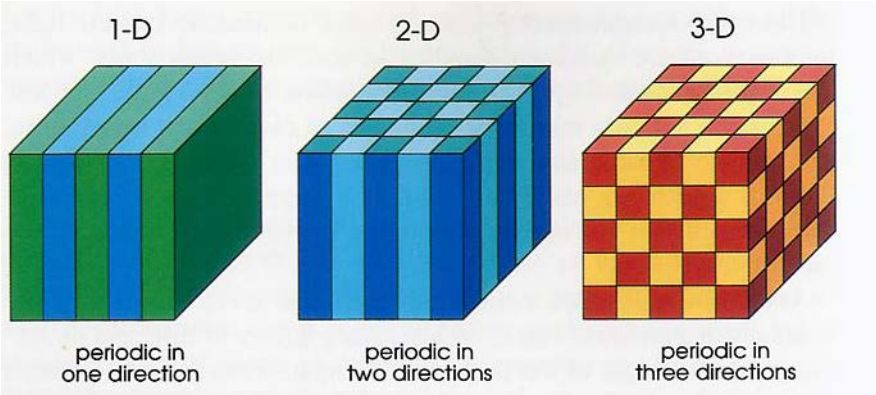
Lecture 5: Photonic crystal: An Introduction

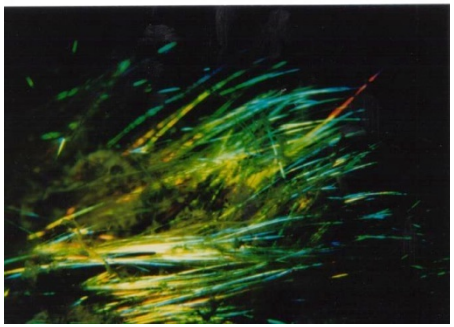


Photonic crystal:

Periodic arrangement of dielectric (metallic, polaritonic…) objects.
Lattice constants comparable to the wavelength of light in the material.

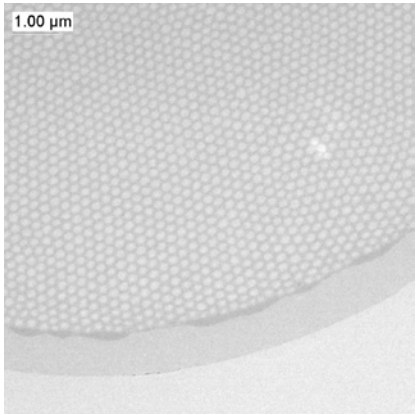
“ A worm ahead of its time”

Sea Mouse

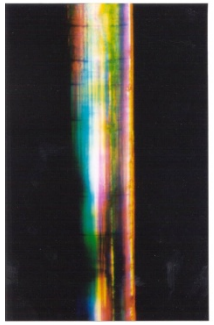


20cm

and its hair



Normal incident light



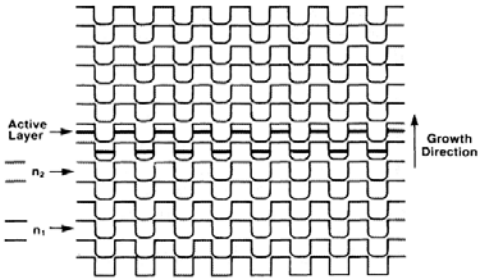
Off-Normal incident light

http://www.physics.usyd.edu.au/~nicolae/seamouse.html

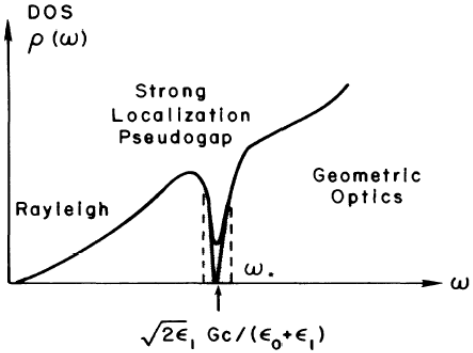
Fast forward to 1987……

E. Yablonovitch
"Inhibited spontaneous emission in solid state physics and electronics"
Physical Review Letters, vol. 58, pp. 2059, 1987

S. John
"Strong localization of photons in certain disordered dielectric superlattices"
Physical Review Letters, vol. 58, pp. 2486, 1987



Face-centered cubic lattice



Complete photonic band gap

The emphasis of recent breakthroughs

•The use of strong index contrast, and the developments of nano-fabrication technologies, which leads to entirely new sets of phenomena.

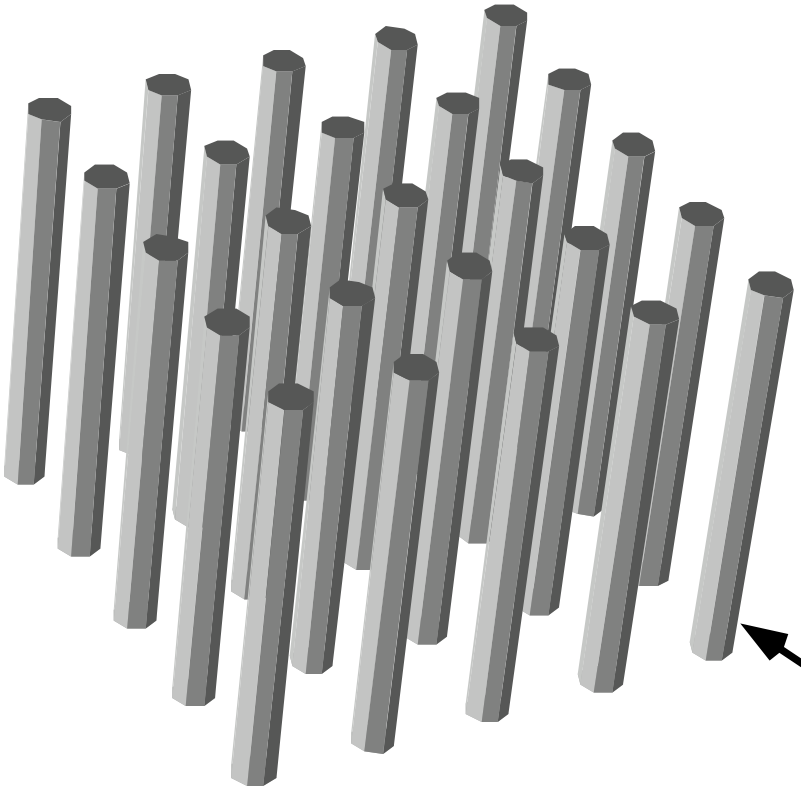
Conventional silica fiber, $\delta n \sim 0.01$, photonic crystal structure, $\delta n \sim 1$

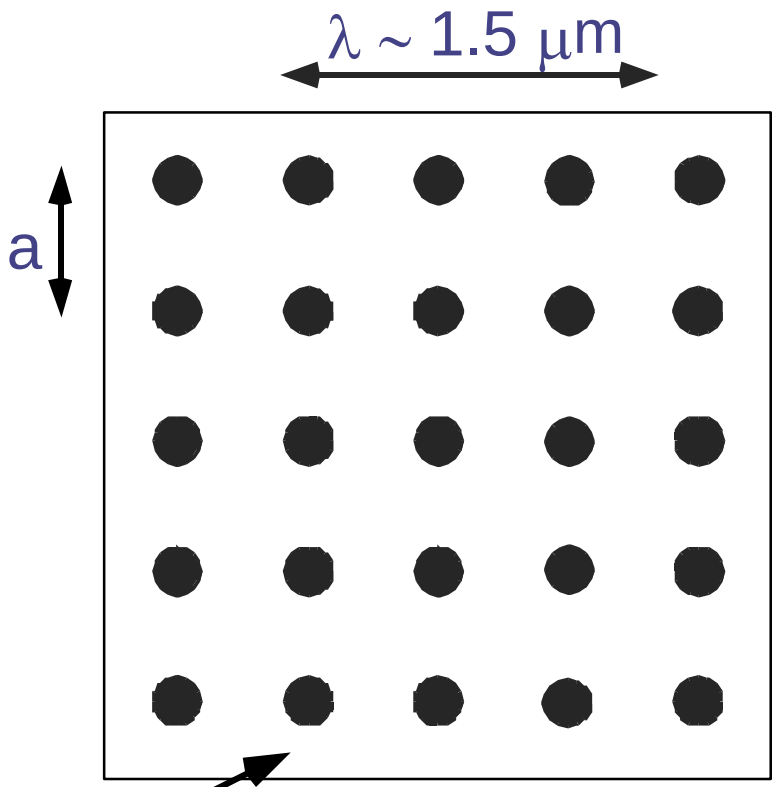
•New conceptual framework in optics

Band structure concepts.
Coupled mode theory approach for photon transport.

•Photonic crystal: semiconductors for light.

Two-dimensional photonic crystal

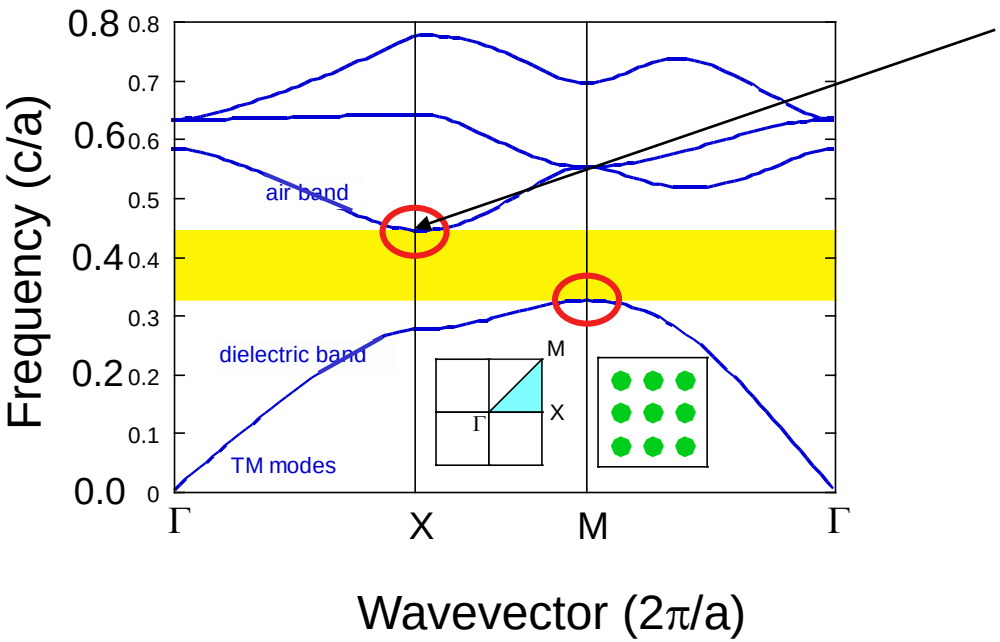


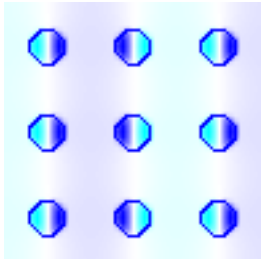


High-index
dielectric material,
e.g. Si or GaAs

Band structure of a two-dimensional crystal

Displacement field parallel to the cylinder



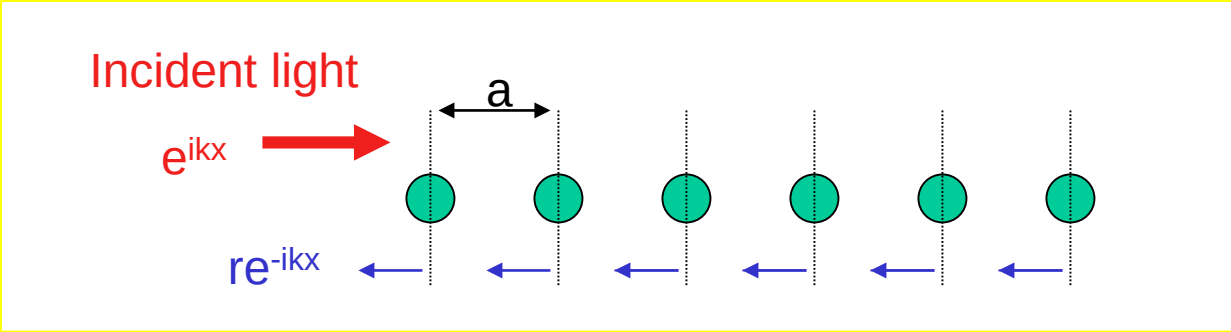


Wavevector determines the phase between nearest neighbor unit cells.

X: (0.5*2π/a, 0): Thus, nearest neighbor unit cell along the x-direction is 180 degree out-of-phase

M: (0.5*2π/a, 0.5*2π/a): nearest neighbor unit cell along the diagonal direction is 180 degree out-of-phase

Bragg scattering



Regardless of how small the reflectivity r is from an individual scatter, the total reflection R from a semi infinite structure:

$$R = re^{-ikx} + re^{-2ika}e^{-ikx} + re^{-4ika}e^{-ikx} + \ldots = re^{-ikx}\frac{1}{1-e^{-2ika}}$$

Diverges if

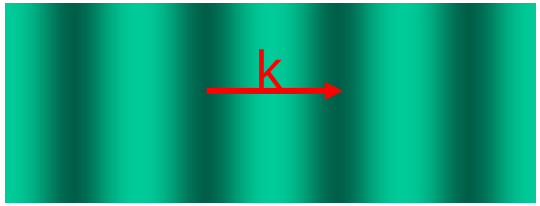
$$\boxed{e^{2ika} = 1 \qquad k = \frac{\pi}{a}}$$

Bragg condition

Light can not propagate in a crystal, when the frequency of the incident light is such that the Bragg condition is satisfied → Origin of the photonic band gap

A simple example of the band-structure: vacuum (1d)

Vacuum: ε=1, μ=1, plane-wave solution to the Maxwell's equation:



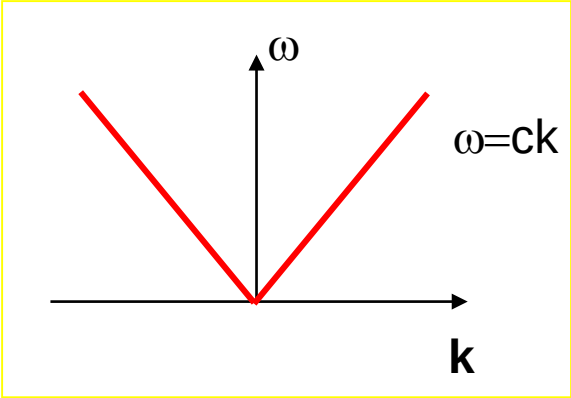
$$\mathbf{H}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

with a transversality constraints: $\mathbf{k}\bullet\mathbf{H}=0$

A band structure, or dispersion relation defines the relation between the frequency ω, and the wavevector k.

$$\omega = c|\mathbf{k}|$$

For a one-dimensional system, the band structure can be simply depicted as:

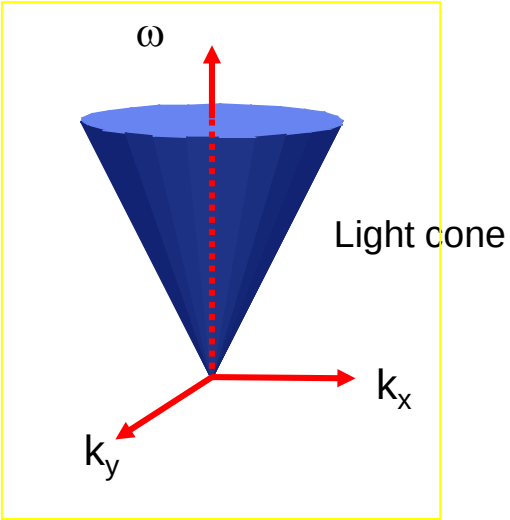


Visualization of the vacuum band structure (2d)

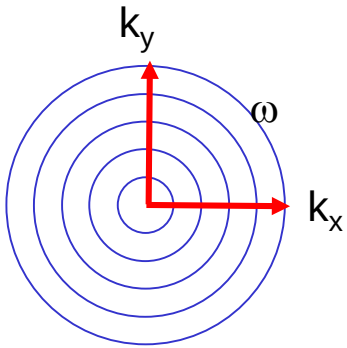
For a two-dimensional system:

$$\omega = c\sqrt{k_x^2 + k_y^2}$$

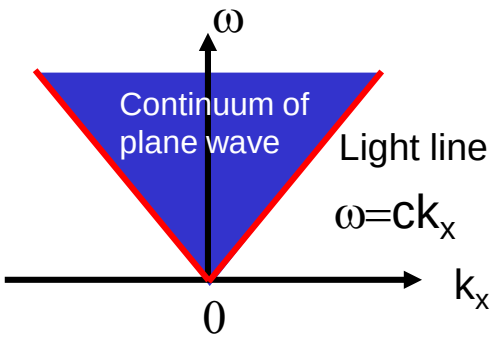
This function depicts a cone: light cone.



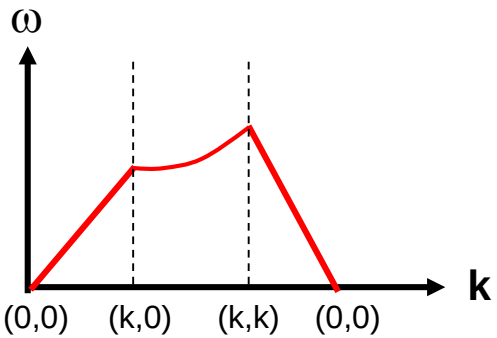
A few ways to visualize this band structure :



Constant frequency contour



Projected band diagram



Band diagram along several “special” directions

Maxwell's equation in the steady state

Time-dependent Maxwell's equation in dielectric media:

$$\nabla \bullet \mathbf{H}(\mathbf{r},t) = 0 \qquad \nabla \times \mathbf{H}(\mathbf{r},t) - \varepsilon(\mathbf{r}) \frac{\partial(\varepsilon_0 \mathbf{E}(\mathbf{r},t))}{\partial \mathbf{t}} = 0$$

$$\nabla \bullet \varepsilon \mathbf{E}(\mathbf{r},t) = 0 \qquad \nabla \times \mathbf{E}(\mathbf{r},t) + \frac{\partial(\mu_0 \mathbf{H}(\mathbf{r},t))}{\partial \mathbf{t}} = 0$$

Time harmonic mode (i.e. steady state):

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r}) e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r}) e^{-i\omega t}$$

Maxwell equation for the steady state:

$$\nabla \times \mathbf{H}(\mathbf{r}) + i\omega(\varepsilon(\mathbf{r})\varepsilon_0 \mathbf{E}(\mathbf{r})) = 0$$

$$\nabla \times \mathbf{E}(\mathbf{r}) - i\omega(\mu_0 \mathbf{H}(\mathbf{r})) = 0$$

Master's equation for steady state in dielectric

Expressing the equation in magnetic field only:

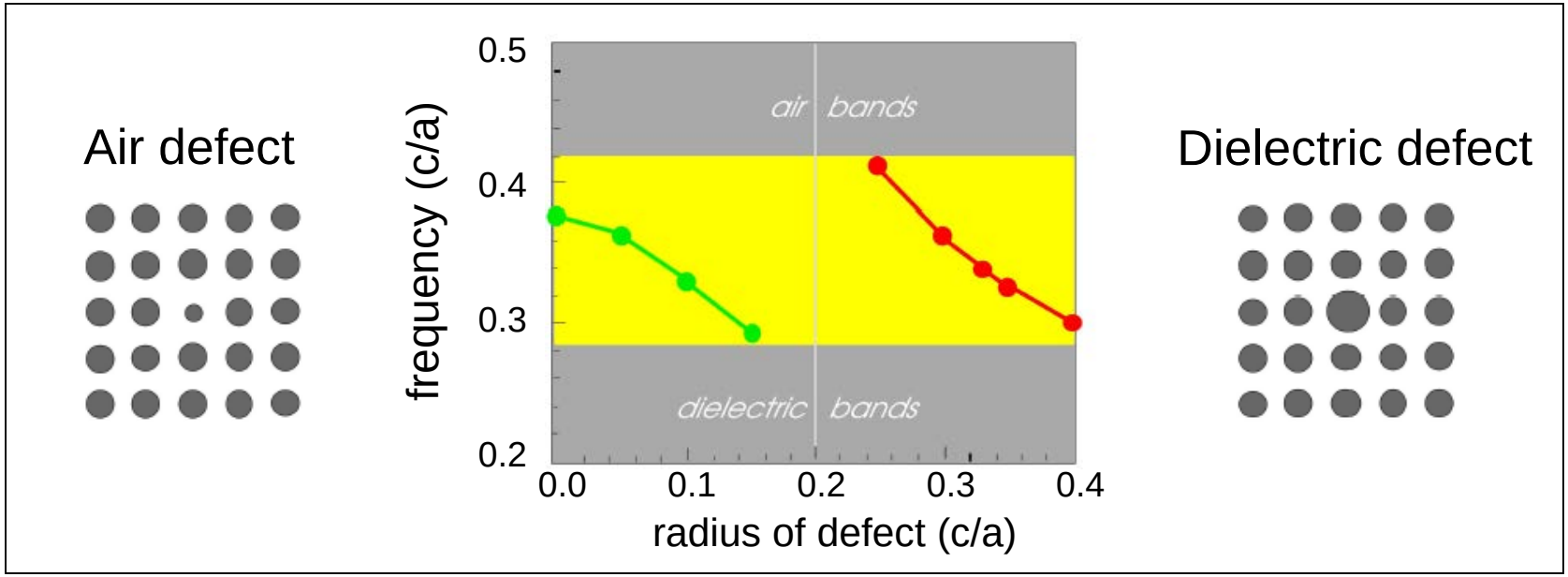
$$\nabla \times \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) = \left(\frac{\omega}{c}\right)^2 \mathbf{H}(\mathbf{r})$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

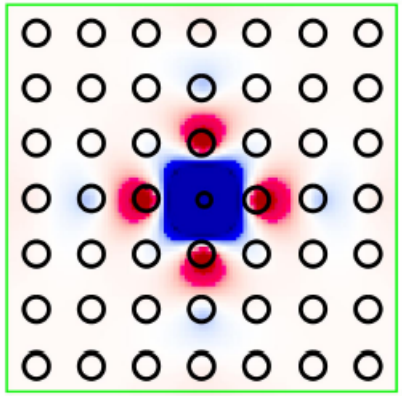
Thus, the Maxwell's equation for the steady state can be expressed in terms of an eigenvalue problem, in direct analogy to quantum mechanics that governs the properties of electrons.

	Quantum mechanics	Electromagnetism
Field	$\Psi(\mathbf{r},t) = \Psi(\mathbf{r}) e^{j\omega t}$	$\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r}) e^{i\omega t}$
Eigen-value problem	$\hat{H}\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$	$\Theta\mathbf{H}(\mathbf{r}) = \left(\frac{\omega^2}{c^2}\right)\mathbf{H}(\mathbf{r})$
Operator	$\hat{H} = \frac{-\hbar^2\nabla^2}{2m} + V(\mathbf{r})$	$\Theta = \nabla \times \frac{1}{\varepsilon(\mathbf{r})} \nabla \times$

Donor and Acceptor States

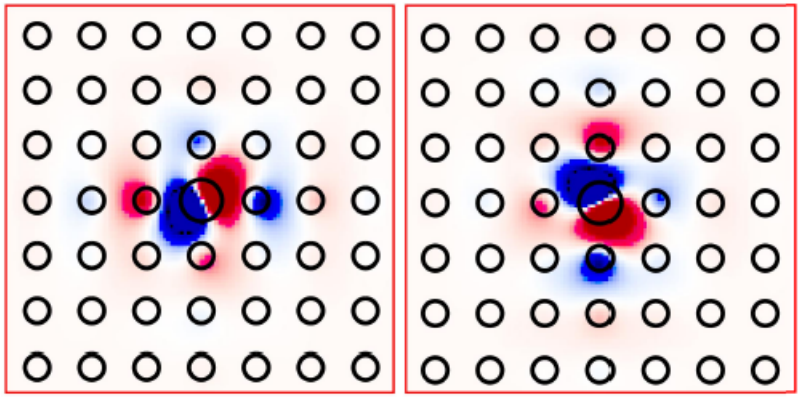


Air defect



s-state

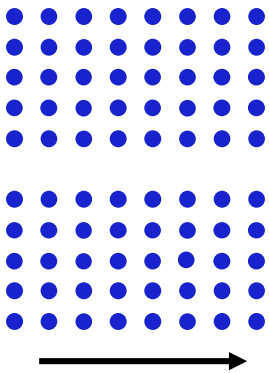
Dielectric defect



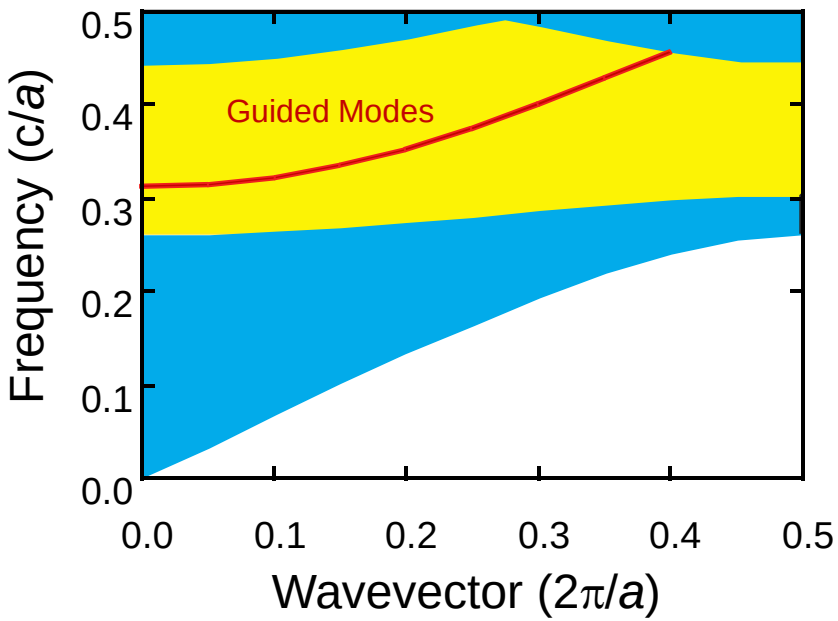
p-state

P. R. Villeneuve, S. Fan, and J. D. Joannopoulos, Phys. Rev. B 54, 7837 (1996)

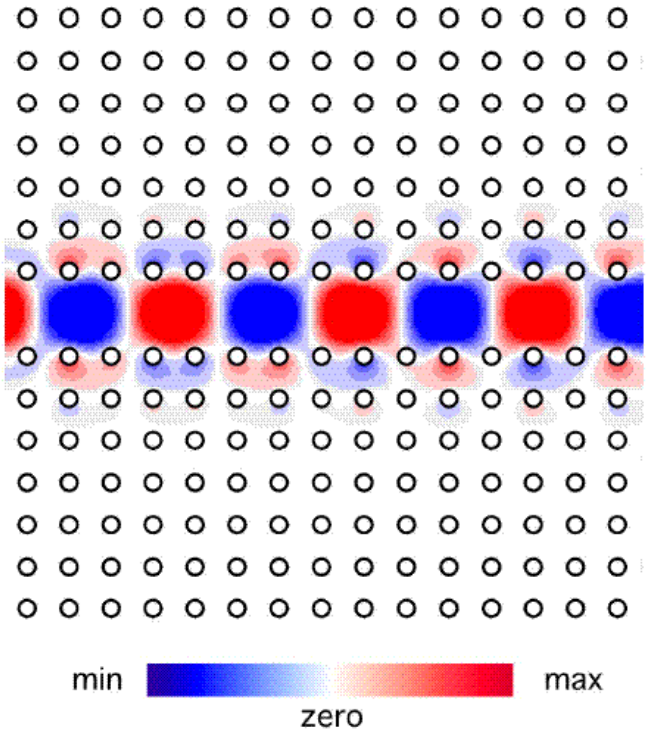
Line defect states: projected band diagram



parallel wavevector

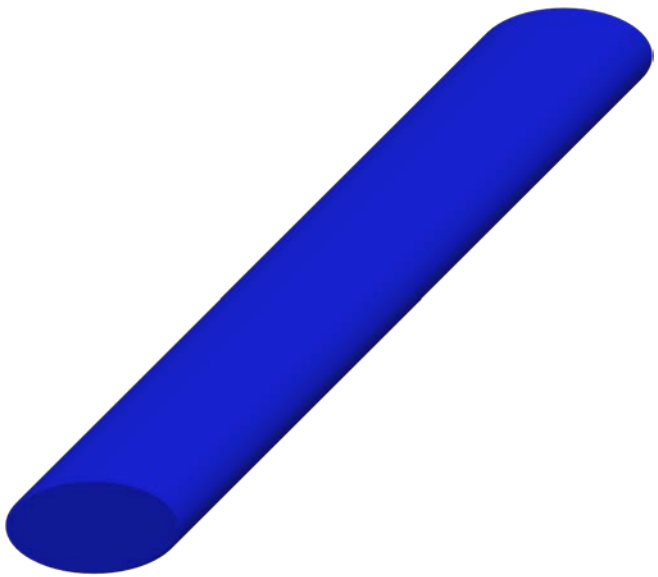


Electric field



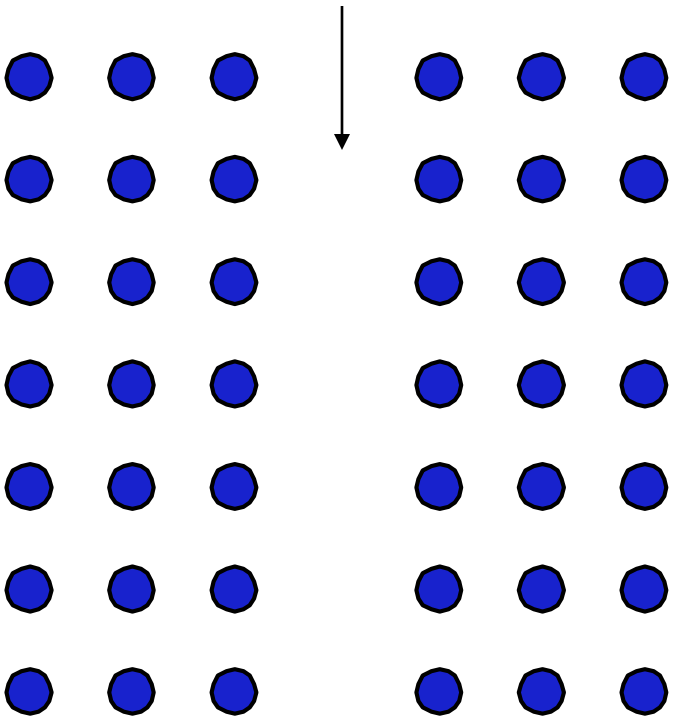
Photonic crystal vs. conventional waveguide

High-index region



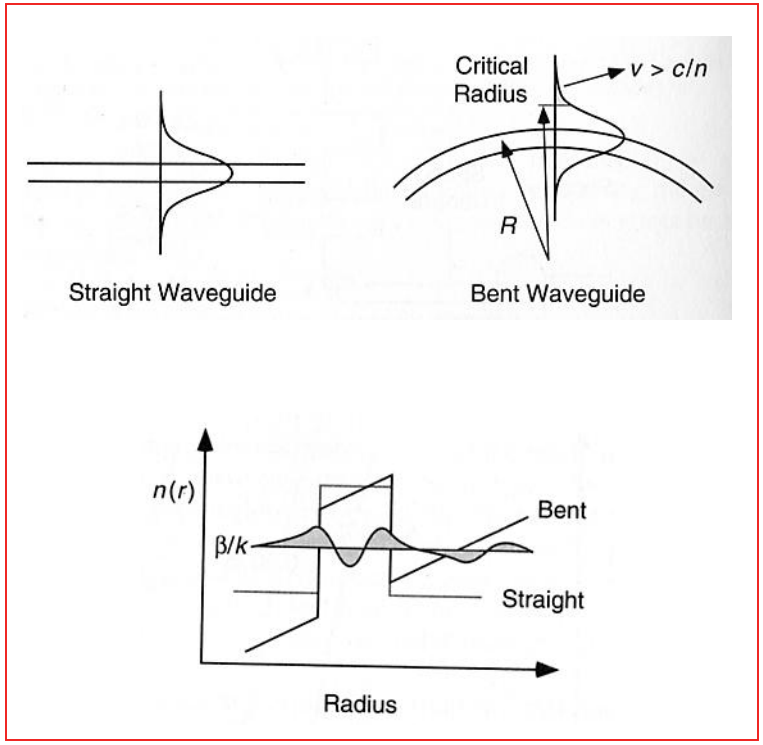
Conventional waveguide

Low index region

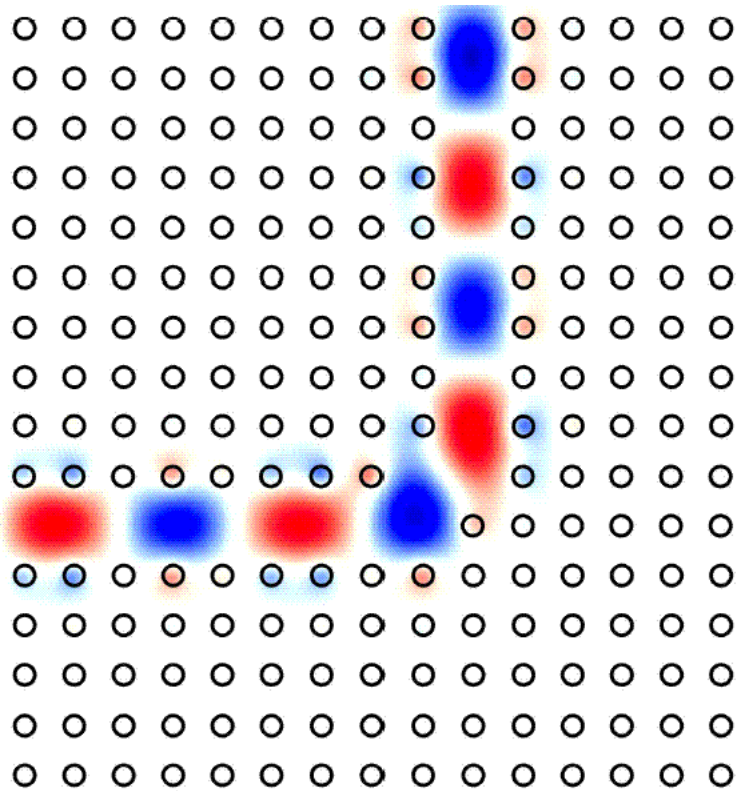


Photonic crystal waveguide

High transmission through sharp bends



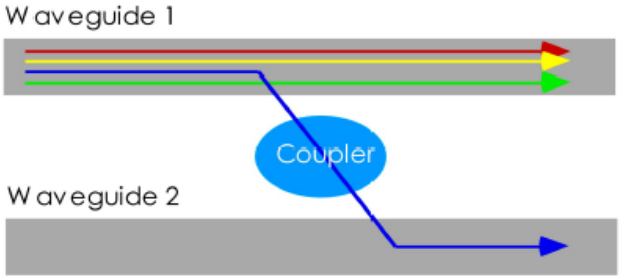
$$\alpha = \frac{1}{2}\left(\frac{\pi}{aV^3}\right)^{1/2}\left[\frac{\kappa a}{\gamma a K_1(\gamma a)}\right]^2 R^{-1/2} e^{-UR}$$



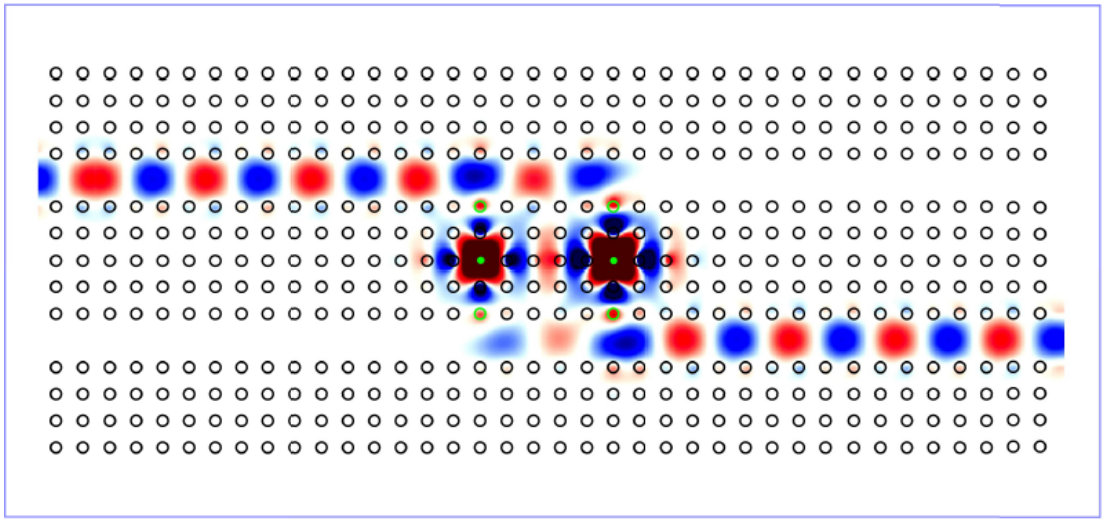
Polluck, Fundamentals of Optoelectronics, 1995

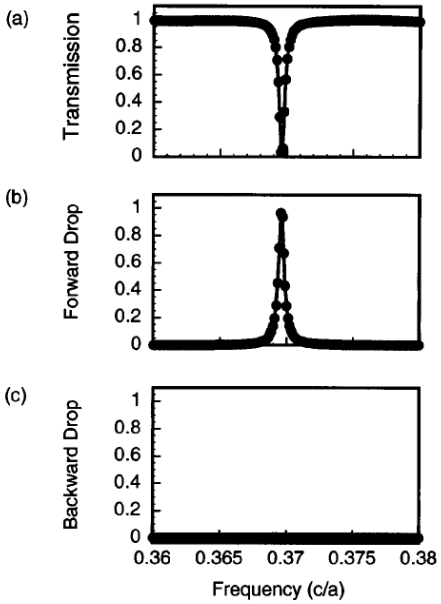
A. Mekis et al, PRL, 77, 3786 (1996)

Micro add/drop filter in photonic crystals



- *Two resonant modes with even and odd symmetry.*
- *The modes must be degenerate.*
- *The modes must have the same decay rate.*





S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, Physical Review Letters 80, 960 (1998).

Summary

•Photonic crystals are artificial media with a periodic index contrast.

•Electromagnetic wave in a photonic crystal is described by a band structure, which relates the frequency of modes to the wavevectors.

•Fundamental properties of modes: scale invariance, orthogonality