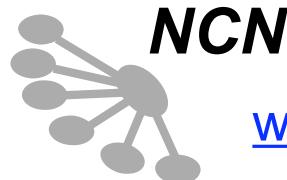


# **EE-612:**

# **Lecture 4**

# **MOS Capacitors**

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West Lafayette, IN USA  
Fall 2006



[www.nanohub.org](http://www.nanohub.org)

Lundstrom EE-612 F06

**PURDUE**  
UNIVERSITY

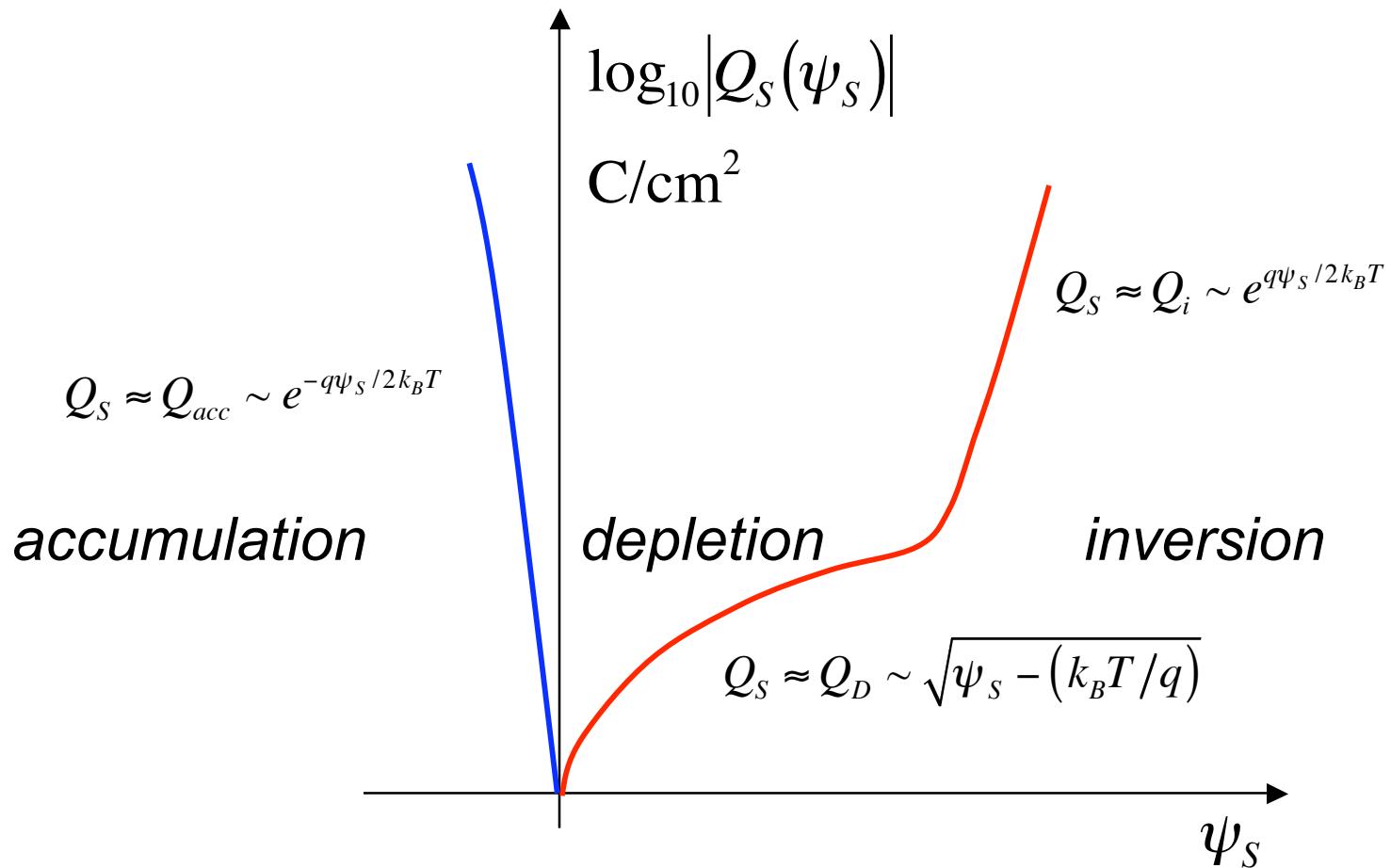
# outline

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- 1) Review
- 2) Gate voltage / surface potential relation
- 3) The flatband voltage
- 4) MOS capacitance vs. voltage
- 5) Gate voltage and inversion layer charge

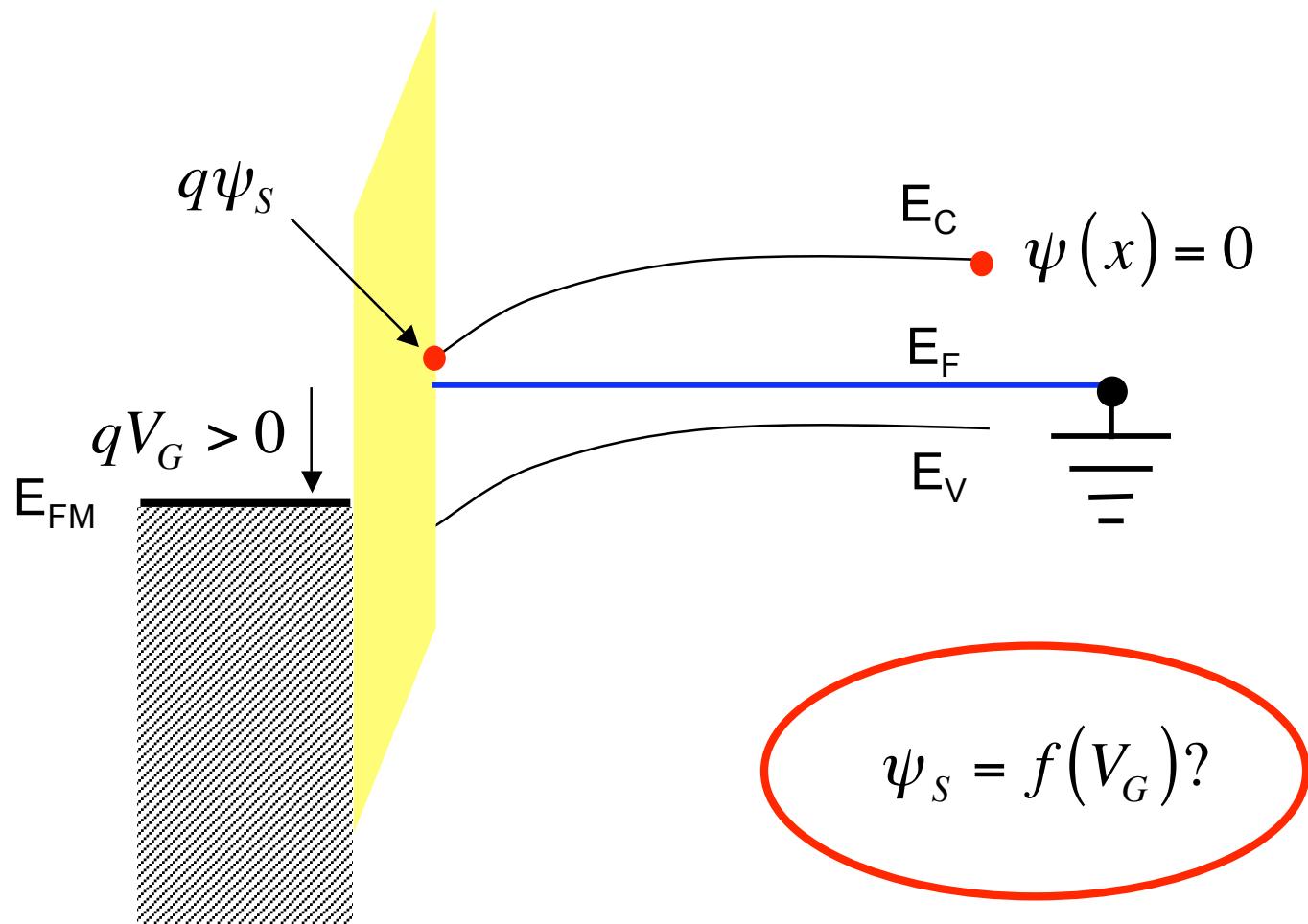
# 1) review

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# 1) review

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# outline

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- 1) Review
- 2) **Gate voltage / surface potential relation**
- 3) The flatband voltage
- 4) MOS capacitance vs. voltage
- 5) Gate voltage and inversion layer charge

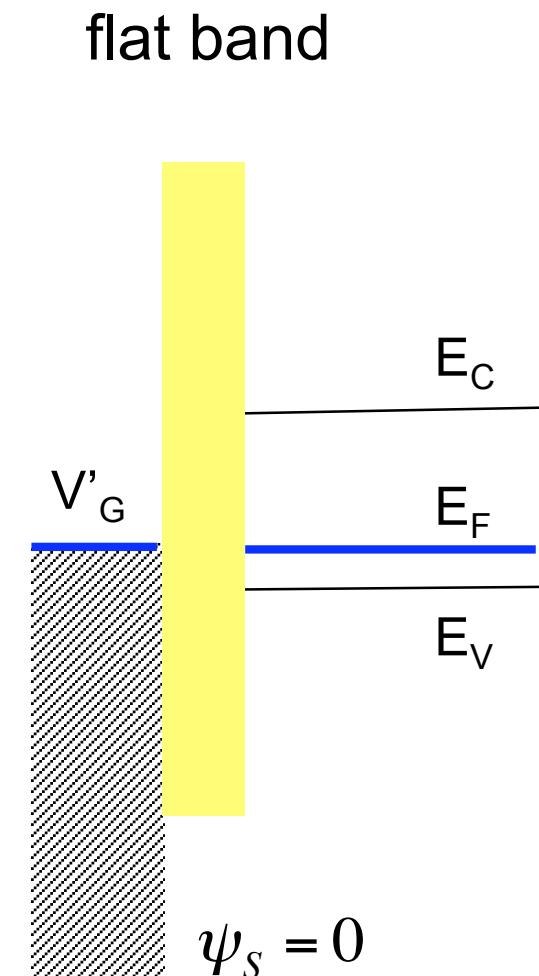
## 2) flat-band voltage

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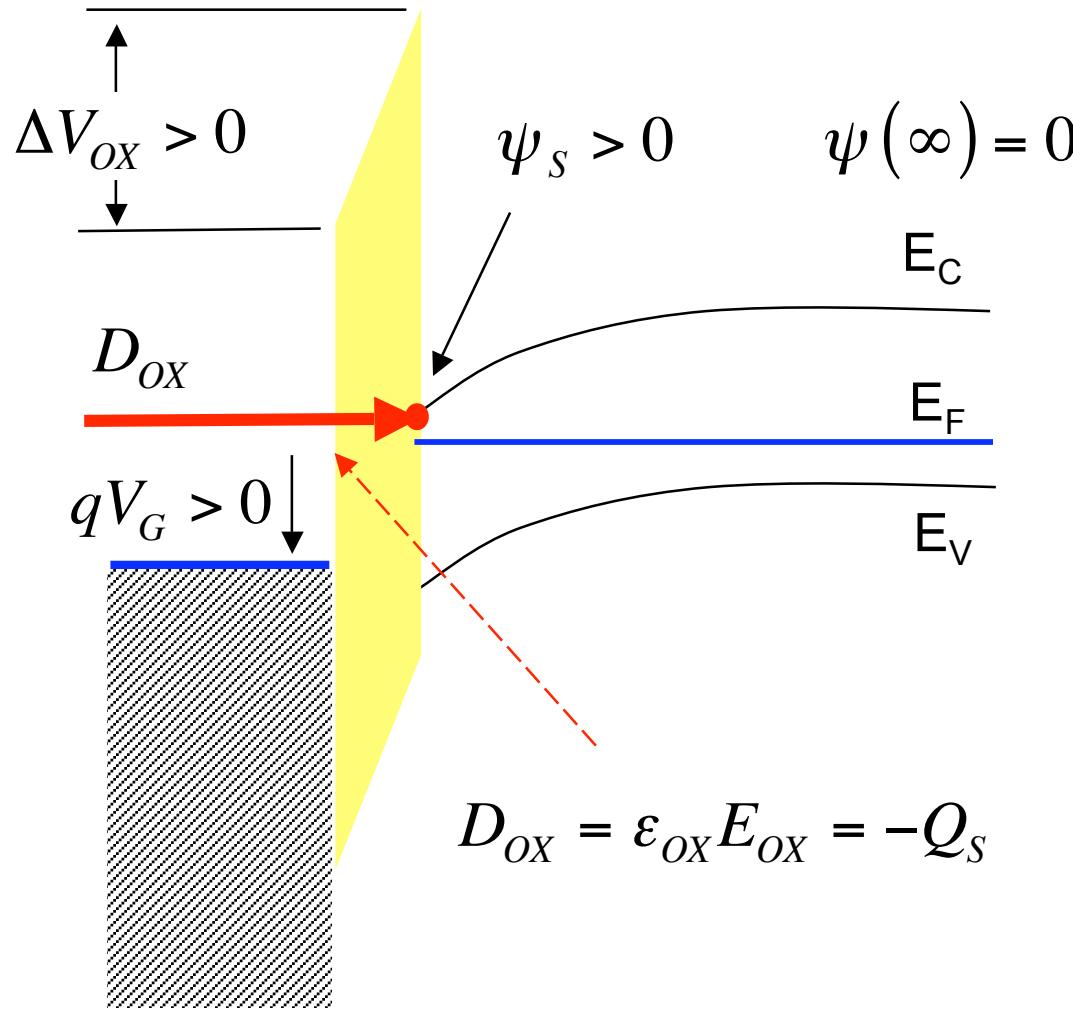
- when the gate Fermi level lines up with the semiconductor Fermi level, the bands are flat in the semiconductor
- this occurs at  $V'_G = 0$  when the gate electrode workfunction equals the semiconductor workfunction

$$\Phi_M = \Phi_S \text{ eV}$$

$$\phi_M = \phi_S \text{ V}$$



# gate voltage and $\psi_S$



$$V'_G = \psi_S + \Delta V_{OX}$$

$$V'_G = \psi_S + E_{OX} t_{OX}$$

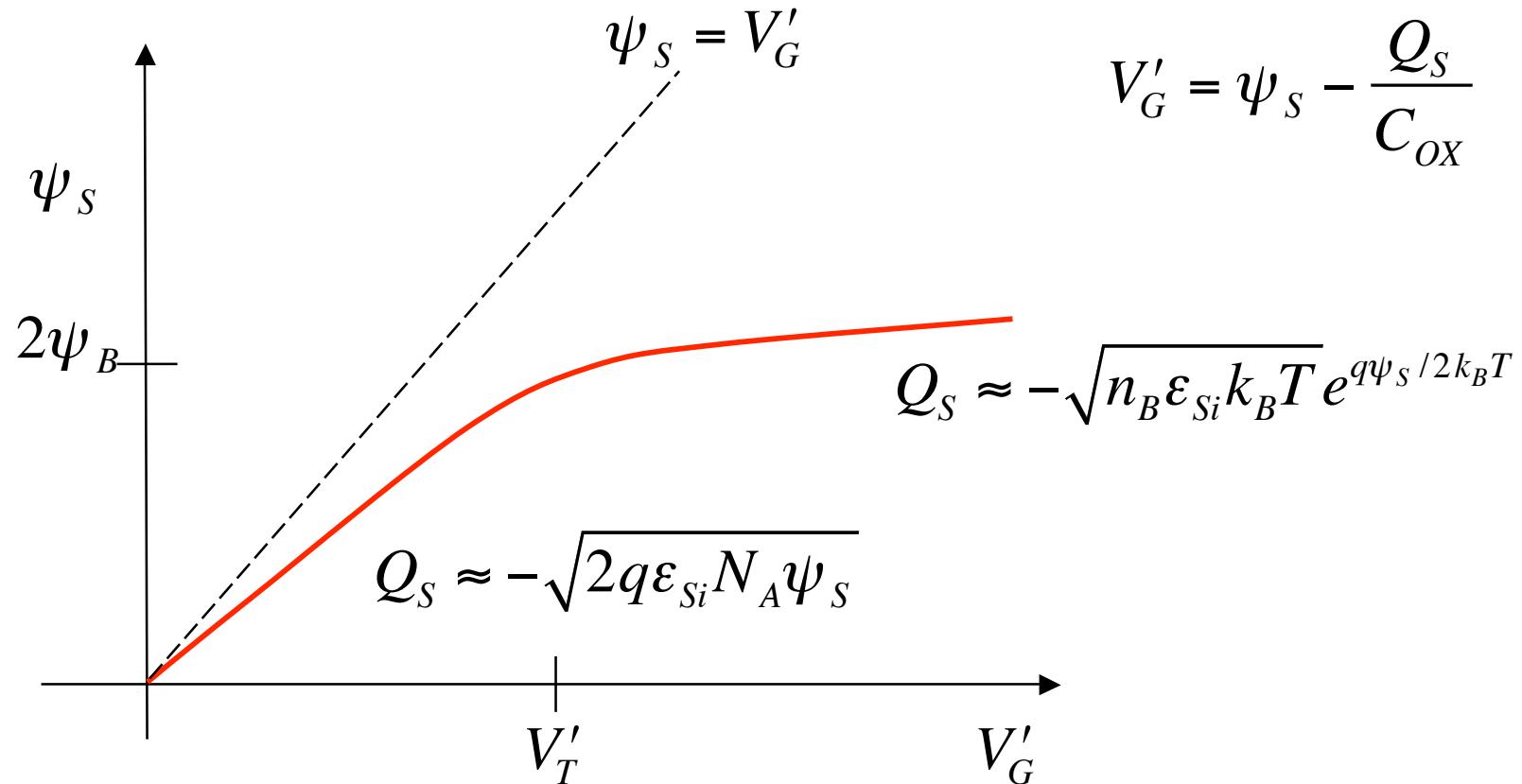
$$V'_G = \psi_S - \frac{Q_S}{\epsilon_{OX}} t_{OX}$$

$$V'_G = \psi_S - \frac{Q_S}{C_{OX}}$$

$$C_{OX} = \frac{\epsilon_{OX}}{t_{OX}} \text{ F/cm}^2$$

# gate voltage and $\psi_S$

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# threshold voltage, $V_T$

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$$\psi_S = 2\psi_B \quad \text{onset of inversion}$$

$$V'_G = V'_T = 2\psi_B - \frac{Q_S(2\psi_B)}{C_{OX}}$$

$$Q_S(2\psi_B) \approx Q_D(2\psi_B) = -\sqrt{2q\epsilon_{Si}N_A(2\psi_B)}$$

$$V'_T = 2\psi_B + \sqrt{2q\epsilon_{Si}N_A(2\psi_B)} / C_{OX}$$

# outline

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- 1) Review
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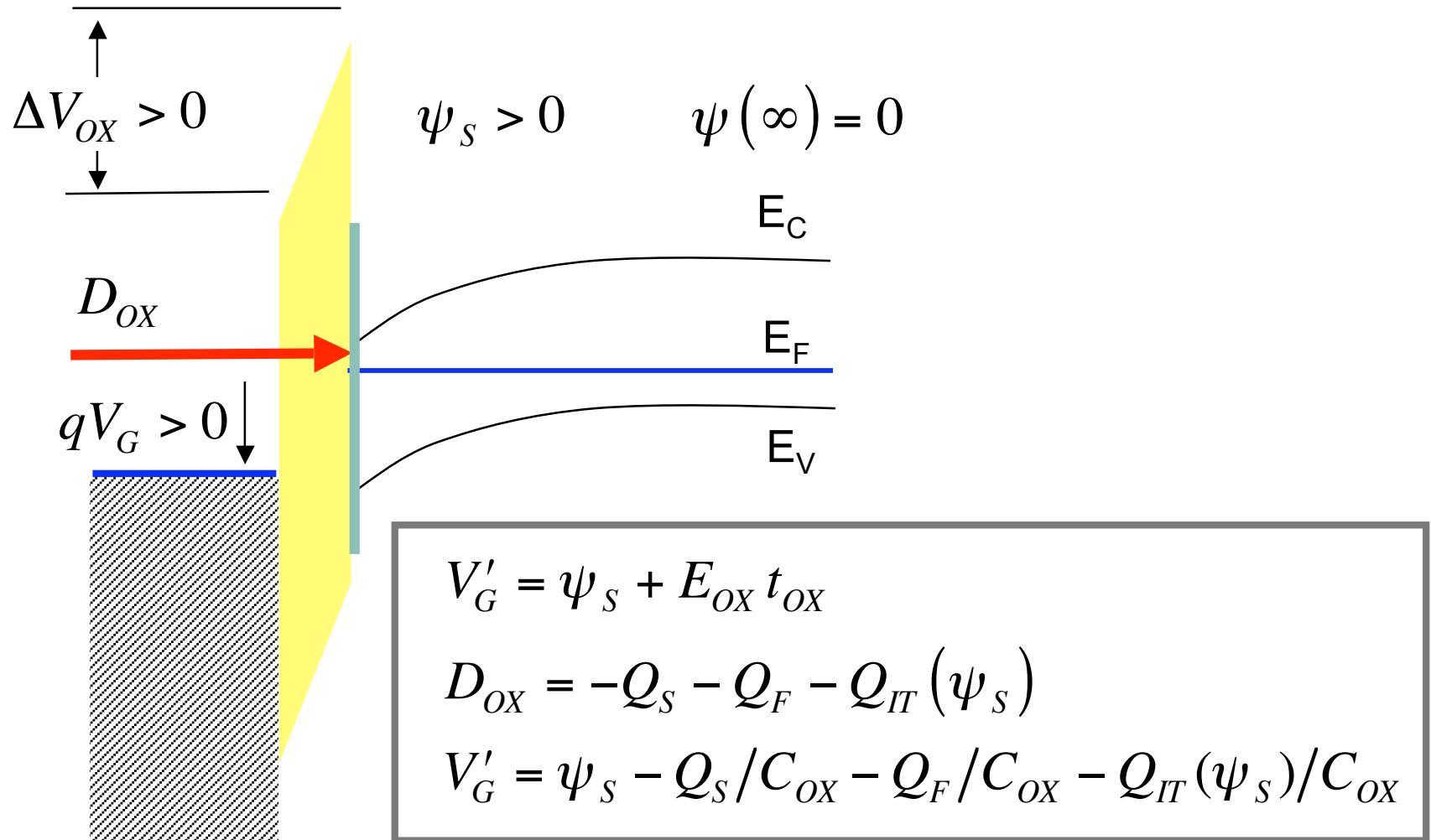
# flatband voltage

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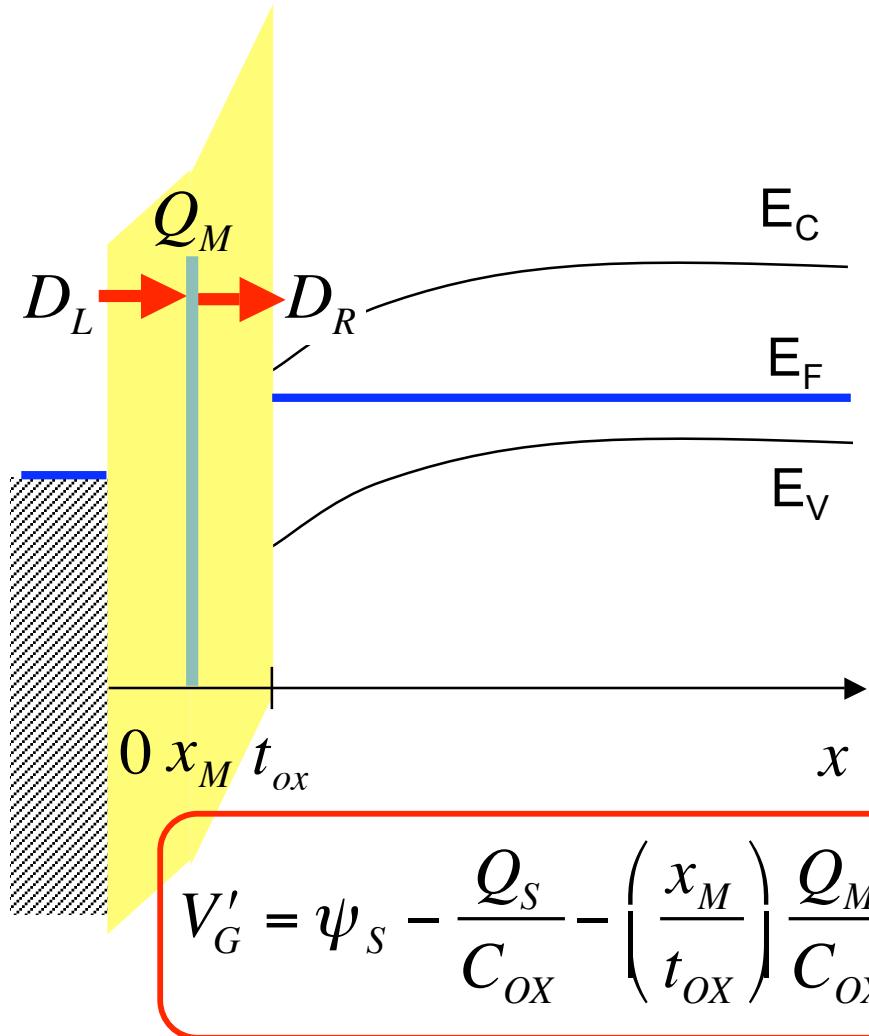
In an ideal MOS-C,  $\psi_s = 0$  when  $V_G' = V_{FB} = 0$ .

In a real MOS-C, ***charges at the oxide-silicon interface***, in the oxide, and gate-semiconductor ***workfunction differences*** shift the flatband voltage.

# interface charge



# charge in the oxide



$$-D_L + D_R = Q_M$$

$$D_R = -Q_S$$

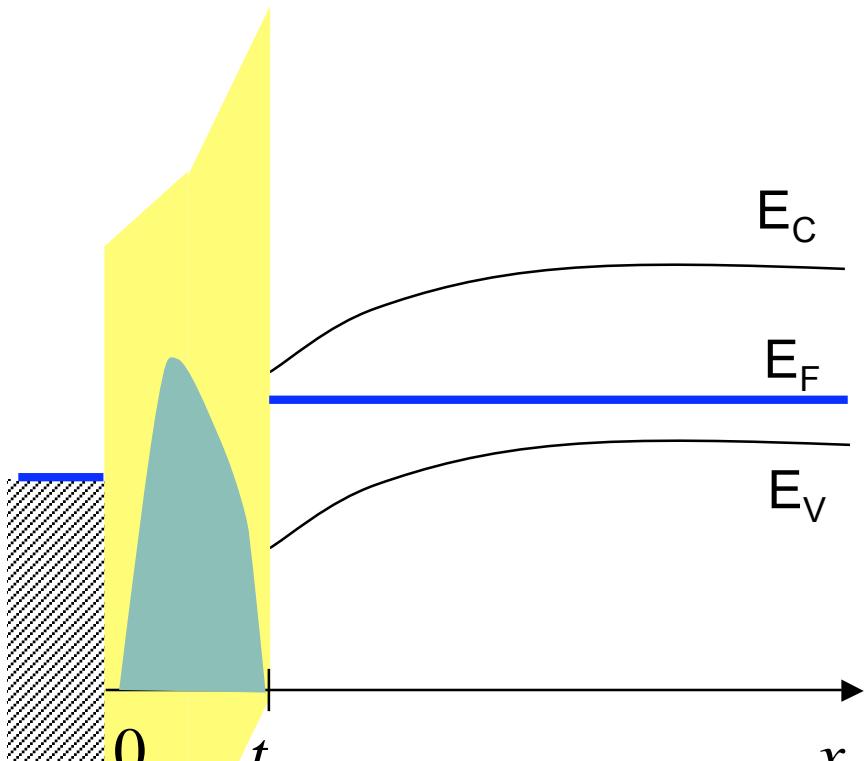
$$E_R = -Q_S / \epsilon_{OX}$$

$$E_L = E_R - Q_M / \epsilon_{OX}$$

$$\Delta V_{OX} = x_M E_L + (t_{OX} - x_M) E_R$$

$$\Delta V_{OX} = -\frac{Q_S}{C_{OX}} - \left( \frac{x_M}{t_{OX}} \right) \frac{Q_M}{C_{OX}}$$

# distributed charge in the oxide



$$V'_G = \psi_s - \frac{Q_s}{C_{OX}} - \left( \frac{x_M}{t_{ox}} \right) \frac{Q_M}{C_{OX}}$$

$$Q_M = \int_0^{t_{ox}} Q(x) dx$$

$$x_M = \frac{\int_0^{t_{ox}} x Q(x) dx}{\int_0^{t_{ox}} Q(x) dx}$$

## additional information

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For more information on the oxide-silicon interface and the origin of the various charges, see:

- 1) R.F. Pierret, *Semiconductor Device Fundamentals*, pp. 650-671  
Addison-Wesley, 1996
  
- 2) J.A. Del Alamo, EE 6720J/ 3.43J Integrated Microelectronic Devices, Fall 2002  
Lecture 22: “The Si Surface and MOS Structure”

Available from MIT OpenCourseWare:

<http://ocw.mit.edu>

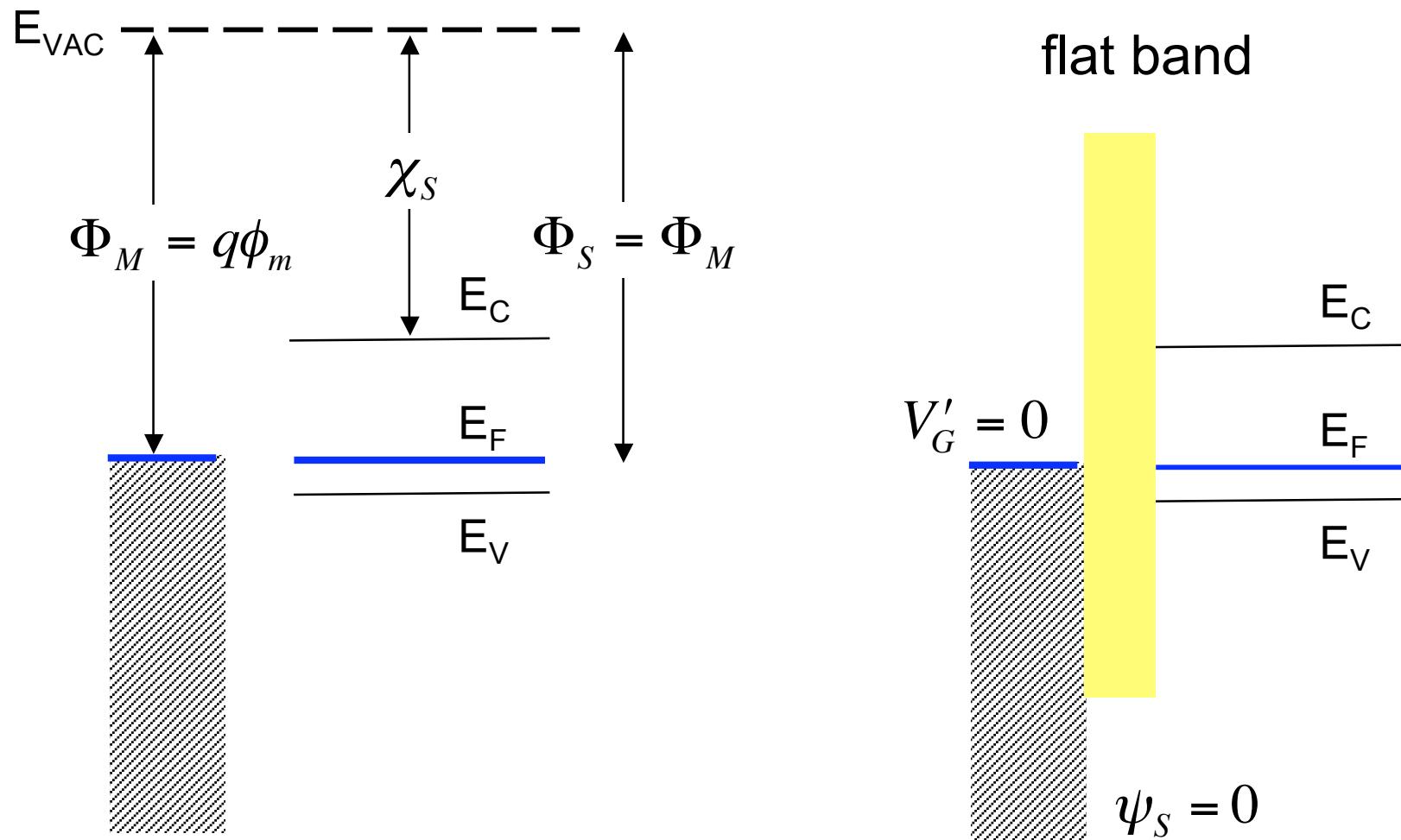
‘Electrical Engineering and Computer Science’  
‘Graduate’

# gate oxides 2006

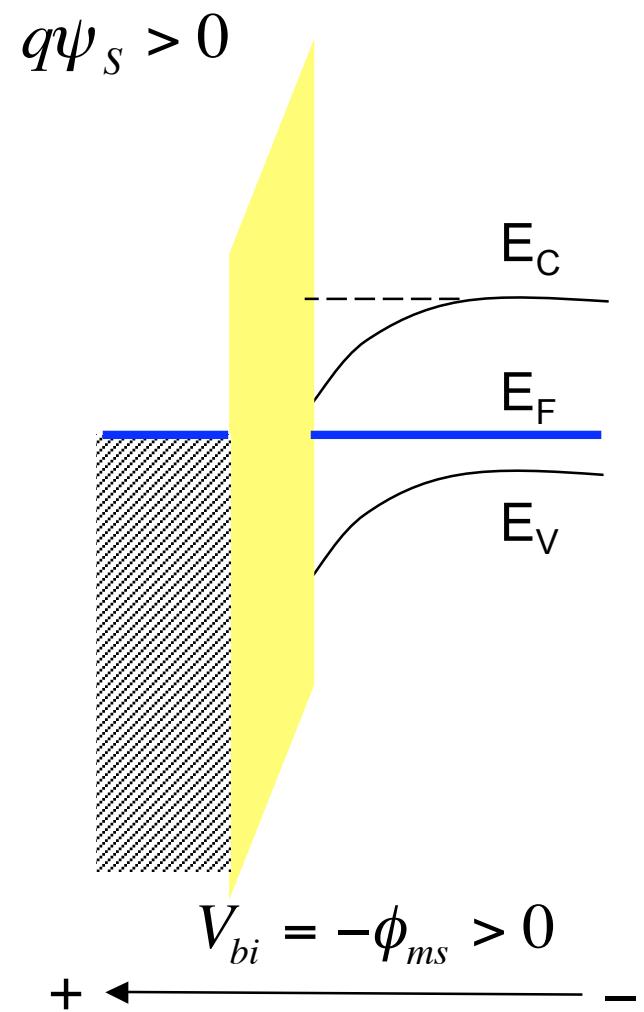
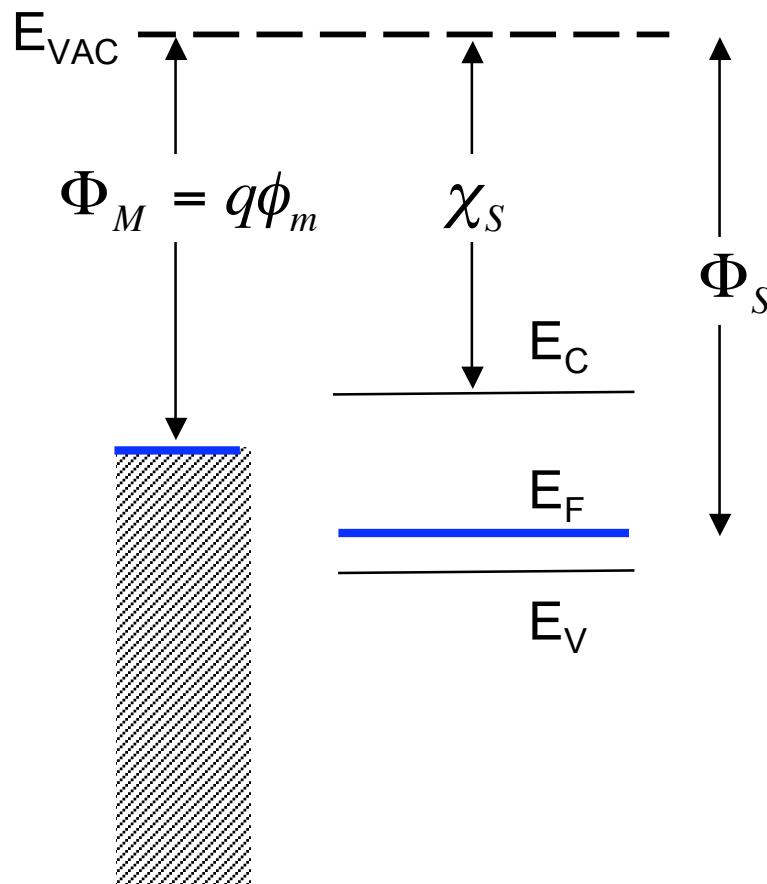
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- Typically SiON with  $k \sim 4.6$  for 15% N<sub>2</sub> (not SiO<sub>2</sub> with  $k = 3.9$ ).
- Use of thicker oxides give less gate leakage.
- SiON is more resistance to boron penetration.
- But, SiON degrades mobility and reliability (NBTI). Engineering the N<sub>2</sub> profile may help.
- Typically grown in dry O<sub>2</sub>, followed by plasma nitridation and rapid thermal anneal. Results in 10-15% N<sub>2</sub>.
- NIT  $\sim 5 \times 10^{10} \text{ cm}^{-2}$
- Oxide scaling has stopped at  $\sim 1.2 \text{ nm}$  due to gate leakage. Introduction of high-k postponed to at least 32 nm node.

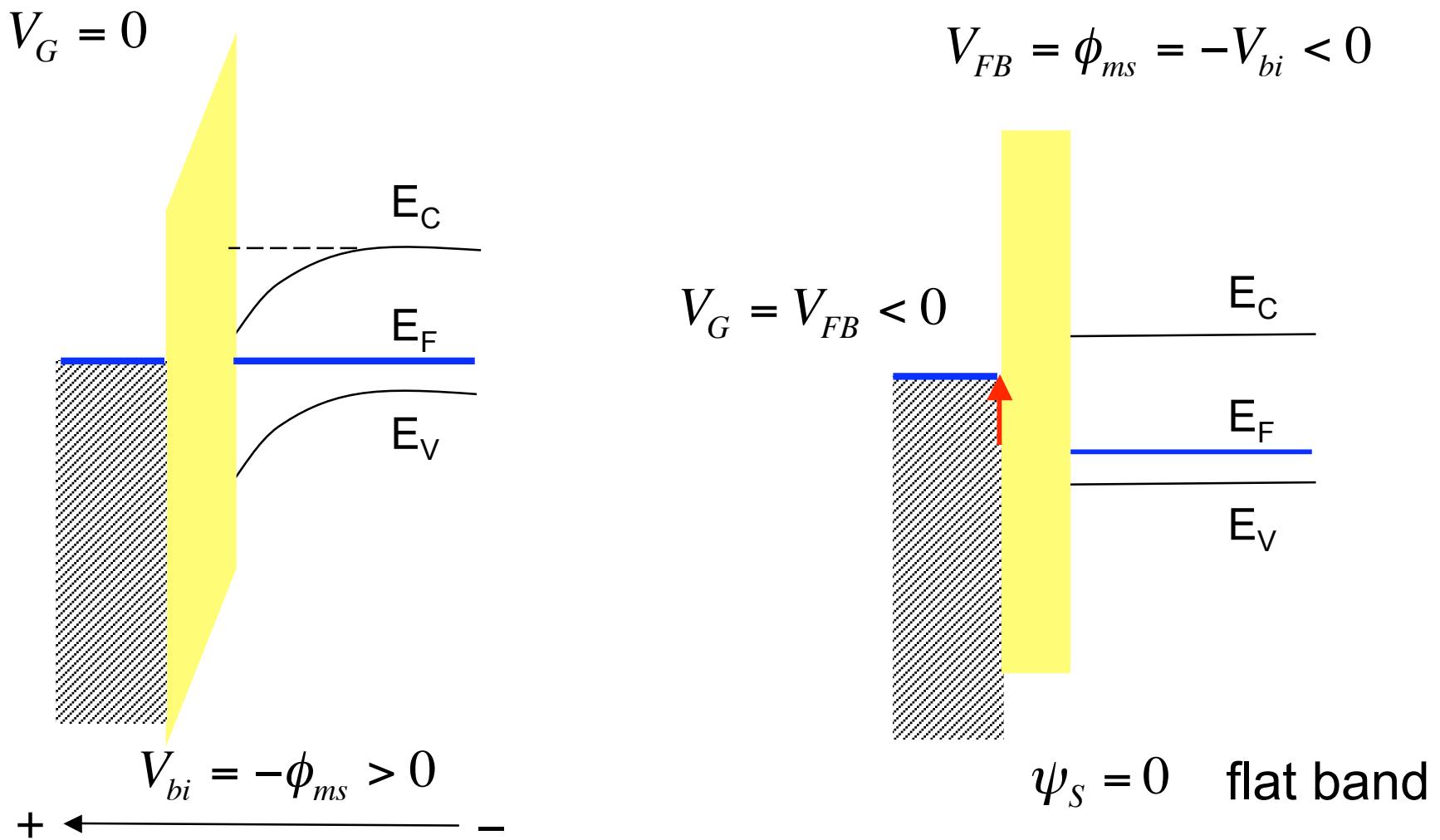
# gate-semiconductor workfunction differences



$$\Phi_M < \Phi_S$$



# flatband voltage



# flatband voltage

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recall:

$$V'_G = \psi_s - \frac{Q_s}{C_{ox}}$$

$$V'_G = V_G - V_{FB}$$

$$V_G = V_{FB} + \psi_s - \frac{Q_s}{C_{ox}}$$

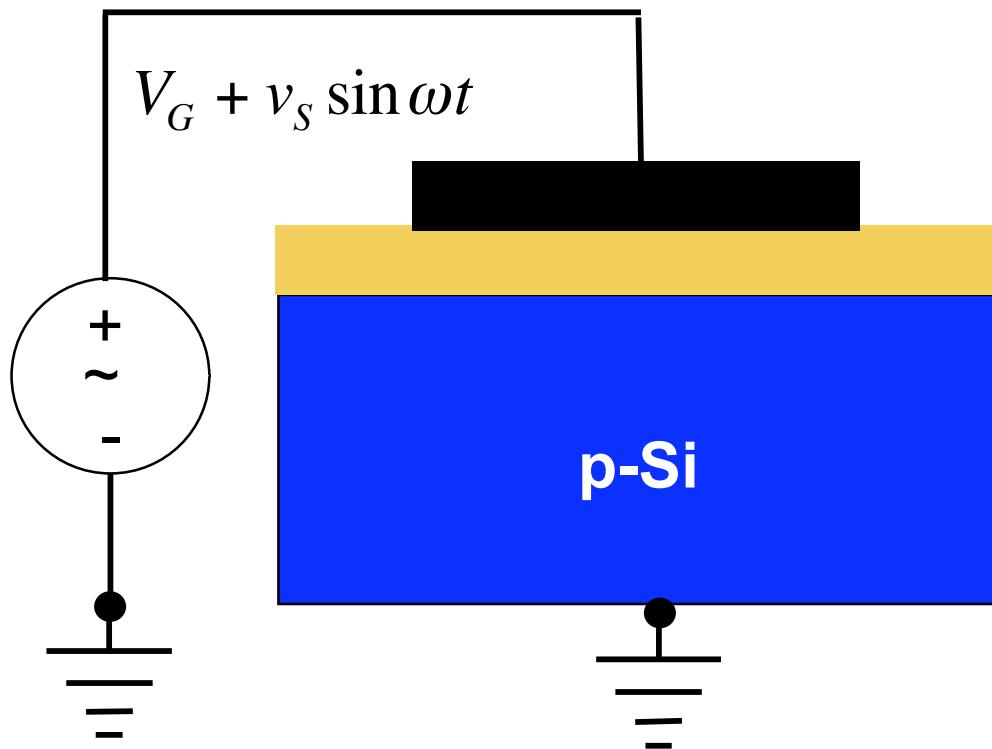
$$V_{FB} = \phi_{ms} - \frac{Q_F}{C_{ox}}$$

# outline

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- 1) Review
- 2) Gate voltage / surface potential relation
- 3) The flatband voltage
- 4) MOS capacitance vs. voltage**
- 5) Gate voltage and inversion layer charge

# capacitance



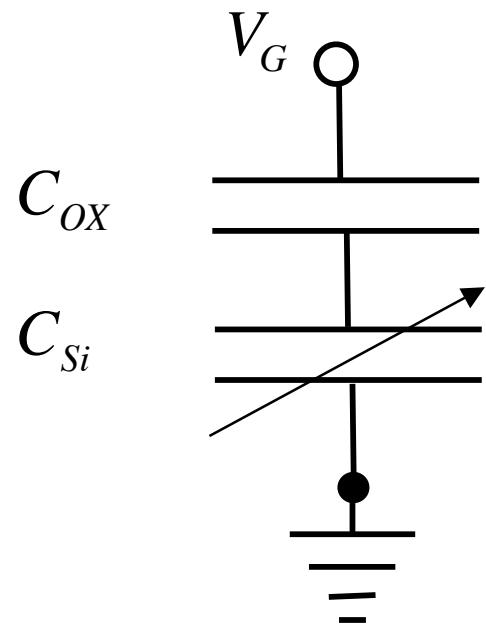
$$C_G \equiv \frac{dQ_G}{dV_G} = \frac{d(-Q_S)}{dV_G}$$

$$V_G = V_{FB} + \psi_S - \frac{Q_S}{C_{OX}}$$

$$\frac{dV_G}{d(-Q_S)} = \frac{d\psi_S}{d(-Q_S)} + \frac{1}{C_{OX}}$$

$$\frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_{OX}}$$

# capacitance



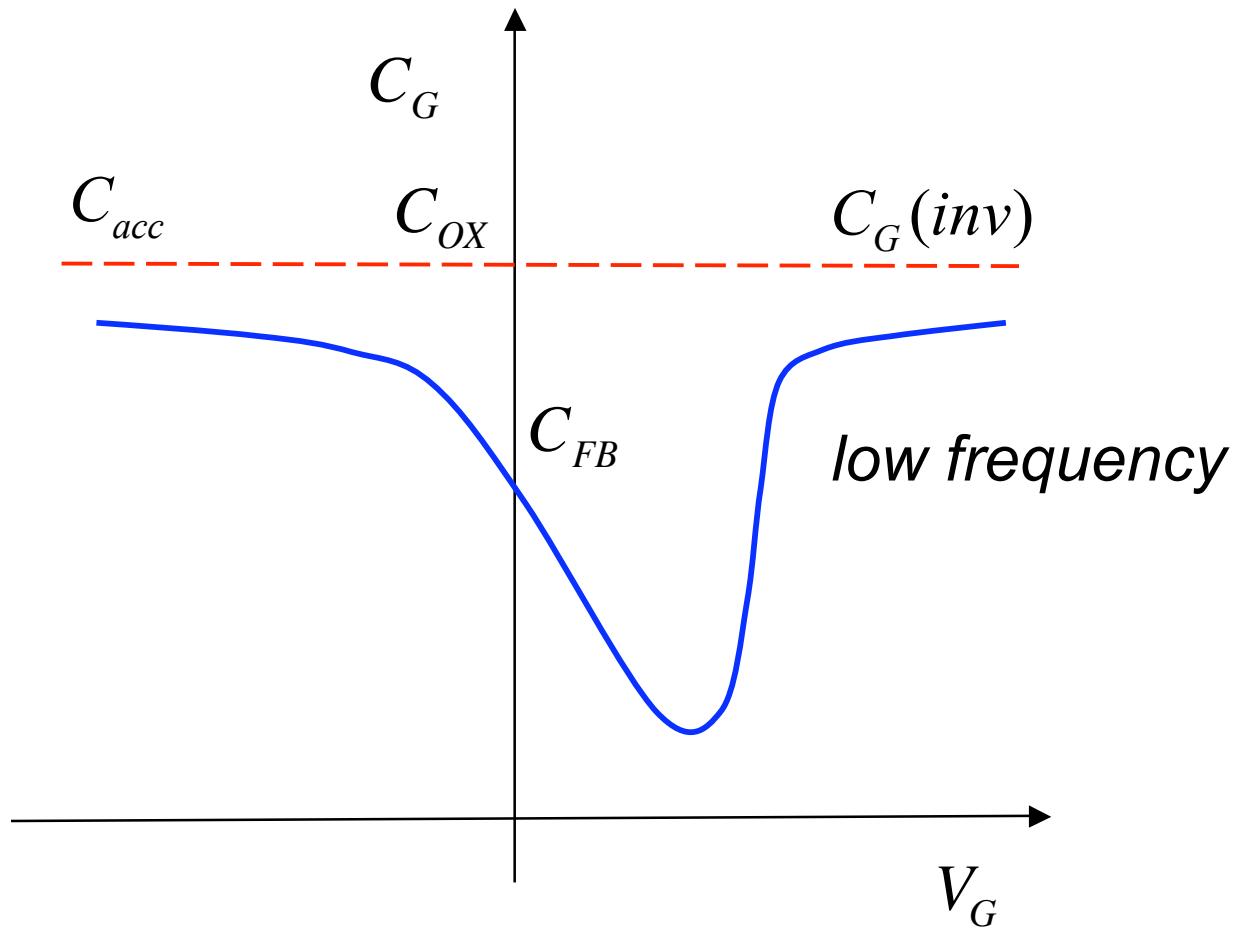
**which we already  
understand!**

$$\frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_{OX}}$$

$$C_S \equiv \frac{d(-Q_S)}{d\psi_S}$$

$$Q_S(\psi_S)$$

# capacitance vs. voltage



## (i) accumulation capacitance

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$$C_S \equiv \frac{d(-Q_{acc})}{d\psi_S}$$

$$Q_{acc} \sim e^{-q\psi_S/2k_B T}$$

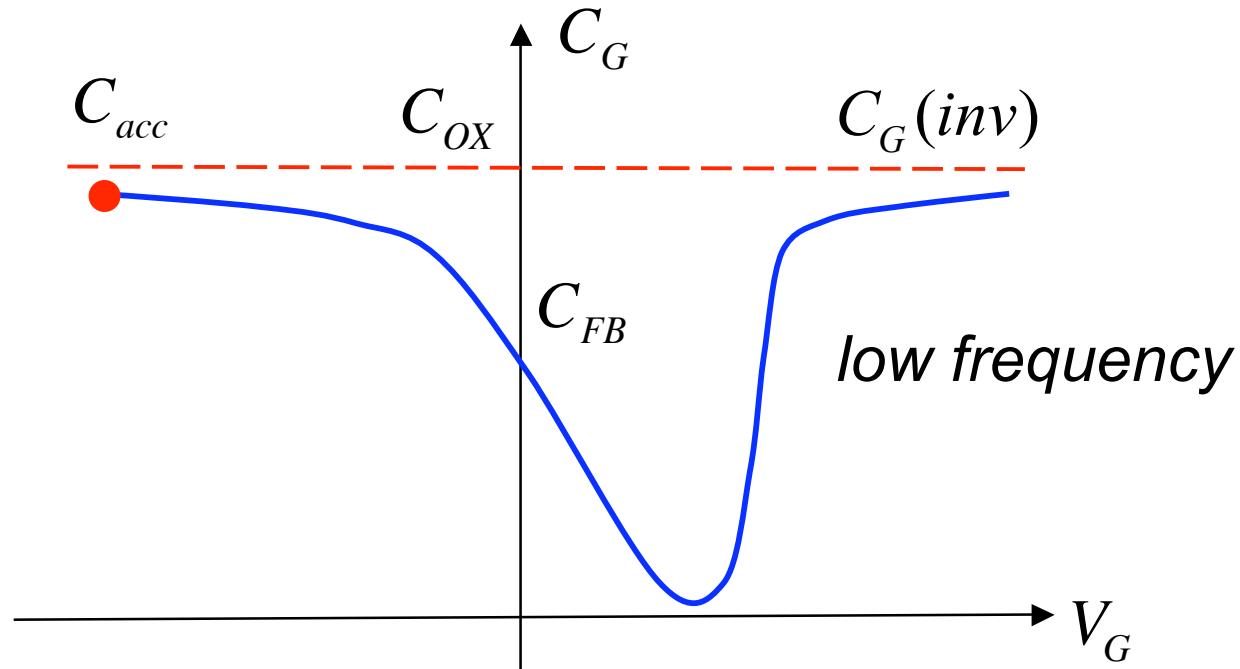
$$V_G = V_{FB} + \psi_S - \frac{Q_{acc}}{C_{OX}}$$

$$C_S \equiv \frac{Q_{acc}}{(2k_B T / q)}$$

$$Q_{acc} = C_{OX} (V_G - V_{FB} - \psi_S)$$

$$C_S \equiv \frac{C_{OX} (V_G - V_{FB} - \psi_S)}{(2k_B T / q)}$$

# accumulation capacitance



$$\frac{1}{C_{acc}} = \frac{1}{C_s} + \frac{1}{C_{OX}} = \frac{1}{C_{OX}} \left( 1 + \frac{2k_B T / q}{(V_G - V_{FB} - \psi_S)} \right) \approx \frac{1}{C_{OX}}$$

# accumulation layer thickness

---

$$C_s \equiv \frac{Q_{acc}}{(2k_B T / q)} = \frac{\varepsilon_{Si} |E_s|}{(2k_B T / q)}$$

$$C_s \equiv \frac{\varepsilon_{Si}}{t_{acc}}$$

$$\Rightarrow t_{acc} = \frac{(2k_B T / q)}{|E_s|}$$

*Note that this is 2 times larger than the expression we obtained in lecture 3.*

# flat band capacitance

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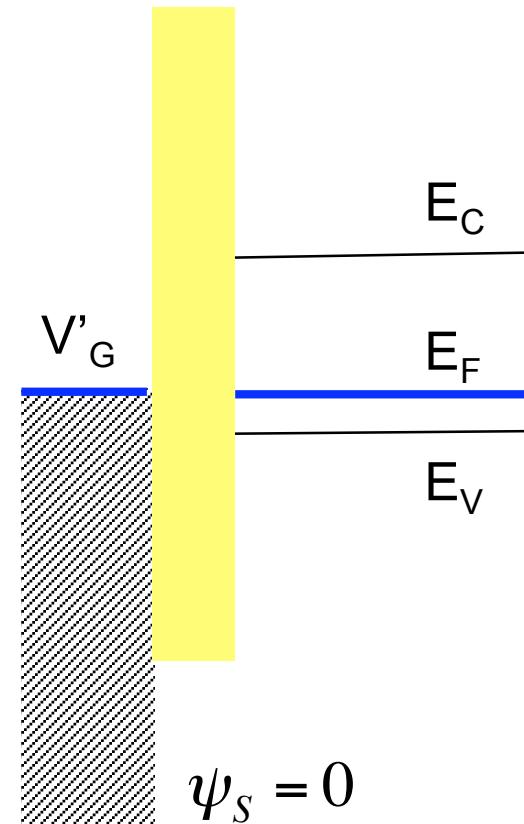
$$Q_S = 0$$

$$C_S(FB) = \frac{-dQ_S}{d\psi_S} = \frac{\epsilon_{Si}}{L_D}$$

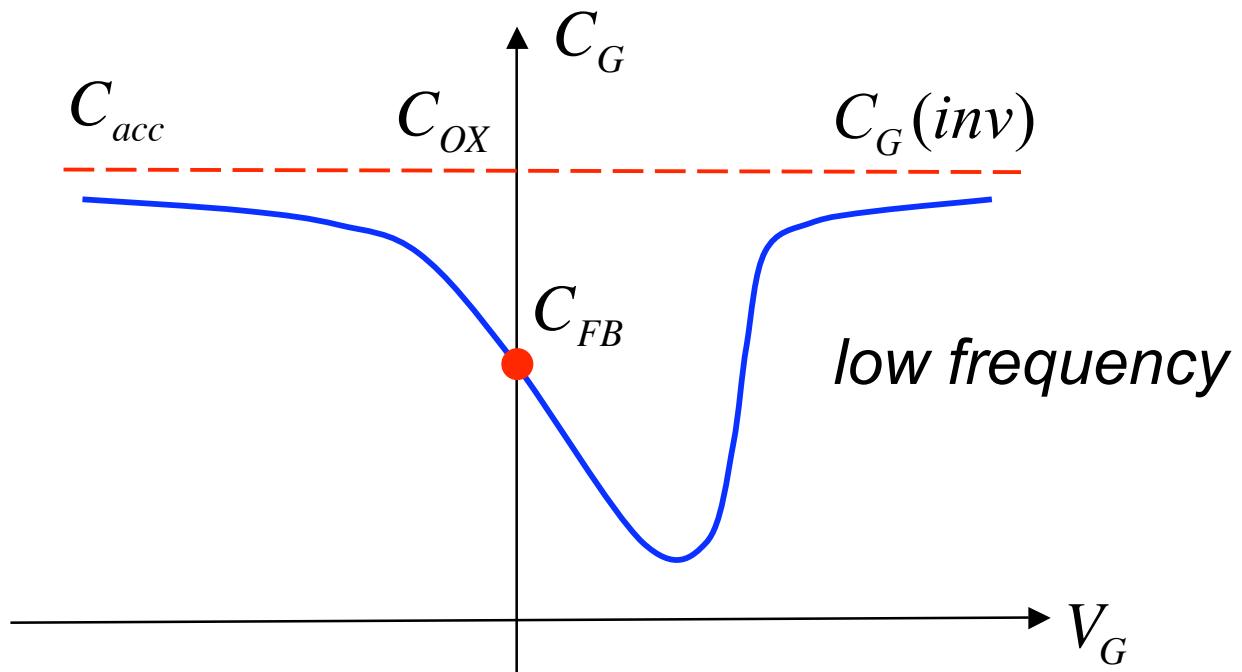
$$L_D = \sqrt{\frac{\epsilon_{Si} k_B T}{q^2 N_A}}$$

See Lundstrom's notes on the Poisson-Boltzmann eqn. for a derivation of  $C_{FB}$ .

flat band



# flat band capacitance



$$\frac{1}{C_{FB}} = \frac{L_D}{\epsilon_{Si}} + \frac{1}{C_{OX}} \quad C_{FB} < C_{OX}$$

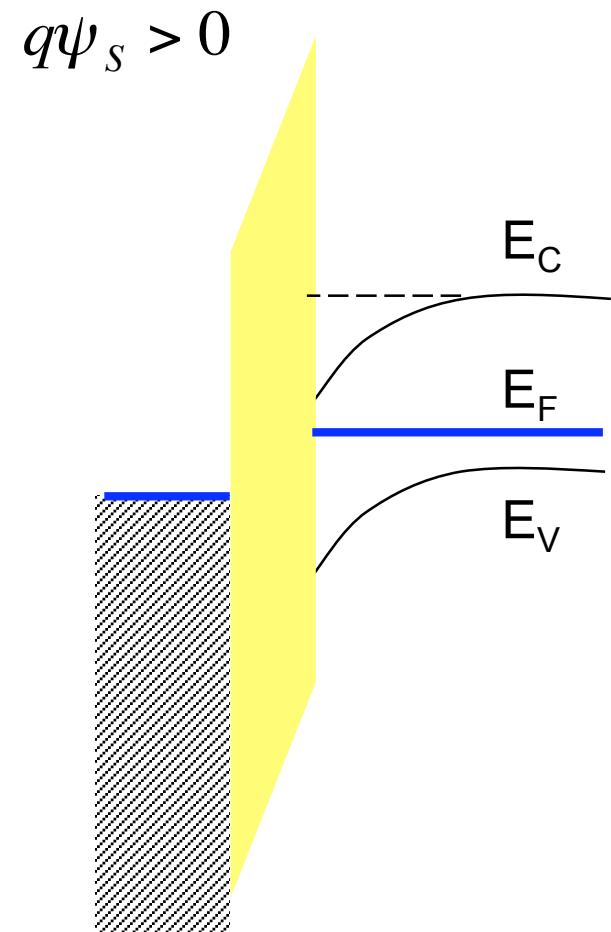
# depletion capacitance

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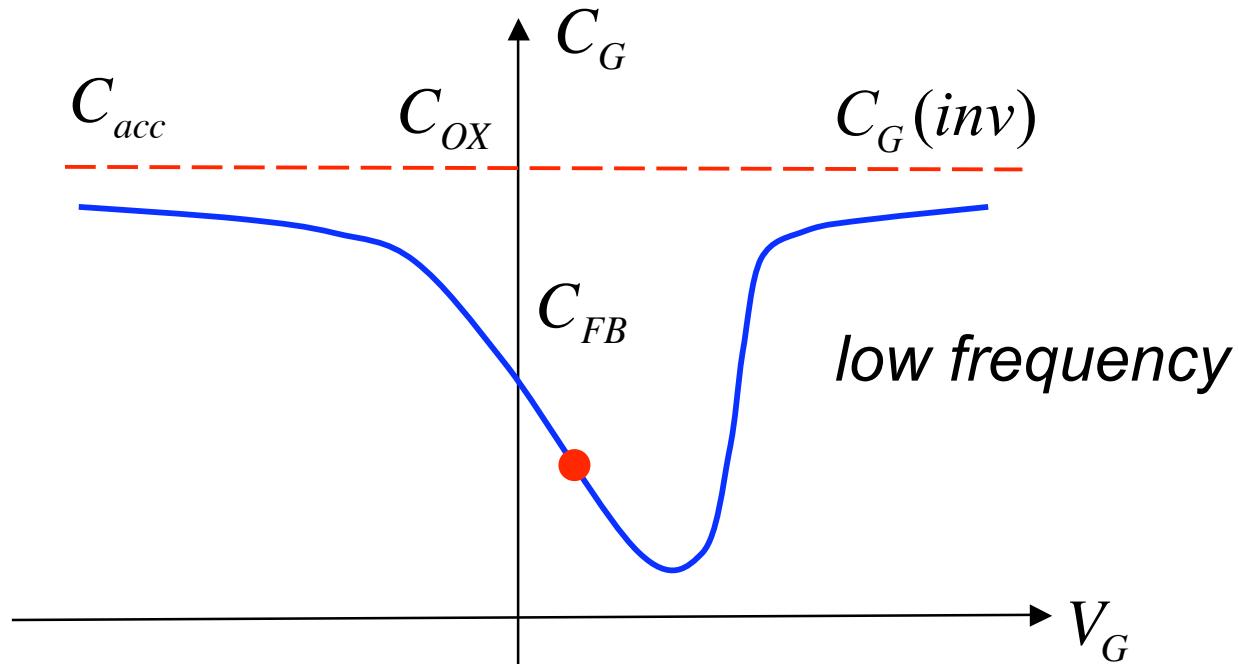
$$Q_S = Q_D = -\sqrt{2q\epsilon_{Si}N_A\psi_S}$$

$$C_S = C_D = \frac{-dQ_S}{d\psi_S} = \frac{\epsilon_{Si}}{W_D}$$

$$W_D = \sqrt{\frac{2\epsilon_{Si}\psi_S}{qN_A}}$$



# depletion capacitance



$$\frac{1}{C_{depl}} = \frac{W_D}{\epsilon_{Si}} + \frac{1}{C_{OX}} \quad C_{depl} < C_{OX}$$

# inversion capacitance

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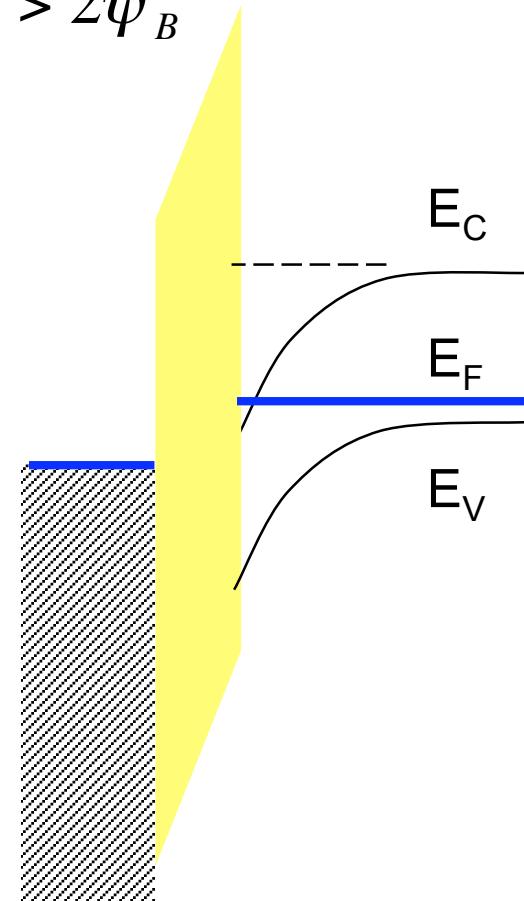
$$C_S \equiv \frac{d(-Q_{inv})}{d\psi_S}$$

$$Q_{inv} \sim e^{q\psi_S / 2k_B T}$$

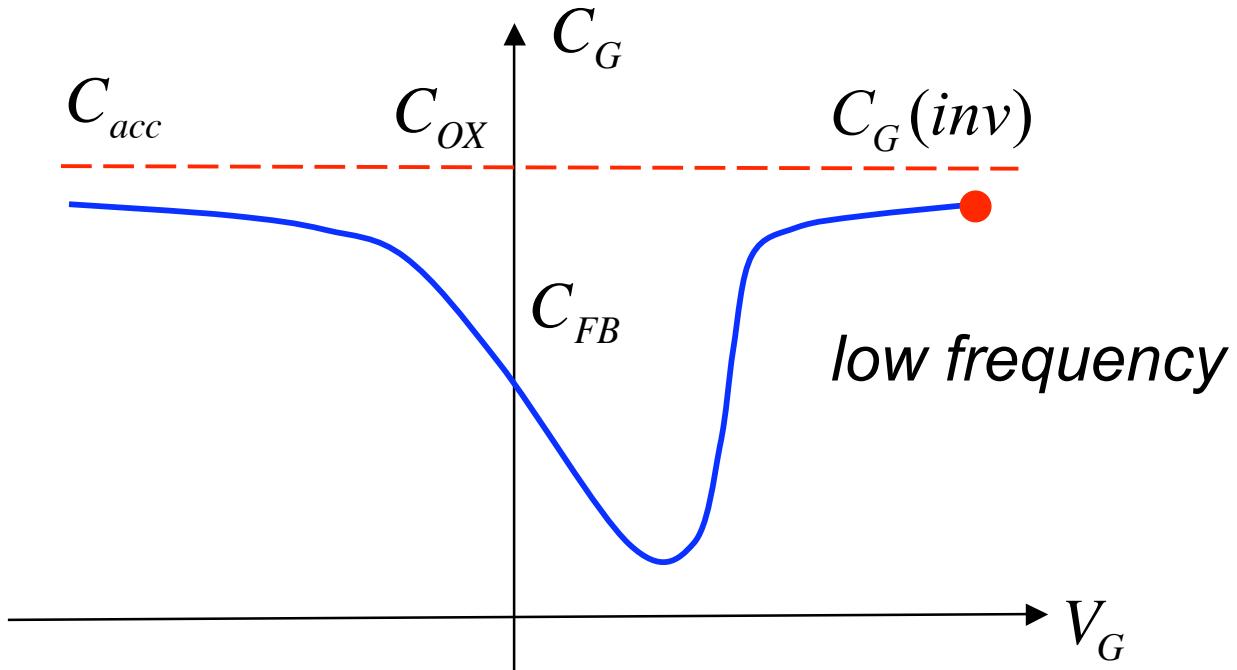
$$C_S \equiv \frac{-Q_{inv}}{(2k_B T / q)}$$

$$C_S = \frac{C_{ox}(V_G - V_T)}{(2k_B T / q)}$$

$$\psi_S > 2\psi_B$$



# inversion capacitance



$$\frac{1}{C_G(\text{inv})} = \frac{1}{C_S} + \frac{1}{C_{OX}} = \frac{1}{C_{OX}} \left( 1 + \frac{2k_B T / q}{(V_G - V_T)} \right) \approx \frac{1}{C_{OX}}$$

# inversion capacitance (high frequency)

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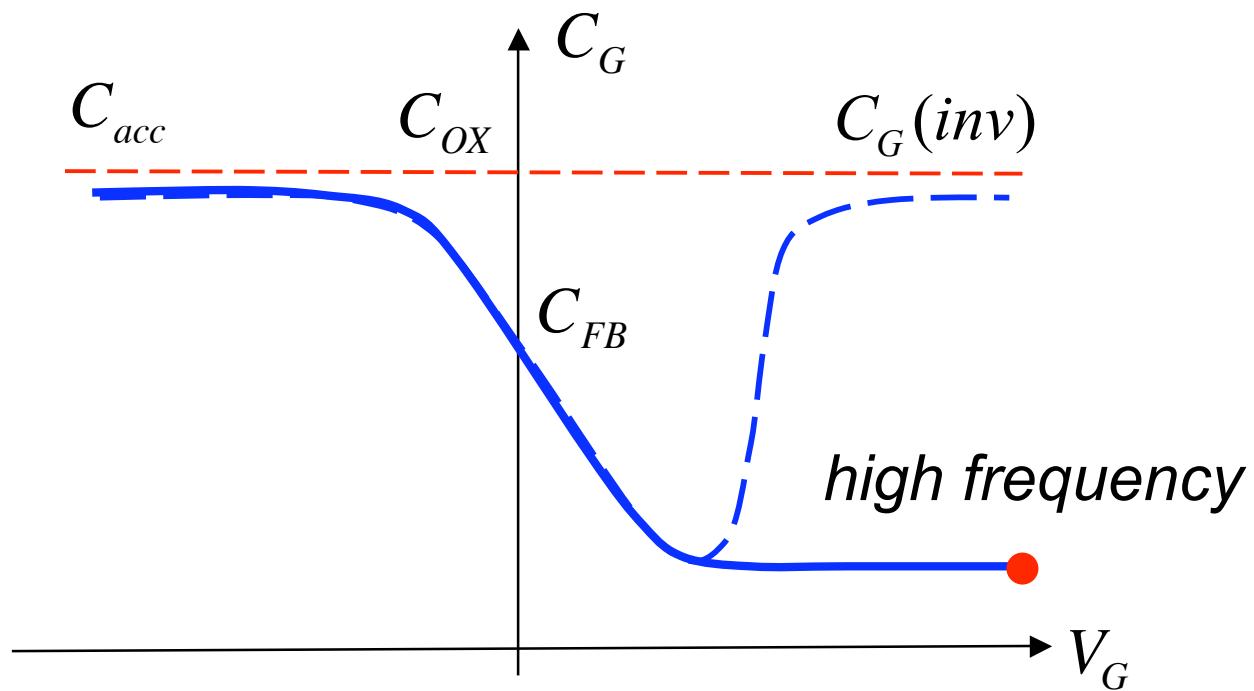
$$Q_S = Q_D(\psi_S) + Q_i(\psi_S)$$

$$C_{inv} = -\frac{dQ_S}{d\psi_S} = -\frac{dQ_D(\psi_S)}{d\psi_S} - \frac{dQ_i(\psi_S)}{d\psi_S}$$

$$C_{inv} \approx -\left. \frac{dQ_D}{d\psi_S} \right|_{\psi_S=2\psi_B} = \frac{\varepsilon_{Si}}{W_{dm}}$$

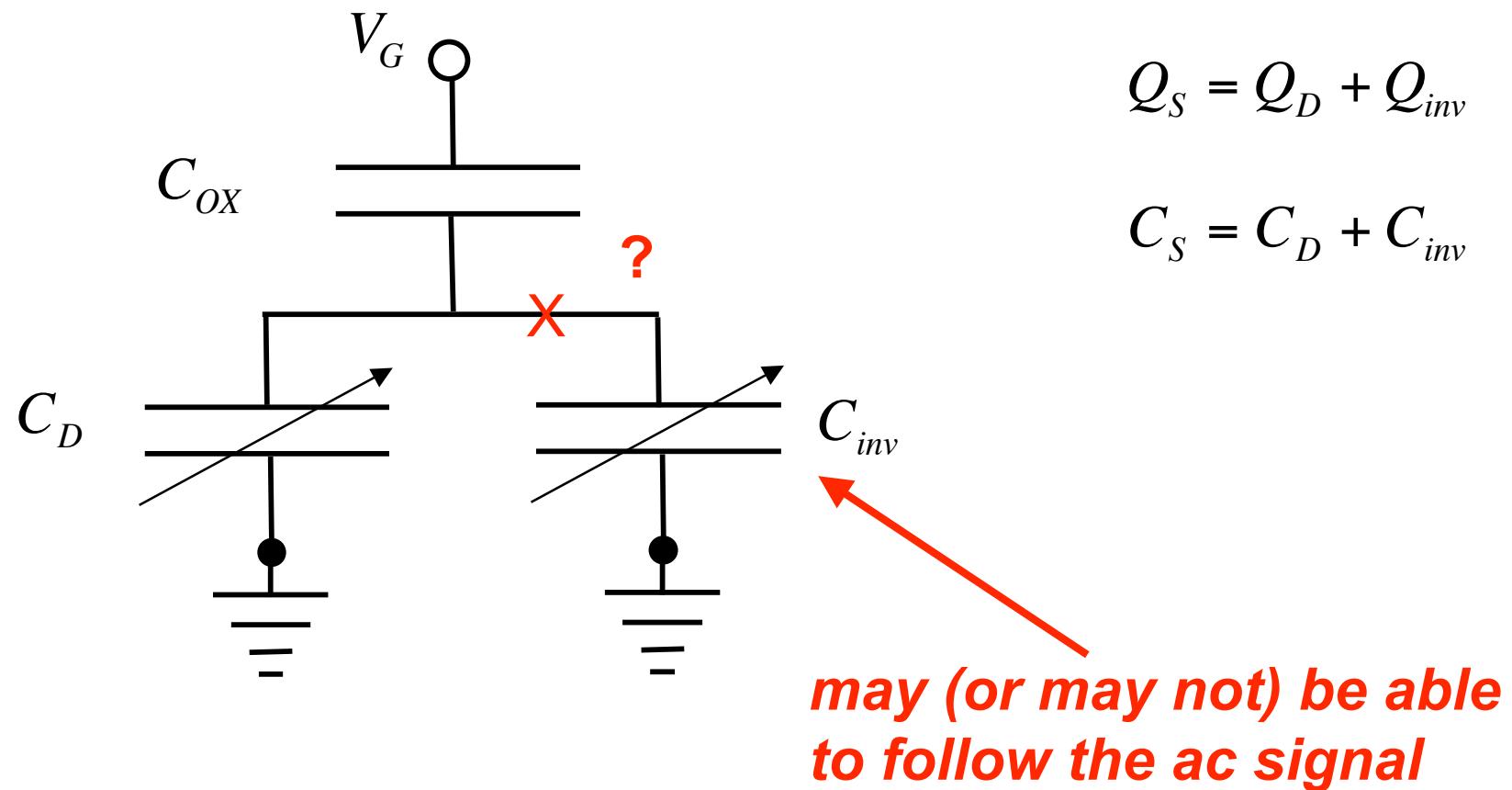
$$W_{dm}(2\psi_B) = \sqrt{\frac{2\varepsilon_{Si}(2\psi_B)}{qN_A}}$$

# inversion capacitance (high frequency)



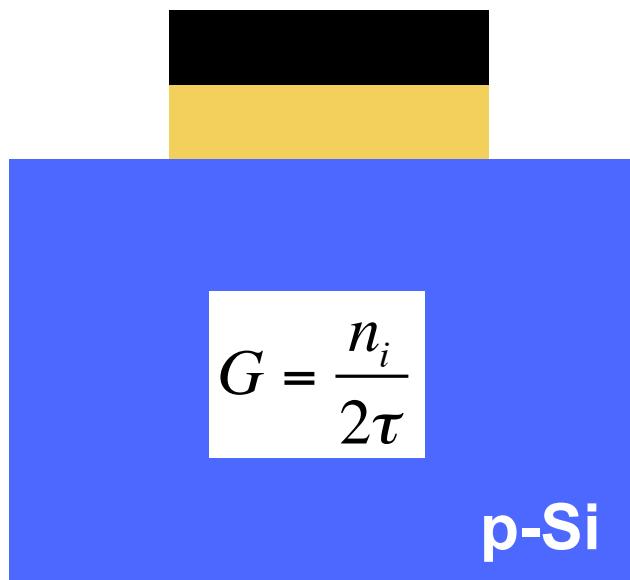
$$\frac{1}{C_G(inv)} = \frac{W_{dm}}{\epsilon_{Si}} + \frac{1}{C_{OX}} \quad C_{inv} < C_{OX}$$

# low vs. high frequency

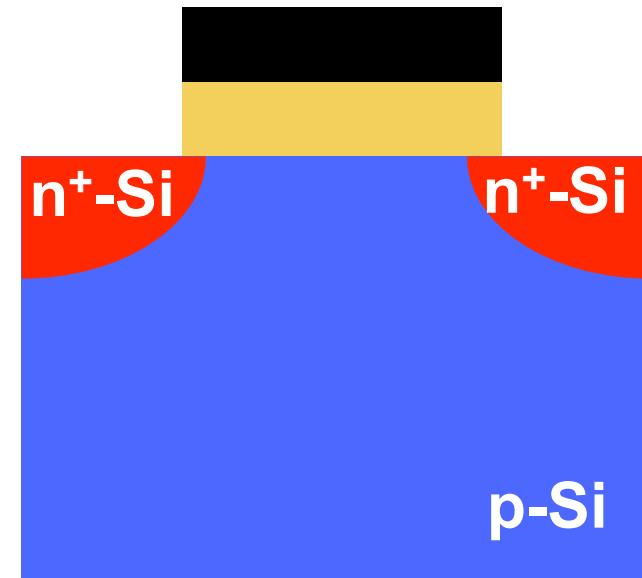


# low or high frequency?

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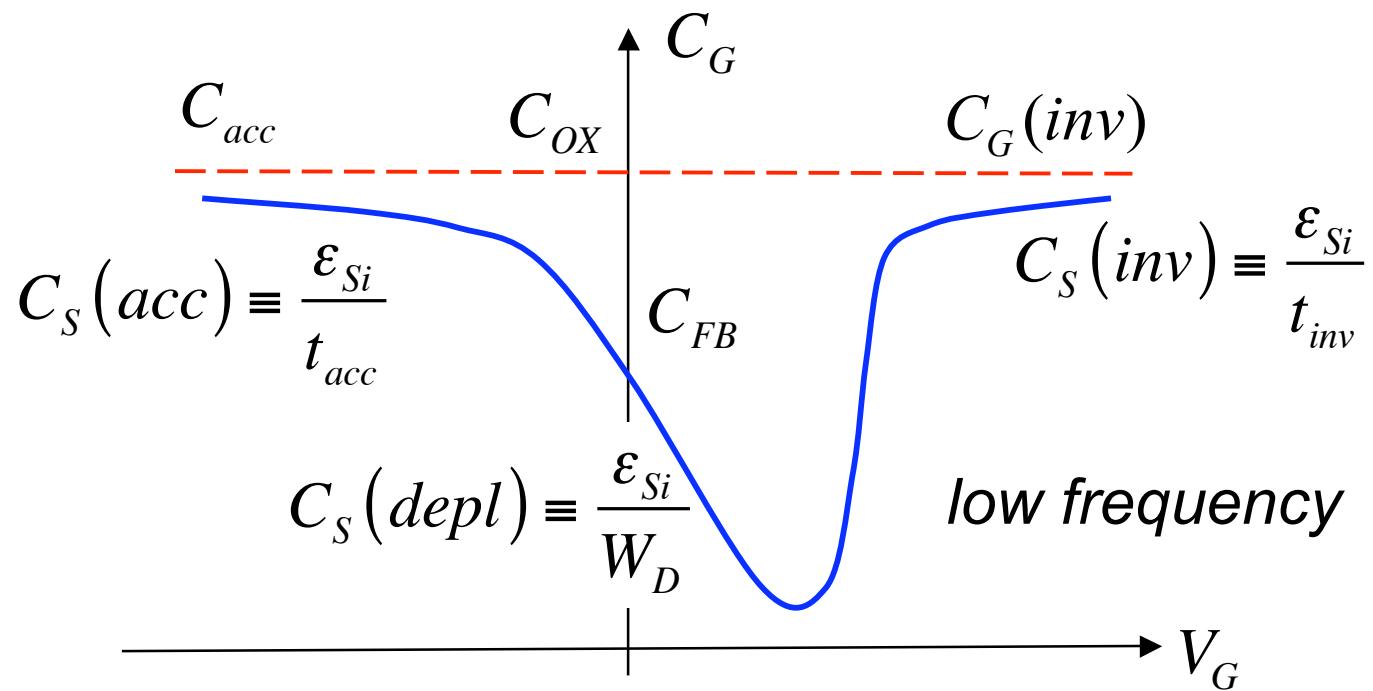


typically observe hi-  
frequency CV



typically observe low-  
frequency CV

# MOS CV recap



# outline

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- 1) Review
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- 3) The flatband voltage
- 4) MOS capacitance vs. voltage
- 5) **Gate voltage and inversion layer charge**

$$Q_i(V_G)$$

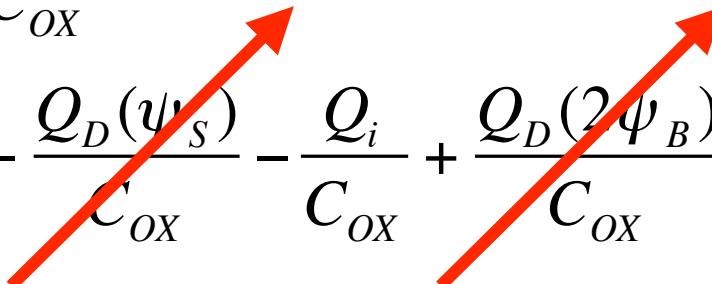
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$$Q_i = -qn(0) \left( \frac{k_B T / q}{E_S} \right) = -qn_B e^{q\psi_S/k_B T} \left( \frac{k_B T / q}{E_S} \right)$$

---

$$V_G = V_{FB} + \psi_S - \frac{Q_S(\psi_S)}{C_{OX}}$$

$$V_T = V_{FB} + 2\psi_B - \frac{Q_D(2\psi_B)}{C_{OX}}$$

$$V_G - V_T = \psi_S - 2\psi_B - \frac{Q_D(\psi_S)}{C_{OX}} - \frac{Q_i}{C_{OX}} + \frac{Q_D(2\psi_B)}{C_{OX}}$$


# $Q_i(V_G)$

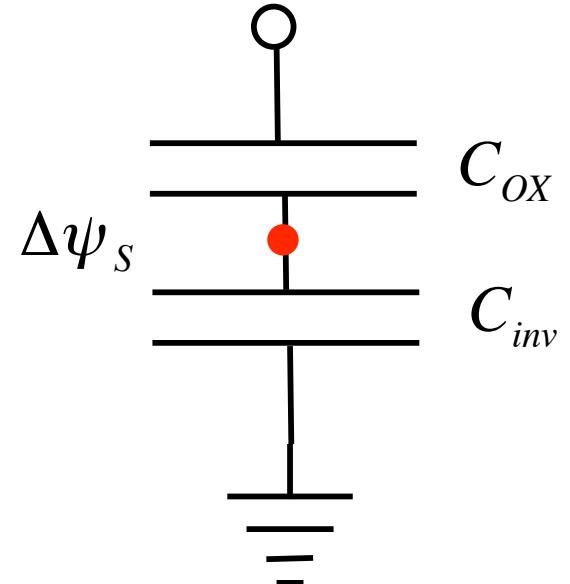
$$V_G - V_T = \Delta\psi_s - \frac{Q_i}{C_{OX}}$$

$$(V_G - V_T) = (V_G - V_T) \frac{C_{OX}}{C_{inv} + C_{OX}} - \frac{Q_i}{C_{OX}}$$

$$Q_i = -C_G (V_G - V_T)$$

$$C_G = \frac{C_{OX} C_{inv}}{C_{inv} + C_{OX}}$$

$$\Delta V = (V_G - V_T)$$



$$\Delta\psi_s = \Delta V \frac{C_{OX}}{C_{inv} + C_{OX}}$$

$$Q_i(V_G)$$

$$Q_i = -C_G (V_G - V_T)$$

$$C_G = \frac{C_{OX} C_{inv}}{C_{inv} + C_{OX}} < C_{OX}$$

$$C_{OX} = \frac{\epsilon_{OX}}{t_{ox}} \quad C_{inv} \equiv \frac{\epsilon_{Si}}{t_{inv}}$$

$$C_G = \frac{\epsilon_{OX}}{EOT_{elec}} \quad EOT_{elec} = t_{OX} + \left( \frac{\epsilon_{OX}}{\epsilon_{Si}} \right) t_{Si} > t_{OX}$$

*'equivalent oxide thickness - electrical '*

# summary

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- 1) Review
- 2) Gate voltage / surface potential relation
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- 4) MOS Capacitance vs. Voltage
- 5) Gate voltage and inversion layer charge