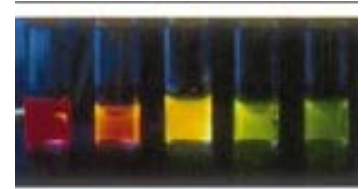


Lecture 8. What happened at the previous lectures ?

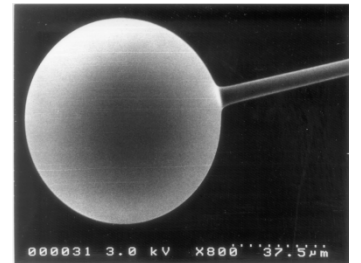
Light interaction with small objects ($d < \lambda$)

- Insulators (Rayleigh Scattering, blue sky..)
- Semiconductors (Size dependent absorption, fluorescence..)
- Metals...Resonant absorption at ω_{sp}



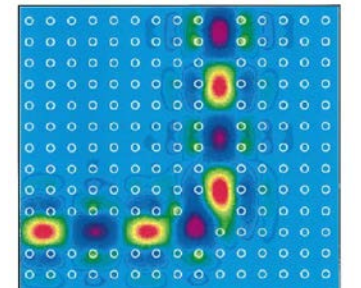
Microparticles

- Particles with $d \approx \lambda$ (λ -independent scattering, white clouds)
- Particles with $d \gg \lambda$ (Intuitive ray-picture useful)



Dielectric photonic crystal

- Molding the flow of light

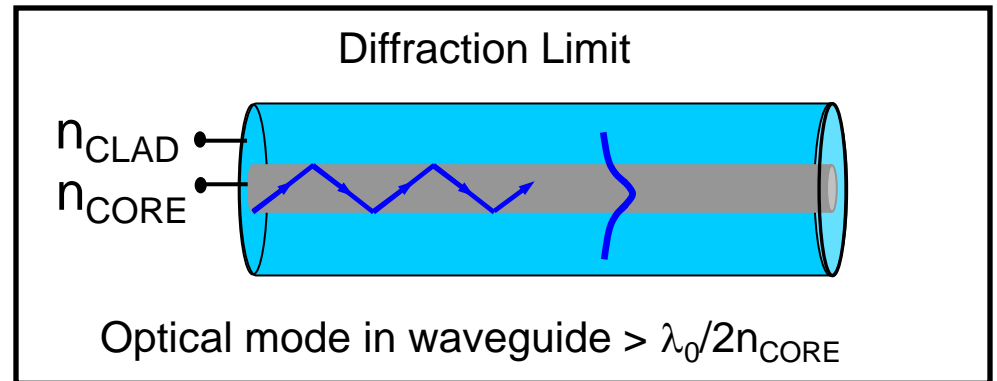
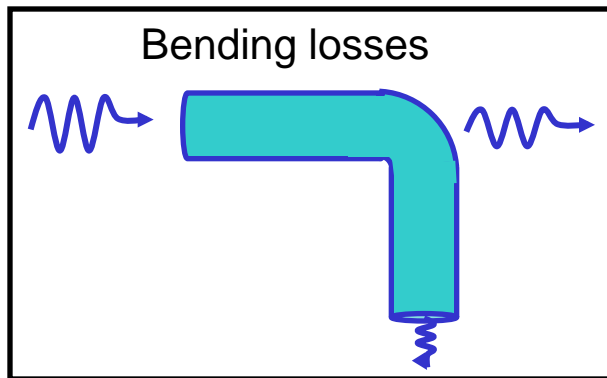


Lecture 8: Metal Optics: An Introduction

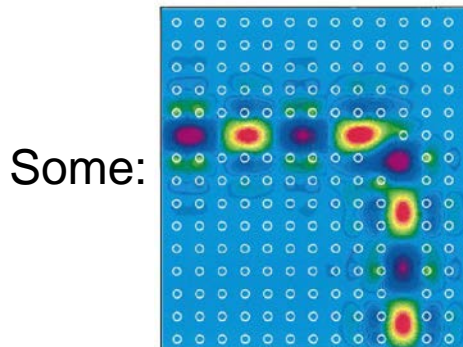
Majority of optical components based on dielectrics

- High speed, high bandwidth (ω), but...
- Does not scale well \Rightarrow Needed for large scale integration

Problems



Solutions ?



Some fundamental problems!



Photonic functionality based on metals?!

Plasmon-Polaritons

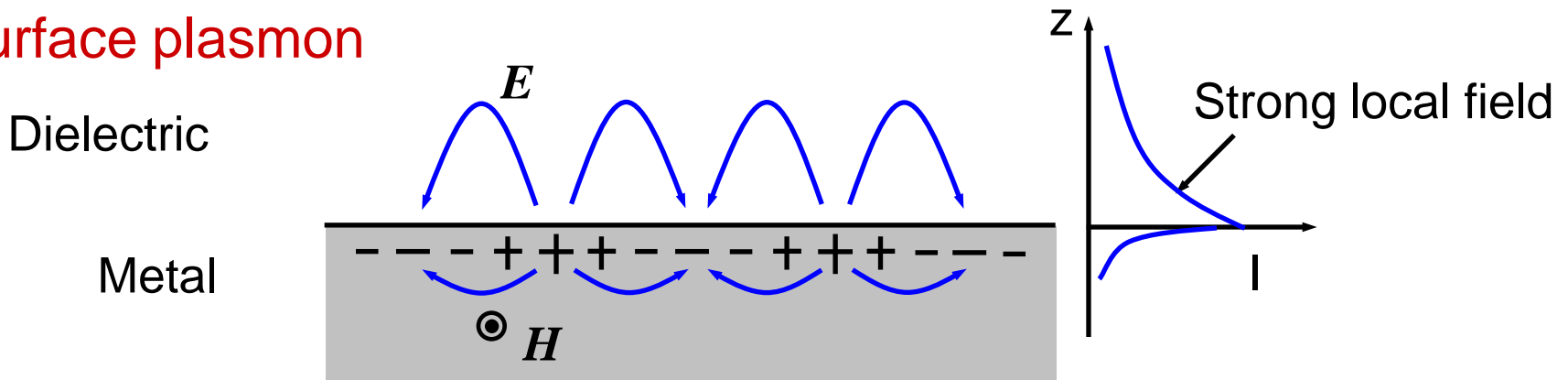
What is a plasmon ?

- Compare electron gas in a metal and real gas of molecules
- Metals are expected to allow for electron density waves: plasmons

Bulk plasmon

- Metals allow for EM wave propagation above the plasma frequency
 ↳ They become transparent!

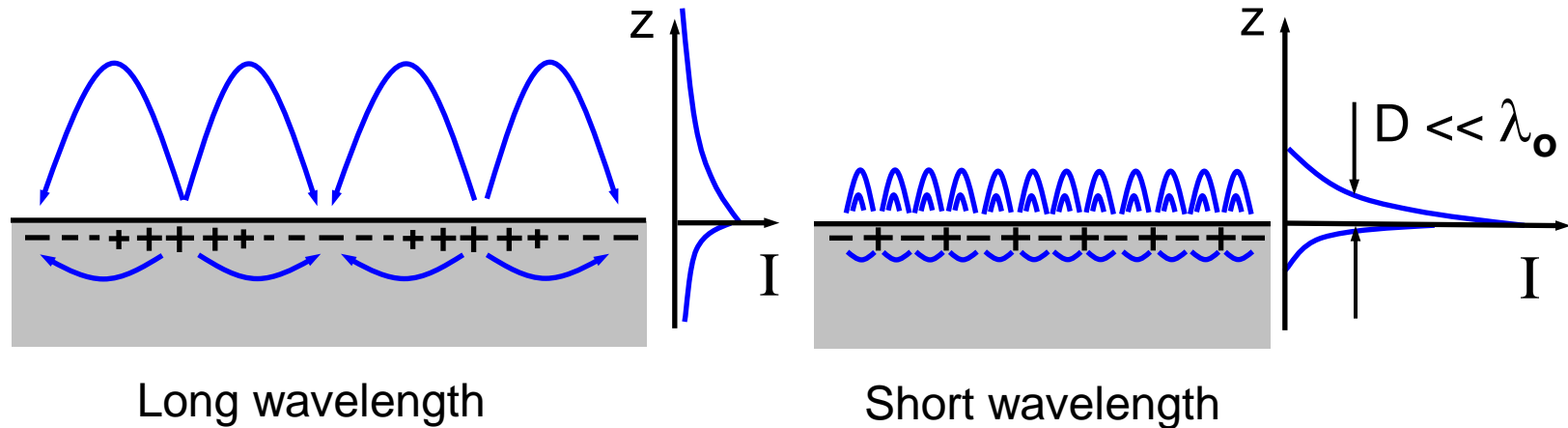
Surface plasmon



Note: This is a TM wave

- Sometimes called a surface plasmon-polariton (strong coupling to EM field)

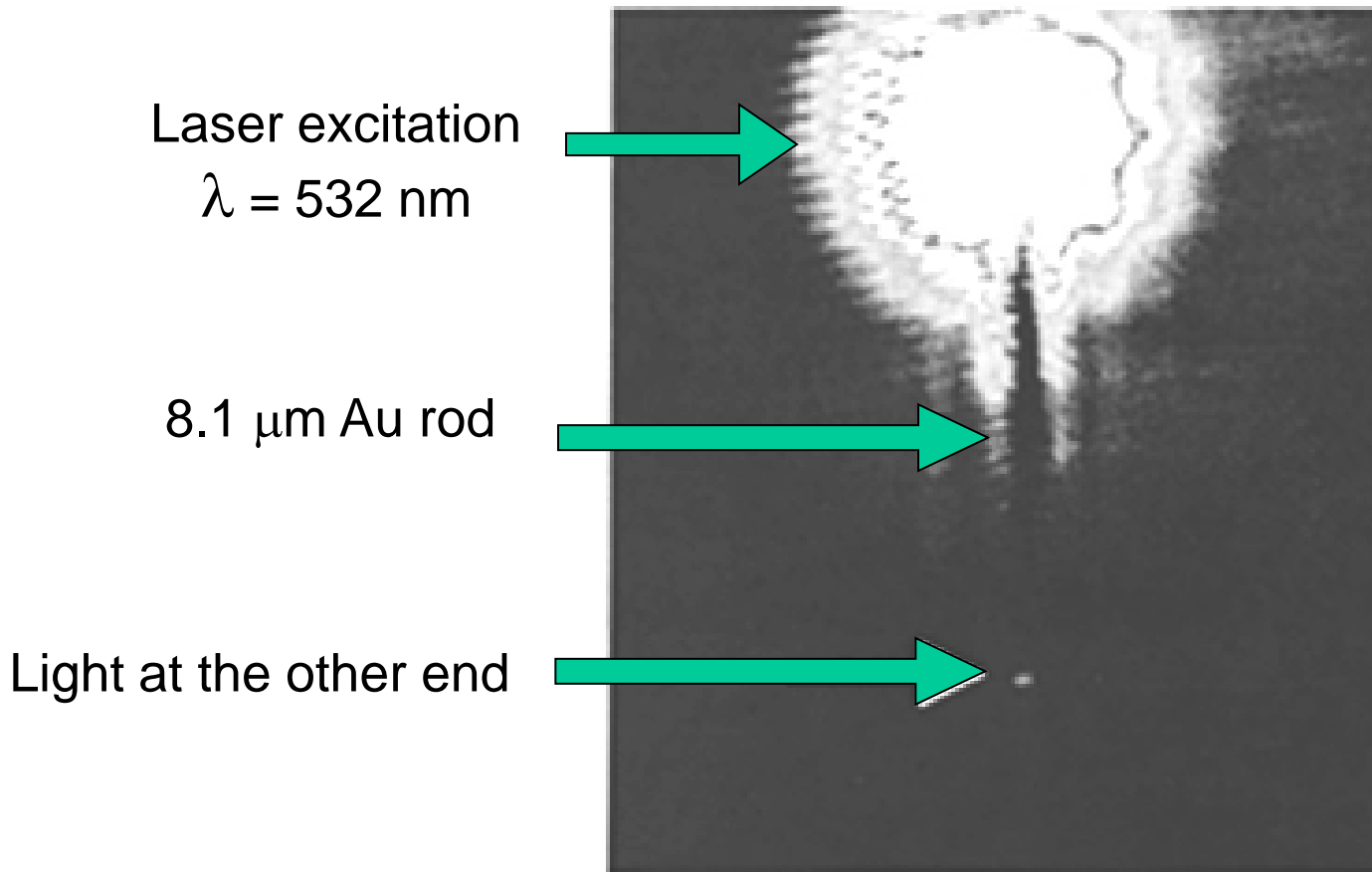
Local Field Intensity Depends on Wavelength



- Characteristics plasmon-polariton**
- Strong localization of the EM field
 - High local field intensities easy to obtain

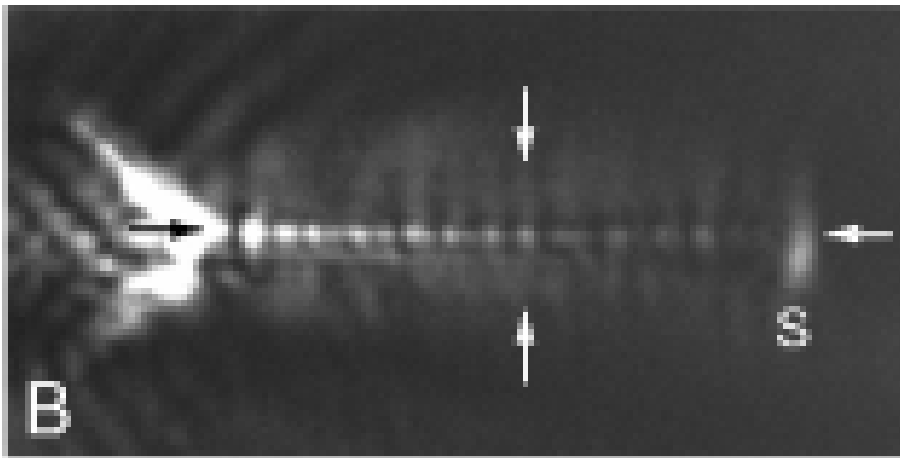
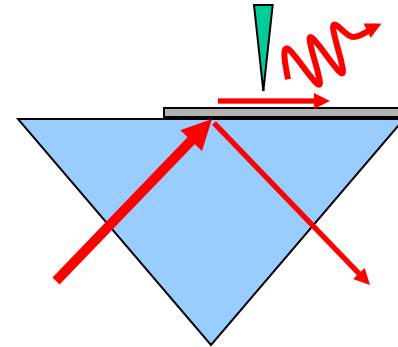
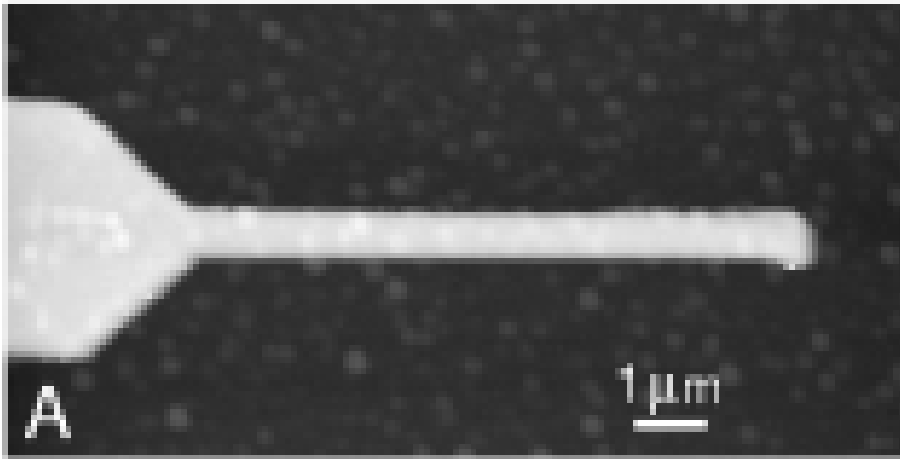
- Applications:**
- Guiding light below the diffraction limit (near-field optics)
 - Non-linear optics
 - Sensitive optical studies of surfaces and interfaces
 - Bio-sensors
 - Study film growth
 -

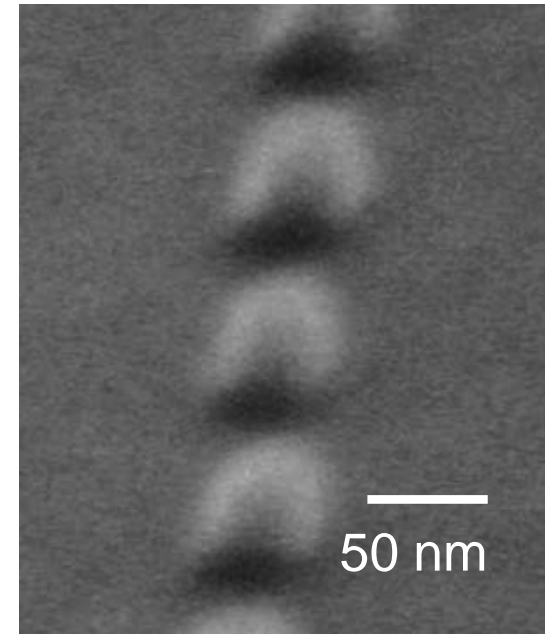
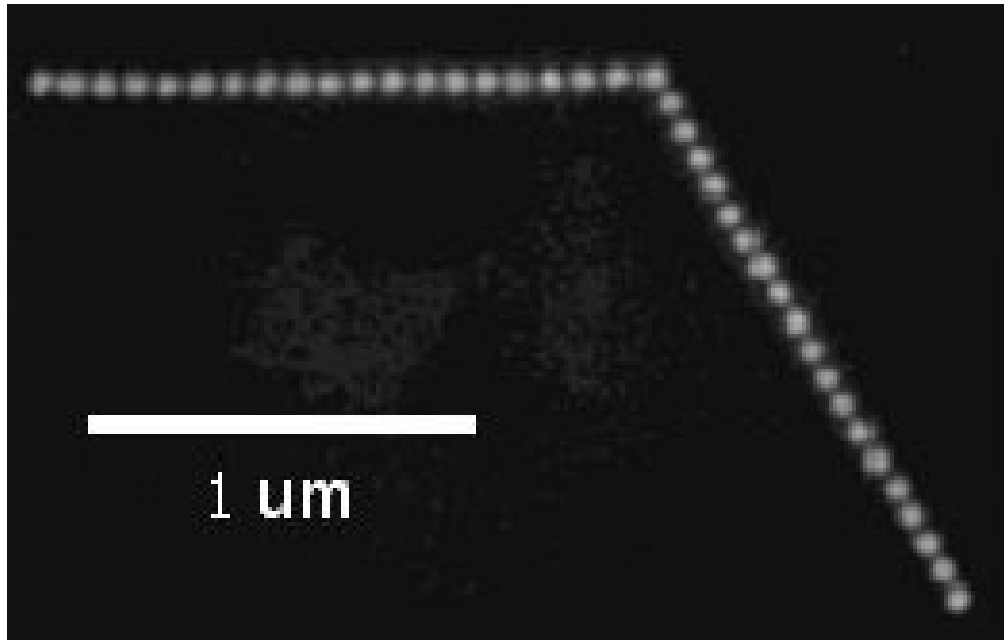
Plasmon-Polariton Propagation in Au rod



R.M. Dickson and L. A. Lyon, J. Phys. Chem. B **104**, 6095-6098 (2000)

Plasmon-Polariton Excitation using a Launch Pad



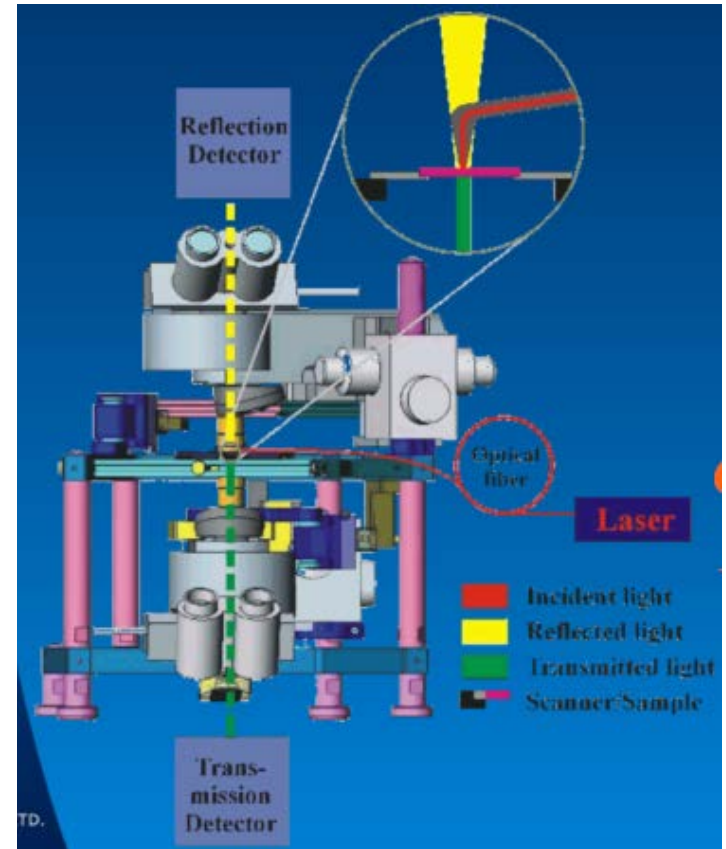


- Array of 50 nanometer diameter Au particles spaced by 75 nanometer
- Guides electromagnetic energy at optical frequency below the diffraction limit
- Enables communication between nanoscale devices
- Information transport at speeds and densities exceeding current electronics

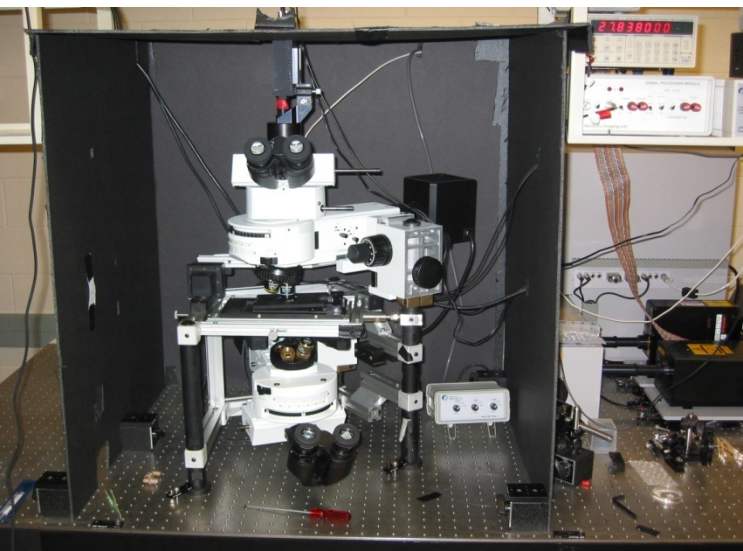
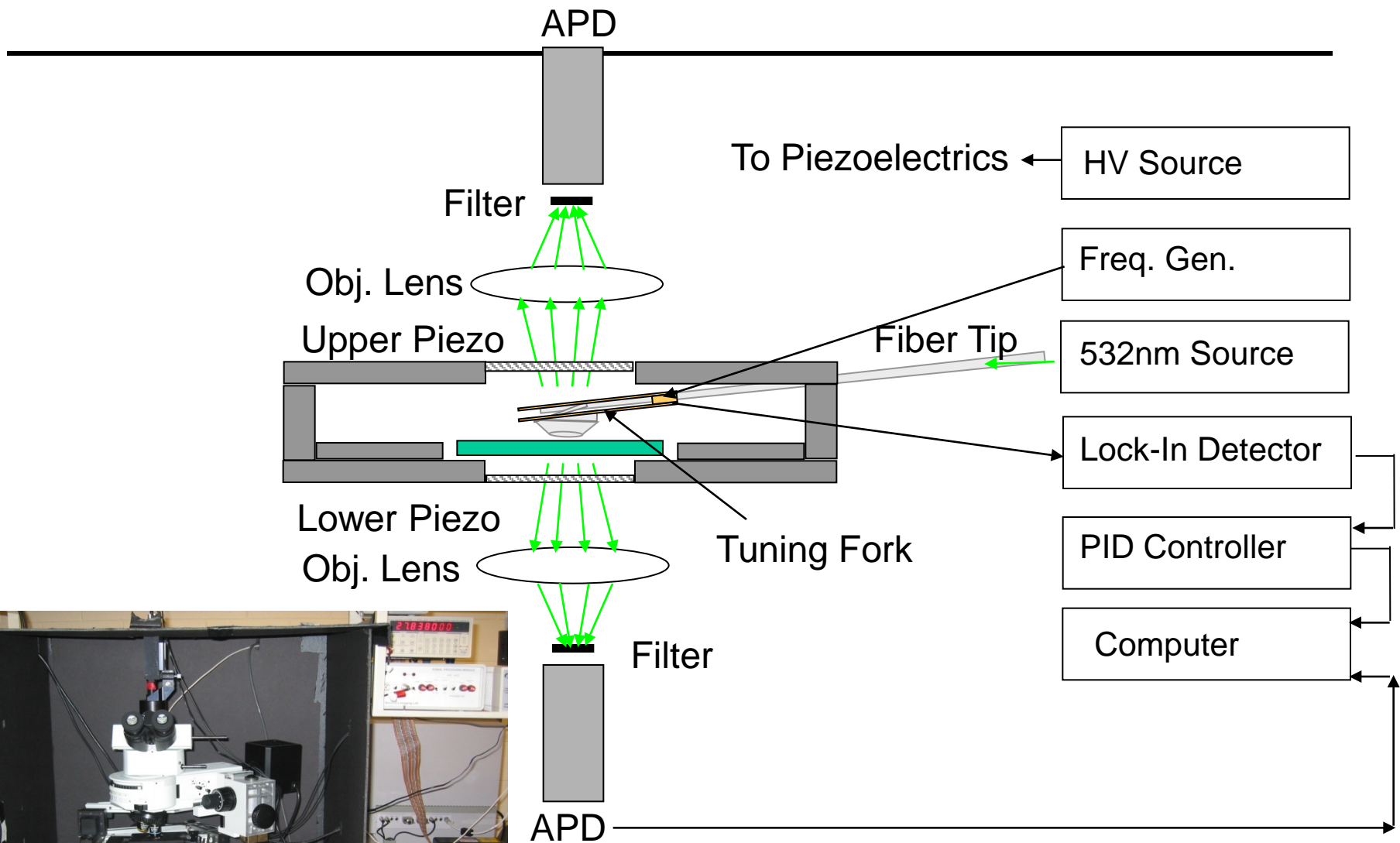
M.L. Brongersma, et al., Phys. Rev. B **62**, R16356 (2000)
S.A. Maier et al., Advanced materials **13**, 1501 (2001)

Purdue Near-Field Optical Microscope

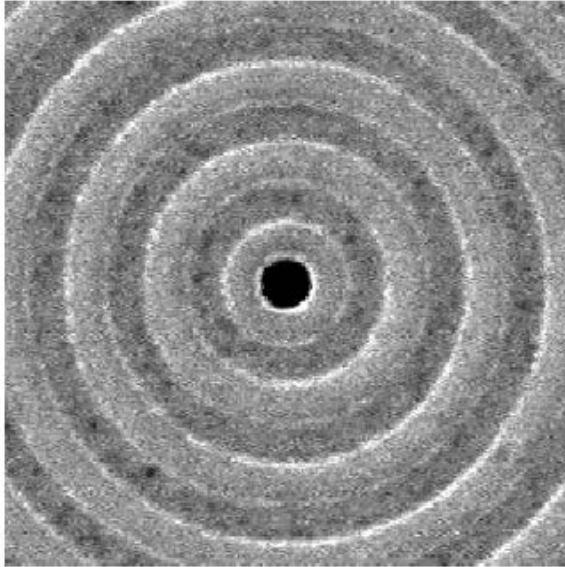
- Nanonics MultiView 2000
- NSOM / AFM
- Tuning Fork Feedback Control
 - Normal or Shear Force
- Aperture tips down to 50 nm
- AFM tips down to 30 nm
- Radiation Source
 - 532 nm



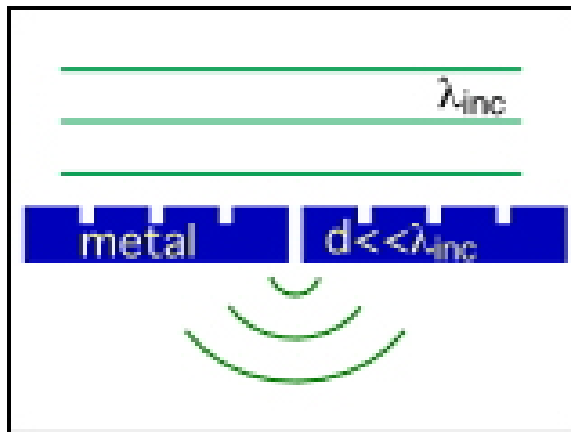
Picture taken from Nanonics



Enhanced Transmission through Sub- λ Apertures



- Ag film with a 440 nm diameter hole surrounded by circular grooves
- Transmission enhancement of 10 x compared to a bare hole
- 3x more light than directly impinging on hole !
- Reason: Excitation of plasmon-polaritons



T.Thio et al., Optics Letters **26**, 1972-1974 (2001).

Optical Properties of an Electron Gas (Metal)

Dielectric constant of a free electron gas (no interband transitions)

- Consider a time varying field:

$$\mathbf{E}(t) = \text{Re} \{ \mathbf{E}(\omega) \exp(-i\omega t) \}$$

- Equation of motion electron (no damping)

$$\left. \begin{aligned} m \frac{d^2 \mathbf{r}}{dt^2} &= -e \mathbf{E} \\ \mathbf{p}(t) &= -e \mathbf{r}(t) \end{aligned} \right\} \Rightarrow m \frac{d^2 \mathbf{p}}{dt^2} = e^2 \mathbf{E}$$

- Dipole moment electron

- Harmonic time dependence

$$\mathbf{p}(t) = \text{Re} \{ \mathbf{p}(\omega) \exp(-i\omega t) \}$$

- Substitution \mathbf{p} into Eq. of motion:

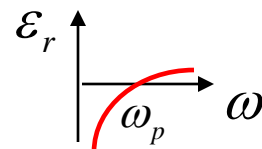
$$-m\omega^2 \mathbf{p}(\omega) = e^2 \mathbf{E}(\omega)$$

- This can be manipulated into:

$$\mathbf{p}(\omega) = -\frac{e^2}{m} \frac{1}{\omega^2} \mathbf{E}(\omega)$$

- The dielectric constant is:

$$\epsilon_r = 1 + \chi = \frac{N\mathbf{p}(\omega)}{\epsilon_0 \mathbf{E}(\omega)} = 1 - \frac{Ne^2}{\epsilon_0 m} \frac{1}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$



Dispersion Relation for EM Waves in Electron Gas

Determination of dispersion relation for bulk plasmons

- The wave equation is given by:

$$\frac{\epsilon_r}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = \nabla^2 \mathbf{E}(\mathbf{r}, t)$$

- Investigate solutions of the form:

$$\mathbf{E}(\mathbf{r}, t) = \text{Re} \{ \mathbf{E}(\mathbf{r}, \omega) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t) \}$$



$$\omega^2 \epsilon_r = c^2 k^2$$

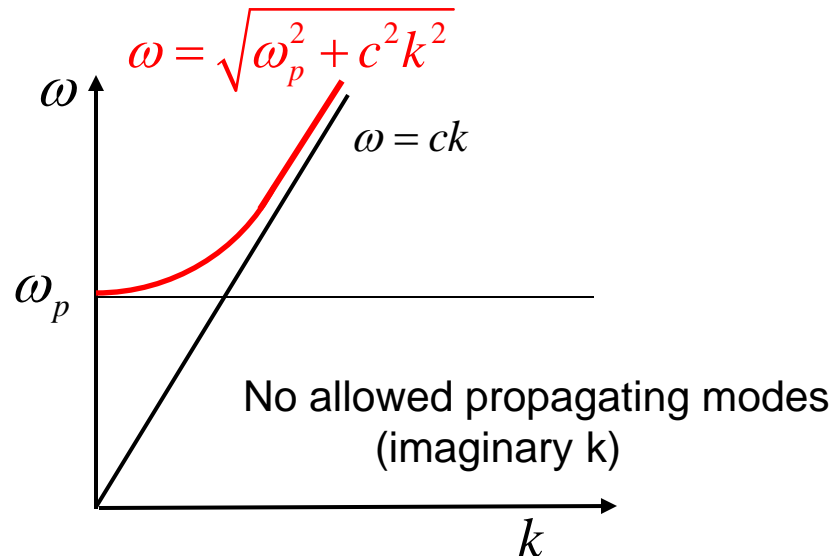
- Dielectric constant:

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$$



$$\omega^2 \left(1 - \frac{\omega_p^2}{\omega^2} \right) = \omega^2 - \omega_p^2 = c^2 k^2$$

- Dispersion relation:



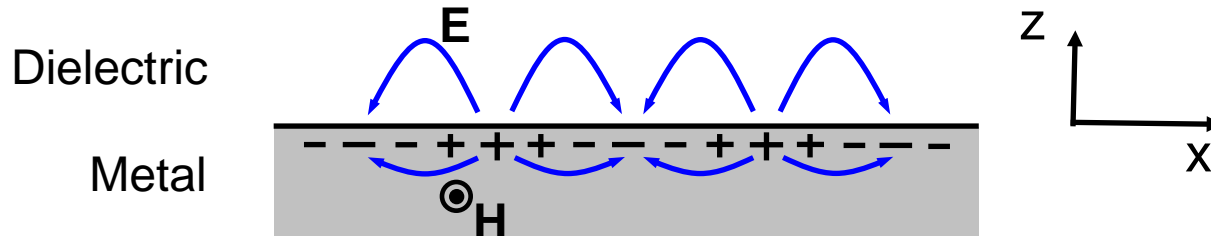
Note1: Solutions lie above light line

Note2: Metals: $\hbar\omega_p \approx 10$ eV; Semiconductors $\hbar\omega_p < 0.5$ eV (depending on dopant conc.)

Dispersion Relation Surface-Plasmon Polaritons

Solve Maxwell's equations with boundary conditions

- We are looking for solutions that look like:



- Mathematically:

$$z < 0 \quad \begin{cases} \mathbf{H}_d = (0, H_{yd}, 0) \exp i(k_{xd}x + k_{zd}z - \omega t) \\ \mathbf{E}_d = (E_{xd}, 0, E_{zd}) \exp i(k_{xd}x + k_{zd}z - \omega t) \end{cases}$$

$$z > 0 \quad \begin{cases} \mathbf{H}_m = (0, H_{ym}, 0) \exp i(k_{xm}x + k_{zm}z - \omega t) \\ \mathbf{E}_m = (E_{xm}, 0, E_{zm}) \exp i(k_{xm}x + k_{zm}z - \omega t) \end{cases}$$

- Maxwell's Equations in medium i ($i = \text{metal or dielectric}$):

$$\nabla \cdot \epsilon_i \mathbf{E} = 0 \quad \nabla \cdot \mathbf{H} = 0 \quad \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad \nabla \times \mathbf{H} = \epsilon_i \frac{\partial \mathbf{E}}{\partial t}$$

- At the boundary:

$$E_{x,m} = E_{x,d} \quad \epsilon_m E_{zm} = \epsilon_d E_{zd} \quad H_{ym} = H_{yd}$$

Dispersion Relation Surface-Plasmon Polaritons

- Start with curl equation for \mathbf{H} in medium i (as we did for EM waves in vacuum)

$$\nabla \times \mathbf{H}_i = \varepsilon_i \frac{\partial \mathbf{E}_i}{\partial t}$$

where $\mathbf{H}_i = (0, H_{yi}, 0) \exp i(k_{xi}x + k_{zi}z - \omega t)$

$\mathbf{E}_i = (E_{xi}, 0, E_{zi}) \exp i(k_{xi}x + k_{zi}z - \omega t)$

$$\left(\frac{\partial H_{zi}}{\partial y} - \frac{\partial H_{yi}}{\partial z}, \frac{\partial H_{xi}}{\partial z} - \frac{\partial H_{zi}}{\partial x}, \frac{\partial H_{yi}}{\partial x} - \frac{\partial H_{xi}}{\partial y} \right) = (\underline{ik_{zi}H_{yi}}, 0, \underline{ik_{xi}H_{yi}}) = (\underline{-i\omega\varepsilon_i E_{xi}}, 0, \underline{i\omega\varepsilon_i E_{zi}})$$

- We will use that: $k_{zi}H_{yi} = -\omega\varepsilon_i E_{xi} \Rightarrow \left\{ \begin{array}{l} k_{zm}H_{yi} = -\omega\varepsilon_m E_{xm} \\ k_{zd}H_{yd} = -\omega\varepsilon_d E_{xd} \end{array} \right\} \Rightarrow \frac{k_{zm}}{\varepsilon_m} H_{ym} = \frac{k_{zd}}{\varepsilon_d} H_{yd}$
- $E_{//}$ across boundary is continuous: $E_{x,m} = E_{x,d}$

- $H_{//}$ across boundary is continuous: $H_{ym} = H_{yd}$

Combine with: $\left. \begin{array}{l} H_{ym} = H_{yd} \\ \frac{k_{zm}}{\varepsilon_m} H_{ym} = \frac{k_{zd}}{\varepsilon_d} H_{yd} \end{array} \right\} \Rightarrow \boxed{\frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d}}$

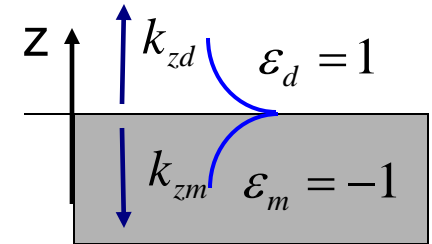
Dispersion Relation Surface-Plasmon Polaritons

Relations between k vectors

- Condition for SP's to exist:

$$\frac{k_{zm}}{\epsilon_m} = \frac{k_{zd}}{\epsilon_d}$$

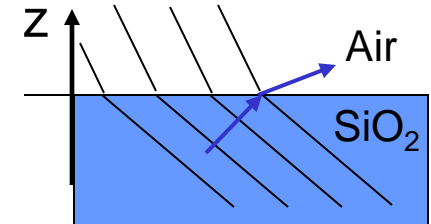
Example



- Relation for k_x (Continuity $E_{||}$, $H_{||}$): $k_{xm} = k_{xd}$

true at any boundary

Example



- For any EM wave:

$$k_x^2 + k_{zi}^2 = \epsilon_i \left(\frac{\omega}{c} \right)^2$$

- Both in the metal and dielectric: $k_{sp} = k_x = \sqrt{\epsilon_i \left(\frac{\omega}{c} \right)^2 - k_{zi}^2}$

$$\frac{k_{zm}}{\epsilon_m} = \frac{k_{zd}}{\epsilon_d}$$

Dispersion relation

$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d} \right)^{1/2}$$

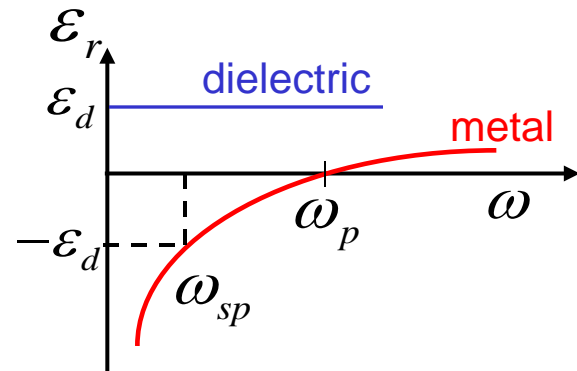
homework 😊

Dispersion Relation Surface-Plasmon Polaritons

Plot of the dispersion relation

- Last page: $k_x = \frac{\omega}{c} \left(\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d} \right)^{1/2}$

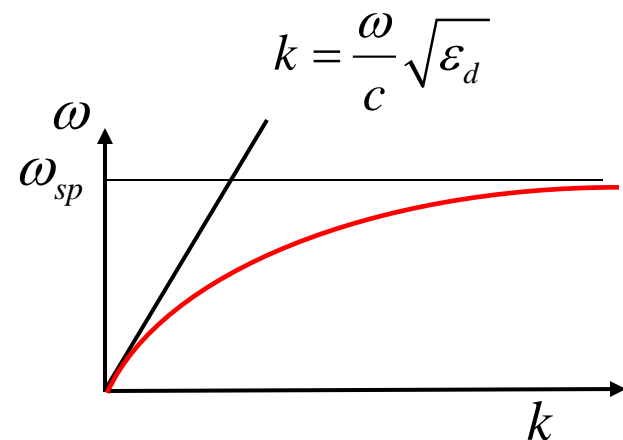
- Plot dielectric constants



- Low ω : $k_x = \frac{\omega}{c} \lim_{\epsilon_m \rightarrow -\infty} \left(\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d} \right)^{1/2} \approx \frac{\omega}{c} \sqrt{\epsilon_d}$

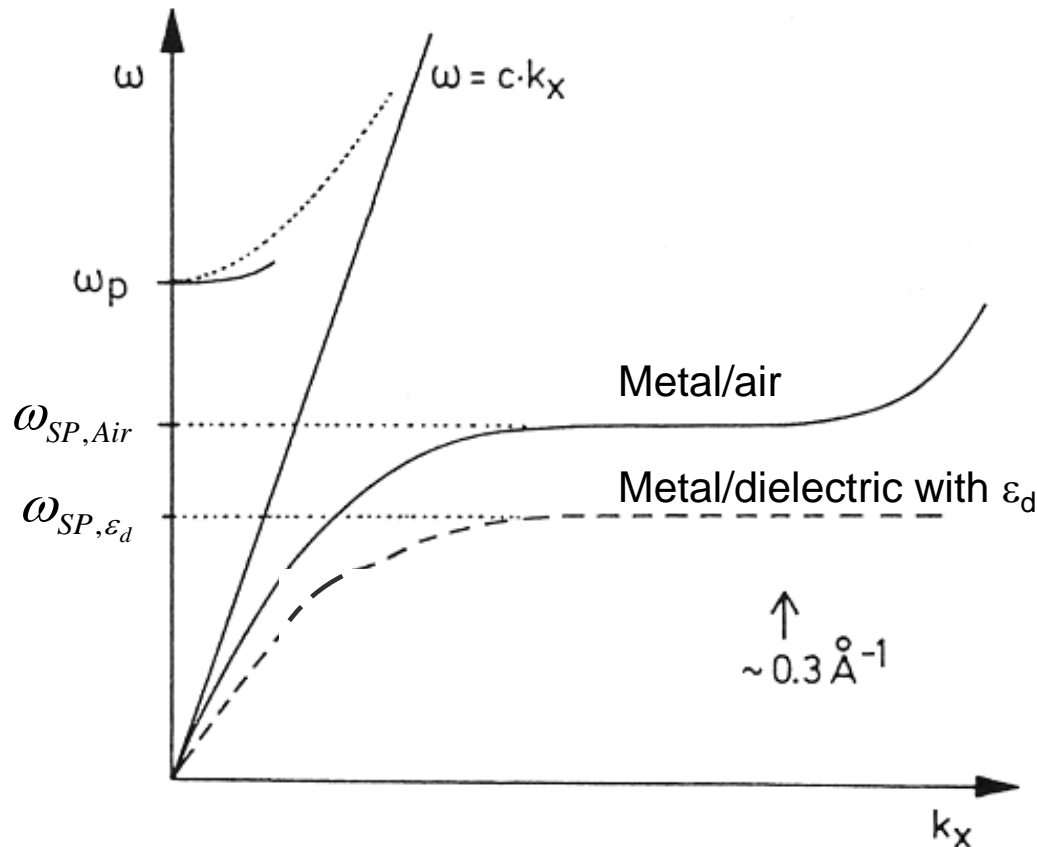
- At $\omega = \omega_{sp}$ (when $\epsilon_m = -\epsilon_d$): $k_x \rightarrow \infty$

- Note: Solution lies below the light line



Dispersion Relation Surface-Plasmon Polaritons

Dispersion relation plasma modes and SPP



- Note: Higher index medium on metal results in lower ω_{sp}

$$\omega = \omega_{sp} \text{ when: } \epsilon_m = 1 - \frac{\omega_p^2}{\omega^2} = -\epsilon_d \Rightarrow \omega^2 - \omega_p^2 = -\epsilon_d \omega^2 \Rightarrow \omega^2 = \frac{\omega_p^2}{1 + \epsilon_d} \Rightarrow \omega = \frac{\omega_p}{\sqrt{1 + \epsilon_d}}$$

Excitation Surface-Plasmon Polaritons with Electrons

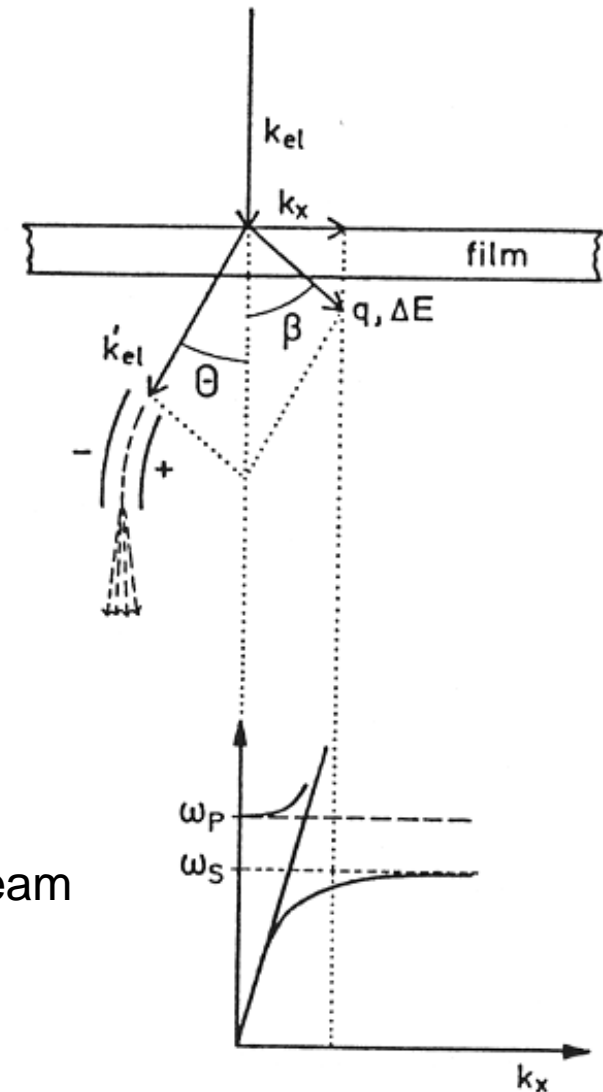
Excitation with electrons

- First experiments with high energy electrons
- Measurement: Energy loss
Direction of e's:
- Whole dispersion relation can be investigated
- Low k's are hard!

Example: 50 keV has a $\lambda = 0.005 \text{ nm} \ll \lambda_{\text{light}}$

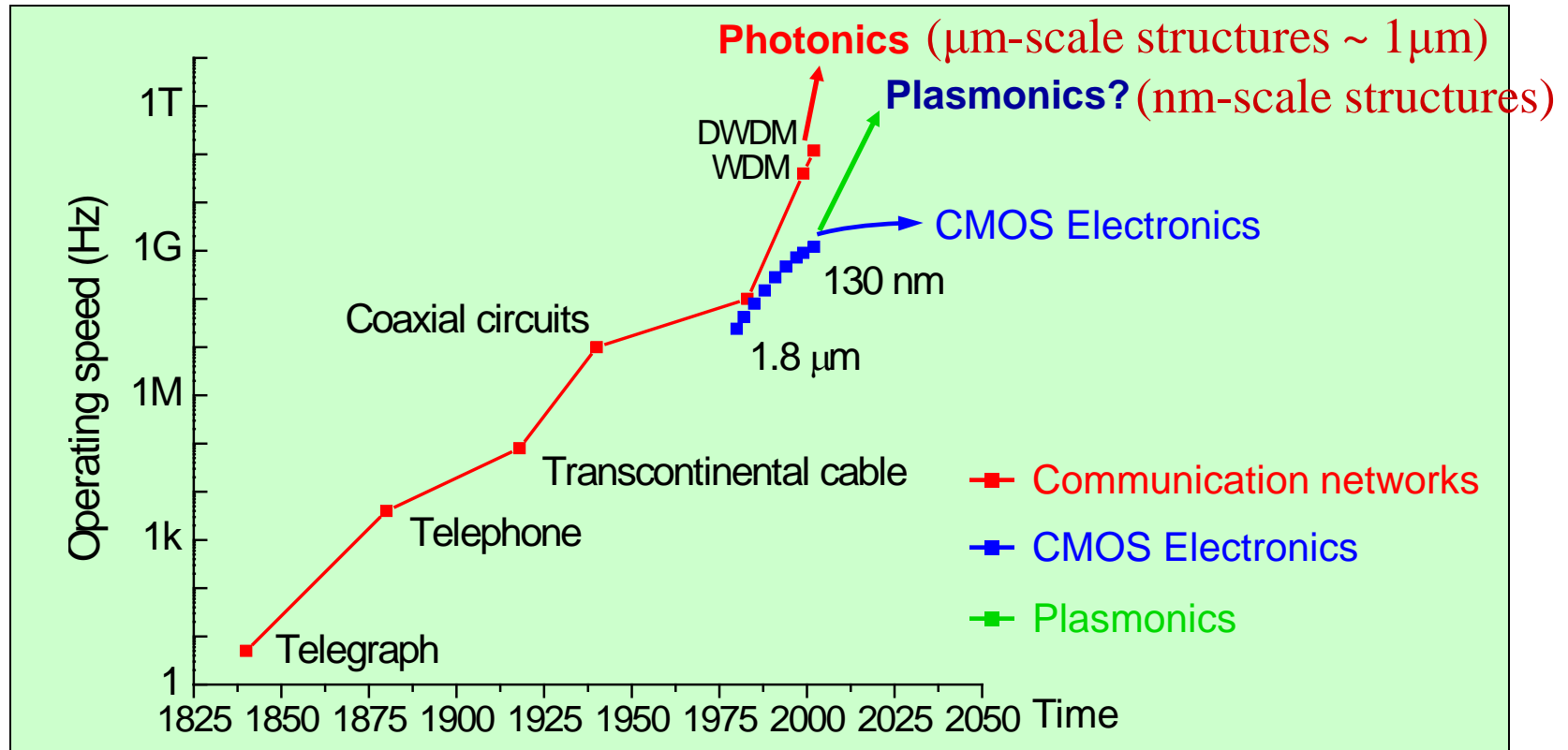
⇒ $k_{\text{electron}} \gg k_{\text{light}}$

⇒ Stringent requirement on divergence e⁻ beam



Nanophotonics with Plasmonics: A logical next step?

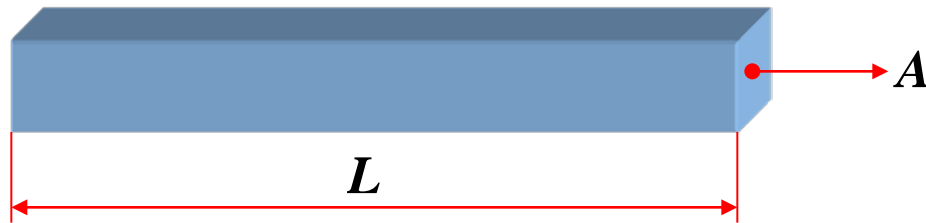
- The operating speed of data transporting and processing systems



- ◆ The ever-increasing need for faster information processing and transport is undeniable
- ◆ Electronic components are running out of steam due to issues with RC-delay times

Why not electronics?

As **data rates** AND component packing **densities** INCREASE, electrical interconnects become progressively limited by **RC**-delay:



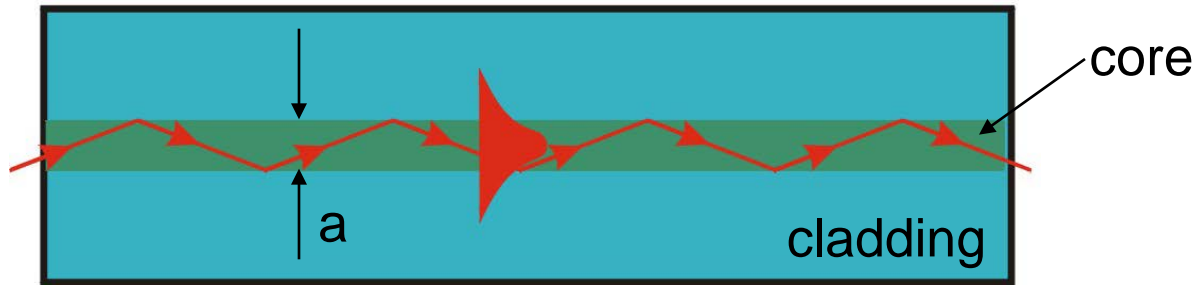
$$R \propto L/A \oplus C \propto L \Rightarrow B_{\max} \propto \frac{1}{RC} \propto \frac{A}{L^2}$$
$$\Rightarrow B_{\max} \leq 10^{15} \times \frac{A}{L^2} \text{ (bit/s)} \quad (A \ll L^2 !)$$



Electronics is aspect-ratio limited in speed!

Why not photonics?

The bit **rate** in optical communications is fundamentally limited **only** by the carrier frequency: $B_{\max} < f \sim 100$ Tbit/s (!),
but **light propagation is subjected to diffraction**:

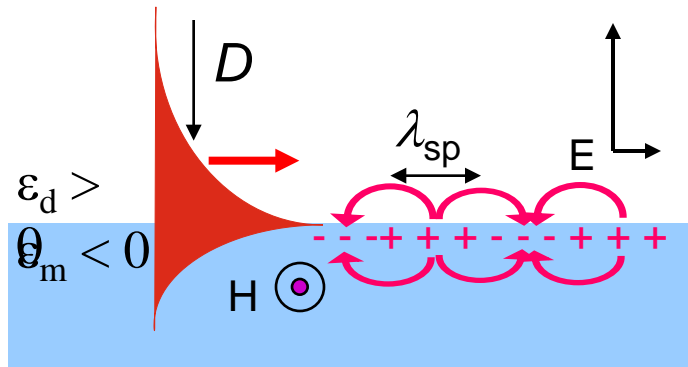


$$n_{core} = n_{clad} + \delta n = n + \delta n \Rightarrow V = \frac{2\pi}{\lambda} a \sqrt{n_{core}^2 - n_{clad}^2} \cong \frac{2\pi}{\lambda} a \sqrt{2n\delta n}$$

well-guided mode: $V \propto \pi \Rightarrow a \cong \lambda / 2\sqrt{2n\delta n}$ – mode size: $\delta n \ll 1$ (!)

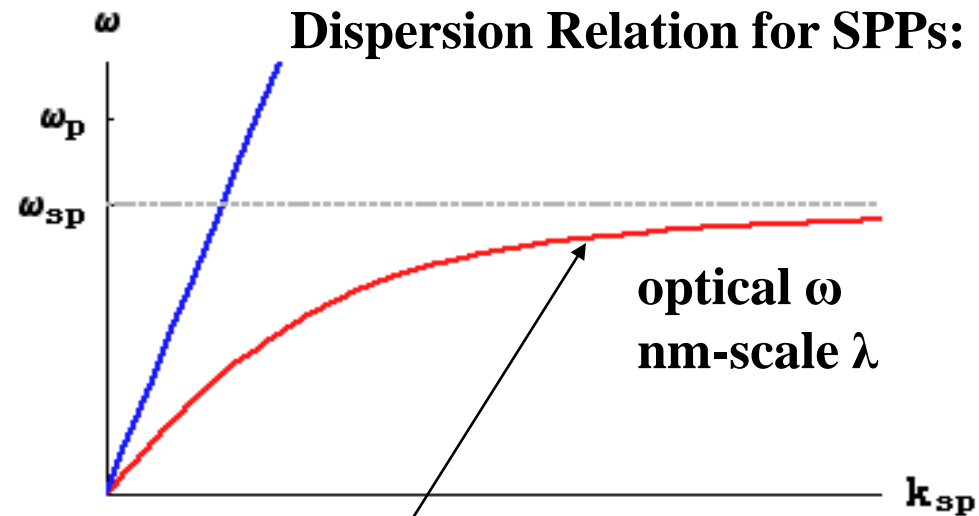
Photonics is diffraction- limited in size!

Why Plasmonics?



$$k_{sp} = \frac{\omega}{c} \sqrt{\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d}}$$

Dispersion Relation for SPPs:



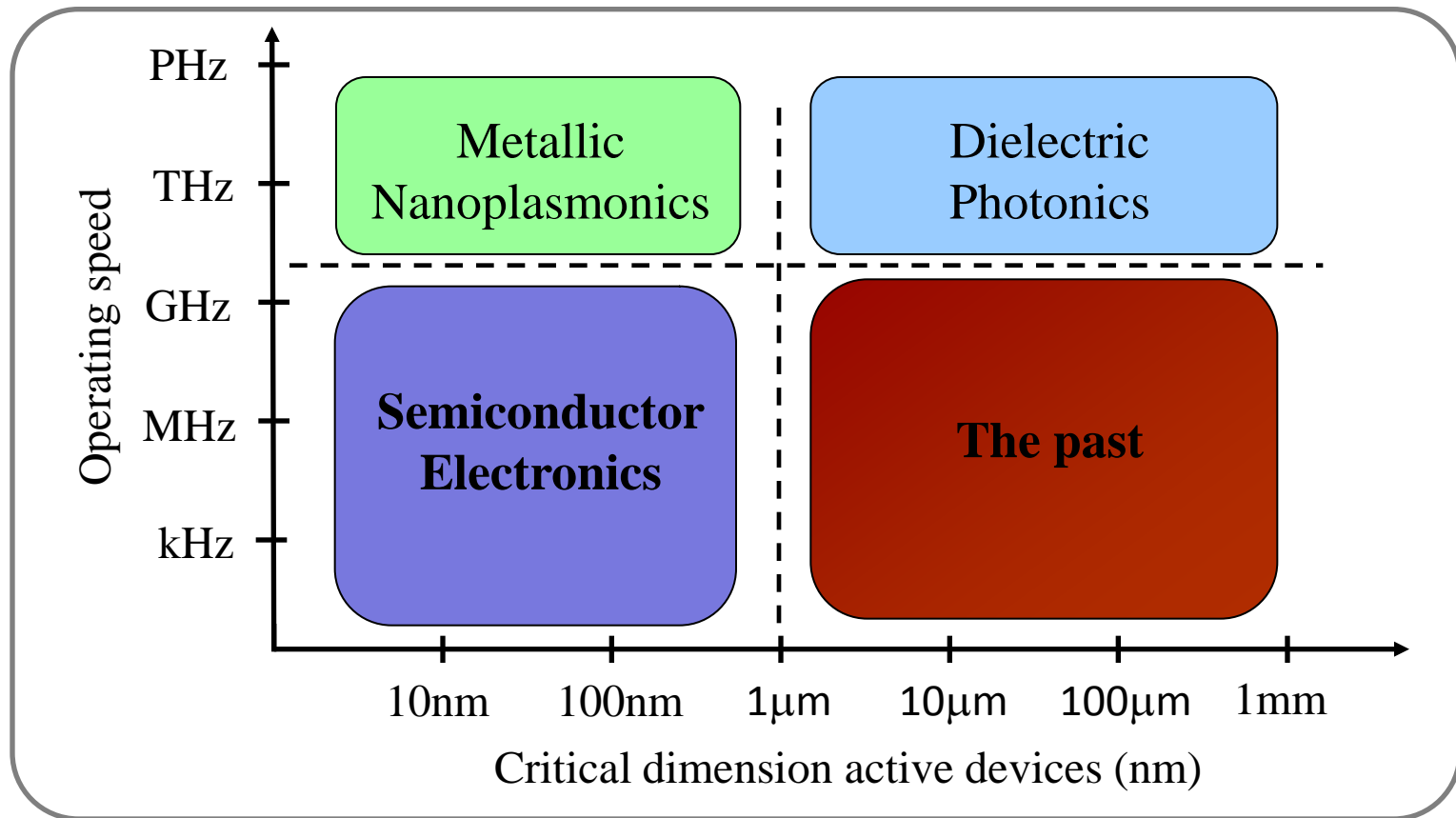
$$\epsilon_m(\omega) \approx -\epsilon_d(\omega)$$

$\lambda_p \sim$ very small



SP wavelengths can reach nanoscale at optical frequencies!
SPPs are “x-ray waves” with optical frequencies

Why Plasmonics/Electric MMs?



- **Plasmonics will enable an improved synergy between electronic and photonic devices**
 - Plasmonics naturally interfaces with similar size electronic components
 - Plasmonics naturally interfaces with similar operating speed photonic networks