outline

1) Brief review
2) Bulk charge theory
3) Discussion
I-V formulation

\[ I_D = W Q_i(y) \nu_y(y) \]

\[ I_D = -W Q_i(y) \mu_{eff} \frac{dV}{dy} \]

\[ I_D dy = -W Q_i(V) \mu_{eff} dV \]

\[ I_D = -\frac{W}{L} \mu_{eff} \int_{0}^{V_{DS}} Q_i(V) dV \]
effect of a reverse bias

strong inversion:

\[ q\psi_s = q(2\psi_B + V_R) \]

\[ qV_R \]

\[ E_C(x) \]

\[ E_I(x) \]

\[ F_p = E_F \]

\[ E_V(x) \]

\[ F_n \]
effect of a reverse bias

Gated doped or p-MOS with adjacent $n^+$ region
  a) gate biased at flat-band
  b) gate biased in inversion
effect of a reverse bias

Gated doped or p-MOS with adjacent, reverse-biased n⁺ region
  a) gate biased at flat-band
  b) gate biased in depletion
  b) gate biased in inversion

the MOSFET

- $V_S = 0$
- $V_G$
- $V_D > 0$

$F_n = F_p = E_F$

$F_n = F_p - qV_D$

$F_n$ increasingly negative from source to drain (reverse bias increases from source to drain)
the MOSFET

2D e-band diagram for an n-MOSFET

(a) device

(b) equilibrium (flat band)

(c) equilibrium ($\psi_S > 0$)

(d) non-equilibrium with $V_G$ and $V_D > 0$ applied

\( \psi_S \) vs. \( y \) in a MOSFET

\( V_{GS} - V_T > 0 \)

\[ \psi_S = 2\psi_B \]

\[ Q_D = -qN_A W_{dm} = -\sqrt{2q\varepsilon_{Si} N_A (2\psi_B)} \]

\[ Q_D = -\sqrt{2q\varepsilon_{Si} N_A (2\psi_B + V_D)} \]
variation of bulk charge

\[ V_{GS} - V_T > 0 \]

depletion layer boundary
square law theory

need $Q_i(y)$ in the channel

\[ MOS - C : \]
\[ Q_i = -C_G \left( V_G - V_T \right) \]

\[ MOSFET : \]
\[ Q_i(y) = -C_G \left[ V_G - V_T - V(y) \right] \]

\[ V_T'(y) \]
outline

1) Brief review
2) Bulk charge theory
3) Discussion
I-V formulation

\[ I_D = W Q_i(y) v_y(y) \]

\[ I_D = -\frac{W}{L} \mu_{\text{eff}} \int_0^{V_{DS}} Q_i[V(y)] dV \]

\[ Q_i(y) = -C_G \left( V_G - V'_{T}(y) \right) \]
local $V_T$ along the channel

no reverse bias:

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q \varepsilon_{Si} N_A (2\psi_B)}/C_{OX}$$

with reverse bias:

$$V'_T(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q \varepsilon_{Si} N_A (2\psi_B + V(y))}/C_{OX}$$

bulk charge
$V_T$ of the MOSFET

$V_T'(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V(y))}/C_{OX}$

$V_T$ for the MOSFET:

$V_T = V_T'(y = 0) = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)}/C_{OX}$
IV relation

\[ I_D = -\frac{W}{L} \mu_{\text{eff}} \int_0^{V_{DS}} Q_i(V) \, dV \quad (1) \]

\[ Q_i(y) = -C_G \left\{ V_G - V_{FB} - 2\psi_B - V(y) - \frac{\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V(y))}}{C_{OX}} \right\} \quad (2) \]

Insert (2) in (1) and integrate, find eqn. (3.18) of Taur and Ning, but 3/2 powers are inconvenient.

(See Pierret, for a more extended discussion of the bulk charge theory.)
I-V formulation

\[ I_D = W Q_i(y) v_y(y) \]

\[ I_D = -\frac{W}{L} \mu_{eff} \int_0^{V_{DS}} Q_i[V(y)] dV \]

\[ Q_i(y) = -C_G \left( V_G - V_T'(y) \right) \]
local $V_T$ along the channel

no reverse bias:

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)} / C_{OX}$$

with reverse bias:

$$V'_T(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V(y))} / C_{OX}$$

*bulk charge*
IV relation

\[ I_D = -\frac{W}{L} \mu_{\text{eff}} \int_0^{V_{DS}} Q_i(V) \, dV \quad (1) \]

\[ Q_i(y) = -C_G \left( V_G - V_{FB} - 2\psi_B - V(y) - \frac{\sqrt{2q\varepsilon_S N_A (2\psi_B + V(y))}}{C_{OX}} \right) \quad (2) \]

Insert (2) in (1) and integrate, find eqn. (3.18) of Taur and Ning, but 3/2 powers are inconvenient.

(See Pierret, for a more extended discussion of the bulk charge theory.)
approximate $Q_D$

$$Q_D(V) = -\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V)}$$

$$Q_D(V) = Q_D(0) + \frac{dQ_D}{dV}igg|_{V=0} V + ...$$

$$\frac{dQ_D}{dV} = -\frac{\varepsilon_{Si}}{W_{DM}} = -C_{DM}$$

$$Q_i(V) = -C_G \left( V_G - V_{FB} - 2\psi_B - V + \frac{Q_D(2\psi_B)}{C_{OX}} - \frac{C_{DM}}{C_{OX}} V \right)$$
IV relation

\[ I_D = -\frac{W}{L} \mu_{\text{eff}} \int_0^{V_{DS}} Q_i(V) \, dV \quad (1) \]

\[ Q_i(y) = -C_G \left( V_G - V_{FB} - 2\psi_B - V(y) - \sqrt{2q\varepsilon_S i N_A(2\psi_B + V(y))} \right) \quad (2) \]

Insert (2) in (1) and integrate, find eqn. (3.18) of Taur and Ning, but 3/2 powers are inconvenient.

(See Pierret, for a more extended discussion of the bulk charge theory.)
approximate $Q_D$

\[ Q_i(V) = -C_G \left( V_G - V_{FB} - 2\psi_B - V + \frac{Q_D(2\psi_B)}{C_{OX}} - \frac{C_{DM}}{C_{OX}} V \right) \]

\[ Q_i(V) = -C_G \left( V_G - V_{FB} - 2\psi_B + \frac{Q_D(2\psi_B)}{C_{OX}} - V - \frac{C_{DM}}{C_{OX}} V \right) \]

\[ -V_T \]

\[ -\left(1 + \frac{C_{DM}}{C_{OX}} \right)V \]

\[ Q_i(y) = -C_G \left( V_G - V_T - mV \right) \]

\[ m = \left(1 + \frac{C_{DM}}{C_{OX}} \right) \]
meaning of $m$

$$m = \left(1 + \frac{C_{DM}}{C_{OX}}\right)$$

'body effect coefficient'

$$m = \left(1 + \frac{3t_{OX}}{W_{DM}}\right)$$

also:

$$V_T = V_{FB} + (2m - 1)2\psi_B$$

in practice:

$$1.1 \leq m \leq 1.4$$

$$\Delta \psi_S = \frac{C_{OX}}{C_{OX} + C_{DM}} \Delta V_G = \frac{\Delta V_G}{m}$$
IV relation

\[ I_D = -\frac{W}{L} \mu_{\text{eff}} \int_0^{V_{DS}} Q_i[V] \, dV \]

\[ I_D = \mu_{\text{eff}} C_G \frac{W}{L} \int_0^{V_D} \left[ V_G - V_T - mV \right] \, dV \]

\[ I_D = \mu_{\text{eff}} C_G \frac{W}{L} \left[ \left( V_{GS} - V_T \right) V_{DS} - \frac{m}{2} V_{DS}^2 \right] \]
pinch-off

\[ Q_i(L) = -C_G \left[ V_G - V_T - mV_D \right] \]

when \( V_D = \left( V_G - V_T \right)/m \),
then \( Q_i(L) = 0 \)

\[ E_y >> E_x \quad \text{GCA fails!} \]

\[ I_D = \mu_{eff} C_G \frac{W}{L} \left[ \left( V_{GS} - V_T \right) V_{DS} - \frac{m}{2} V_{DS}^2 \right] \]

\( V_{GS} > V_T \)

\( V_{DS} < \left( V_{GS} - V_T \right)/m \)
meaning of $m$

\[ m = \left( 1 + \frac{C_{DM}}{C_{OX}} \right) \]

‘body effect coefficient’

\[ m = \left( 1 + 3\frac{t_{OX}}{W_{DM}} \right) \]

also:

\[ V_T = V_{FB} + (2m - 1)2\psi_B \]

in practice:

\[ 1.1 \leq m \leq 1.4 \]
beyond pinch-off, $V_{DS} > V_{DSAT}$

channel is pinched-off near the drain but current still flows.

$$I_D = -\frac{W}{L} \mu_{eff} \int_0^{V_{DSAT}} Q_i[V]dV$$

$$I_D \approx I_D \left(V_{DS} = \frac{V_{GS} - V_T}{m}\right)$$

$$I_D = \mu_{eff} C_G \frac{W}{2L'} \frac{(V_{GS} - V_T)^2}{m}$$

$V_{GS} > V_T$

$V_{DS} > \frac{(V_{GS} - V_T)}{m}$
outline

1) Brief review
2) Bulk charge theory
3) Discussion
IV summary

\[ I_D = \mu_{\text{eff}} C_G \frac{W}{2L'} \left( V_{GS} - V_T \right)^2 / m \]

\[ V_{DSAT} = \left( V_{GS} - V_T \right) / m \]
linear region (low $V_{DS}$)

$$I_D = \mu_{eff} C_G \frac{W}{L'} (V_{GS} - V_T) V_{DS}$$

$$I_D = \frac{V_{DS}}{R_{CH}}$$

slope gives mobility

mobility degradation at high $V_{GS}$

$V_{DS}$ small

actual

subthreshold conduction

intercept gives $V_T$
saturated region (high $V_{DS}$)

\[ I_{DSAT} = \mu_{eff} C_G \frac{W}{2L'} (V_{GS} - V_T)^2 / m \]

\[ V_{DS} > V_{DSAT} \]

in practice:

\[ I_{DSAT} \left( V_{GS} - V_T \right)^\alpha \]

\[ \alpha \approx 1.3 \]

\[ V_T(SAT) < V_T(LIN) \]
MOSFET IV approaches

\[ I_D = -\frac{W}{L} \mu_{\text{eff}} \int_0^{V_{DS}} Q_i(V) \, dV \]

1) “exact” (Pao-Sah or Pierret-Shields)
   see p. 117 Taur and Ning

2) Square Law
   \[ Q_i(V) = -C_G \left( V_G - V_T - V \right) \]

3) Bulk Charge
   \[ Q_i(V) = -C_G \left( V_G - V_{FB} - 2\psi_B - V - \sqrt{2q\varepsilon_S N_A \left( 2\psi_B + V \right)} \right) \]

4) Simplified Bulk Charge
   \[ Q_i(V) = -C_G \left( V_G - V_T - mV \right) \]
physics of drain current saturation

1) Low $V_{DS}$:
\[ Q_i(V) \approx -C_G \left[ V_G - V_T \right] \]
\[ E_y \approx V_{DS}/L \]

2) Larger $V_{DS}$:
\[ Q_i(L) = -C_G \left[ V_G - V_T - V(L) \right] < Q_i(0) \]
\[ E_y(L) > E_y(0) \]

3) Larger $V_{DS}$:
\[ Q_i(L) \approx 0 \]
\[ E_y(L) >> E_y(0) \]
outline

1) Brief review
2) Bulk charge theory
3) Discussion