Outline

• Recap from Monday
• Thermal transfer overview
  – Convection
  – Conduction
  – Radiative transfer
Recap from Monday

• Applications of Beam Propagation Method
  – Tunable Photonic Crystal Fibers
  – Electro-Optic Modulator
  – Electro-Optic Switch
Thermal Transport Mechanisms

- Convection: heat transfer by surface contact with gas or fluid molecules
- Conduction: volumetric heat transfer by propagation of phonons
- Radiative thermal transfer: emission of thermal photons from source to receiver
Thermal Transport: Convection

• Heat transfer by gas or fluid molecules
• Transfer rate per unit area given by:
  \[ Q = h(T_1 - T_2) \]
• Heat transfer constant \( h \) determined by many factors, including material choice, microstructures, fluid flow environment, etc.
Thermal Transport: Conduction

• Volumetric heat transfer through phonon transfer

• Heat transfer rate quantified by Fourier’s law:
  \[ Q = -k \nabla T \]

• Conservation of energy yields:
  \[ \frac{\partial u}{\partial t} = L - \nabla \cdot Q \]
  \[ \frac{\partial u}{\partial t} - L = k \nabla^2 T = \frac{k}{\rho c_v} \nabla^2 u \equiv \alpha \nabla^2 u \]
Thermal Transport: Radiative Transfer

• Heat transfer via photon emission
• For a blackbody, total emission follows Stefan-Boltmann law:
  \[ P = \sigma T^4 \]
• Net thermal transfer between two infinite surfaces becomes:
  \[ Q = \sigma (T_1^4 - T_2^4) \]
Thermal Transport: Radiative Transfer

- Emission for real materials depends on emissivity.
- In thermal equilibrium, Kirchoff’s law states emissivity = absorptivity at each wavelength.
- Emission spectrum is given by:
  \[
  \frac{dQ}{d\lambda} = \frac{2\pi hc^2 \, \varepsilon(\lambda)}{\lambda^5 \left[ e^{\frac{hc}{\lambda kT}} - 1 \right]}
  \]
- Blackbody result recovered by setting \( \varepsilon(\lambda) = 1 \) and integrating.
Thermal Transport: Modeling

• Convection amounts to a boundary condition in most problems
• Will thus be first combined with conduction
• Strategy:
  – Create FEM grid for thermal conduction
  – Impose BC’s from convection
  – (Optionally) include radiative transfer from disconnected bodies
Thermal Transport FEM

• Employ Galerkin method to reduce to linear algebra problem as before (see Petr Krysl’s step-by-step introduction, Chapter 6):

\[ \dot{C}T + (K + H)T = \sum L_i \]

• Where:

\[
\begin{align*}
C_{ji} &= \int_{S_c} N_j c_V N_i \, dS \\
K_{ij} &= \int_{S_c} (\text{grad}N_j) \kappa (\text{grad}N_i)^T \, dS \\
H_{ji} &= \int_{C_c,3} N_j h N_i \, dC \\
L_{Q,j} &= \int_{S_c} N_j Q \, dS \\
L_{q2,j} &= -\int_{C_c,2} N_j \bar{q}_n \, dC \\
L_{q3,j} &= \int_{C_c,3} N_j hT_a \, dC
\end{align*}
\]
SOFEA: MATLAB FEM Toolbox

• 1D Meshing routine:

```matlab
for j = 1:n+1
    fens=[fens fenode(struct ('id',j,'xyz',[x]))];
    x = x+(L/n);
end

gcells = [];
for j = 1:n
    gcells = [gcells gcell_l2(struct('id',j,'conn',[j j+1]))];
end
```

• Construct finite element block:

```matlab
feb = feblock_defor_taut_wire(struct ('mater', mater_defor, ...
    'gcells', gcells, ...
    'integration_rule', simpson_1_3_rule, ...
    'P',P));

geom = field(struct ('name', ['geom'], 'dim', 1, 'fens', fens));
w = 0*clone(geom,'w');
```
SOFEA: MATLAB FEM Toolbox

• Apply boundary conditions:

```matlab
fenids=[1]; prescribed=[1]; component=[1]; val=0;
w  = set_ebc(w, fenids, prescribed, component, val);
w  = apply_ebc (w);
w  = numbereqns (w);
```

• Assemble and solve equations:

```matlab
K = start (dense_sysmat, get(w, 'neqns'));
K = assemble (K, stiffness(feb, geom, w));
bl = body_load(struct ('magn',inline(num2str(q))));
F = start (sysvec, get(w, 'neqns'));
F = assemble (F, body_loads(feb, geom, w, bl));
w = scatter_sysvec(w, get(K,'mat')\get(F,'vec'));
```
SOFEA: MATLAB FEM Toolbox

• Note: latest version is now called FAESOR:
  
  http://hogwarts.ucsd.edu/~pkrysl/faesor/faesor_publish.html
Results

• Steady-state solution for a thermally insulating medium, with a variable temperature placed along one surface
Thermal Conduction: Results

- Steady-state thermal gradient between two adjoining walls with different temperatures
Thermal Conduction: Results

- Transient cooling of a shrink-fitted assembly:
  - In red: highest temperature vs. cooling time
  - In blue: lowest temperature vs. cooling time
Error Evaluation

- Error for linear elements T3 higher overall than quadratic elements T6; both decrease almost quadratically with mesh size
Next Class

• Is on Friday, Feb. 22
• Next time, we will cover other FEM applications in electronic transport