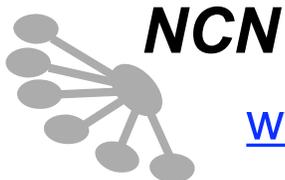


# EE-612: Lecture 12 Subthreshold Conduction

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Fall 2006



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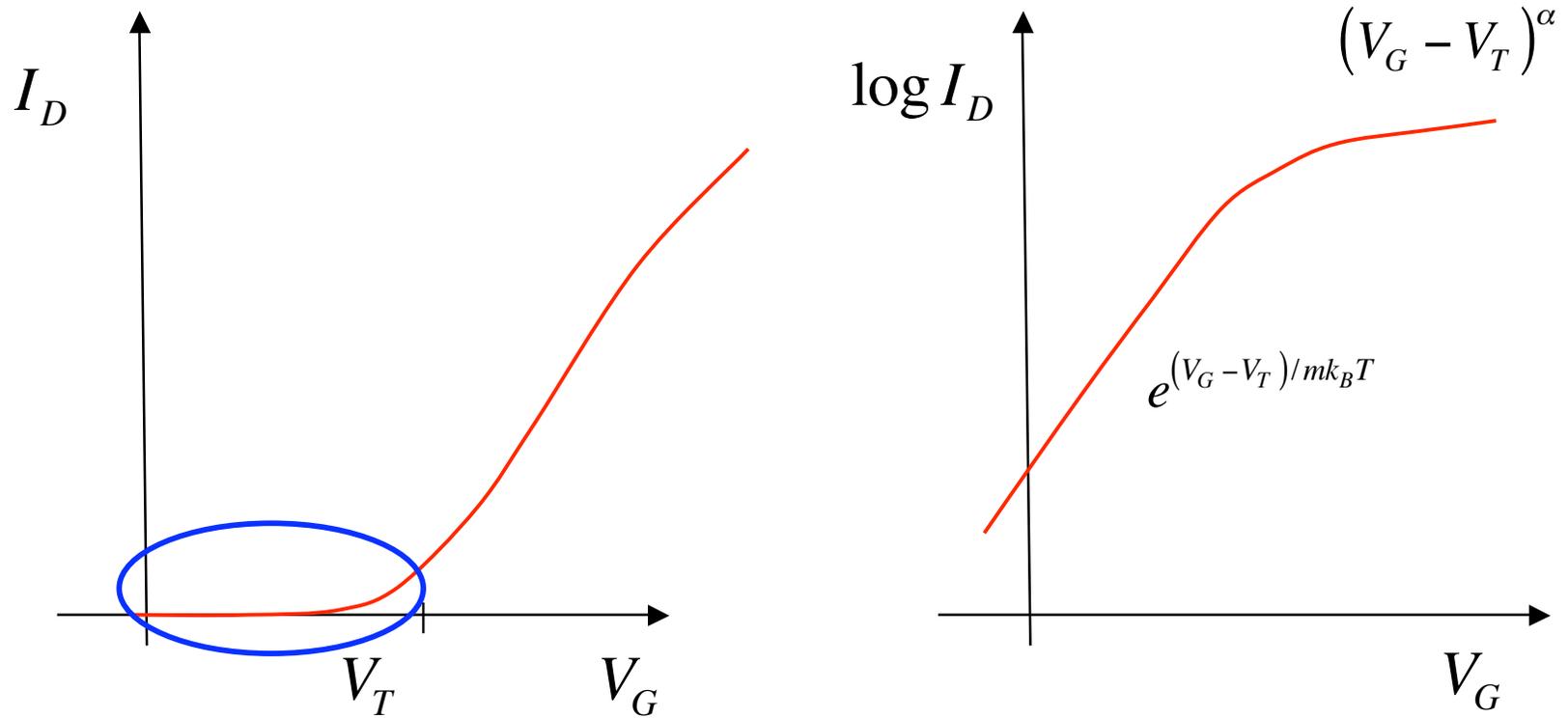
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# outline

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- 1) Introduction
- 2) Bipolar transistors
- 3) Derivation
- 4) Discussion

# Introduction

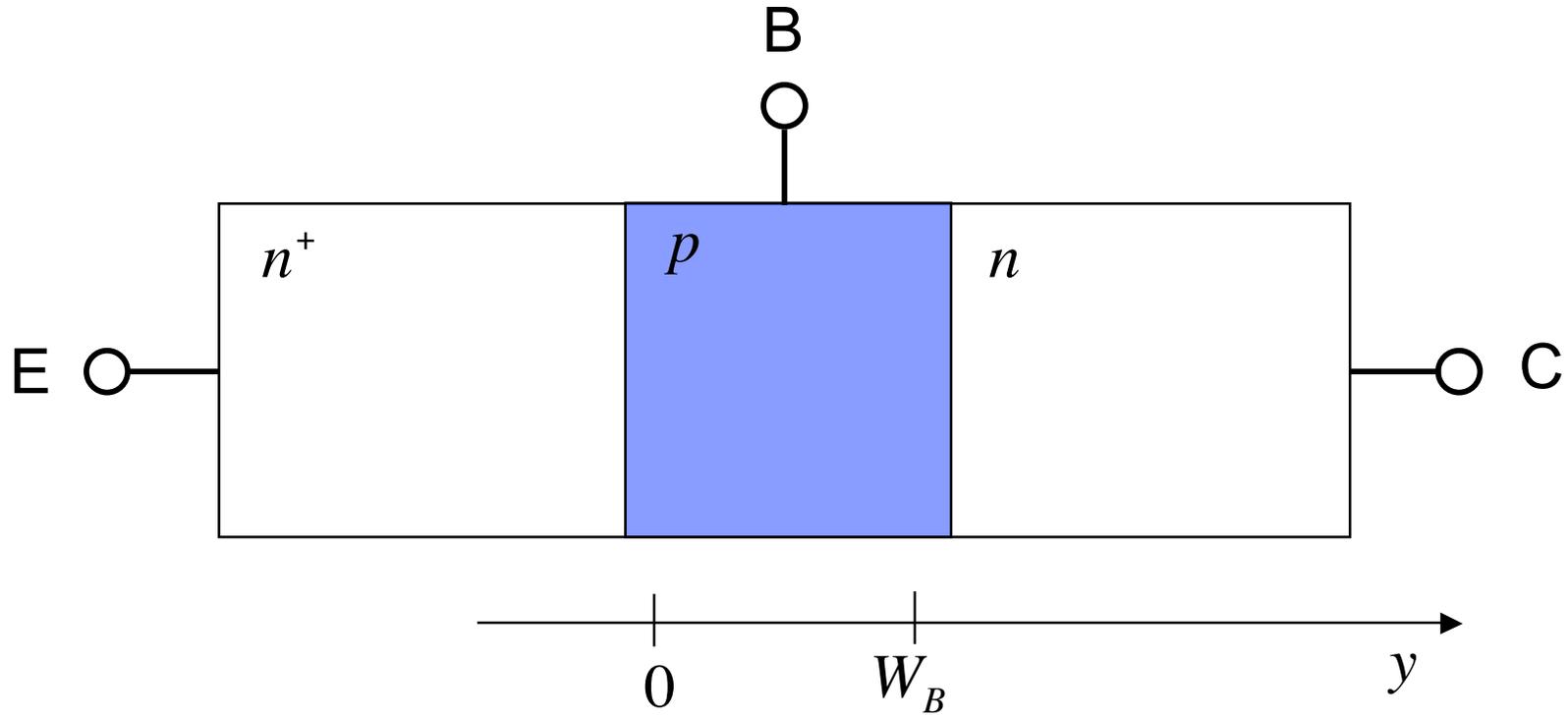


# outline

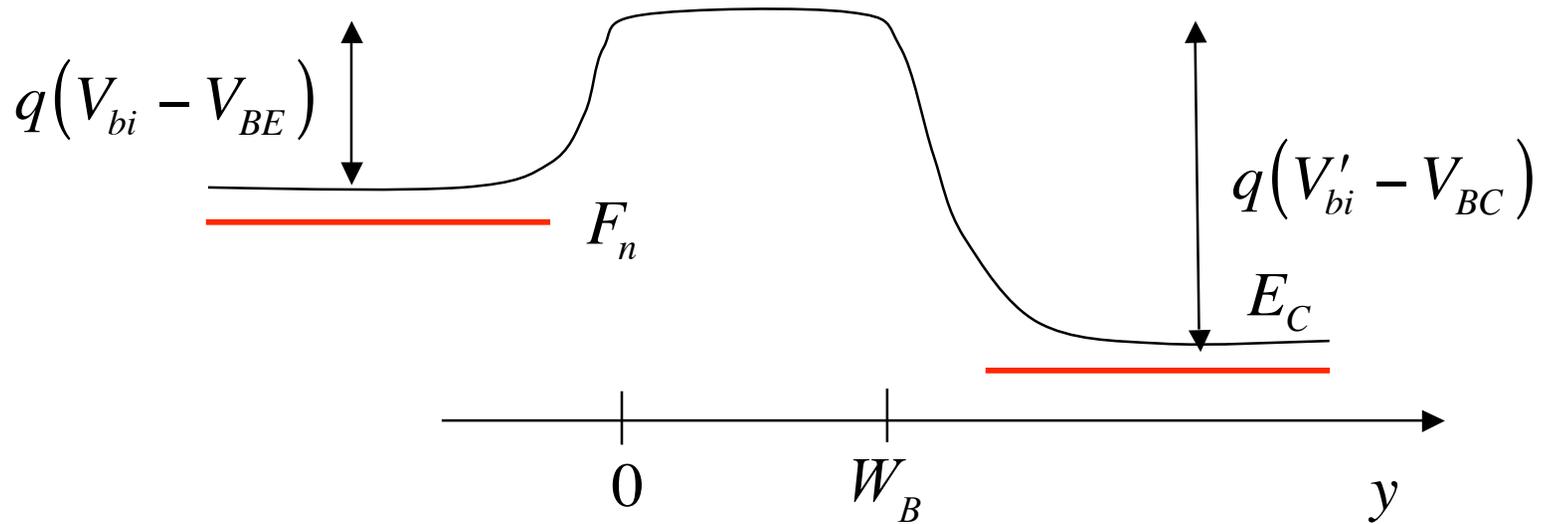
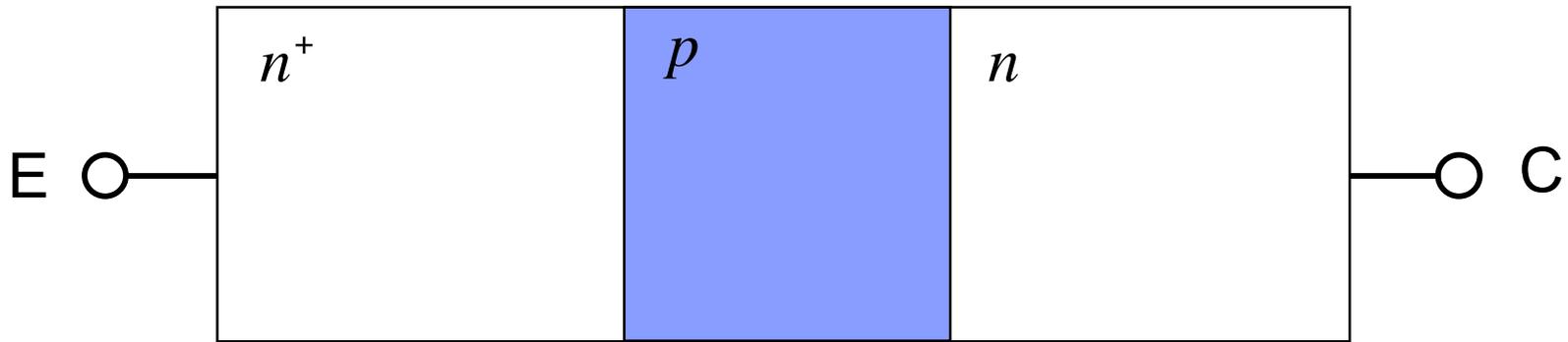
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- 1) Introduction
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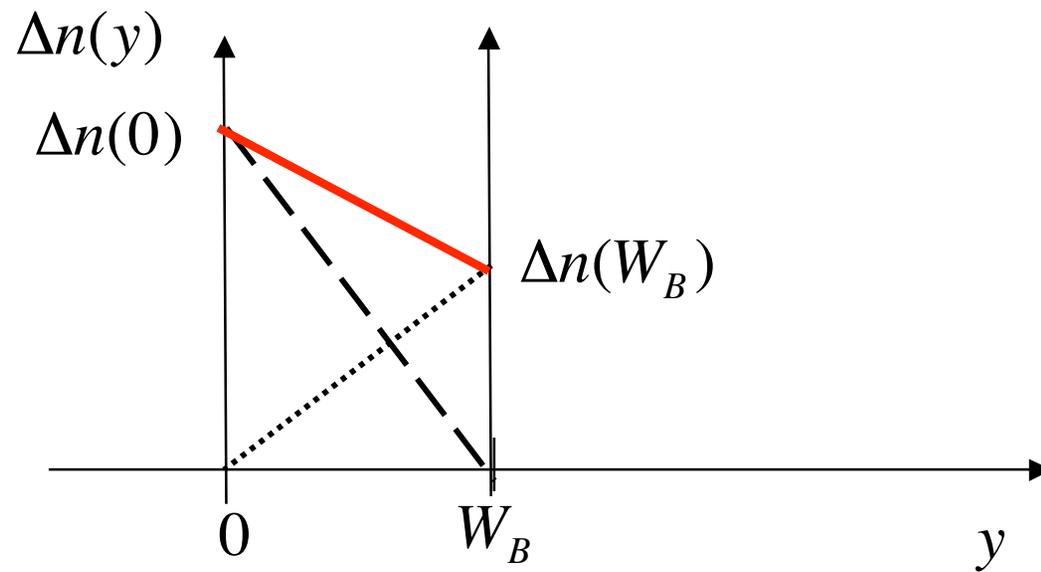
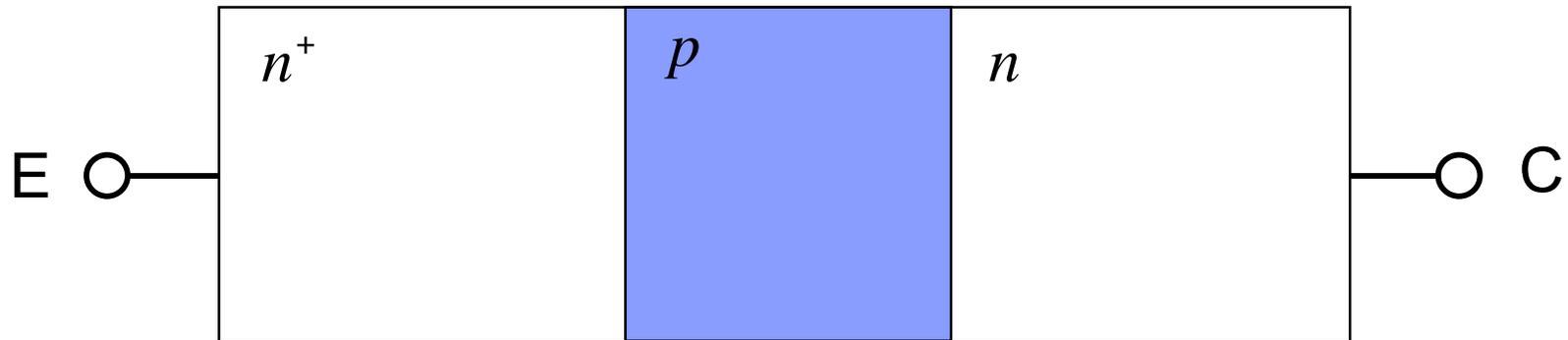
# BJTs



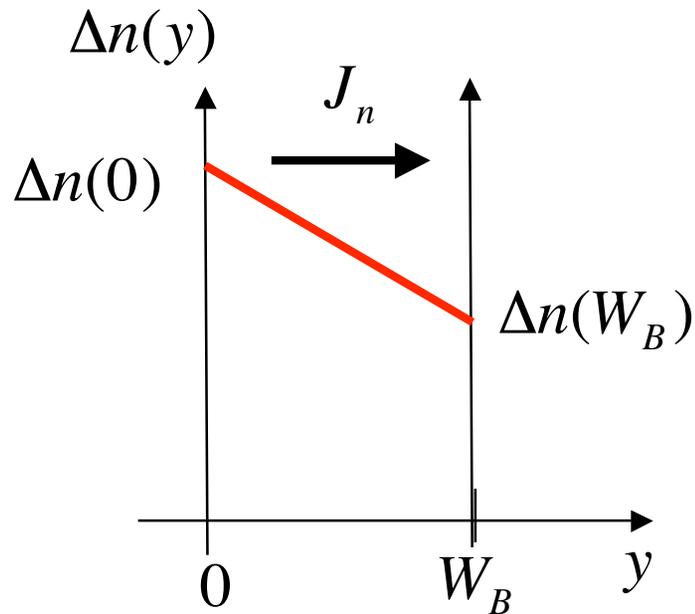
# BJTs



# BJTs



# BJTs



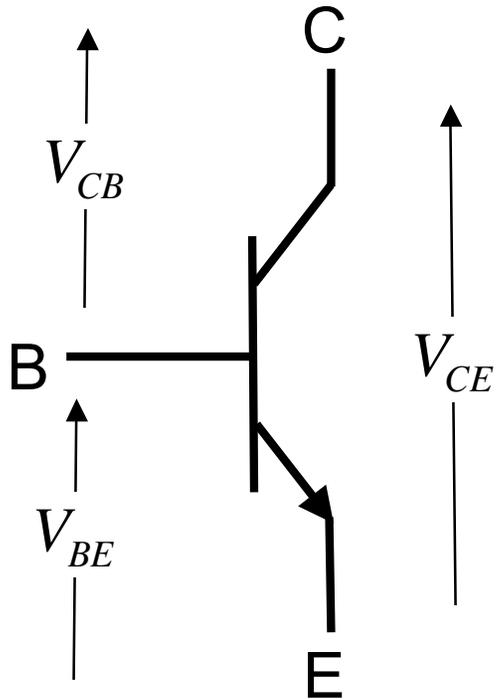
$$J_n = -qD_n \frac{dn}{dx}$$

$$J_C = qD_n \frac{[\Delta n(0) - \Delta n(W_B)]}{W_B}$$

$$\Delta n(0) = \frac{n_i^2}{N_A} e^{qV_{BE}/k_B T}$$

$$\Delta n(W_B) = \frac{n_i^2}{N_A} e^{qV_{BC}/k_B T}$$

# BJTs



$$J_C = q \frac{D_n}{W_B} \frac{n_i^2}{N_A} \left( e^{qV_{BE}/k_B T} - e^{qV_{BC}/k_B T} \right)$$

$$J_C = q \frac{D_n}{W_B} \frac{n_i^2}{N_A} e^{qV_{BE}/k_B T} \left( 1 - e^{q(V_{BC} - V_{BE})/k_B T} \right)$$

$$V_{CE} = V_{BE} + V_{CB} = V_{BE} - V_{BC}$$

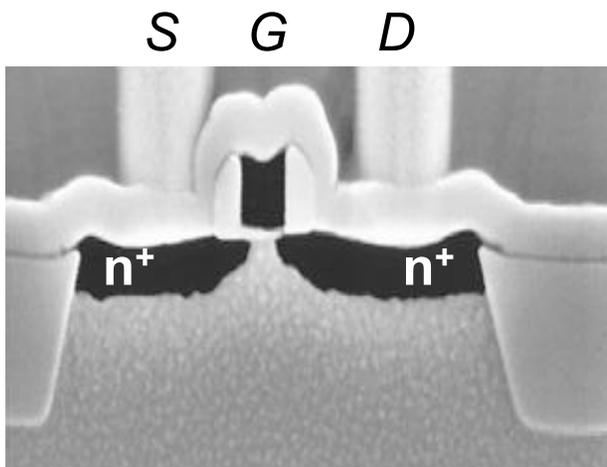
$$J_C = q \frac{D_n}{W_B} \frac{n_i^2}{N_A} e^{qV_{BE}/k_B T} \left( 1 - e^{-qV_{CE}/k_B T} \right)$$

# outline

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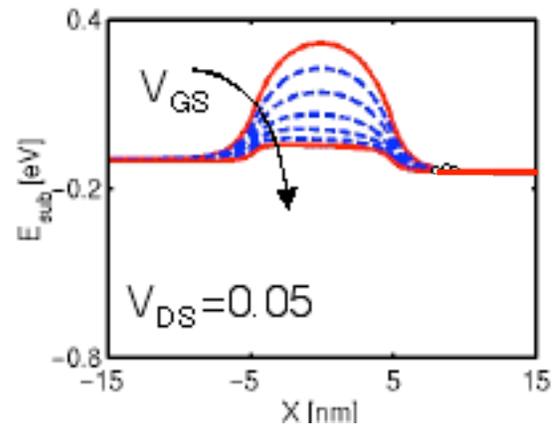
- 1) Introduction
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- 3) Derivation**
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# MOSFETs and BJTs

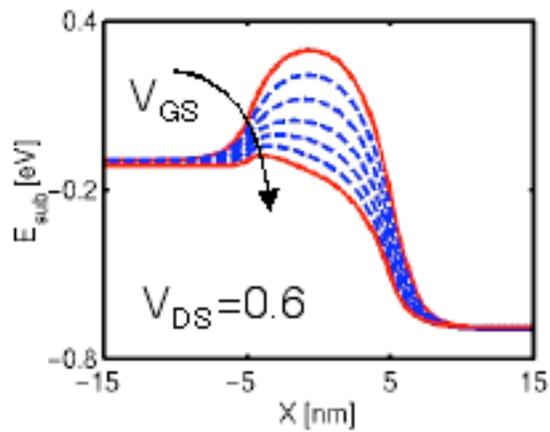


$$q(V_{bi} - \psi_s)$$

electron energy vs. position



$V_D \approx 0V$



$V_D = V_{DD}$

# MOSFETs below $V_T$

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$$J_C = q \frac{D_n}{W_B} \frac{n_i^2}{N_A} e^{qV_{BE}/k_B T} \left( 1 - e^{-qV_{CE}/k_B T} \right) \text{ A/cm}^2$$

---

$$I_D = J_C W t_n$$

$$V_{BE} \rightarrow \psi_S$$

$$W_B \rightarrow L$$

$$t_n = \frac{k_B T / q}{E_S}$$

$$V_{CE} \rightarrow V_{DS}$$

$$D_n \rightarrow (k_B T / q) \mu_{eff}$$

## MOSFETs below $V_T$ (ii)

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$$J_C = q \frac{D_n}{W_B} \frac{n_i^2}{N_A} e^{qV_{BE}/k_B T} \left( 1 - e^{-qV_{CE}/k_B T} \right) \text{ A/cm}^2$$

$$I_D = q \frac{W}{L} \left( \frac{n_i^2}{N_A} \right) \frac{(k_B T / q)^2}{E_S} \mu_{\text{eff}} e^{q\psi_S / k_B T} \left( 1 - e^{-qV_{DS} / k_B T} \right) \text{ A}$$

Now make this look like a MOSFET expression.  
Relate  $\psi_S$  to  $V_G$ .

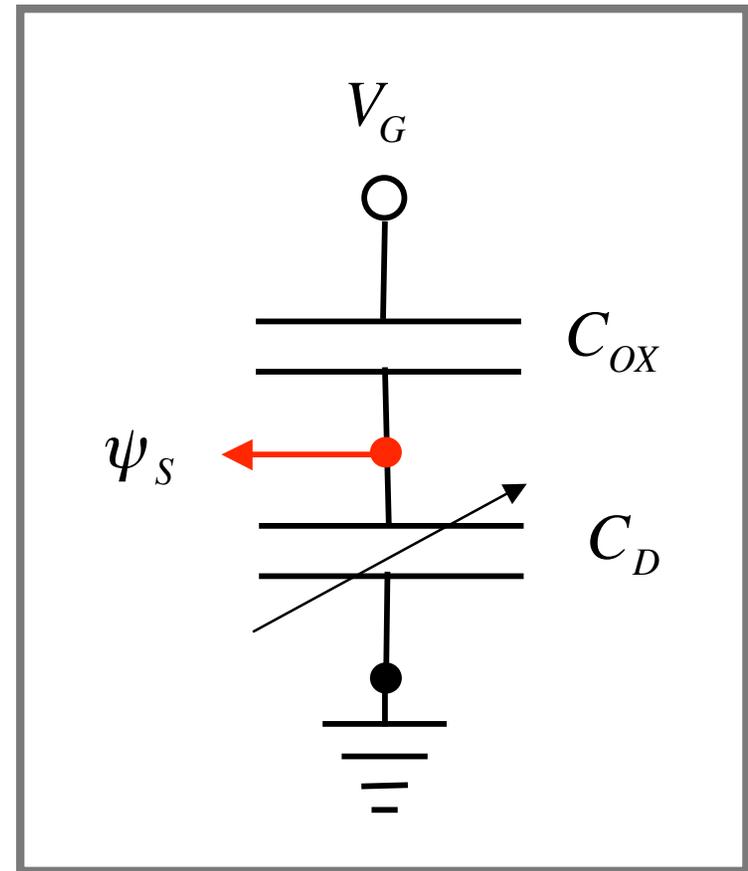
# Surface potential and gate voltage

$$V_{GS} = V_{FB} + \psi_S + \sqrt{2q\epsilon_{Si}N_A\psi_S}/C_{OX}$$

$$\psi_S = V_{GS} \frac{C_{OX}}{C_{OX} + C_D} = \frac{V_{GS}}{1 + C_D / C_{OX}}$$

$$\psi_S = \frac{V_{GS}}{m}$$

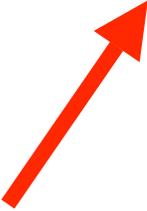
$$m = 1 + C_D / C_{OX}$$



## MOSFETs below $V_T$ (iii)

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$$I_D = q \frac{W}{L} \left( \frac{n_i^2}{N_A} \right) \frac{(k_B T / q)^2}{E_S} \mu_{eff} e^{q\psi_S / k_B T} \left( 1 - e^{-qV_{DS} / k_B T} \right) A$$

$$I_D = q \frac{W}{L} \left( \frac{n_i^2}{N_A} \right) \frac{(k_B T / q)^2}{E_S} \mu_{eff} e^{qV_{GS} / mk_B T} \left( 1 - e^{-qV_{DS} / k_B T} \right) A$$


# Surface electric field

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$$E_S = \frac{qN_A W_D}{\epsilon_{Si}} = \frac{qN_A}{C_D}$$

$$m = 1 + C_D / C_{OX} \quad (m - 1)C_{OX} = C_D$$

$$E_S = \frac{qN_A}{(m - 1)C_{OX}}$$

## MOSFETs below $V_T$ (iv)

---

$$I_D = q \frac{W}{L} \left( \frac{n_i^2}{N_A} \right) \frac{(k_B T / q)^2}{E_S} \mu_{eff} e^{qV_{GS}/mk_B T} \left( 1 - e^{-qV_{DS}/k_B T} \right) \text{ A}$$

$$E_S = \frac{qN_A}{(m-1)C_{OX}}$$

$$I_D = \frac{W}{L} \left( \frac{n_i}{N_A} \right)^2 (m-1)C_{OX} (k_B T / q)^2 \mu_{eff} e^{qV_{GS}/mk_B T} \left( 1 - e^{-qV_{DS}/k_B T} \right) \text{ A}$$

# MOSFETs below $V_T$ (v)

$$(n_i/N_A) = ?$$

$$\psi_B = \frac{k_B T}{q} \ln \left( \frac{N_A}{n_i} \right) \quad \left( \frac{N_A}{n_i} \right) = e^{q\psi_B / k_B T} \quad \left( \frac{n_i}{N_A} \right)^2 = e^{-q2\psi_B / k_B T}$$

$$\psi_S = \frac{V_{GS}}{m}$$

$$2\psi_B = \frac{V_T}{m}$$

$$\left( \frac{n_i}{N_A} \right)^2 = e^{-qV_T / mk_B T}$$

## MOSFETs below $V_T$ (iv)

$$I_D = \frac{W}{L} \left( \frac{n_i}{N_A} \right)^2 (m-1) C_{OX} (k_B T / q)^2 \mu_{eff} e^{qV_{GS}/mk_B T} \left( 1 - e^{-qV_{DS}/k_B T} \right)$$

$$\left( \frac{n_i}{N_A} \right)^2 = e^{-qV_T/mk_B T}$$

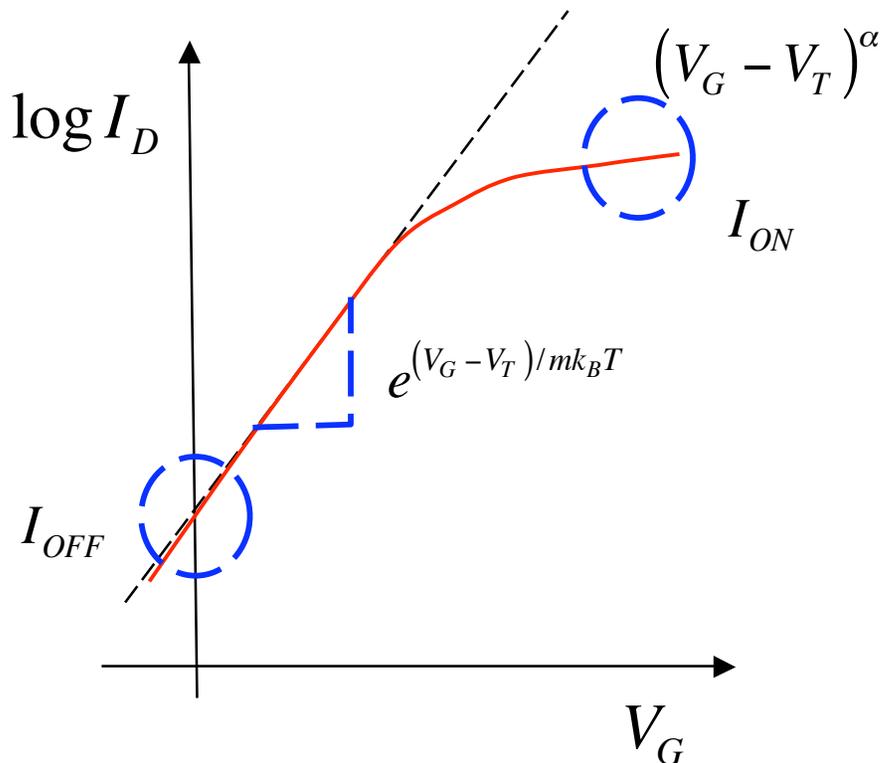
$$I_D = \mu_{eff} C_{OX} \left( \frac{W}{L} \right) (m-1) \left( \frac{k_B T}{q} \right)^2 e^{q(V_{GS} - V_T)/mk_B T} \left( 1 - e^{-qV_{DS}/k_B T} \right)$$

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- 1) Introduction
- 2) Bipolar transistors
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- 4) **Discussion**

# log $I_D$ vs. $V_{GS}$



- 1) subthreshold slope
- 2) off-current
- 3) on-current

$$I_D = \mu_{eff} C_{OX} \left( \frac{W}{L} \right) (m - 1) \left( \frac{k_B T}{q} \right)^2 e^{q(V_{GS} - V_T)/mk_B T} \left( 1 - e^{-qV_{DS}/k_B T} \right)$$

# subthreshold swing

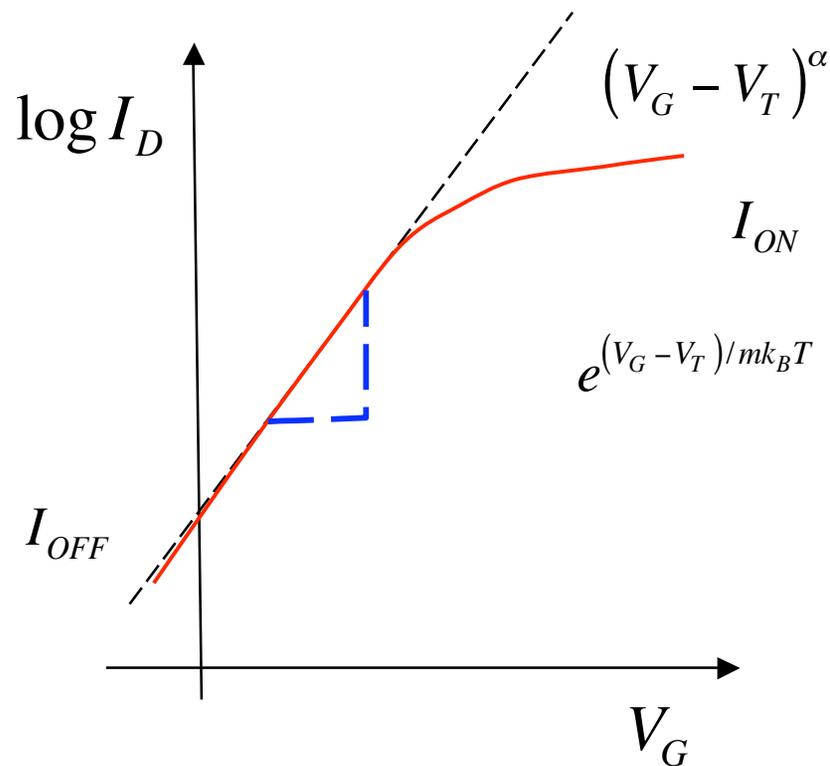
$$I_D \sim e^{q(V_{GS} - V_T)/mk_B T}$$

$$\ln I_D = \text{constant} + \frac{V_{GS}}{m(k_B T / q)} \quad \log_{10} I_D = \text{constant} + \frac{V_{GS}}{2.3m(k_B T / q)}$$

$$\frac{\partial(\log_{10} I_D)}{V_{GS}} = \frac{1}{2.3m(k_B T / q)} \quad \frac{\text{Decades of } I_D}{\text{Volts of } V_{GS}}$$

$$S = \left( \frac{\partial(\log_{10} I_D)}{V_{GS}} \right)^{-1} = 2.3m(k_B T / q) \frac{\text{mV}}{\text{dec}}$$

## subthreshold swing (ii)



$$S = 2.3m \left( k_B T / q \right) \frac{\text{mV}}{\text{dec}}$$

$$m = \left( 1 + C_D / C_{OX} \right) \geq 1$$

$$(1.1 - 1.4 \text{ typ})$$

$$S > 60 \frac{\text{mV}}{\text{dec}} \quad (T = 300 \text{ K})$$

$$S \approx 100 \frac{\text{mV}}{\text{dec}} \quad (\text{typically})$$

## what controls $S$ ?

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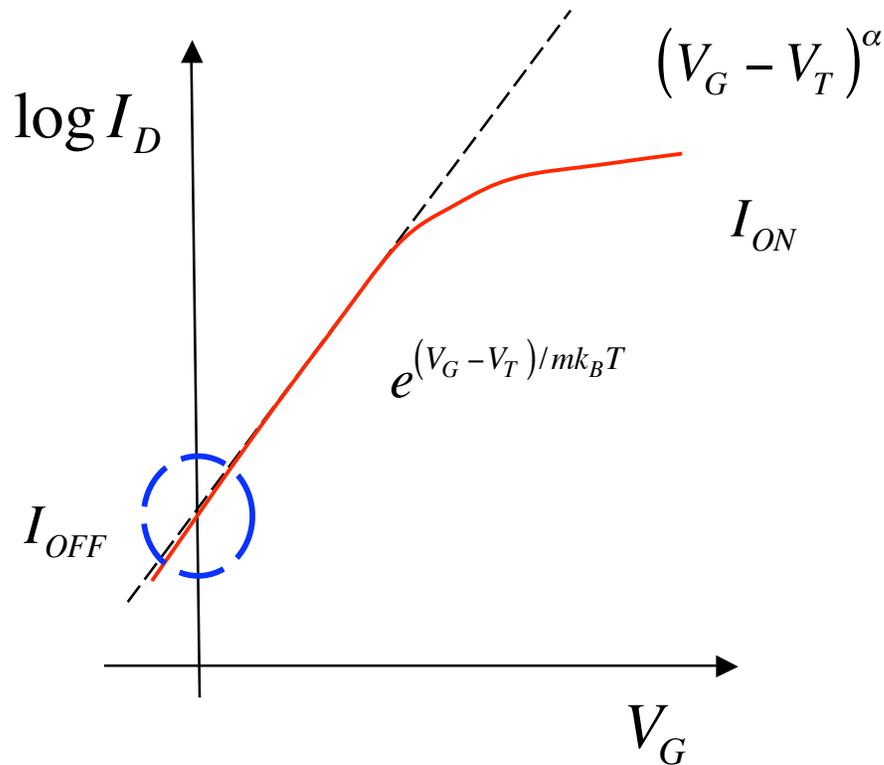
$$S = 2.3m(k_B T / q) \frac{\text{mV}}{\text{dec}}$$

- 1) Temperature
- 2) Interface traps

$$m = 1 + (C_D + C_{IT}) / C_{OX}$$

### 3) ***2D electrostatics***

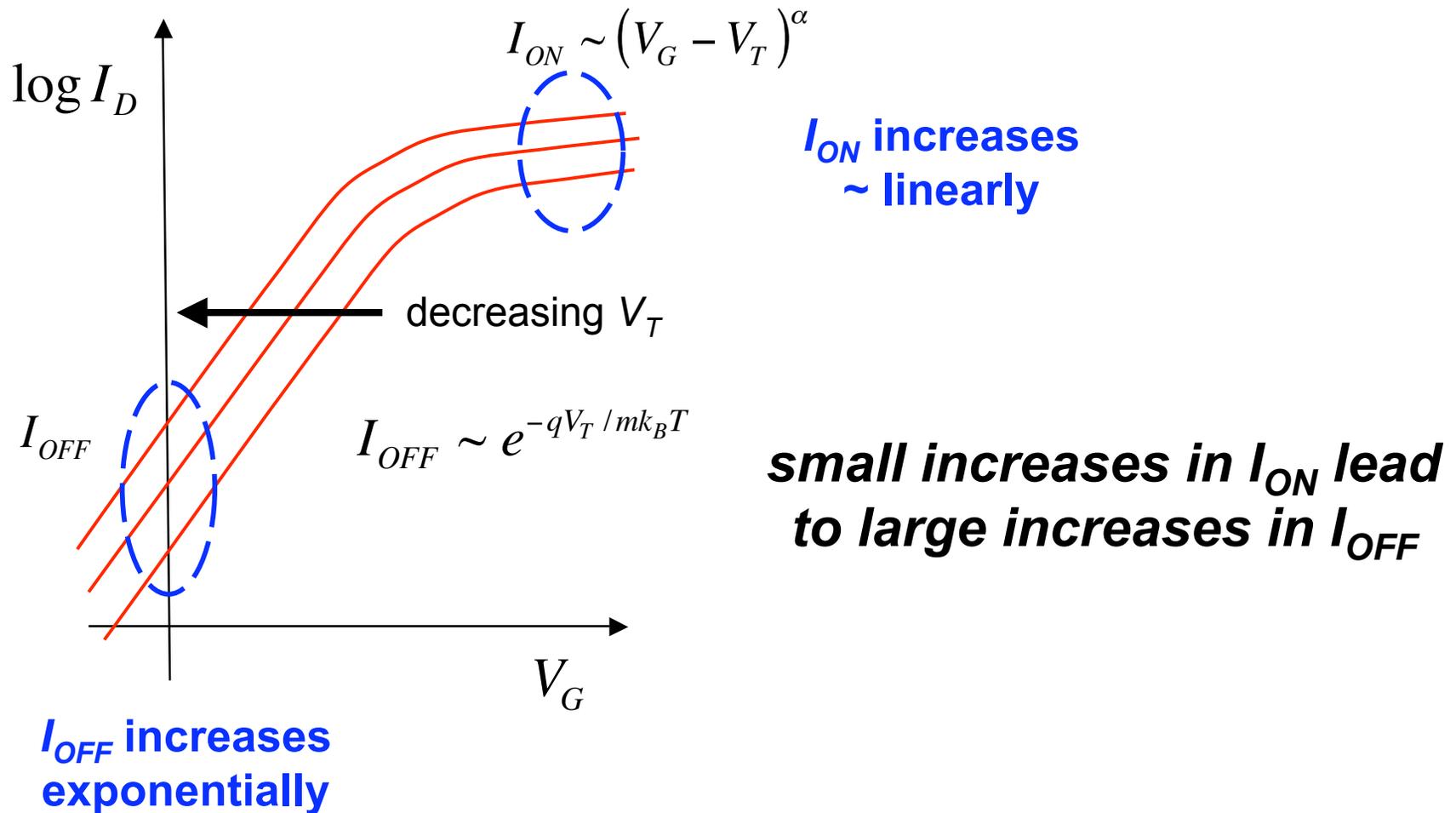
# off current



$$I_D = e^{-qV_T / mk_B T}$$

$$I_D = \mu_{eff} C_{OX} \left( \frac{W}{L} \right) (m - 1) \left( \frac{k_B T}{q} \right)^2 e^{q(V_{GS} - V_T) / mk_B T} \left( 1 - e^{-qV_{DS} / k_B T} \right)$$

# off current and $V_T$



# $I_{ON}$ and $I_{OFF}$

$$I_{OFF} = I_0 e^{-qV_T / mk_B T}$$

$$I_{ON} = G_0 (V_{DD} - V_T) \quad (\alpha = 1)$$

$$\left( \begin{aligned} I_{ON} &= (mk_B T / q) G_0 (V_{DD} - V_T) q / mk_B T \\ qI_{ON} / (mk_B T G_0) &= q(V_{DD} - V_T) / mk_B T \\ e^{-qV_T / mk_B T} &= e^{-qV_{DD} / mk_B T} e^{qI_{ON} / (G_0 mk_B T)} \end{aligned} \right)$$

$$I_{OFF} = I_1 e^{I_{ON} / I_2}$$

$$\ln I_{OFF} = \ln I_1 + I_{ON} / I_2$$

$$I_1 = I_0 e^{-qV_{DD} / mk_B T}$$

$$I_2 = G_0 (mk_B T / q)$$

# $\log I_{OFF}$ vs. $I_{ON}$

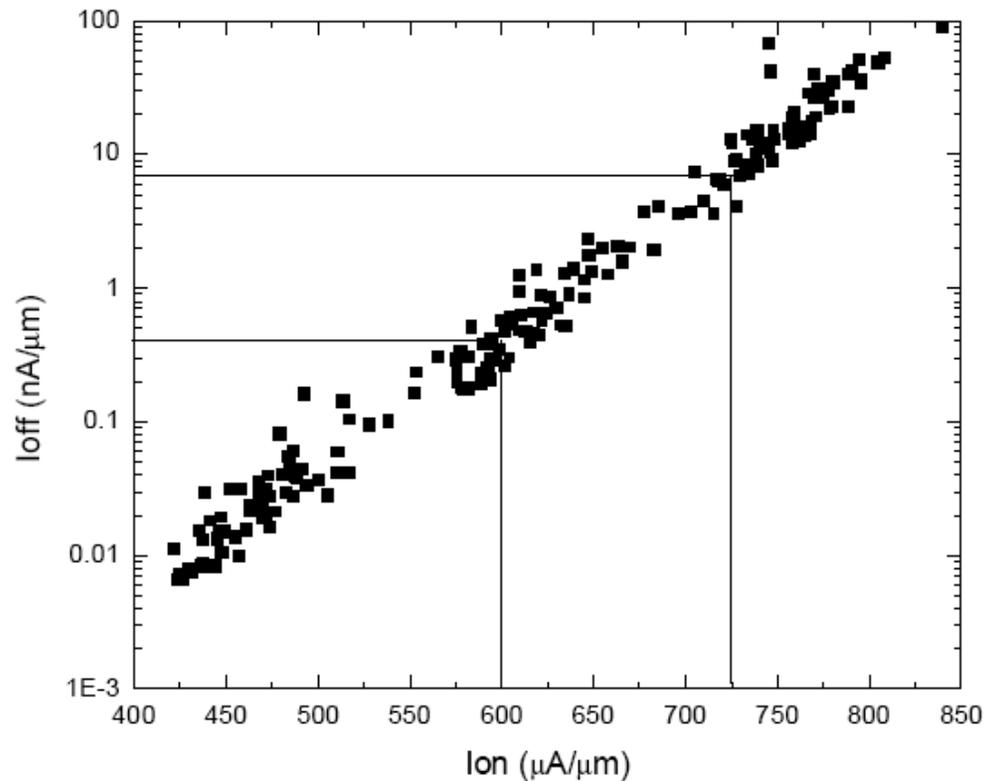
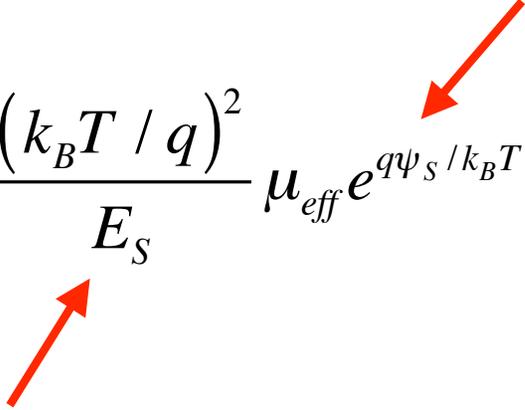


Fig.10a: Nfet  $I_{ON}/I_{OFF}$  performance.

“65nm CMOS Technology for low power applications,”  
A. Steegen, et al., 2005 International Electron Devices Meeting

## *below vs. above threshold*

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$$I_D = q \frac{W}{L} \left( \frac{n_i^2}{N_A} \right) \frac{(k_B T / q)^2}{E_S} \mu_{eff} e^{q\psi_S / k_B T} \left( 1 - e^{-qV_{DS} / k_B T} \right) \quad (1)$$


Why was our derivation restricted to subthreshold?

$$\text{i) } E_S = \frac{qN_A}{(m-1)C_{OX}} \quad \text{ii) } \psi_S = \frac{V_{GS}}{1 + C_{OX} / C_D}$$

## *below vs. above threshold*

---

$$I_D = q \frac{W}{L} \left( \frac{n_i^2}{N_A} \right) \frac{(k_B T / q)^2}{E_S} \mu_{eff} e^{q\psi_S / k_B T} \left( 1 - e^{-qV_{DS} / k_B T} \right) \quad (1)$$

$$\text{i) } E_S = \frac{Q_i}{\epsilon_{Si}} \quad \text{ii) } e^{q\psi_S / k_B T} = \frac{Q_i^2}{\epsilon_{Si} k_B T n_B}$$

$$I_D = q \frac{W}{L} \left( \frac{n_i^2}{N_A} \right) \frac{(k_B T / q)^2}{Q_i} \epsilon_{Si} \mu_{eff} \frac{Q_i^2}{\epsilon_{Si} k_B T n_B} \left( 1 - e^{-qV_{DS} / k_B T} \right)$$

## *below vs. above threshold*

---

$$I_D = WC_{OX}\mu_{eff} \frac{(k_B T / q)}{L} (V_{GS} - V_T) \quad (V_{DS} \gg k_B T / q)$$

reasonable above threshold result, except

$$L \rightarrow \mathcal{L}$$

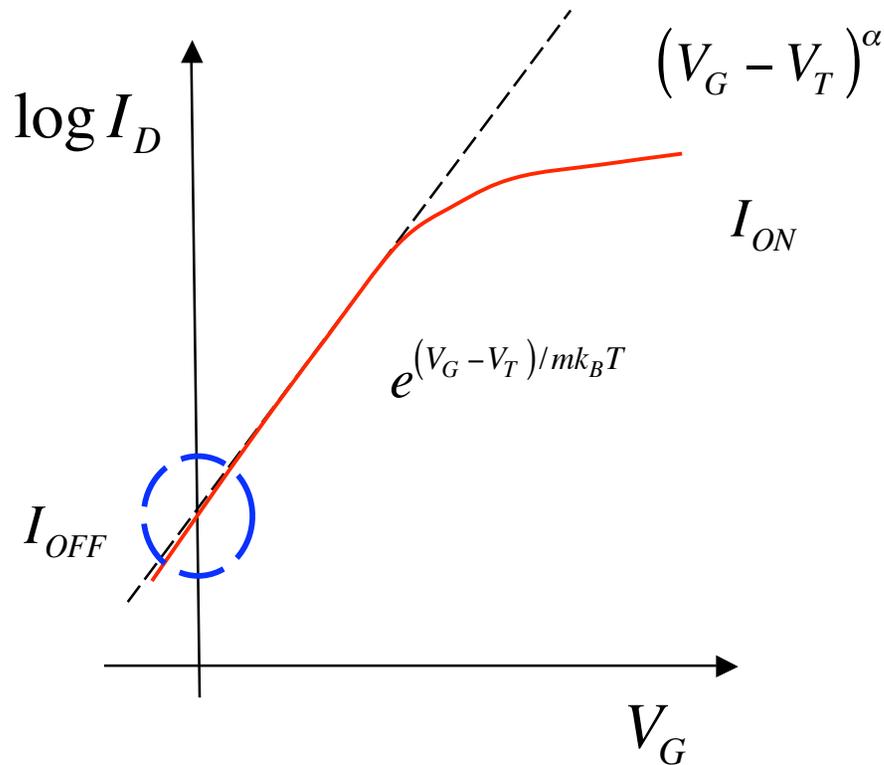
$$\mathcal{L} \ll L$$

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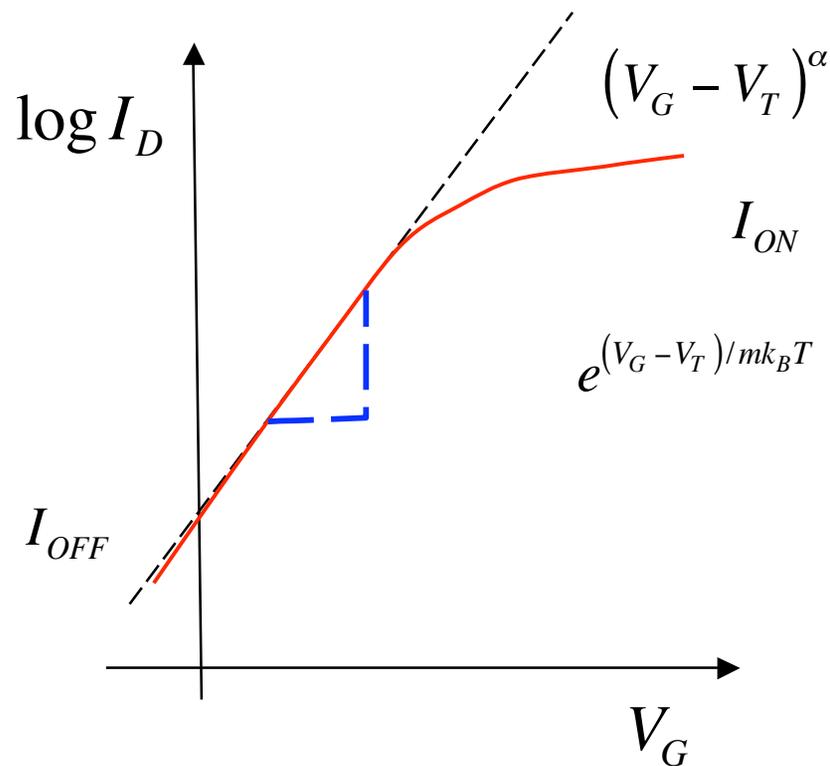
# off current



$$I_D = e^{-qV_T / mk_B T}$$

$$I_D = \mu_{eff} C_{OX} \left( \frac{W}{L} \right) (m - 1) \left( \frac{k_B T}{q} \right)^2 e^{q(V_{GS} - V_T) / mk_B T} \left( 1 - e^{-qV_{DS} / k_B T} \right)$$

## subthreshold swing (ii)



$$S = 2.3m \left( k_B T / q \right) \frac{\text{mV}}{\text{dec}}$$

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