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ECE695: Reliability Physics of Nano-Transistors

Lecture 34: Scaling Theory of Design of Experiments

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Outline

1. Introduction
2. Buckingham PI Theorem
3. An Illustrative Example
4. Recall the scaling theory of HCI, NBTI, and TDDB
5. Conclusions

Problem definition

Many temperature dependent degradation rate depend on temperature, barrier height, etc.

$$R = f(T, E_B; k_B, \hbar)$$

If I need to perform 10 experiment for T and 10 for EB, etc. the number of experiments will be 100 – too expensive.

Can the same information be obtained with fewer experiments?

$$\frac{R}{(k_B T/\hbar)} = f_1\left(\frac{E_B}{k_B T}\right)$$

Variables do not matter individually,
They only matter in combination.
Fewer experiments are sufficient.

Buckingham PI Theorem

Assume that a function g depends on parameters $q_1, q_2, \ldots\ q_n$, such that

$$g(q_1, q_2, \ldots, q_n) = 0$$

Here g could a differential equation

$$q_1 \frac{d^2 y}{dx^2} + q_2 \frac{dy}{dx} + q_3 y + q_4 = 0$$

Or, it could be a unknown blackbox, with control parameters q_1, q_2, q_3, etc.

Buckingham PI Theorem

If the function g depends on parameters $q_1, q_2, \ldots\ q_n$, then

$$g(q_1, q_2, \ldots, q_n) = 0$$

The same expression can be expressed in terms of (n-m) independent dimensionless ration, or Π parameters.

$$G(\Pi_1, \Pi_2, \ldots, \Pi_{n-m}) = 0$$

m= minimum number of independent dimension typically given by r, where r is the rank of the matrix

To determine PI …

Determine

$$A = \begin{bmatrix} P & R \\ Q & S \end{bmatrix}$$

P is a $r \times r$ nonsingular matrix

Find the exponent matrix

$$E = (-QP^{-1}, I)$$

Finally,

$$\Pi_i = q_1^{e_{i1}} q_2^{e_{i2}} \ldots\ldots q_N^{e_{iN}}$$

Recall the dimensions of variables

$$\text{Variable} \rightarrow M^{a} \times L^{b} \times t^{c} \times \Theta^{d}$$

$$E_B \rightarrow M^{1} \times L^{2} \times t^{-2} \times \Theta^{0} \qquad \left(0.5m\upsilon^{2}\right)$$

$$T \rightarrow M^{0} \times L^{0} \times t^{0} \times \Theta^{1} \qquad \text{(kelvin)}$$

$$R \rightarrow M^{0} \times L^{0} \times t^{-1} \times \Theta^{0} \qquad (\text{sec}^{-1})$$

$$k_B \rightarrow M^{1} \times L^{2} \times t^{-2} \times \Theta^{-1} \qquad \text{(energy/kelvin)}$$

$$\hbar \rightarrow M^{1} \times L^{2} \times t^{-1} \times \Theta^{0} \qquad \text{(energy-sec)}$$

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Illustrative Example

$$R = f(T, E_B; k_B, \hbar) \Rightarrow 0 = g(R, T, E_B; k_B, \hbar)$$

$$\begin{array}{cc} & \begin{array}{cccc} M & L & t & \Theta \end{array} \\ A = & \left[\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 1 & 2 & -2 & -1 \\ 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 2 & -2 & 0 \end{array}\right] \begin{array}{l} \mathrm{T} \\ \mathrm{k_B} \\ \mathrm{h} \\ \mathrm{R} \\ \mathrm{E_B} \end{array} \end{array}$$

- Number of unknowns
  $n = 5$
- Rank of the matrix
  $r = 3$ (independent rows)
- Number of parameters $(\pi_1, \pi_2\ )$
  $n - r = 2$
- Number of repeating variable
  $r = m = 3$

Example: Any nonzero determinant for P

$$A = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} M & L & t & \Theta \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 2 & -2 & -1 \\ 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 2 & -2 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \mathrm{T} \\ \mathrm{k_B} \\ \mathrm{h} \\ \mathrm{R} \\ \mathrm{E_B} \end{array}$$

$$P_1 = \begin{array}{c} \begin{array}{ccc} L & T & \Theta \end{array} \\ \begin{bmatrix} 0 & 0 & -1 \\ 2 & -2 & -1 \\ 2 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ T \\ k_B \\ h \end{array}$$

Rank 3 …

$$Q_1 = \begin{array}{c} \begin{array}{ccc} L & T & \Theta \end{array} \\ \begin{bmatrix} 0 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ R \\ E_B \end{array}$$

$$E = \left[-QP^{-1}, I\right] = \begin{array}{c} \begin{array}{ccccc} \mathrm{T} & \mathrm{k_B} & \mathrm{h} & \mathrm{R} & \mathrm{E_B} \end{array} \\ \begin{bmatrix} -1 & -1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array} \Rightarrow \Pi_1 = \frac{\hbar R}{kT}, \Pi_2 = \frac{E_B}{kT}$$

Physical Meaning of the exponent matrix

$$E=\left[-QP^{-1},I\right]=\begin{array}{c} \begin{array}{ccccc} T & k_B & h & R & E_B \end{array} \\ \left[\begin{array}{ccc|cc} -1 & -1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 \end{array}\right] \end{array} \Rightarrow \Pi_1=\frac{\hbar R}{kT},\Pi_2=\frac{E_B}{kT}$$

$$T^a \times k_B{}^b \times h^c \times R = L^0 \times t^0 \times \Theta^0$$

$\Rightarrow$ 3 equations for a,b,c

$$T^a \times k_B{}^b \times h^c \times E_B = L^0 \times t^0 \times \Theta^0$$

$\Rightarrow$ 3 equations for a,b,c

$\text{Variable} \rightarrow M^a \times L^b \times t^c \times \Theta^d$

$E_B \rightarrow M^1 \times L^2 \times t^{-2} \times \Theta^0 \quad \left(0.5m\upsilon^2\right)$

$T \rightarrow M^0 \times L^0 \times t^0 \times \Theta^1 \quad$ (kelvin)

$R \rightarrow M^0 \times L^0 \times t^{-1} \times \Theta^0 \quad$ (sec^{-1})

$k_B \rightarrow M^1 \times L^2 \times t^{-2} \times \Theta^{-1} \quad$ (energy/kelvin)

$\hbar \rightarrow M^1 \times L^2 \times t^{-1} \times \Theta^0 \quad$ (energy-sec)

The dimensionless parameters are the Pi parameters

Example continued …

$$E = \left[-QP^{-1}, I\right] = \begin{array}{c} \begin{array}{ccccc} T & k_B & h & R & E_B \end{array} \\ \begin{bmatrix} -1 & -1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array} \Rightarrow \Pi_1 = \frac{\hbar R}{kT}, \Pi_2 = \frac{E_B}{kT}$$

$$G\left(\Pi_1 \equiv \frac{\hbar R}{kT}, \Pi_2 \equiv \frac{E_B}{kT}\right) = 0$$

$$\Rightarrow \frac{\hbar R}{kT} = f\left(\frac{E_B}{kT}\right)$$

If you assumed $\hbar$ to be absent (you did not know about it before Quantum mechanics), then

$$\frac{cR}{kT} = f\left(\frac{E_B}{kT}\right)$$

Example: Any nonzero determinant for P

$$
A = \begin{array}{c c}
\begin{array}{cccc} M & L & t & \Theta \end{array} & \\
\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 2 & -2 & -1 \\ 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 2 & -2 & 0 \end{bmatrix} & \begin{array}{c} \mathrm{T} \\ \mathrm{k_B} \\ \mathrm{h} \\ \mathrm{R} \\ \mathrm{E_B} \end{array}
\end{array}
$$

$$
P_2 = \begin{array}{c c}
\begin{array}{ccc} L & T & \Theta \end{array} & \\
\begin{bmatrix} 2 & -2 & -1 \\ 2 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} & \begin{array}{c} T \\ k_B \\ R \end{array}
\end{array}
$$

Rank 3 …

$$
Q_2 = \begin{array}{c c}
\begin{bmatrix} 2 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} E_B \\ T \end{array}
\end{array}
$$

$$
E = \left[-QP^{-1}, I \right] = \begin{array}{c}
\begin{array}{ccccc} \mathrm{T} & \mathrm{k_B} & \mathrm{h} & \mathrm{R} & \mathrm{E_B} \end{array} \\
\begin{bmatrix} +0 & -1 & -1 & 1 & 0 \\ +1 & -1 & -1 & 0 & 1 \end{bmatrix}
\end{array}
\Rightarrow \Pi_1 = \frac{E_B}{k_B R}, \Pi_2 = \frac{k_B T}{\hbar R}
$$

Example: Any nonzero determinant for P

$$E = \left[-QP^{-1}, I \right] = \begin{array}{c} \begin{array}{ccccc} T & k_B & h & R & E_B \end{array} \\ \begin{bmatrix} +0 & -1 & -1 & 1 & 0 \\ +1 & -1 & -1 & 0 & 1 \end{bmatrix} \end{array} \Rightarrow \Pi_1 = \frac{E_B}{k_B R}, \Pi_2 = \frac{k_B T}{\hbar R}$$

$$G\left(\Pi_1 \equiv \frac{E_B}{k_B R}, \Pi_2 \equiv \frac{k_B T}{\hbar R} \right) = 0$$

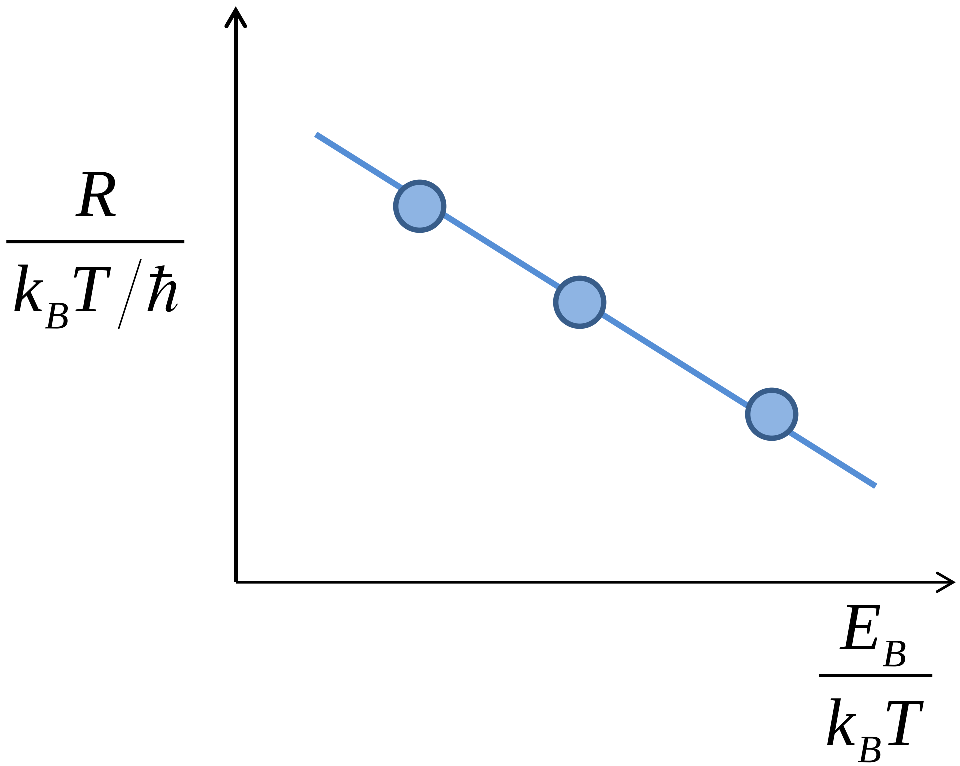
$$\Rightarrow \frac{E_B}{k_B R} = f\left(\frac{k_B T}{\hbar R} \right)$$

$$\Pi_3 \equiv \frac{\Pi_1}{\Pi_2} = \frac{E_B}{k_B T}$$

$$G\left(\Pi_3 \equiv \frac{E_B}{k_B T}, \Pi_2 \equiv \frac{k_B T}{\hbar R} \right) = 0$$

$$\Rightarrow \frac{k_B T}{\hbar R} = f_2\left(\frac{E_B}{k_B T} \right)$$

Plotting with dimensionless variables



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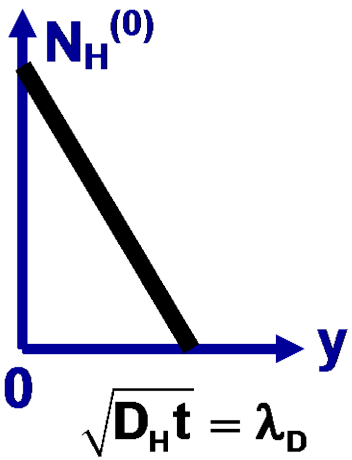
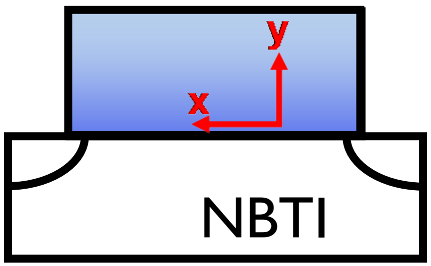
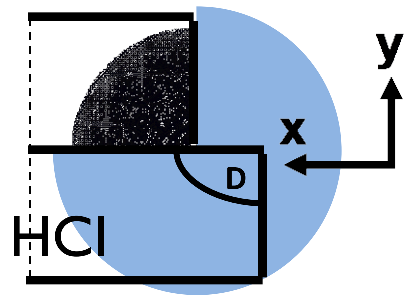
Recall: Time Exponent of Si-H dissociation

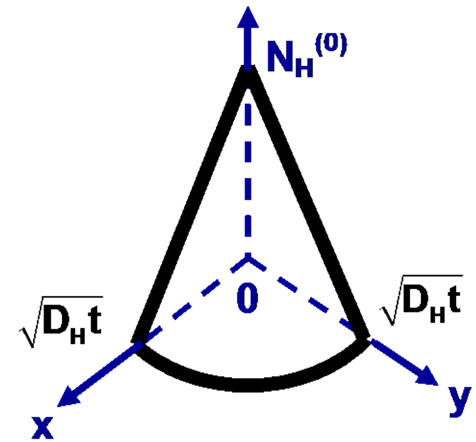
$$\frac{dN_{IT}}{dt} = k_f\left[N_0 - N_{IT}\right] - k_r N_{IT} N_H(0) \qquad \Rightarrow k_F N_0 / k_R = N_{IT} N_H(0)$$



$$N_{IT}(t) \propto \int N_H(r,t)\,dV$$







$$N_{IT}^{NBTI}(t) = N_H^{(0)} \times \sqrt{D_H t}$$

$$N_{IT}^{HCI}(t) = \left(\frac{\pi}{12}\right) N_H^{(0)} \times \left(\sqrt{D_H t}\right)^2$$

$$N_{IT}(t) = \sqrt{\frac{k_f N_0}{k_r}}\left(D_H t\right)^{1/4}$$

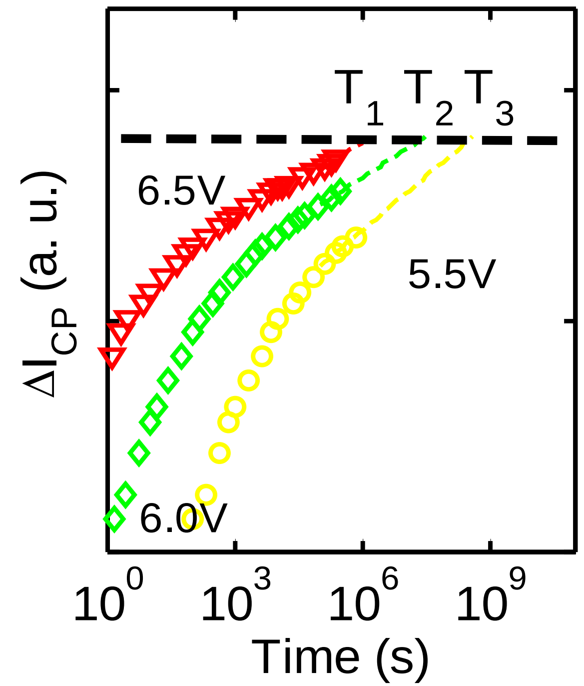
$$N_{IT}(t) = \sqrt{\frac{k_f N_0}{k_r}}\left(D_H t\right)^{1/2}$$

Universal Scaling for SiH Bonds

$$N_{IT}^{SiH} = \left(\frac{k_F\left(V_G, V_D\right) N_0}{k_R} \right)^{\alpha} \times t^n$$

$$\equiv \left(\frac{t}{t_0} \right)^n = f_{SiH}\left(\frac{t}{t_0} \right)$$

with $t_0\left(V_G, V_D\right) = g\left(\frac{k_F N_0}{k_R} \right)$



Theory provided the scaling variable.

Discussion

1. Widely used in fluid mechanics (Rayleigh, Reynold, Pandl numbers), percolation theory, reliability problems, etc. Newton predicted bending of light by gravitational field simply by dimensional analysis. He was off by a factor of 2 compared to Einstein.

   HW. $F = f(D, V, \rho, \mu)$, show that $\frac{F}{\rho V^2 D^2} = f(\frac{\mu}{\rho V D})$.

   We did not solve for Navier-Stokes equation.
   Similitude explains why Wind-tunnels work. And why Wright brothers succeeded why others failed.

2. If you add extra variables, which are unimportant – they will other disappear or appear as normalized variable that will be shown to be irrelevant experimentally.

3. Related to the principle component analysis in a interesting way (e.g. 'Recommended for you' by Amazon and Netflix)

Conclusions

1. Scaling of variables is a very important way of reducing the number of variables in an experiment. However, scaling requires that we have some idea about the key variables of the problem.

2. There are many applications of the scaling theory, especially in fluid mechanics. The problem is complex, similar to that of reliability, and therefore scaling provides enormous simplification.

3. Some of the problems may not be fully specified in terms of explicitly stated variables. The Fisher/Taguchi method help design those experiments.

References

Dimensional Analysis:

Most books on Fluid mechanics has a chapter on "Dimensional Analysis". See for example,http://en.wikibooks.org/wiki/Fluid_Mechanics/Dimensional_Analysis

One of the best articles on this topic is by D. Bolster, R. Hershberger, and R. J. Donnelly, Physics Today, p. 42, 2011. http://astro.berkeley.edu/~eliot/Astro202/dimensional_PhysicsToday.pdf

The reliability example I used is from a bookchapter on "Some Unifying Concepts in Reliability Physics, Mathematical Models, and Statistics" by R. E. Thomas,

Principal Component Analysis

A tutorial on Principle Component Analysis, J. Shlens, arxib 2009. (shlens@salk.edu)

For an interesting application in PCA, see "Recommended for you", J. A. Konstan and J. Riedl, IEEE Spectrum, p. 55, Oc. 2012.