

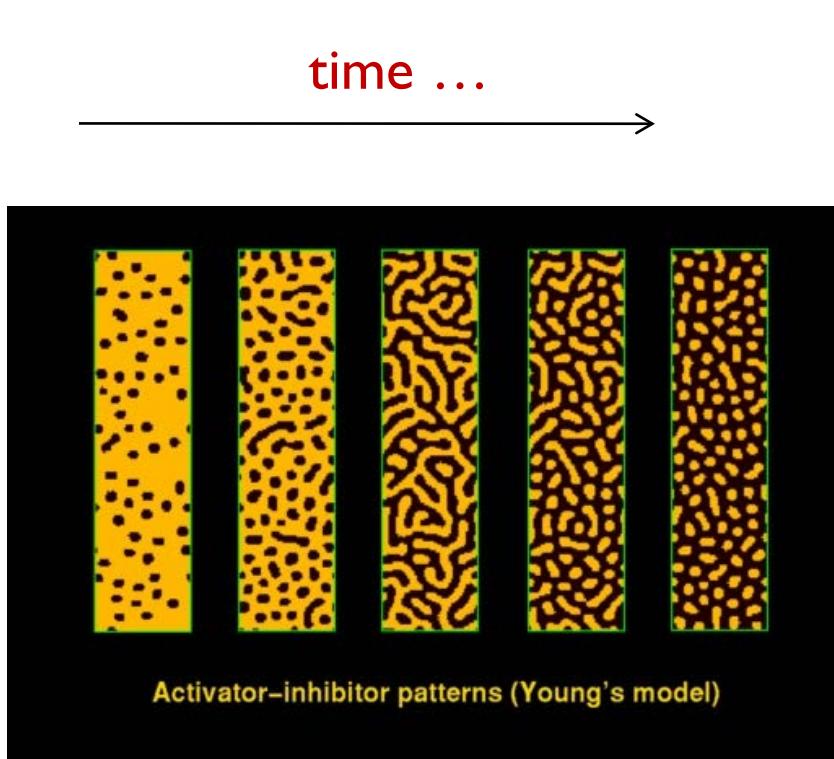
ECE695: Reliability Physics of Nano-Transistors

Lecture 10A: Appendix

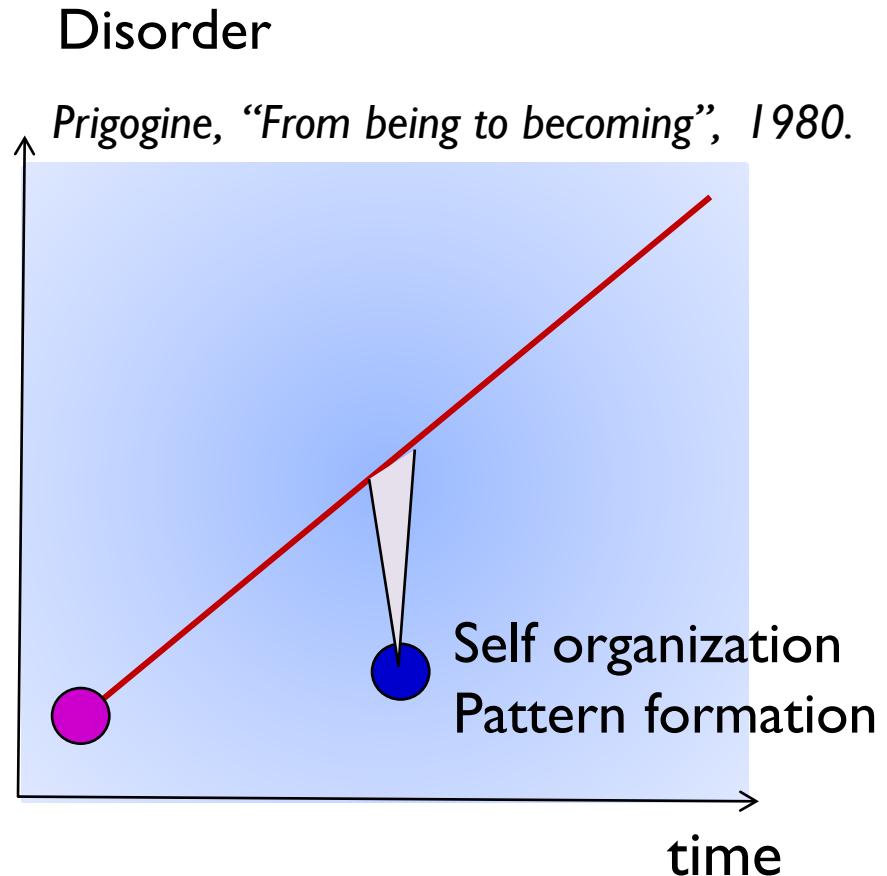
- Reflection on R-D Equation

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Appendix: Reflection on R-D equation



Turing's model of Morphogenesis, 1953.



Reaction-diffusion model produces complex structures out of homogenous systems

Turing's reaction-diffusion model

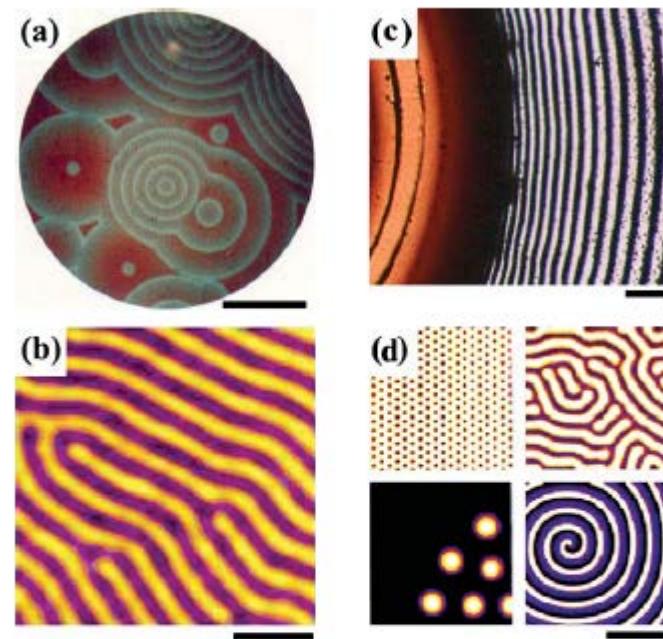
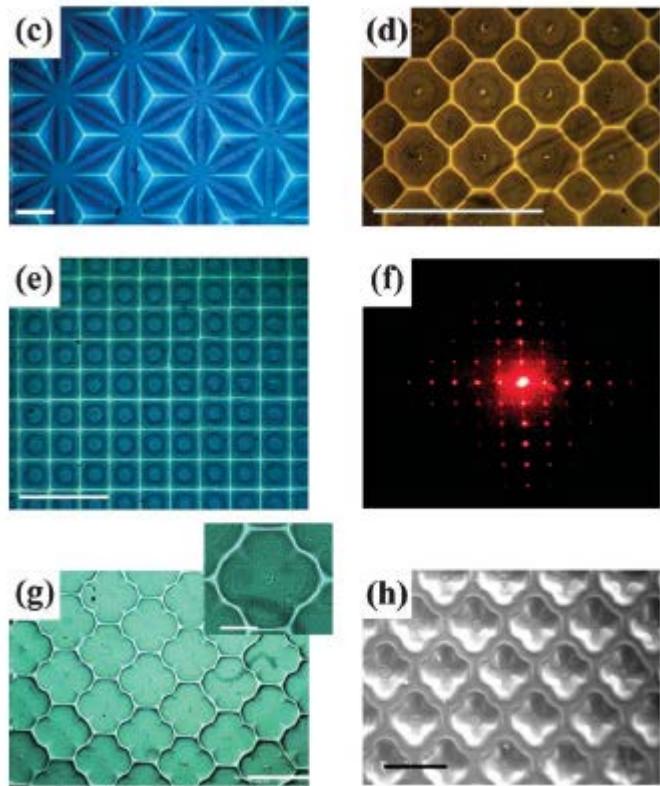


Fig. 2 Man-made reaction–diffusion systems. (a) Traveling waves in the Belousov–Zhabotinsky chemical system (scale bar: 30 mm). (b) Turing pattern formed by CIMA reaction (scale bar: 5 mm). (c) Periodically-precipitated Liesegang rings of silver dichromate in a layer of gelatin (scale bar: 400 μ m). (d) Patterns formed by discharge filaments (scale bar: 2 mm) [Image credits: (a) Courtesy of I. Epstein, Brandeis University. (b) Courtesy of J. Boissonade, CRPP Bordeaux. (c) Courtesy of A. Bitner, Northwestern University. (d) Courtesy of H.G. Purwins, University of Münster.]

Micro- and nanotechnology via reaction–diffusion

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Pattern formation and Reaction-Diffusion model

One variable Reaction-diffusion model

$$\frac{\partial U}{\partial t} = D \frac{\partial^2 U}{\partial x^2} + R(U)$$

$R(U) = U(1-U)$ Fisher (biological population)

$R(U) = U(1-U^2)$ Newell-Whitehead-Segel (Rayleigh-Bernard flow)

$R(U) = U(1-U)(U-a)$ Zeldovich (Combustion)

Two variable Reaction-diffusion model

Scott-gray model: $U+2V=3V$

$$\frac{\partial U}{\partial t} = D_U \frac{\partial^2 U}{\partial x^2} - UV^2 + f(1-U)$$

$$\frac{\partial V}{\partial t} = D_V \frac{\partial^2 V}{\partial x^2} + UV^2 - kV$$

NBTI Model

$$\frac{\partial U}{\partial t} = D_U \frac{\partial^2 U}{\partial x^2} + \frac{\delta}{2} \frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial t} = k_F(a-V) - k_R UV$$

Patterns and Equations

I) Reaction-diffusion model for pattern formation

2) Soliton equations

Korteweg-de Vries equation

$$\partial_t \phi + \partial_x^3 \phi - 6 \phi \partial_x \phi = 0,$$

$$\phi(x, t) = \frac{1}{2} c \operatorname{sech}^2 \left[\frac{\sqrt{c}}{2} (x - ct - a) \right]$$

Sine Gordon equation

$$\varphi_{tt} - \varphi_{xx} + \sin \varphi = 0.$$

$$\varphi_{\text{soliton}}(x, t) := 4 \arctan e^{m\gamma(x-vt)+\delta}$$

$$\gamma^2 = \frac{1}{1 - v^2}.$$

Nonlinear Schrodinger equation

$$i \partial_t \psi = -\frac{1}{2} \partial_x^2 \psi + \kappa |\psi|^2 \psi$$

3) Game of life (Conway), Cellular automata