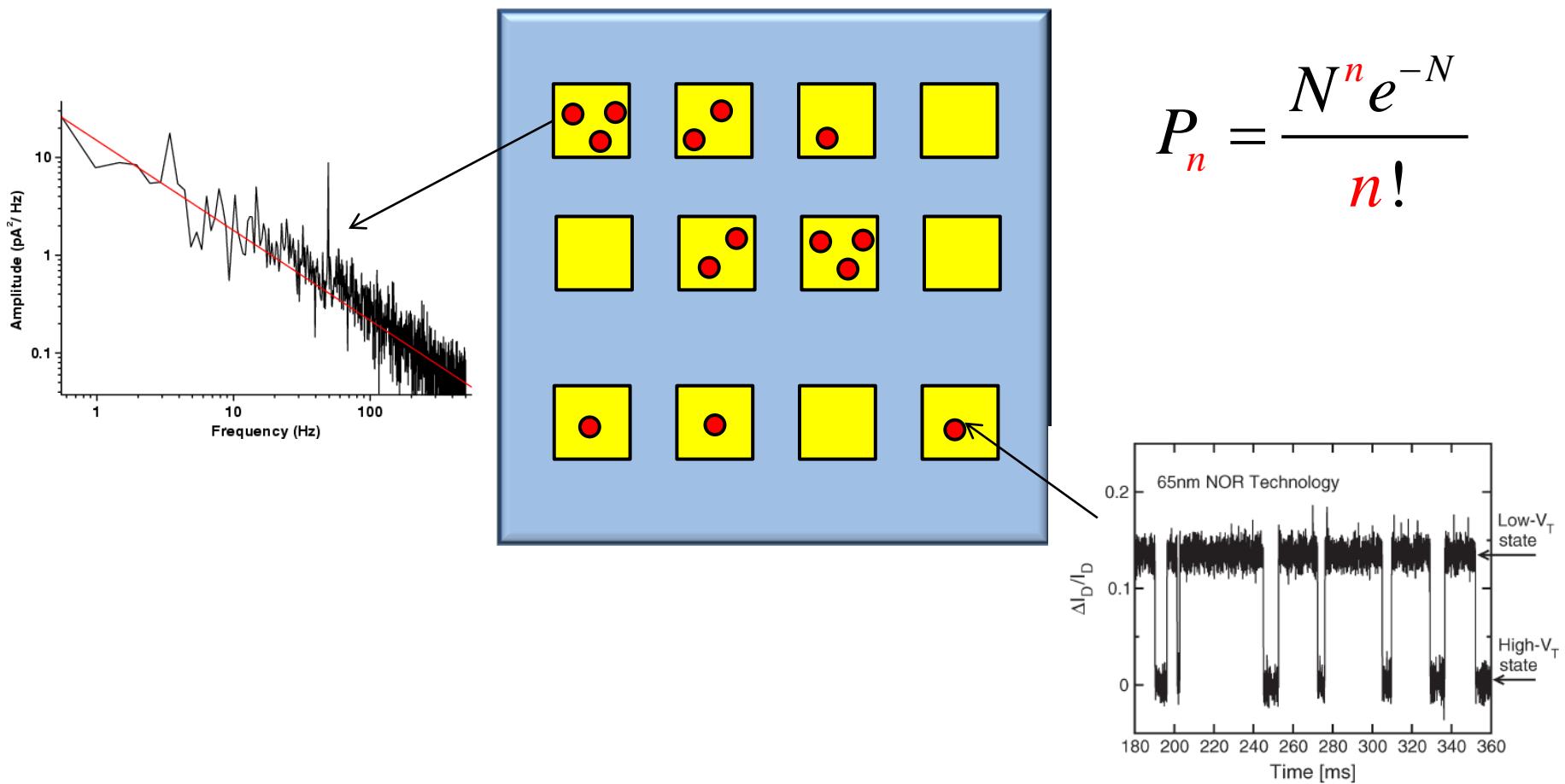
$$f(t; v, x_0) = \frac{x_0}{\sqrt{4\pi D t^3}} e^{-(x_0 + vt)^2 / 4Dt}$$

3. Characterization D, x_0

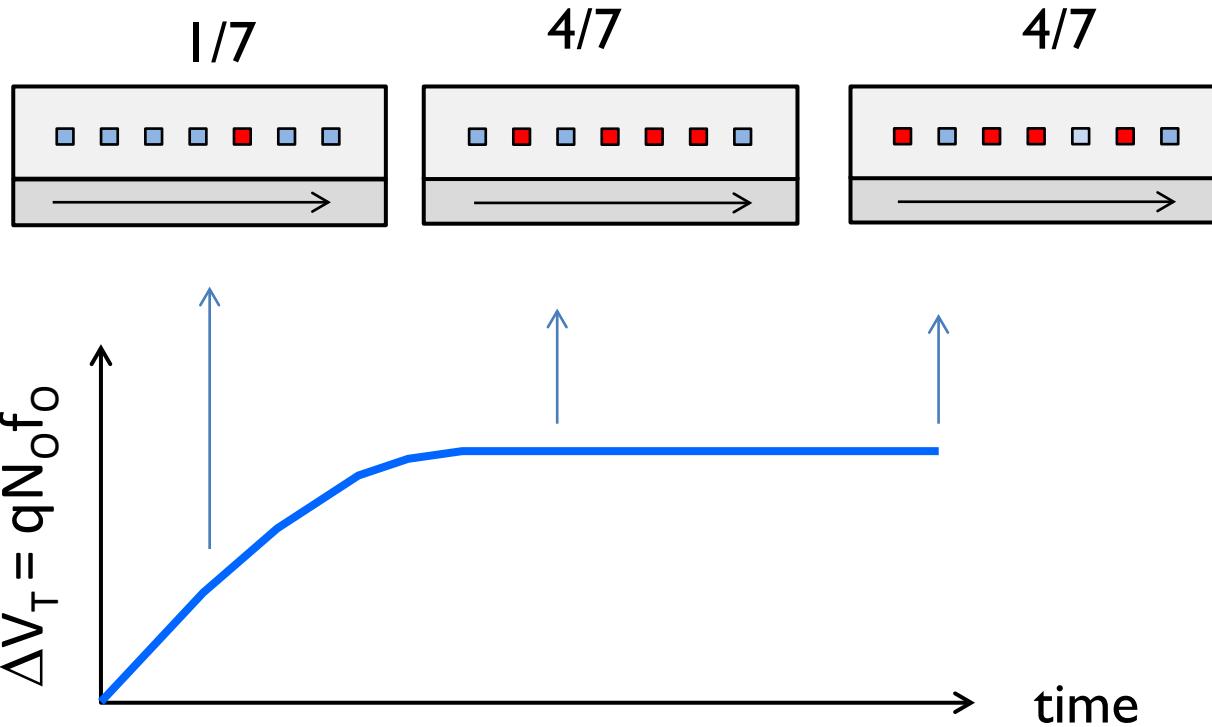
4. Analysis of Statistical data

Statistics of trapping

$$10^{17} \text{ cm}^{-3} \times 2\text{nm} \times 100\text{nm} \times 100\text{nm} = 2 \text{ traps/device}$$



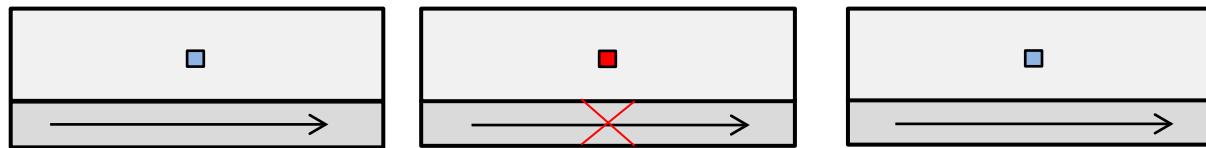
Fluctuation in trap occupation



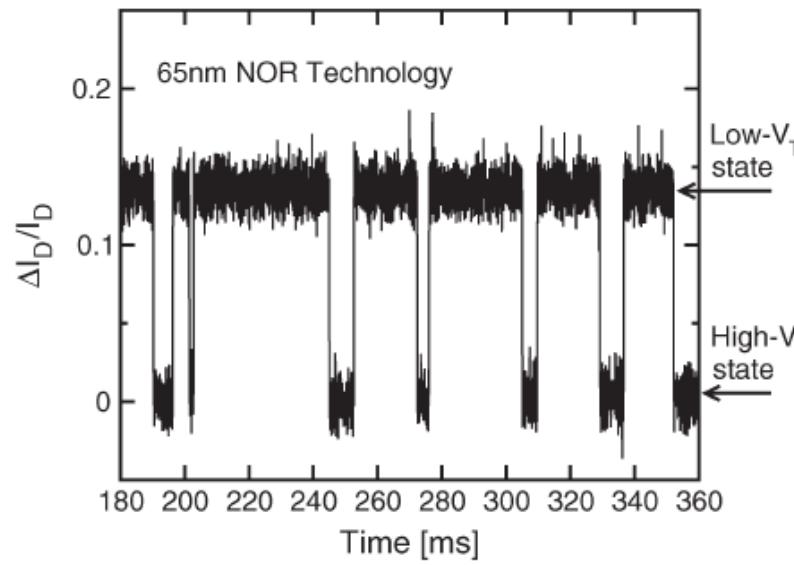
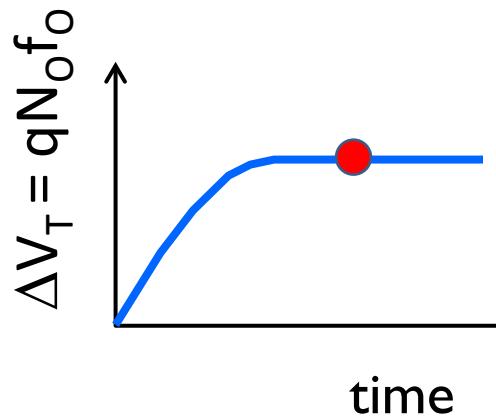
$$f_o = \frac{\textcolor{blue}{T}_1 \left[1 - \exp(-\sigma v_{th} (n_e \textcolor{blue}{T}_1 + p_s \textcolor{blue}{T}_1 + p_G \textcolor{red}{T}_2) t) \right]}{(1 + p_s/n_e) \textcolor{blue}{T}_1 + p_G \textcolor{red}{T}_2 / n_e} \equiv b \left[1 - \exp(-t/\tau_c) \right]$$

$$\tau_c^{-1} = \tau_{\text{ON}}^{-1} + \tau_{\text{OFF}}^{-1}$$

Fluctuation in single trap occupation



Sodini, IEDM, 2000



$$\begin{aligned} &\equiv b[1 - \exp(-t/t_0)] \\ &= \frac{\langle \tau_{\text{off}} \rangle}{\langle \tau_{\text{off}} \rangle + \langle \tau_{\text{on}} \rangle} \end{aligned}$$

$$(t, t_k) = \sum_k N_0 e^{-\left(\frac{t-t_k}{\tau_c}\right)}$$

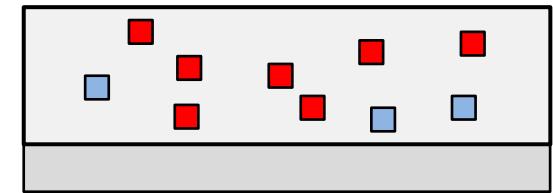
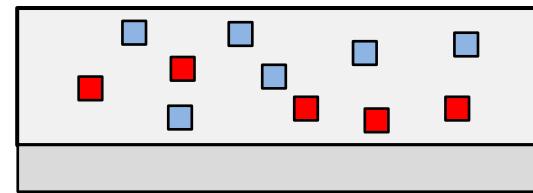
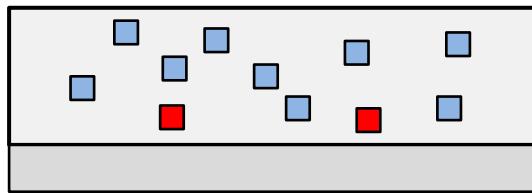
$$\tau_c \propto T_1^{-1} \propto e^{+\frac{x}{x_0}}$$

$$F(\omega) = \left[\frac{N_0}{\tau_c + j\omega} \right] \sum_k e^{j\omega t_k}$$

$$S(\omega) = \frac{1}{T \rightarrow \infty} \left\langle |F(\omega)|^2 \right\rangle = \left[\frac{N_0^2 \tau_c}{1 + (\tau_c \omega)^2} \right]$$

Fluctuation for distributed traps

$$\Delta V_T = A \ln t + B$$



$$S_I = A \int_0^{x_{\max}} dx \int_0^{\Phi_B} dE S(\omega(x, E)) \times N_O(x, E) \times k_B T \times \delta(E - E_F)$$

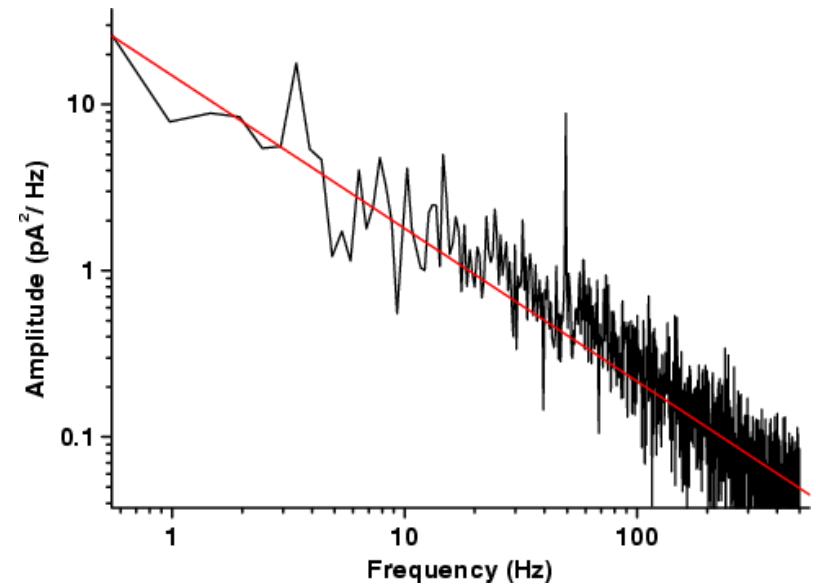
$$\propto N_0 \int_0^{x_{\max}} dx \left[\frac{\tau_c}{1 + (\tau_c \omega)^2} \right]$$

$$\tau_{\min} = \tau_0, \tau_{\max} = \tau_0 \exp(x_{\max} / x_0)$$

$$\propto N_0 \int_{\tau_{\min}}^{\tau_{\max}} d\tau \left[\frac{\tau_c}{1 + (\tau_c \omega)^2} \right] N_0$$

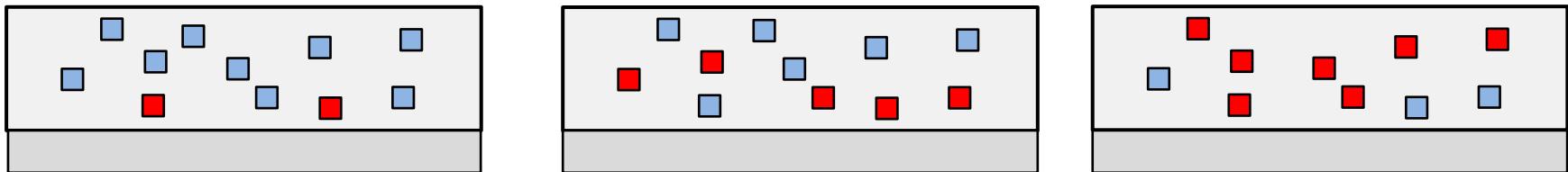
$$\propto \frac{A}{\omega} \left[\tan^{-1}(\omega \tau_{\max}) - \tan^{-1}(\omega \tau_{\min}) \right]$$

$$\propto \frac{A}{f} \quad (\tau_{\max}^{-1} \square \tau_{\min}^{-1})$$



$1/f$ noise and time-spectra

Noise – a few comments



Johnson discovers it accidentally in 1925 in Shottky barrier diode

Pink noise ... Filtered broadband light looks reddish

Popcorn Noise If you amplify, sounds like popcorn in microwave

Self-similar and scale invariant

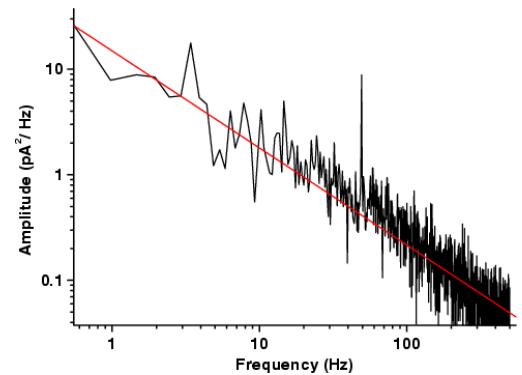
(e.g. 2.5 year long experiment shows no deviation)

What happens at $f=0$ at DC?

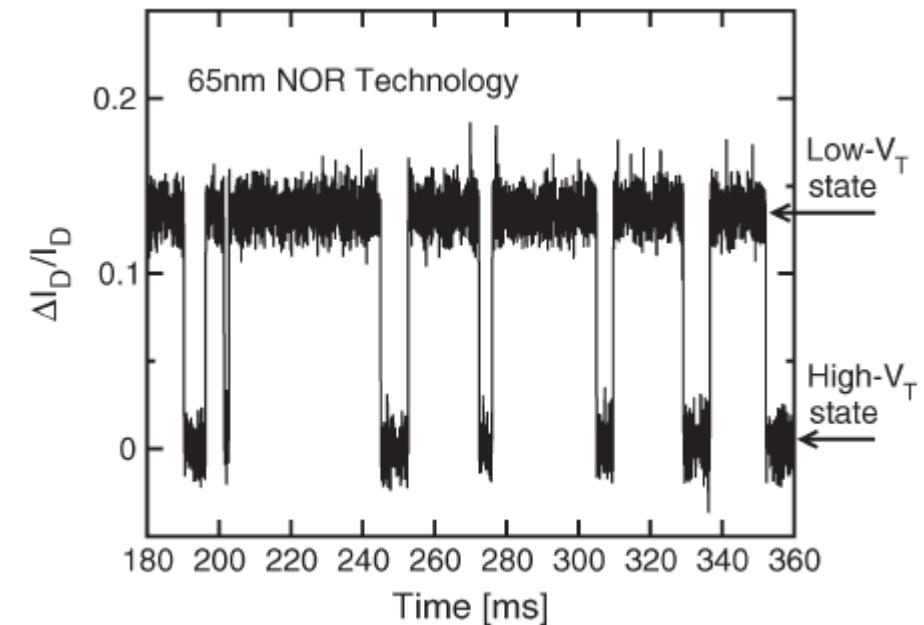
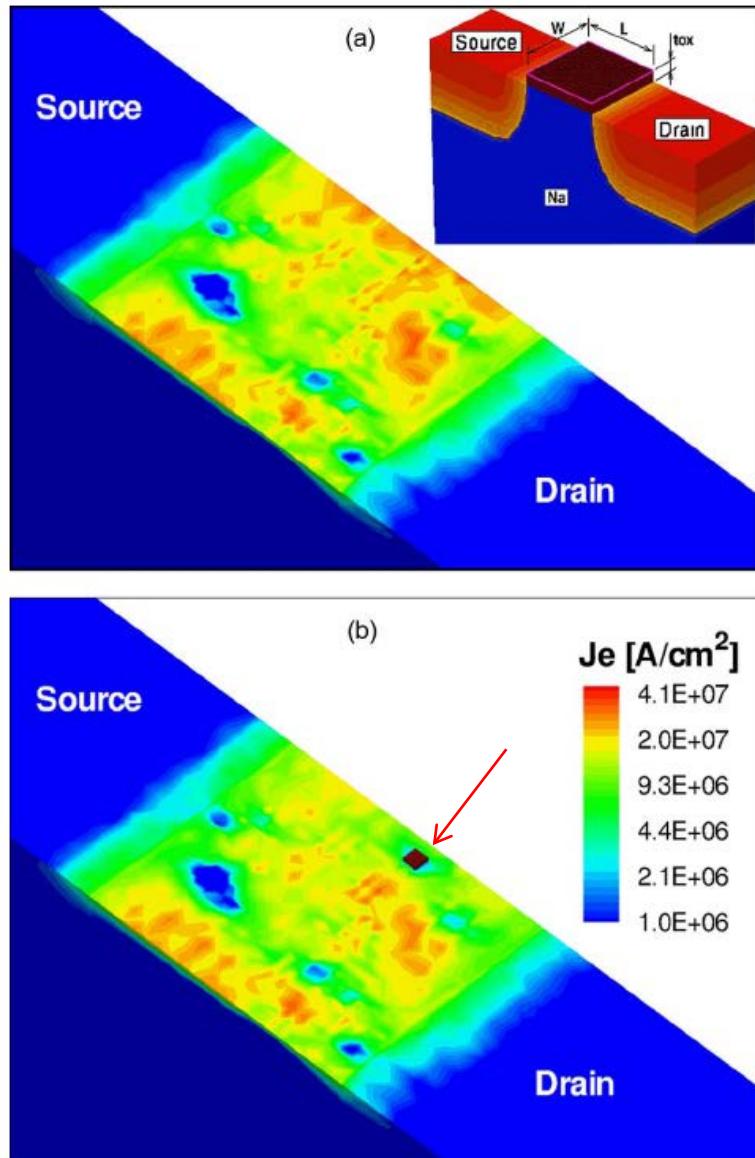
$$\text{Noise} = \int_{f_{\min}}^{f_{\max}} S_I df = A \times \log \frac{f_{\max}}{f_{\min}}$$

Cut-off by thickness:

For 1 dec, $A=10 \text{ uV}$, even for 10 year tunneling time, i.e. $f_{\min} \sim 10^{-8} \text{ sec}$,
 $V \sim 80 \text{ uV}$ Only a factor of 8 larger.



Trapping, doping, and channel conduction



Ghetti, TED, 56(8), 1746, 2009