

INFO AND EQUATION SHEETS

Sheets with relevant equations and a listing of Si material parameters, physical constants, and key conversion factors were routinely supplied at the end of closed-book test booklets.

Rather than provide the info/equation sheets with each test, we have collected on the following pages[†] a set of sheets that can be used with any of the closed-book Module-B test booklets.

[†] Pages B0-2 through B0-4 and the equations on pages B0-5 through B0-8 were reproduced from *PIERRET, ROBERT F., SEMICONDUCTOR DEVICE FUNDAMENTALS, 1st edition, © 1996. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.*

SILICON MATERIAL PARAMETERS

$$E_G = 1.12 \text{ eV} \quad (T = 300\text{K})$$

$$n_i = 10^{10}/\text{cm}^3 \quad (T = 300\text{K})$$

$$K_S = 11.8$$

$$K_O = 3.9$$

PHYSICAL CONSTANTS

Symbol	Name	Value
q	Electronic charge (magnitude)	1.60×10^{-19} coul
ϵ_0	Permittivity of free space	8.85×10^{-14} farad/cm
k	Boltzmann constant	8.617×10^{-5} eV/K
h	Planck constant	6.63×10^{-34} joule-sec
m_0	Electron rest mass	9.11×10^{-31} kg
kT	Thermal energy	0.0259 eV ($T = 300$ K)
kT/q	Thermal voltage	0.0259 V ($T = 300$ K)

CONVERSION FACTORS

$$1 \text{ \AA} = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$$

$$1 \text{ \mu m} = 10^{-4} \text{ cm} = 10^{-6} \text{ m}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ joules}$$

Table 2.4 Carrier Modeling Equation Summary.

<i>Density of States and Fermi Function</i>		
$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}, \quad E \geq E_c$		$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$
$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}, \quad E \leq E_v$		
<i>Carrier Concentration Relationships</i>		
$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c)$	$N_C = 2 \left[\frac{m_n^* kT}{2\pi \hbar^2} \right]^{3/2}$	$n = N_C e^{(E_F - E_c)/kT}$
$p = N_V \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v)$	$N_V = 2 \left[\frac{m_p^* kT}{2\pi \hbar^2} \right]^{3/2}$	$p = N_V e^{(E_v - E_F)/kT}$
		$n = n_i e^{(E_F - E_i)/kT}$
		$p = n_i e^{(E_i - E_F)/kT}$
<i>n_i, np-Product, and Charge Neutrality</i>		
$n_i = \sqrt{N_C N_V} e^{-E_G/2kT}$	$np = n_i^2$	$p - n + N_D - N_A = 0$
<i>n, p, and Fermi Level Computational Relationships</i>		
$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2}$		$E_i = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln \left(\frac{m_p^*}{m_n^*} \right)$
$n \approx N_D$	$N_D \gg N_A, N_D \gg n_i$	$E_F - E_i = kT \ln(n/n_i) = -kT \ln(p/n_i)$
$p \approx n_i^2/N_D$		
$p \approx N_A$	$N_A \gg N_D, N_A \gg n_i$	$E_F - E_i = kT \ln(N_D/n_i) \quad N_D \gg N_A, N_D \gg n_i$
$n \approx n_i^2/N_A$		$E_i - E_F = kT \ln(N_A/n_i) \quad N_A \gg N_D, N_A \gg n_i$

Table 3.3 Carrier Action Equation Summary.

<i>Equations of State</i>	
$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_N + \left. \frac{\partial n}{\partial t} \right _{\text{thermal R-G}} + \left. \frac{\partial n}{\partial t} \right _{\text{other processes}}$	$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$
$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot \mathbf{J}_P + \left. \frac{\partial p}{\partial t} \right _{\text{thermal R-G}} + \left. \frac{\partial p}{\partial t} \right _{\text{other processes}}$	$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$
<i>Current and R-G Relationships</i>	
$\mathbf{J}_N = \mathbf{J}_{N \text{drift}} + \mathbf{J}_{N \text{diff}} = q\mu_n n \mathcal{E} + qD_N \nabla n$ <p style="text-align: center;"> \Downarrow drift \Downarrow diffusion </p>	$\left. \frac{\partial n}{\partial t} \right _{\text{i-thermal R-G}} = -\frac{\Delta n}{\tau_n}$
$\mathbf{J}_P = \mathbf{J}_{P \text{drift}} + \mathbf{J}_{P \text{diff}} = q\mu_p p \mathcal{E} - qD_P \nabla p$	$\left. \frac{\partial p}{\partial t} \right _{\text{i-thermal R-G}} = -\frac{\Delta p}{\tau_p}$
$\mathbf{J} = \mathbf{J}_N + \mathbf{J}_P$	
<i>Key Parametric Relationships</i>	
$L_N \equiv \sqrt{D_N \tau_n}$	$\frac{D_N}{\mu_n} = \frac{kT}{q}$
$L_P \equiv \sqrt{D_P \tau_p}$	$\frac{D_P}{\mu_p} = \frac{kT}{q}$
	$\tau_n = \frac{1}{c_n N_T}$
	$\tau_p = \frac{1}{c_p N_T}$
<i>Resistivity and Electrostatic Relationships</i>	
$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$	$\rho = \frac{1}{q\mu_n N_D} \quad \dots n\text{-type semiconductor}$
	$\rho = \frac{1}{q\mu_p N_A} \quad \dots p\text{-type semiconductor}$
$\mathcal{E} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx} = \frac{1}{q} \frac{dE_i}{dx}$	$V = -\frac{1}{q} (E_c - E_{\text{ref}})$
<i>Quasi-Fermi Level Relationships</i>	
$F_N \equiv E_i + kT \ln\left(\frac{n}{n_i}\right)$	$\mathbf{J}_N = \mu_n n \nabla F_N$
$F_P \equiv E_i - kT \ln\left(\frac{p}{n_i}\right)$	$\mathbf{J}_P = \mu_p p \nabla F_P$

DIODES

BUILT-IN VOLTAGE

$$J_N = q\mu_n n \mathcal{E} + qD_N \frac{dn}{dx} = 0$$

$$\mathcal{E} = -\frac{D_N}{\mu_n} \frac{dn/dx}{n} = -\frac{kT}{q} \frac{dn/dx}{n}$$

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E} dx = \frac{kT}{q} \int_{n(-x_p)}^{n(x_n)} \frac{dn}{n} = \frac{kT}{q} \ln \left[\frac{n(x_n)}{n(-x_p)} \right]$$

$$n(x_n) = N_D$$

$$n(-x_p) = n_i^2 / N_A$$

$$V_{bi} = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

For $-x_p \leq x \leq 0 \dots$

STEP JUNCTION

$$\mathcal{E}(x) = -\frac{qN_A}{K_S \epsilon_0} (x_p + x)$$

$$V(x) = \frac{qN_A}{2K_S \epsilon_0} (x_p + x)^2$$

$$x_p = \left[\frac{2K_S \epsilon_0}{q} \frac{N_D}{N_A(N_A + N_D)} (V_{bi} - V_A) \right]^{1/2}$$

For $0 \leq x \leq x_n \dots$

$$\mathcal{E}(x) = -\frac{qN_D}{K_S \epsilon_0} (x_n - x)$$

$$V(x) = V_{bi} - V_A - \frac{qN_D}{2K_S \epsilon_0} (x_n - x)^2$$

$$x_n = \left[\frac{2K_S \epsilon_0}{q} \frac{N_A}{N_D(N_A + N_D)} (V_{bi} - V_A) \right]^{1/2}$$

and

$$W = \left[\frac{2K_S \epsilon_0}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) (V_{bi} - V_A) \right]^{1/2}$$

LINEARLY GRADED JUNCTION

$$\rho(x) = \begin{cases} qa & \dots -W/2 \leq x \leq W/2 \\ 0 & \dots x \leq -W/2 \text{ and } x \geq W/2 \end{cases}$$

$$x_p = x_n = \frac{W}{2}$$

$$\mathcal{E}(x) = \frac{qa}{2K_S \epsilon_0} \left[x^2 - \left(\frac{W}{2} \right)^2 \right] \quad \dots -\frac{W}{2} \leq x \leq \frac{W}{2}$$

$$V(x) = \frac{qa}{6K_S \epsilon_0} \left[2 \left(\frac{W}{2} \right)^3 + 3 \left(\frac{W}{2} \right)^2 x - x^3 \right] \quad \dots -\frac{W}{2} \leq x \leq \frac{W}{2}$$

$$W = \left[\frac{12K_S \epsilon_0}{qa} (V_{bi} - V_A) \right]^{1/3}$$

IDEAL DIODE EQUATION - 1

$$n(x_n)p(x_n) = p(x_n)N_D = n_i^2 e^{qV_A/kT}$$

$$p(x_n) = \frac{n_i^2}{N_D} e^{qV_A/kT}$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$$

$$0 = D_P \frac{d^2 \Delta p_n}{dx'^2} - \frac{\Delta p_n}{\tau_p} \quad \dots x' \geq 0$$

$$\Delta p_n(x' \rightarrow \infty) = 0$$

$$\Delta p_n(x'=0) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$$

IDEAL DIODE EQUATION - 2

$$\Delta p_n(x') = A_1 e^{-x'/L_P} + A_2 e^{x'/L_P} \quad \dots x' \geq 0$$

$$L_P = \sqrt{D_P \tau_p}$$

$$\Delta p_n(x') = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1) e^{-x'/L_P} \quad \dots x' \geq 0$$

$$J_p(x') = -q D_P \frac{d \Delta p_n}{dx'} = q \frac{D_P}{L_P} \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1) e^{-x'/L_P} \quad \dots x' \geq 0$$

$$\Delta n_p(x'') = \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1) e^{-x''/L_N} \quad \dots x'' \geq 0$$

IDEAL DIODE EQ. - 3

$$J_N(x'') = -q D_N \frac{d \Delta n_p}{dx''} = q \frac{D_N}{L_N} \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1) e^{-x''/L_N} \quad \dots x'' \geq 0$$

$$J_N(x=-x_p) = J_N(x''=0) = q \frac{D_N}{L_N} \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1)$$

$$J_p(x=x_n) = J_p(x'=0) = q \frac{D_P}{L_P} \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$$

$$I = AJ = qA \left(\frac{D_N}{L_N} \frac{n_i^2}{N_A} + \frac{D_P}{L_P} \frac{n_i^2}{N_D} \right) (e^{qV_A/kT} - 1)$$

$$I = I_0 (e^{qV_A/kT} - 1)$$

$$I_0 = qA \left(\frac{D_N}{L_N} \frac{n_i^2}{N_A} + \frac{D_P}{L_P} \frac{n_i^2}{N_D} \right)$$

JUNCTION CAPACITANCE

$$Y = G + j\omega C$$

$$C_J = \frac{K_S \epsilon_0 A}{W}$$

$$W = \left[\frac{2K_S \epsilon_0}{qN_B} (V_{bi} - V_A) \right]^{1/2} \quad \dots \text{asymmetrical step junction}$$

$$W = \left[\frac{12K_S \epsilon_0}{qa} (V_{bi} - V_A) \right]^{1/3} \quad \dots \text{linearly-graded junction}$$

$$N_B(x) = bx^m \quad \dots x > 0$$

SCHOTTKY I-V

$$I_{M \rightarrow S}(V_A) = I_{M \rightarrow S}(V_A = 0)$$

$$I_{M \rightarrow S}(V_A = 0) = -I_{S \rightarrow M}(V_A = 0) = -Aq^* T^2 e^{-\Phi_B/kT}$$

$$I = I_{S \rightarrow M} + I_{M \rightarrow S} = I_{S \rightarrow M} + I_{M \rightarrow S}(V_A = 0)$$

$$I = I_s (e^{qV_A/kT} - 1)$$

$$I_s \equiv Aq^* T^2 e^{-\Phi_B/kT}$$

BJT

$$I_E = I_B + I_C$$

$$V_{EB} + V_{BC} + V_{CE} = 0 \quad (V_{CE} = -V_{EC})$$

$$\gamma = \frac{I_{EP}}{I_E} = \frac{I_{EP}}{I_{EP} + I_{En}} \quad \dots \text{npn BJT}$$

$$\alpha_T = \frac{I_{CP}}{I_{EP}} \quad \dots \text{npn BJT}$$

$$\alpha_{dc} = \gamma \alpha_T$$

$$\beta_{dc} = \frac{I_C}{I_B}$$

$$\beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}}$$

$$I_{En} = -qAD_E \left. \frac{d\Delta n_E}{dx} \right|_{x=W} \quad I_{Cp} = -qAD_B \left. \frac{d\Delta p_B}{dx} \right|_{x=W}$$

$$I_{Ep} = -qAD_B \left. \frac{d\Delta p_B}{dx} \right|_{x=0} \quad I_{Cn} = qAD_C \left. \frac{d\Delta n_C}{dx} \right|_{x=0}$$

$$\Delta p_B(x) = \Delta p_B(0) \frac{\sinh[(W-x)/L_B]}{\sinh(W/L_B)} + \Delta p_B(W) \frac{\sinh(x/L_B)}{\sinh(W/L_B)}$$

$$\Delta p_B(x) = \Delta p_B(0) + [\Delta p_B(W) - \Delta p_B(0)] \frac{x}{W}$$

$$I_{En} = qA \frac{D_E}{L_E} n_{E0} (e^{qV_{EB}/kT} - 1)$$

$$I_{Ep} = qA \frac{D_B}{L_B} p_{B0} \left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)} (e^{qV_{EB}/kT} - 1) - \frac{1}{\sinh(W/L_B)} (e^{qV_{CB}/kT} - 1) \right]$$

$$I_{Cn} = -qA \frac{D_C}{L_C} n_{C0} (e^{qV_{CB}/kT} - 1)$$

$$I_{Cp} = qA \frac{D_B}{L_B} p_{B0} \left[\frac{1}{\sinh(W/L_B)} (e^{qV_{EB}/kT} - 1) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)} (e^{qV_{CB}/kT} - 1) \right]$$

$$\gamma = \frac{1}{1 + \frac{D_E N_B W}{D_B N_E L_E}}$$

$$\alpha_T = \frac{1}{1 + \frac{1}{2} \left(\frac{W}{L_B} \right)^2}$$

$$\alpha_{dc} = \frac{1}{1 + \frac{D_E N_B W}{D_B N_E L_E} + \frac{1}{2} \left(\frac{W}{L_B} \right)^2}$$

$$\beta_{dc} = \frac{1}{\frac{D_E N_B W}{D_B N_E L_E} + \frac{1}{2} \left(\frac{W}{L_B} \right)^2}$$

$$I_B = I_{F0} (e^{qV_{EB}/kT} - 1) - \alpha_R I_{R0} (e^{qV_{CB}/kT} - 1)$$

$$I_C = \alpha_F I_{F0} (e^{qV_{EB}/kT} - 1) - I_{R0} (e^{qV_{CB}/kT} - 1)$$

$$V_{CB0} = V_{CB0} (1 - \alpha_{dc})^{1/m} = \frac{V_{CB0}}{(\beta_{dc} + 1)^{1/m}}$$