

# **EE-612:**

# **Lecture 9**

# **MOSFET IV: Part 3**

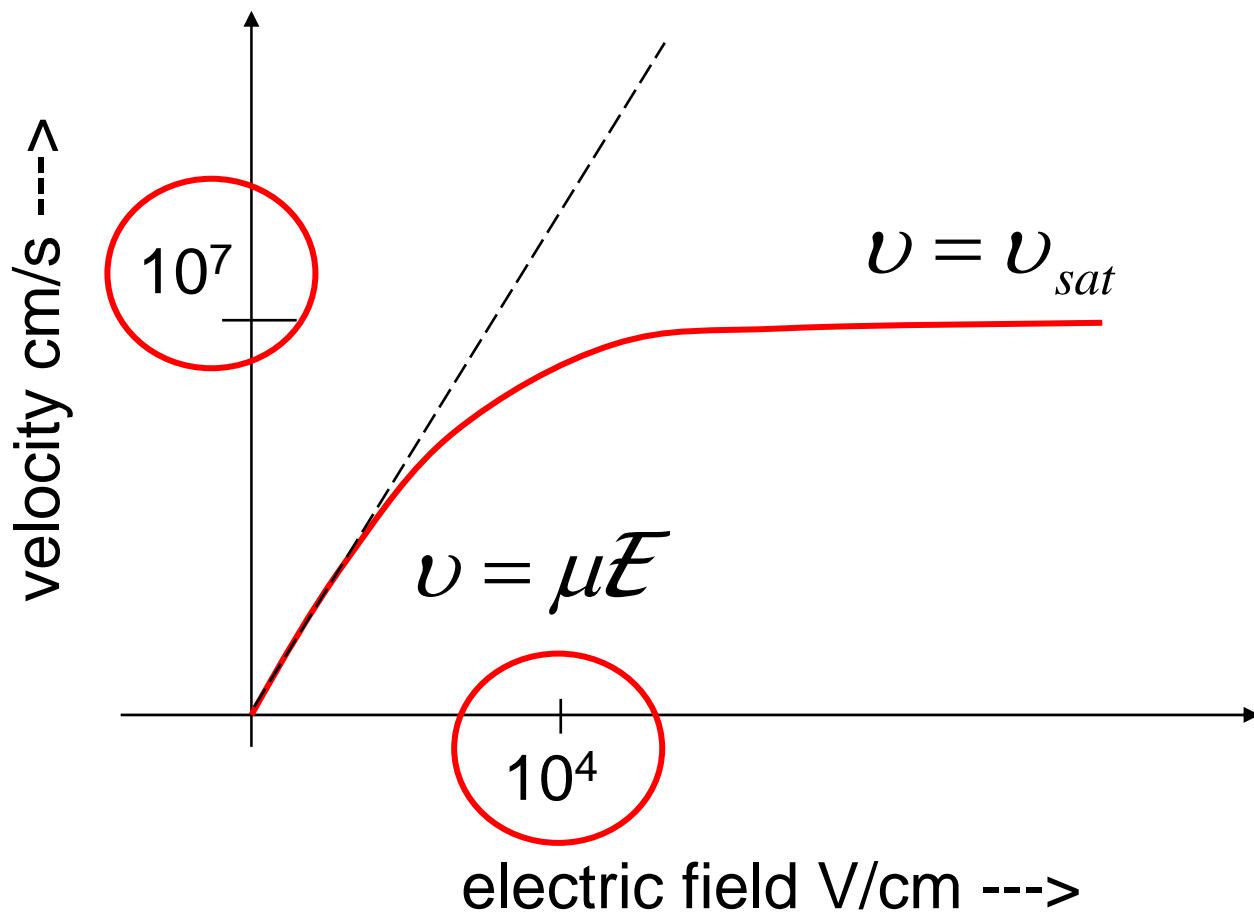
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# outline

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- 1) Quick review
- 2) Velocity saturation theory
- 3) Discussion

# velocity saturation in bulk silicon



# velocity saturation and MOSFETs

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$$I_D = W Q_i(y) v_y(y)$$

$$v_y(y) = \mu_{eff} E_y(y) ?$$

$$E_y \sim \frac{V_{DD}}{L} \ll 10^4 \text{ V/cm}$$

**OK for  $L > 1$  micrometer**

$$L \gg \frac{V_{DD}}{10^4}$$

# bulk charge theory of MOSFETs

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$$I_D = \mu_{eff} C_G \frac{W}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{m}{2} V_{DS}^2 \right]$$

$$V_{GS} > V_T$$

$$V_{DS} < (V_{GS} - V_T)/m$$

**before**

**channel pinch-off**

$$I_D = \mu_{eff} C_G \frac{W}{2L'} \frac{(V_{GS} - V_T)^2}{m}$$

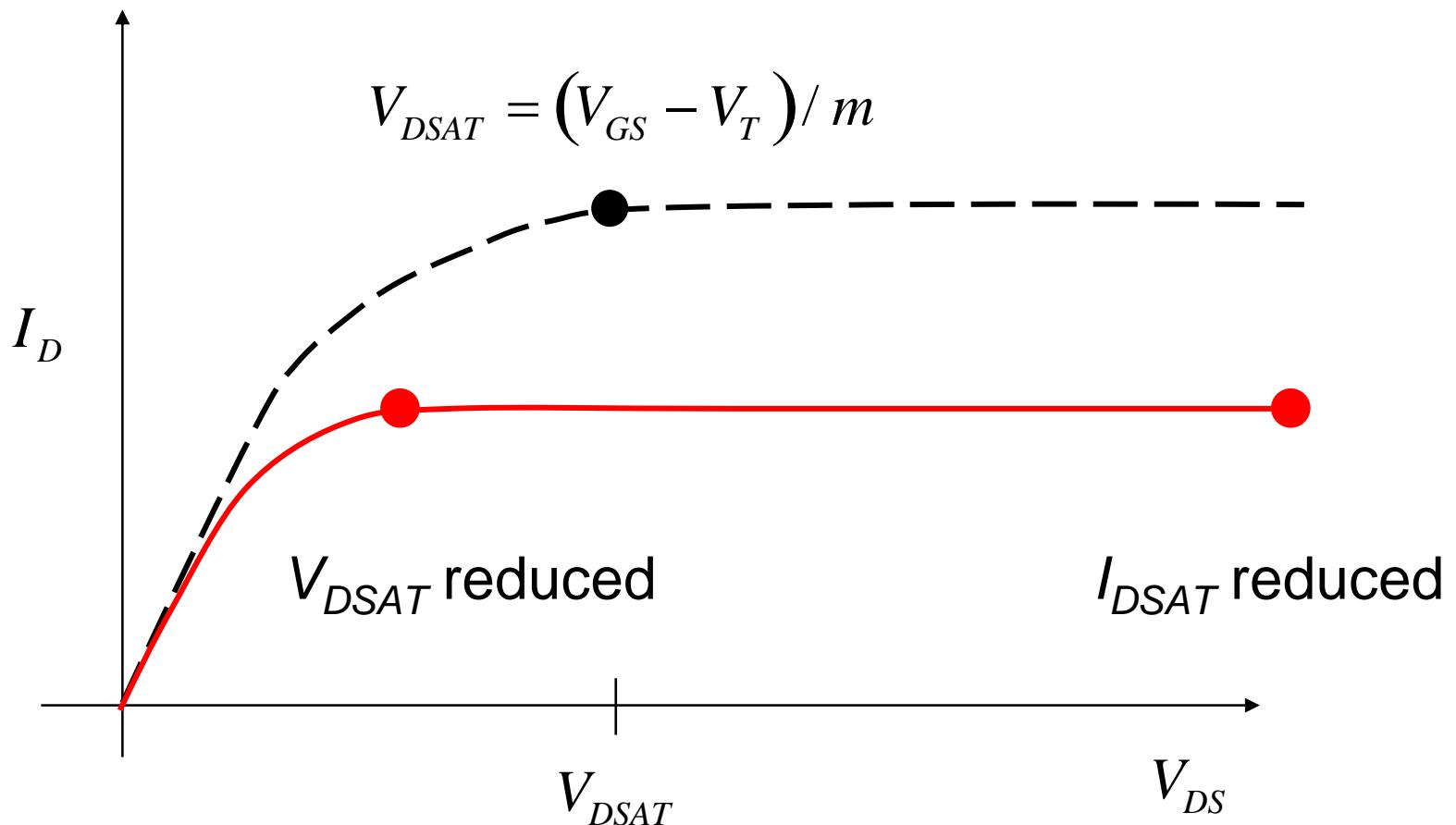
$$V_{GS} > V_T$$

$$V_{DS} > (V_{GS} - V_T)/m$$

**beyond**

**channel pinch-off**

# expected result

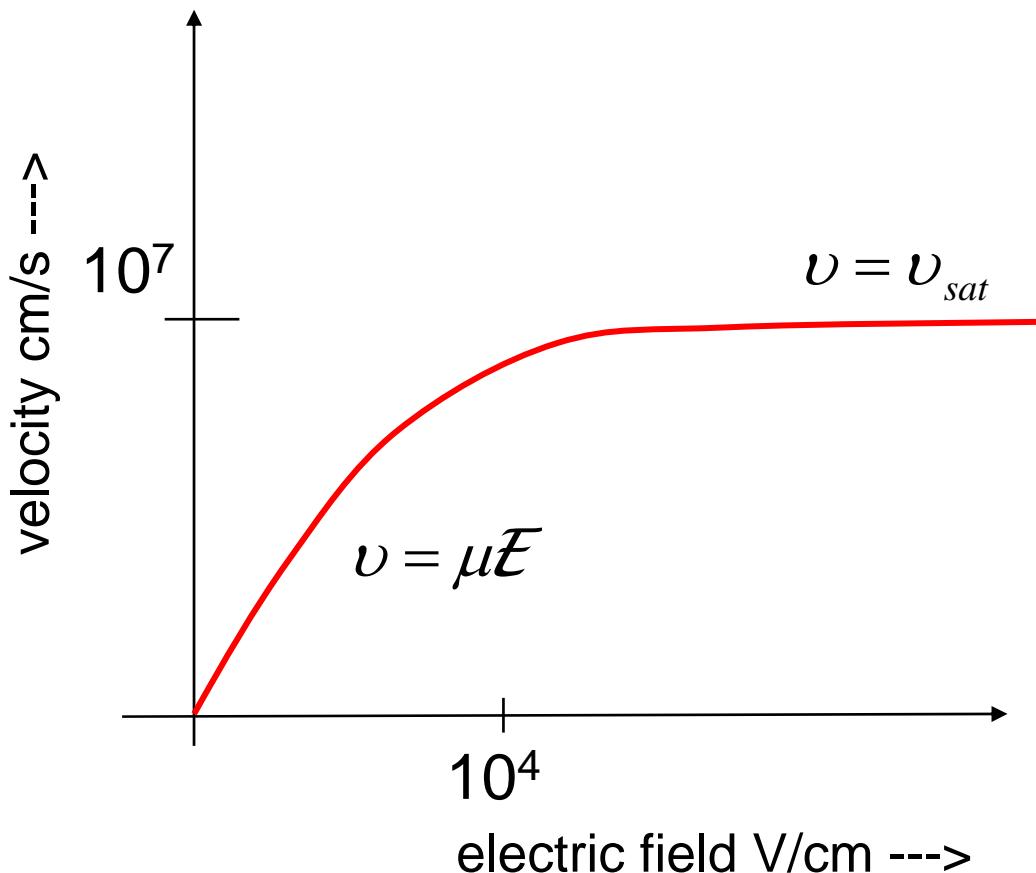


# outline

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- 1) Brief review
- 2) **Velocity saturation theory**
- 3) Discussion

# velocity vs. field characteristic (electrons)

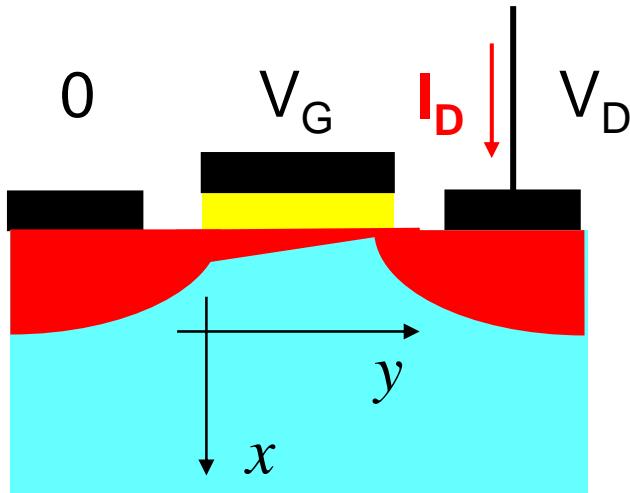


$$v_d = \frac{-\mu E}{[1 + (E/E_c)^2]^{1/2}}$$

$$v_d = \frac{-\mu E}{1 + (|E|/E_c)}$$

$$\mu E_C = v_{sat}$$

# I-V derivation



$$I_D = W Q_i(y) v_y(y)$$

$$v(y) = \frac{-\mu_{eff} E}{1 + (|E|/E_c)}$$

$$I_D = \frac{-WQ_i\mu_{eff}E_y}{1 + |E_y|/E_c}$$

$$I_D \left( 1 + \frac{1}{E_c} \frac{dV}{dy} \right) = -WQ_i\mu_{eff} \frac{dV}{dy}$$

## derivation (ii)

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$$I_D \left( dy + \frac{1}{E_c} dV \right) = -WQ_i(V)\mu_{eff}dV$$

$$\int_0^L I_D dy + \int_0^{V_{DS}} \frac{I_D}{E_c} dV = - \int_0^{V_{DS}} WQ_i(V)\mu_{eff}dV$$

$$I_D \left( L + V_{DS} / E_c \right) = - \int_0^{V_{DS}} WQ_i(V)\mu_{eff}dV$$


**exactly the same** as for the bulk charge theory

## derivation (iii)

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$$I_D = F_v \mu_{eff} C_{OX} \frac{W}{L} \left[ (V_{GS} - V_T) V_{DS} - m \frac{V_{DS}^2}{2} \right] \quad (1)$$

$$F_v = \frac{1}{(1 + V_{DS} / LE_c)} = \frac{1}{(1 + \mu_{eff} V_{DS} / v_{sat} L)}$$

$V_{DS} / L$  = average electric field in the channel

when  $V_{DS} / L > E_c$  then  $F_v < 1$

(1) valid when:

$$V_{GS} > V_T \quad V_{DS} < ?$$

$$V_{DSAT}$$

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$$\frac{dI_D}{dV_{DS}} = 0$$

$$I_D = F_v \mu_{eff} C_{ox} \frac{W}{L} \left[ (V_{GS} - V_T) V_{DS} - m \frac{V_{DS}^2}{2} \right]$$

$$V_{DSAT} = \frac{2(V_{GS} - V_T)/m}{1 + \sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L}} < \frac{(V_{GS} - V_T)}{m}$$

eqn. (3.77) of Taur and Ning

$$I_{DSAT}$$

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$$I_{DSAT} = W C_G v_{sat} \left( V_{GS} - V_T \right) \frac{\sqrt{1 + 2\mu_{eff} (V_{GS} - V_T)/m v_{sat} L} - 1}{\sqrt{1 + 2\mu_{eff} (V_{GS} - V_T)/m v_{sat} L} + 1}$$

eqn. (3.78) of Taur and Ning

Examine two limits:

- i)  $L \rightarrow \infty$
- ii)  $L \rightarrow 0$

$L \rightarrow \infty$

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$$V_{DSAT} = \frac{2(V_{GS} - V_T)/m}{1 + \sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L}}$$

$$V_{DSAT} \rightarrow \frac{(V_{GS} - V_T)}{m}$$

$$I_{DSAT} = W C_G \upsilon_{sat} (V_{GS} - V_T) \frac{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} - 1}{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} + 1}$$

$$I_{DSAT} \rightarrow \mu_{eff} C_G \frac{W}{2L} \frac{(V_{GS} - V_T)^2}{m}$$

$$L \rightarrow 0$$

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$$V_{DSAT} = \frac{2(V_{GS} - V_T)/m}{1 + \sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L}}$$

$$V_{DSAT} \rightarrow \sqrt{2\upsilon_{sat}L(V_{GS} - V_T)/m\mu_{eff}}$$

$$I_{DSAT} = W C_G \upsilon_{sat} (V_{GS} - V_T) \frac{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} - 1}{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} + 1}$$

$$I_{DSAT} = W C_G \upsilon_{sat} (V_{GS} - V_T)$$

**“complete velocity saturation”  
current independent of  $L$**

## *near threshold*

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$$V_{DSAT} = \frac{2(V_{GS} - V_T)/m}{1 + \sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L}}$$
$$\frac{2\mu_{eff}(V_{GS} - V_T)}{m\upsilon_{sat}L} \ll 1$$

$$V_{DSAT} \rightarrow (V_{GS} - V_T)/m$$

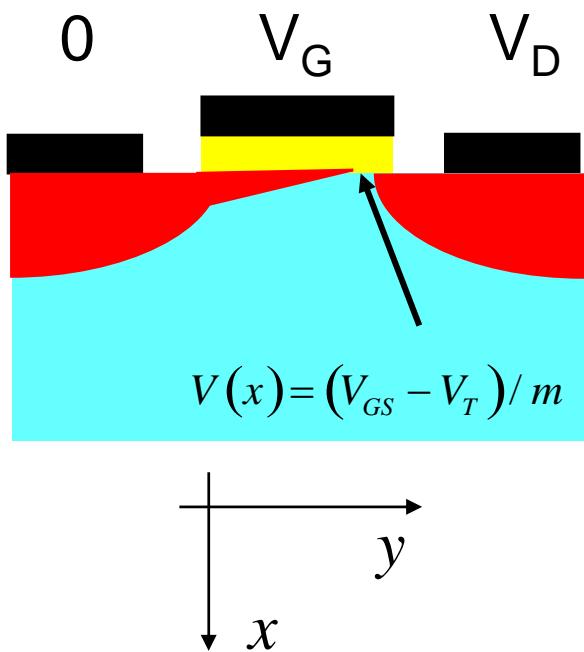
$$I_{DSAT} = W C_G \upsilon_{sat} (V_{GS} - V_T) \frac{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} - 1}{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} + 1}$$

$$I_{DSAT} \rightarrow \mu_{eff} C_G \frac{W}{2L} \frac{(V_{GS} - V_T)^2}{m}$$

***near threshold is  
like long channel***

# *near threshold*

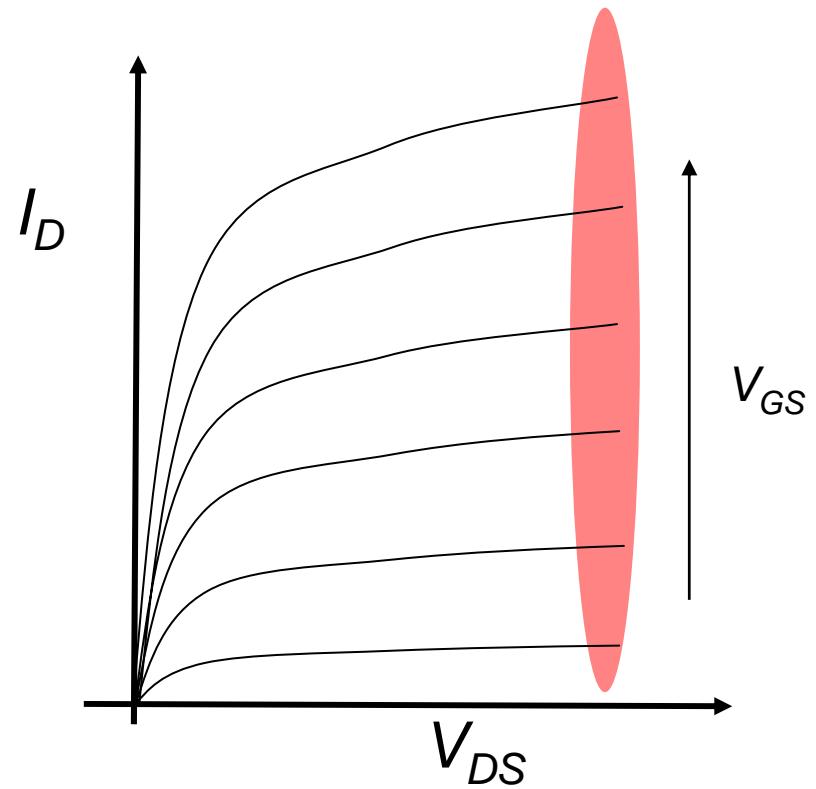
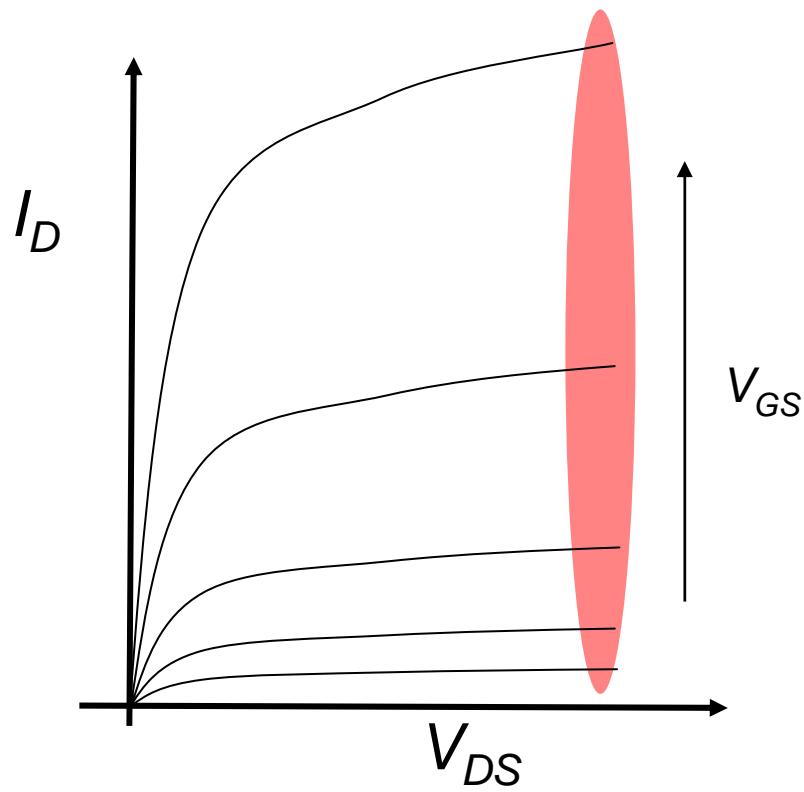
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$$\frac{2\mu_{eff}(V_{GS} - V_T)}{m v_{sat} L} \ll 1$$

$$\frac{(V_{GS} - V_T)/m}{L} < \frac{E_c}{2}$$

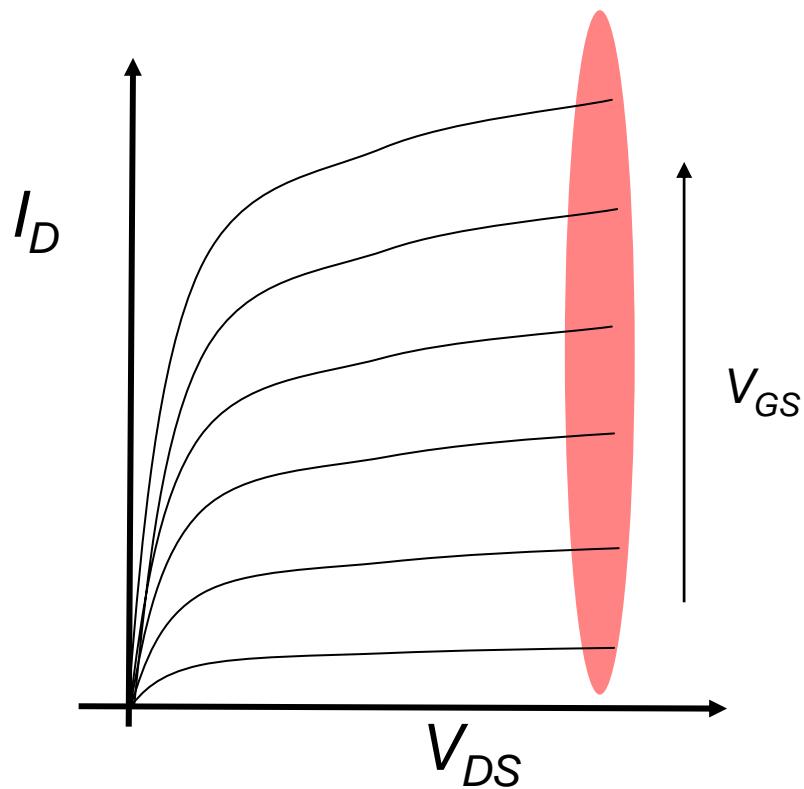
# ‘signature’ of velocity saturation



$$I_D = \frac{W}{2L} \mu_{eff} C_{ox} \frac{(V_{GS} - V_T)^2}{m}$$

$$I_D = W \nu_{sat} C_{ox} (V_{GS} - V_T)$$

# $I_D$ and $(V_{GS} - V_T)$



$$I_D(V_{DS} = V_{DD}) \sim (V_{GS} - V_T)^\alpha$$

$$1 < \alpha < 2$$

complete  
velocity  
saturation

long channel

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# what happens near the drain?

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$$Q_i(y) = -C_G [V_G - V_T - mV(y)]$$

$$Q_i(y = L) = -C_G [V_G - V_T - mV_{DSAT}]$$

$$V_{DSAT} = \frac{2(V_{GS} - V_T)/m}{1 + \sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L}}$$

$$Q_i(y = L) = -C_G (V_{GS} - V_T) \frac{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} - 1}{\sqrt{1 + 2\mu_{eff}(V_{GS} - V_T)/m\upsilon_{sat}L} + 1}$$

$$|Q_i(y = L)| > 0$$

# what happens near the drain?

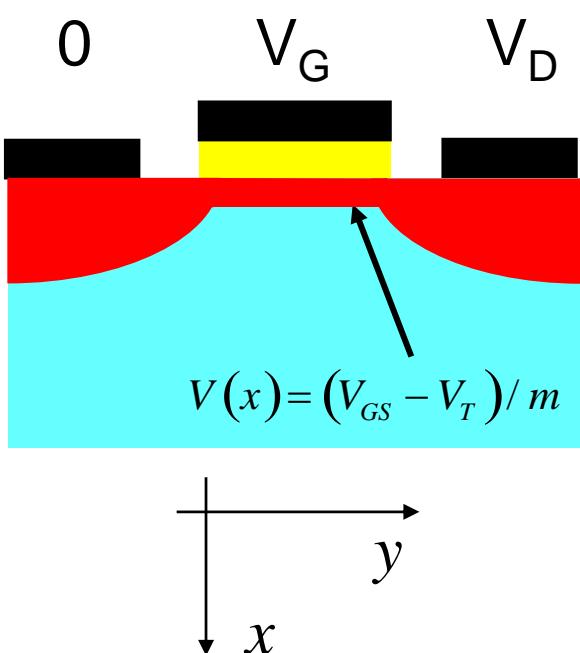
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$$Q_i(y = L) = -C_G (V_{GS} - V_T) \frac{\sqrt{1 + 2\mu_{eff} (V_{GS} - V_T)/m v_{sat} L} - 1}{\sqrt{1 + 2\mu_{eff} (V_{GS} - V_T)/m v_{sat} L} + 1}$$

$$I_{DSAT} = W C_G v_{sat} (V_{GS} - V_T) \frac{\sqrt{1 + 2\mu_{eff} (V_{GS} - V_T)/m v_{sat} L} - 1}{\sqrt{1 + 2\mu_{eff} (V_{GS} - V_T)/m v_{sat} L} + 1}$$

$$I_{DSAT} = W v_{sat} Q_i(y = L)$$

# what happens when $L \rightarrow 0$ ?

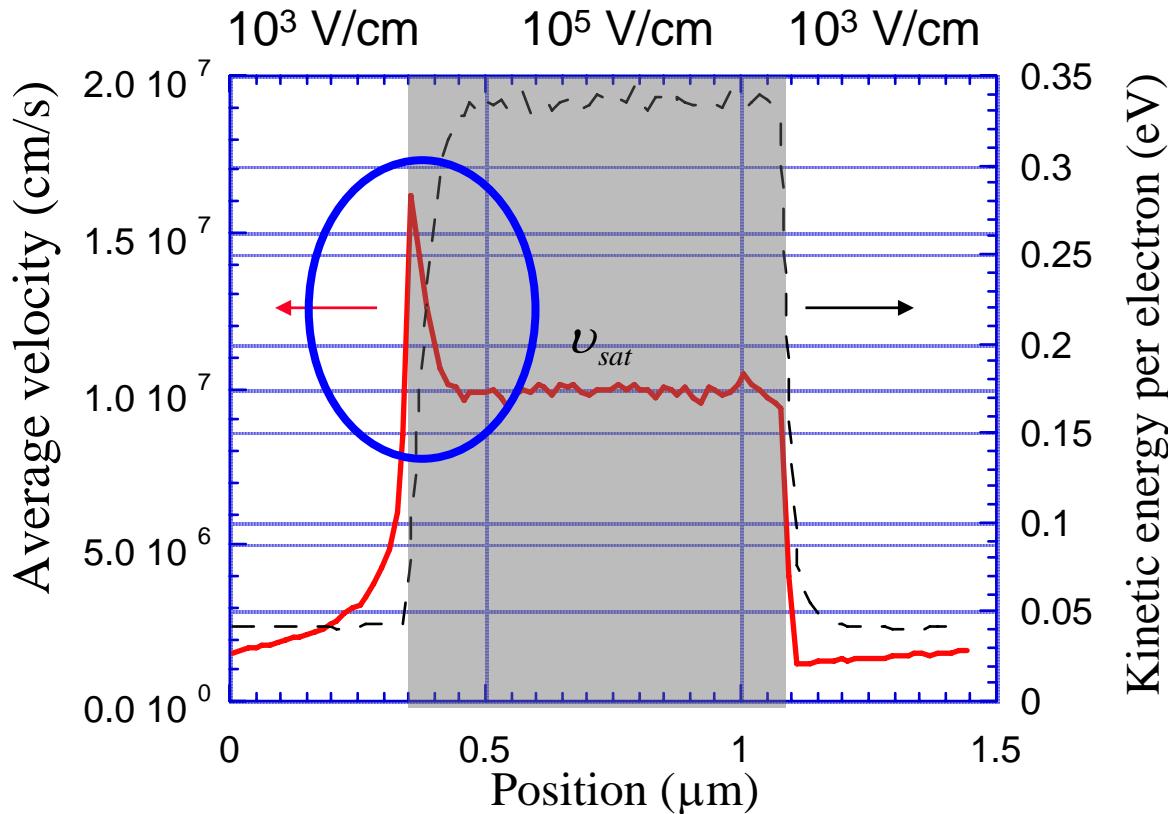


$$Q_i(y = L) > 0$$

$$L \rightarrow 0$$

$$Q_i(y = L) \rightarrow Q_i(y = 0) = C_G (V_{GS} - V_T)$$

# *velocity overshoot*



$$v \neq \mu_n(E)E$$

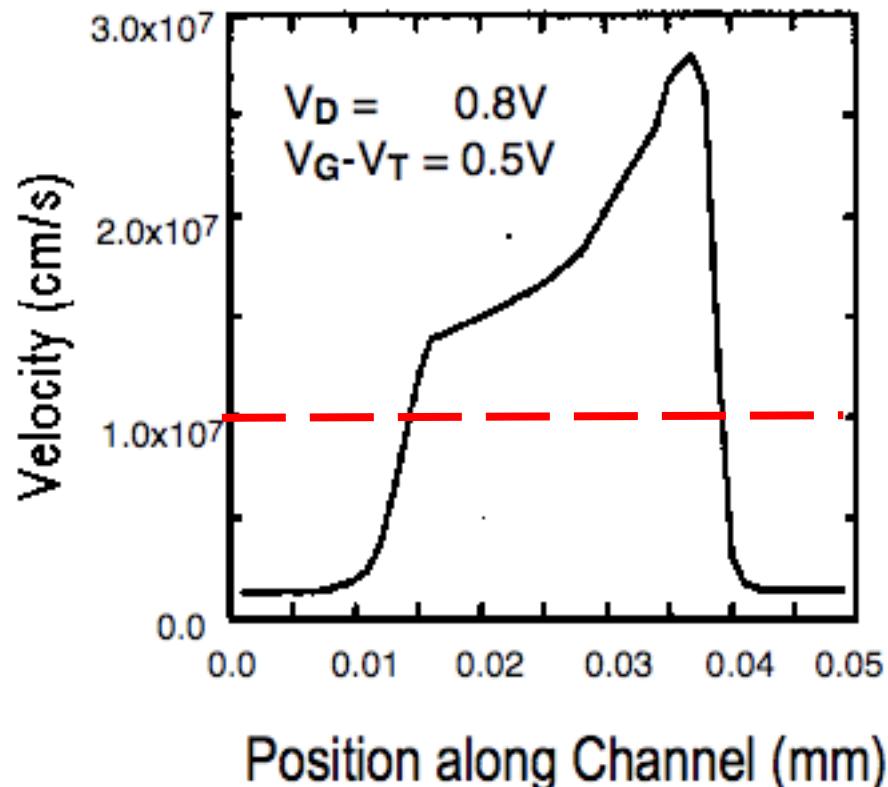
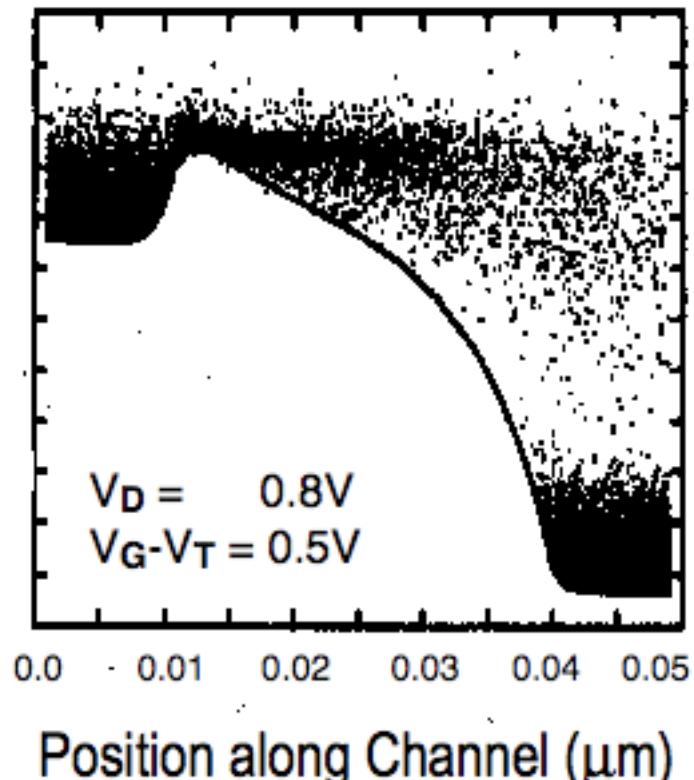
# *velocity and transconductance*

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$$I_{DSAT} = W C_G v_{sat} (V_{GS} - V_T)$$

$$g_m \equiv \frac{dI_{DSAT}}{dV_G} = W C_G v_{sat}$$

# *velocity overshoot in a MOSFET*



Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

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