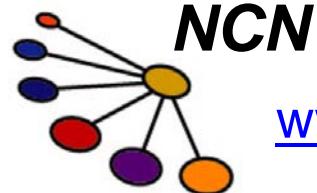


EE-612:

Lecture 29:

SOI Electrostatics

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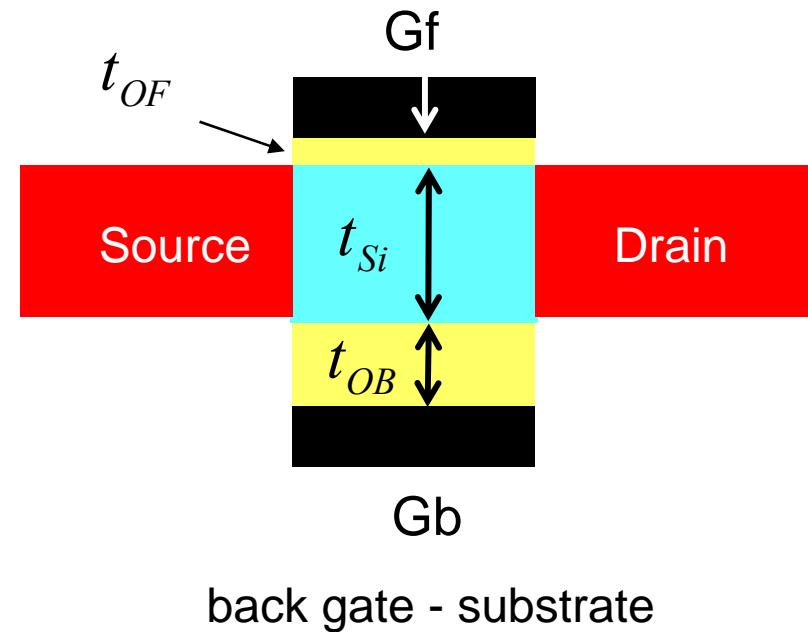
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outline

1. Introduction
2. General solution
3. V_{TF} vs. V_{GB}
4. Subthreshold slope
5. Double gate (DG) SOI

SOI MOSFETs

- Two separate gates
- Upper oxide thermal
- Lower gate oxide is the BOX (e.g. produced by SIMOX)
- Typically $t_{OF} \ll t_{OB}$



Goal: To understand the effect of back gate on MOSFET operation

band diagram: PD SOI

- Maximum depletion width for bulk Si:

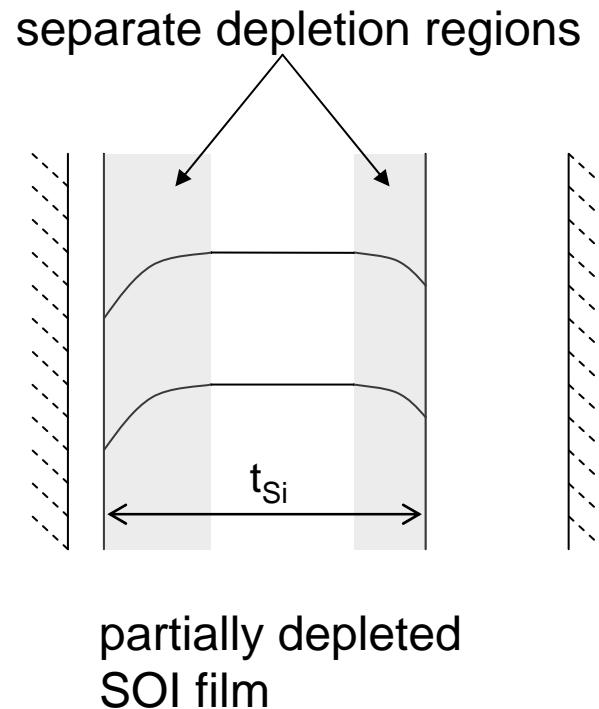
$$W_{DM} = \sqrt{\frac{4\epsilon_{Si}\psi_B}{qN_A}} \quad \psi_B = \frac{k_B T}{q} \ln\left(\frac{N_A}{n_i}\right)$$

- Partially depleted (PD) SOI:

$$t_{Si} > 2W_{DM}$$

- Front and back gates are decoupled electrostatically: ψ_{SF} independent of ψ_{SB}

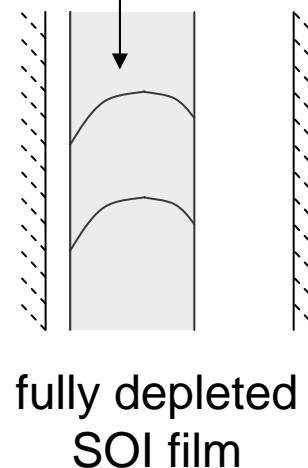
- Device operation similar to bulk MOSFETs



band diagram: FD SOI

- Fully depleted (FD) SOI: $t_{Si} < W_{DM}$
- SOI layer is fully depleted, irrespective of back gate bias
- Front and back gates are electrostatically coupled: ψ_{SF} is a function of ψ_{SB}
- Back gate bias plays important role in device operation
- The rest of this lecture will focus on fully depleted SOI

Depletion regions merged

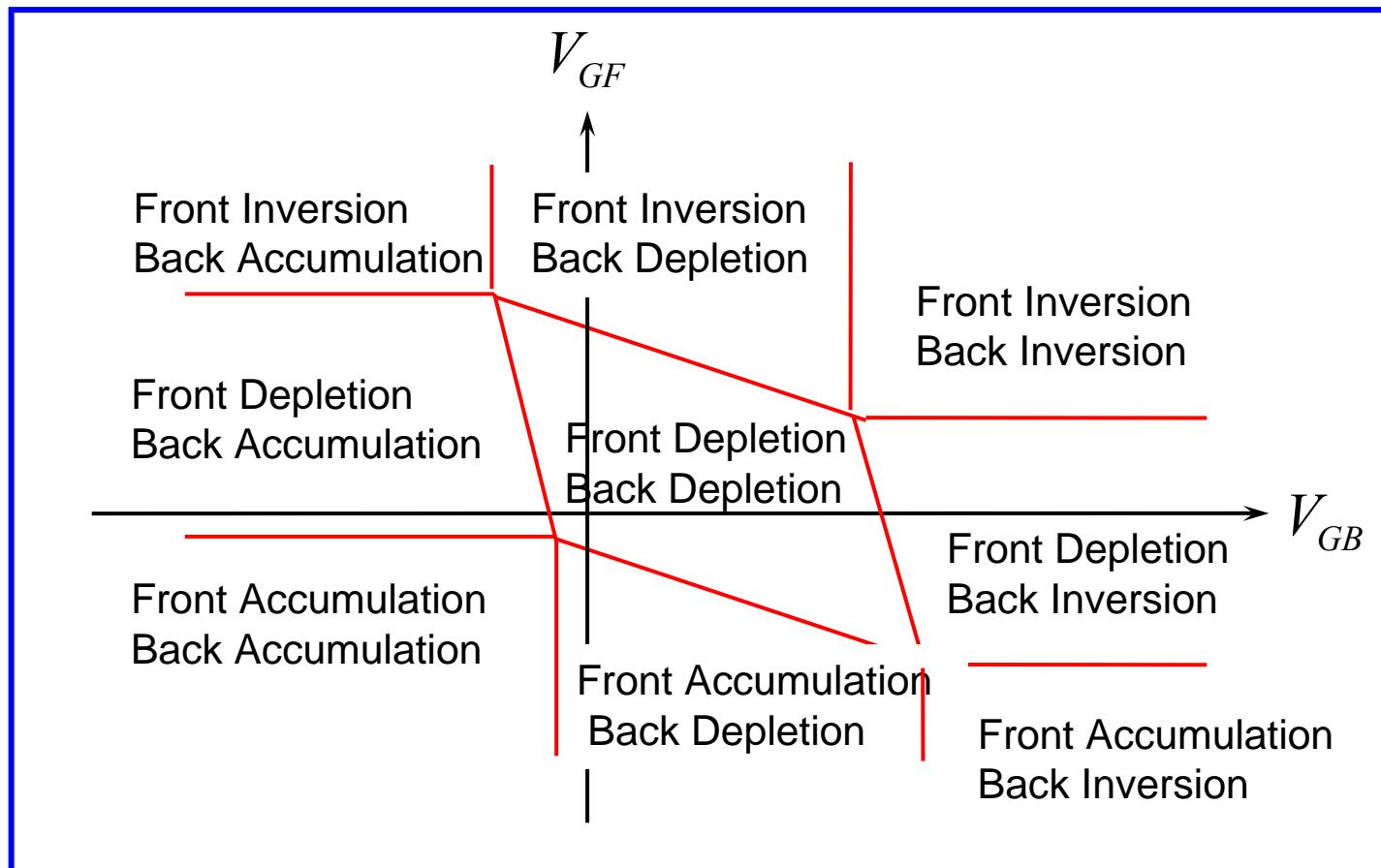


FD SOI nMOSFET operating regions

Nine operating regions:

| Front gate: | Back gate: |
|--------------------|-------------------|
| depleted | depleted |
| inverted | inverted |
| accumulated | accumulated |

FD SOI nMOSFET operating regions



Key references for SOI 1D Electrostatics

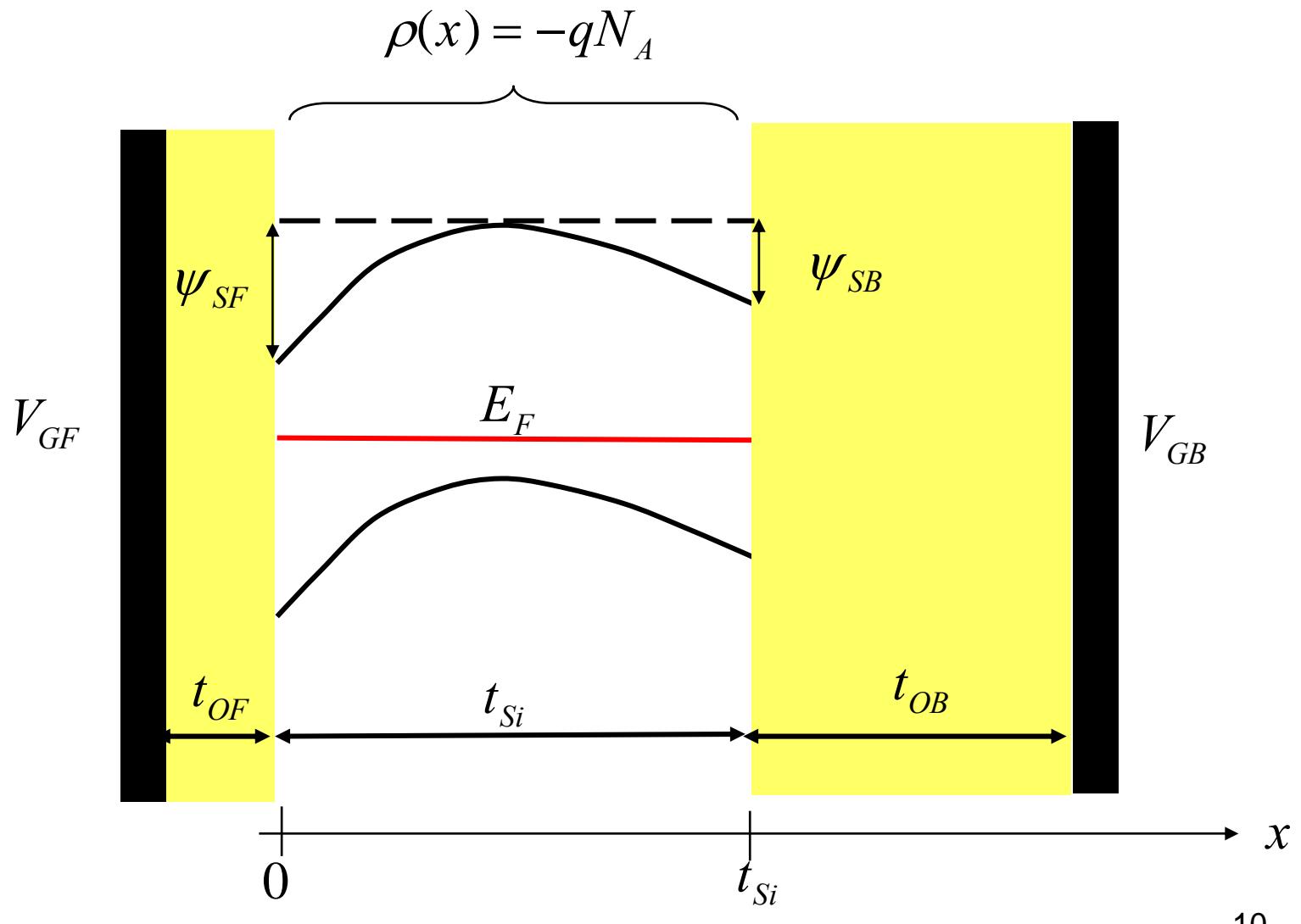
H.-K Lim and J.G. Fossum, “Threshold Voltage of Thin-Film Silicon-on-Insulator (SOI) MOSFETs,” *IEEE Trans. Electron Devices*, **30**, 1244-1251, 1983.

V.P. Trivedi, J.G. Fossum, and W. Zhang, “Threshold Voltage in Nonclassical CMOS Devices with Undoped Ultra-Thin Bodies,” to appear in *IEEE Trans. Electron Devices*, 2006.

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FD SOI band diagram



objectives

for bulk MOSFETS, we know:

$$V'_G = \psi_s - Q_s / C_{OX}$$

for FDSOI MOSFETS, determine:

$$\psi_{SF} = f(V_{GF}, V_{GB})$$

$$\psi_{SB} = f(V_{GF}, V_{GB})$$

electric field in SOI

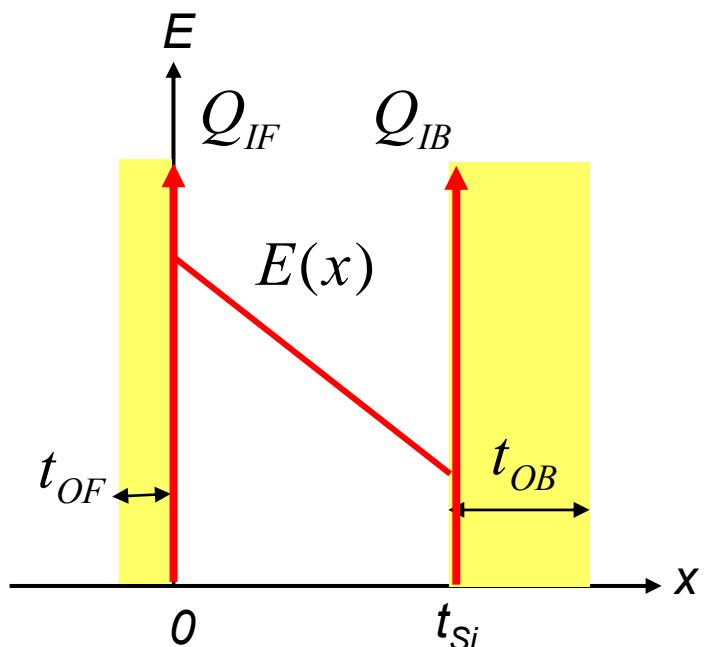
Delta-depletion Approximation: Assume that any mobile charge is at the Si surface in a delta function

Apply Gauss' Law to FD bulk:

$$\frac{dE}{dx} = \frac{-qN_A}{\epsilon_{Si}}$$

$$\int_{E(0^+)}^{E(t_{Si}^-)} dE = \frac{-qN_A}{\epsilon_{Si}} \int_{0^+}^{t_{Si}} dx$$

$$E(t_{Si}^-) - E(0^+) = -qN_A t_{Si} / \epsilon_{Si}$$



electric field in SOI (ii)

from:

$$E(t_{Si}^-) - E(0^+) = -qN_A t_{Si} / \epsilon_{Si}$$

we get:

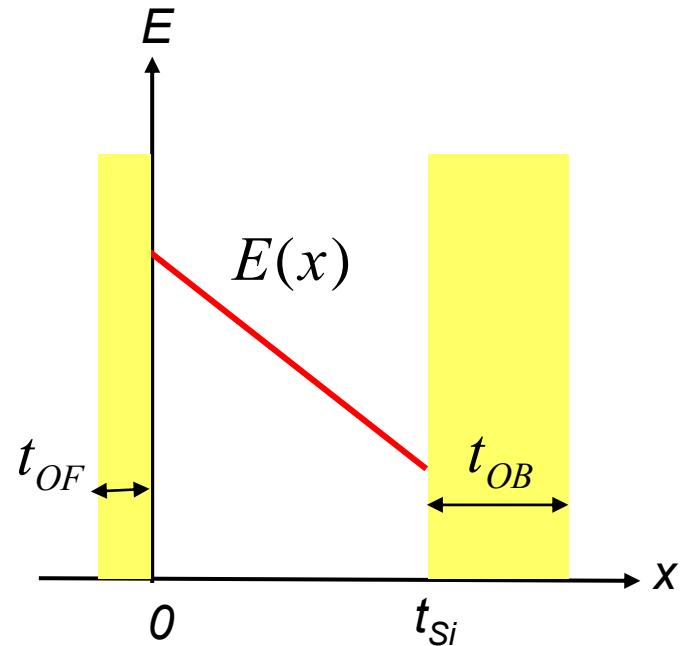
$$E(0^+) = E(t_{Si}^-) + qN_A t_{Si} / \epsilon_{Si} \quad (1)$$

also:

$$\Delta\psi = \psi_{SF} - \psi_{SB} = \frac{1}{2} [E(0^+) + E(t_{Si}^-)] t_{Si}$$

from which, we obtain:

$$E(0^+) = 2(\psi_{SF} - \psi_{SB}) / t_{Si} - E(t_{Si}^-) \quad (2)$$



electric field in SOI (iii)

solve (1) and (2) for:

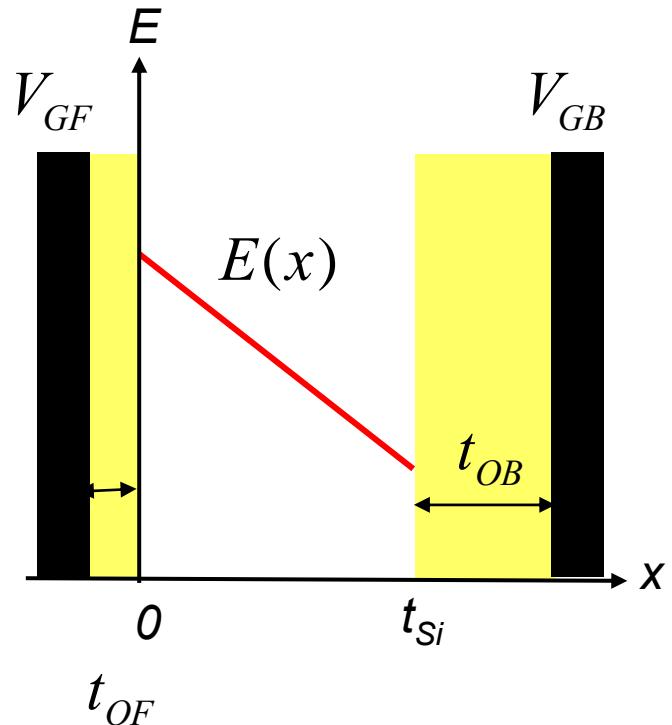
$$E(0^+) = \left(\frac{\psi_{SF} - \psi_{SB}}{t_{Si}} \right) + \frac{qN_A t_{Si}}{2\epsilon_{Si}} \quad (3)$$

$$E(t_{Si}^-) = \left(\frac{\psi_{SF} - \psi_{SB}}{t_{Si}} \right) - \frac{qN_A t_{Si}}{2\epsilon_{Si}} \quad (4)$$

we can also relate:

$E(0^+)$ to V_{GF}

$E(t_{Si}^-)$ to V_{GB}



effect of front and back gate voltages

The field in the front gate oxide is:

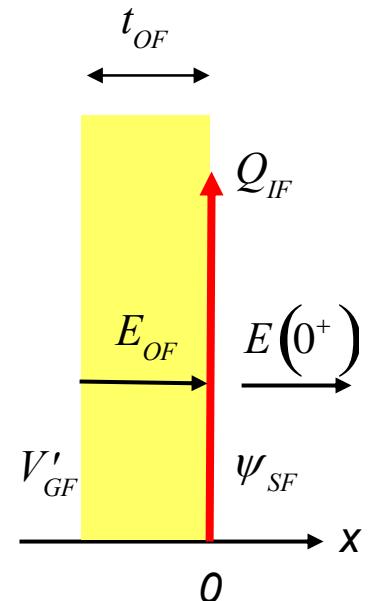
$$E_{OF} = \left(V'_{GF} - \psi_{SF} \right) / t_{OF}, \text{ where } V'_{GF} = V_{GF} - \phi_{msf}$$

Taking the inversion charge into account:

$$\epsilon_{ox} E_{OF} = \epsilon_{Si} E(0^+) - Q_{IF}$$

$$E(0^+) = \frac{\epsilon_{ox}}{\epsilon_{Si}} E_{OF} + \frac{Q_{IF}}{\epsilon_{Si}}$$

$$E(0^+) = \frac{\epsilon_{ox}}{\epsilon_{Si}} \frac{(V'_{GF} - \psi_{SF})}{t_{OF}} + \frac{Q_{IF}}{\epsilon_{Si}} \quad (5)$$



effect of front and back gate voltages (ii)

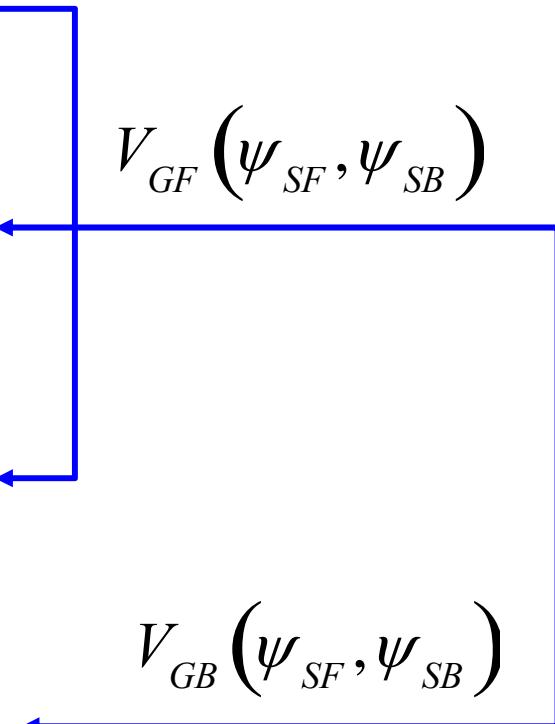
after a similar analysis for the back gate:

$$E(0^+) = \frac{\epsilon_{ox}}{\epsilon_{Si}} \frac{(V'_{GF} - \psi_{SF})}{t_{OF}} + \frac{Q_{IF}}{\epsilon_{Si}} \quad (5)$$

$$E(t_{Si}^-) = \frac{\epsilon_{ox}}{\epsilon_{Si}} \frac{(\psi_{SB} - V'_{GB})}{t_{OB}} - \frac{Q_{IB}}{\epsilon_{Si}} \quad (6)$$

$$E(0^+) = \left(\frac{\psi_{SF} - \psi_{SB}}{t_{Si}} \right) + \frac{qN_A t_{Si}}{2\epsilon_{Si}} \quad (3)$$

$$E(t_{Si}^-) = \left(\frac{\psi_{SF} - \psi_{SB}}{t_{Si}} \right) - \frac{qN_A t_{Si}}{2\epsilon_{Si}} \quad (4)$$



general solution

$$V_{GF} = \phi_{msf} + \psi_{SF} - \frac{Q_{IF} + Q_B/2}{C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (\psi_{SF} - \psi_{SB}) \quad (7)$$

$$V_{GB} = \phi_{msb} + \psi_{SB} - \frac{Q_{IB} + Q_B/2}{C_{OB}} + \frac{C_{Si}}{C_{OB}} \times (\psi_{SB} - \psi_{SF}) \quad (8)$$

$$C_{Si} \equiv \frac{\epsilon_{Si}}{t_{Si}} \quad Q_B \equiv -qN_A t_{Si}$$

extra volt drop across oxide
due to different surface
potentials

compare to: $V'_G = \psi_S - Q_S/C_{OX}$

extra term due to different surface potentials

For bulk silicon MOS structure, the gate voltage is given by:

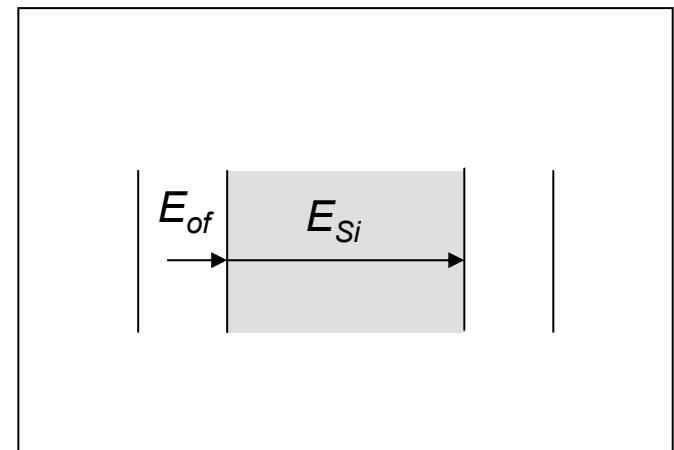
$$V_G = \psi_S - (Q_B + Q_I) / C_{ox}$$

Comparing with the bulk, the DGSOI gate voltage has an extra term which accounts for the voltage drop across oxide due to different surface potentials.

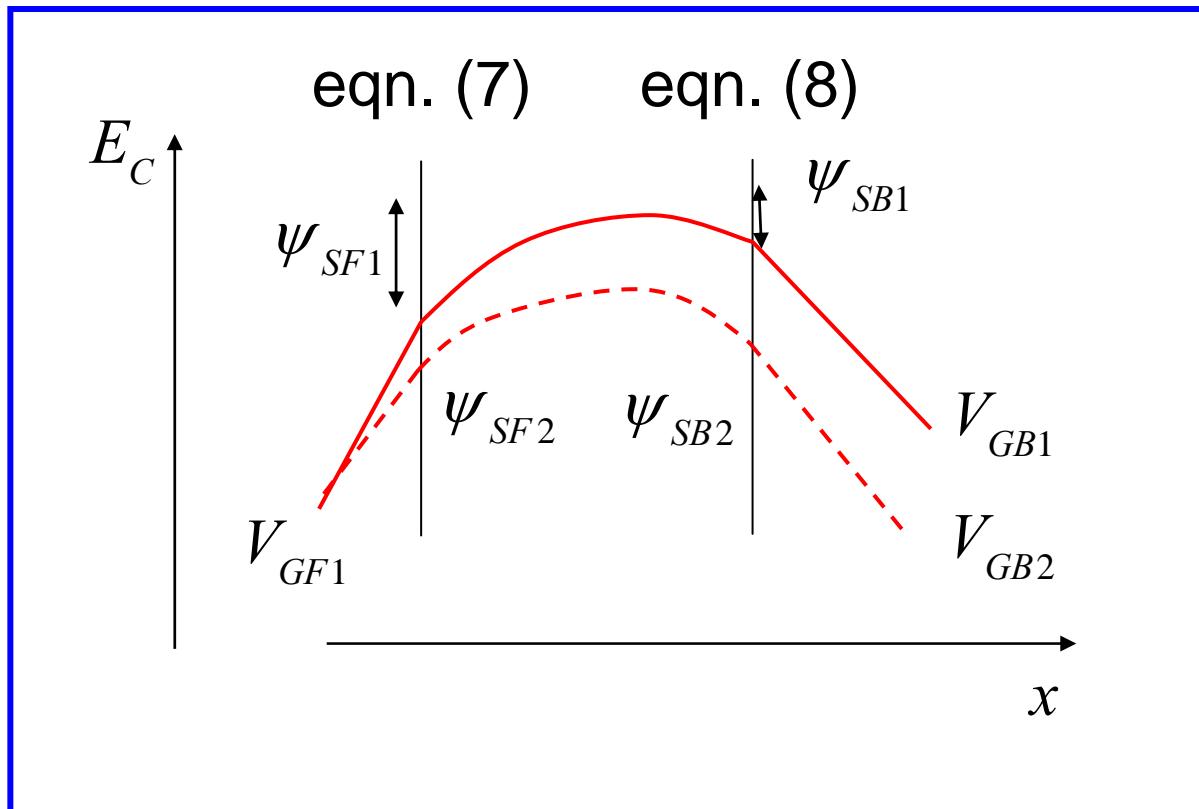
$$E_{Si} = (\psi_{SF} - \psi_{SB}) / t_{Si}$$

$$\epsilon_{of} E_{OF} = \epsilon_{Si} E_{Si} \Rightarrow \epsilon_{of} E_{OF} = \epsilon_{Si} (\psi_{SF} - \psi_{SB}) / t_{Si}$$

$$\Delta V_{OX} = t_{OF} E_{OF} = \frac{C_{Si}}{C_{OF}} (\psi_{SF} - \psi_{SB})$$



front and back coupled electrostatics

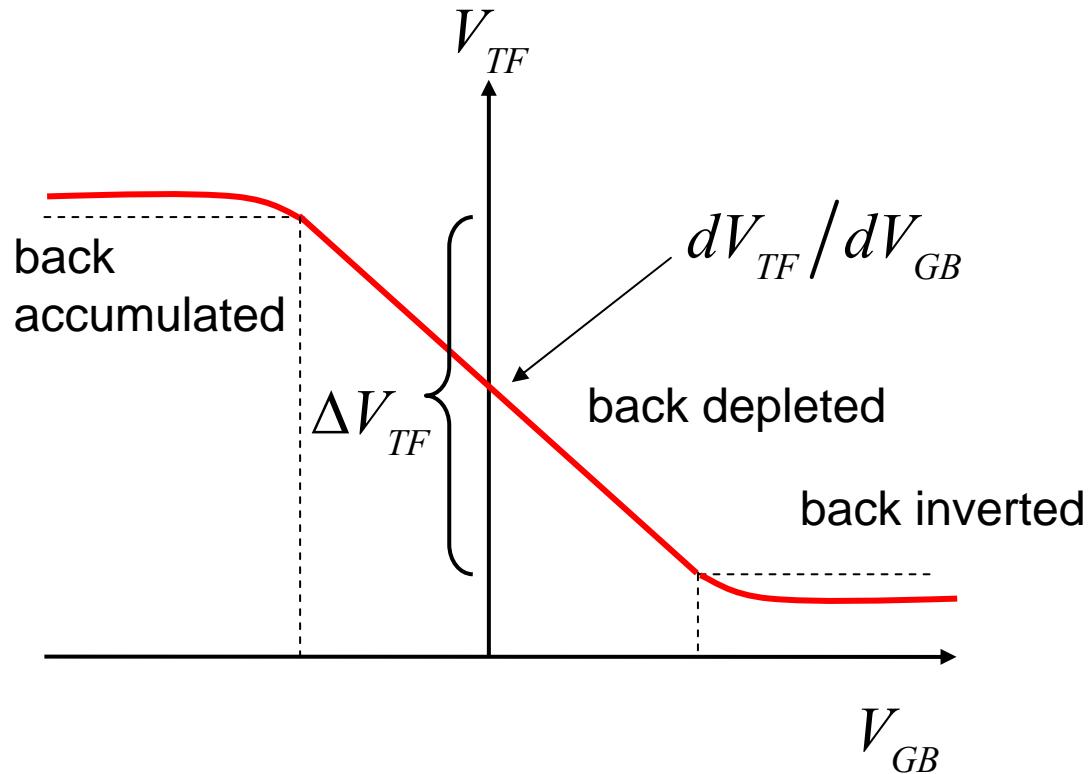


for a fixed V_{GF} , increasing V_{GB} increases ψ_{SF} (lowers V_{TF})

outline

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V_{TF} vs. V_{GB}



threshold voltage (V_{TF})

If V_{GF} is fixed, raising V_{GB} increases ψ_{SF} , so the front gate threshold voltage V_{TF} should **decrease**.

$$V_{GF} = \phi_{msf} + \psi_{SF} - \frac{Q_{IF} + Q_B/2}{C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (\psi_{SF} - \psi_{SB})$$

At threshold, $Q_{IF} = Q_B/2$ and $\psi_{SF} = 2\psi_B$

$$V_{TF} = \phi_{msf} + 2\psi_B - \frac{Q_B}{2C_{OF}} + \frac{C_{Si}}{C_{OF}}(2\psi_B - \psi_{SB})$$

V_{TF} is a function of ψ_{SB} , and hence can be varied using back gate bias (V_{GB})

1) back inverted

$$V_{TF} = \phi_{msf} + 2\psi_B - \frac{Q_B}{2C_{OF}} + \frac{C_{Si}}{C_{OF}}(2\psi_B - \psi_{SB})$$

Back side inversion:

$$\psi_{SB} = 2\psi_B$$

$$V_{TF} (\text{back inv}) = \phi_{msf} + 2\psi_B - \frac{Q_B}{2C_{OF}}$$

Current flows even when $V_{GF} < V_{TF}$ because the back surface is inverted. Since the device doesn't turn off, this mode of operation is not useful.

2) back accumulated

$$V_{TF} = \phi_{msf} + 2\psi_B - \frac{Q_B}{2C_{OF}} + \frac{C_{Si}}{C_{OF}}(2\psi_B - \psi_{SB})$$

Back accumulation:

$$\psi_{SB} \approx 0$$

$$V_{TF} (\text{back acc}) = \phi_{msf} + 2\psi_B - \frac{Q_B}{2C_{OF}} + \frac{C_{Si}}{C_{OF}}(2\psi_B)$$

Net V_{TF} shift:

$$\begin{aligned}\Delta V_{TF} &= V_{TF} (\text{back acc}) - V_{TF} (\text{back inv}) \\ &= \frac{C_{Si}}{C_{OF}}(2\psi_B) = \frac{\varepsilon_{Si}}{\varepsilon_{ox}} \frac{t_{OF}}{t_{Si}} (2\psi_B)\end{aligned}$$

3) back depleted

$$V_{GF} = \phi_{msf} + \psi_{SF} - \frac{Q_{IF} + Q_B/2}{C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (\psi_{SF} - \psi_{SB}) \quad (7)$$

$$V_{TF} = \phi_{msf} + 2\psi_{SB} - \frac{Q_B}{2C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (2\psi_B - \psi_{SB}) \quad (9)$$

We need to relate ψ_{SB} to V_{GB} ...

3) back depleted (ii)

$$V_{GB} = \phi_{msb} + \psi_{SB} - \frac{Q_{IB} + Q_B/2}{C_{OB}} + \frac{C_{Si}}{C_{OB}} \times (\psi_{SB} - \psi_{SF}) \quad (8)$$

at front threshold:

$$V_{GB} = \phi_{msb} + \psi_{SB} - \frac{Q_B}{2C_{OB}} + \frac{C_{Si}}{C_{OB}} \times (\psi_{SB} - 2\psi_B) \quad (10)$$

at the start of back accumulation ($\psi_{SB} = 0$):

$$V_{GB}(\text{acc}) = \phi_{msb} - \frac{Q_B}{2C_{OB}} - \frac{C_{Si}}{C_{OB}} 2\psi_B \quad (11)$$

3) back depleted (iii)

The back surface is depleted when $V_{GB} > V_{GB}(\text{acc})$

From (10) and (11):

$$V_{GB} - V_{GB}(\text{acc}) = \left(1 + \frac{C_{Si}}{C_{OB}} \right) \psi_{SB}$$

$$\psi_{SB} = \frac{C_{ob}}{C_{OB} + C_{Si}} [V_{GB} - V_{GB}(\text{acc})] \quad (12)$$

3) back depleted (iv)

recap:

$$V_{TF} = \phi_{msf} + 2\psi_{SB} - \frac{Q_B}{2C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (2\psi_B - \psi_{SB}) \quad (9)$$

$$\psi_{SB} = \frac{C_{ob}}{C_{OB} + C_{Si}} [V_{GB} - V_{GB}(\text{acc})] \quad (12)$$

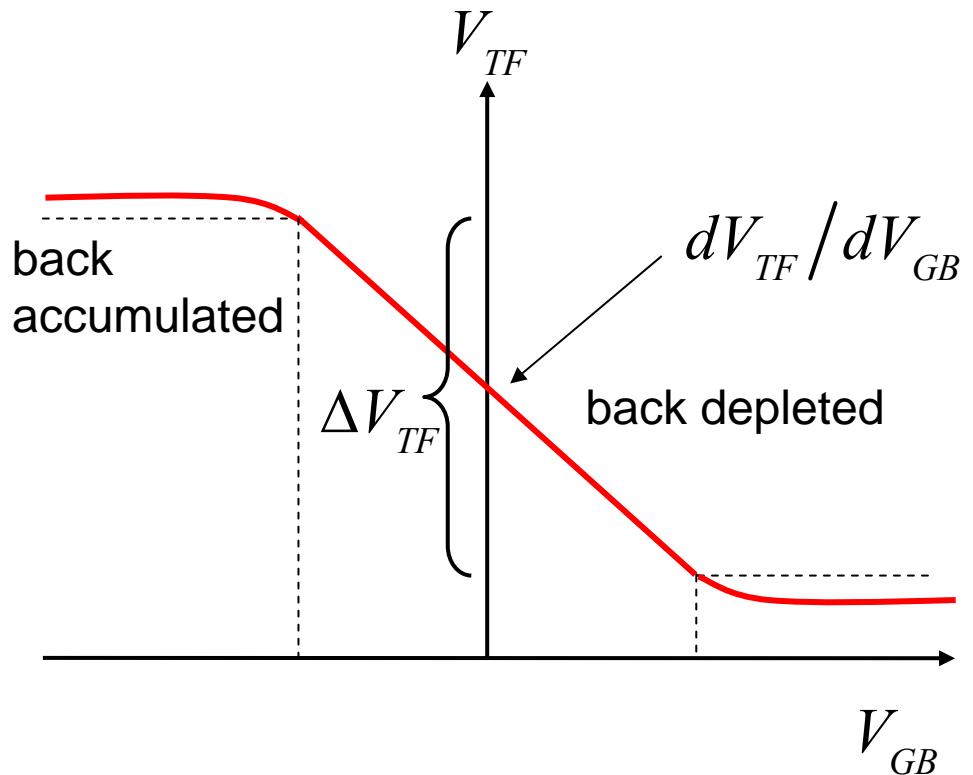
$$V_{TF} = \phi_{msf} + 2\psi_B - \frac{Q_B}{2C_{OF}} + \frac{C_{Si}}{C_{OF}} 2\psi_B - \frac{C_{Si} C_{OB}}{C_{OF} (C_{OB} + C_{Si})} [V_{GB} - V_{GB}(\text{acc})]$$

$$\frac{dV_{TF}}{dV_{GB}} = - \frac{C_{Si} C_{OB}}{C_{OF} (C_{OB} + C_{Si})} = - \left(\frac{t_{OF}}{t_{OB}} \right) \frac{1}{\left(1 + C_{OB}/C_{Si} \right)}$$

V_T summary

$$\Delta V_{TF} = \frac{\varepsilon_{Si}}{\varepsilon_{ox}} \frac{t_{OF}}{t_{Si}} (2\psi_B)$$

$$\frac{dV_{TF}}{dV_{GB}} = -\frac{t_{OF}/t_{OB}}{1 + C_{OB}/C_{Si}}$$



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subthreshold swing (bulk MOSFET review)

subthreshold current:

$$I_D \sim e^{q\psi_S/k_B T}$$

$$\ln I_D = q\psi_S / k_B T$$

$$2.303 \log I_D = q\psi_S / k_B T$$

subthreshold slope:

$$\frac{\partial \log I_D}{\partial V_{GS}} = \frac{1}{2.3(k_B T / q)} \frac{\partial \psi_S}{\partial V_{GS}} = S^{-1}$$

subthreshold swing:

$$S = 2.3(k_B T / q) \left(\frac{\partial \psi_S}{\partial V_{GS}} \right)^{-1} = 2.3m(k_B T / q)$$

$$m = (\partial \psi_S / \partial V_{GS})^{-1}$$

subthreshold swing (SOI MOSFET)

subthreshold current: $I_D \sim e^{q\psi_{SF}/k_B T}$

subthreshold swing: $S = 2.3(k_B T / q) \left(\frac{\partial \psi_{SF}}{\partial V_{GF}} \right)^{-1} = 2.3m(k_B T / q)$

'body effect parameter': $m = \left(\frac{\partial \psi_{SF}}{\partial V_{GF}} \right)^{-1}$

SOI subthreshold swing derivation

return to general solution:

$$V_{GF} = \phi_{msf} + \psi_{SF} - \frac{Q_{IF} + Q_B/2}{C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (\psi_{SF} - \psi_{SB}) \quad (7)$$

$$V_{GB} = \phi_{msb} + \psi_{SB} - \frac{Q_{IB} + Q_B/2}{C_{OB}} + \frac{C_{Si}}{C_{OB}} \times (\psi_{SB} - \psi_{SF}) \quad (8)$$

compute $dV_{GF} / d\psi_{SF}$ from (7)

assume Q_B is constant (FD)

SOI subthreshold swing derivation (ii)

$$\frac{\partial V_{GF}}{\partial \psi_{SF}} = 1 + \frac{C_{Si}}{C_{OF}} \left(1 - \frac{\partial \psi_{SB}}{\partial \psi_{SF}} \right) \quad (\odot)$$

To get $\frac{\partial \psi_{SB}}{\partial \psi_{SF}}$, differentiate (8) assuming V_{GB} is constant

$$0 = \frac{\partial \psi_{SB}}{\partial \psi_{SF}} + \frac{C_{Si}}{C_{OB}} \left(\frac{\partial \psi_{SB}}{\partial \psi_{SF}} - 1 \right)$$

$$\frac{\partial \psi_{SB}}{\partial \psi_{SF}} = \frac{C_{Si}/C_{OB}}{1 + C_{Si}/C_{OB}} = \frac{C_{Si}}{C_{Si} + C_{OB}} \quad \text{insert in } (\odot)$$

SOI subthreshold swing derivation (iii)

$$\frac{dV_{GF}}{d\psi_{SF}} = m = 1 + \frac{C_{Si} C_{OB}}{C_{OF} (C_{Si} + C_{OB})} \quad \left[m = 1 + \frac{C_D}{C_{OX}} \quad \text{bulk} \right]$$

$$C_D(\text{eff}) = \frac{C_{Si} C_{OB}}{(C_{Si} + C_{OB})} \quad m = 1 + \frac{C_D(\text{eff})}{C_{OF}}$$

if the bottom oxide is thick, $C_{OB} \ll C_{Si}, C_{OF}$ $m \rightarrow 1$

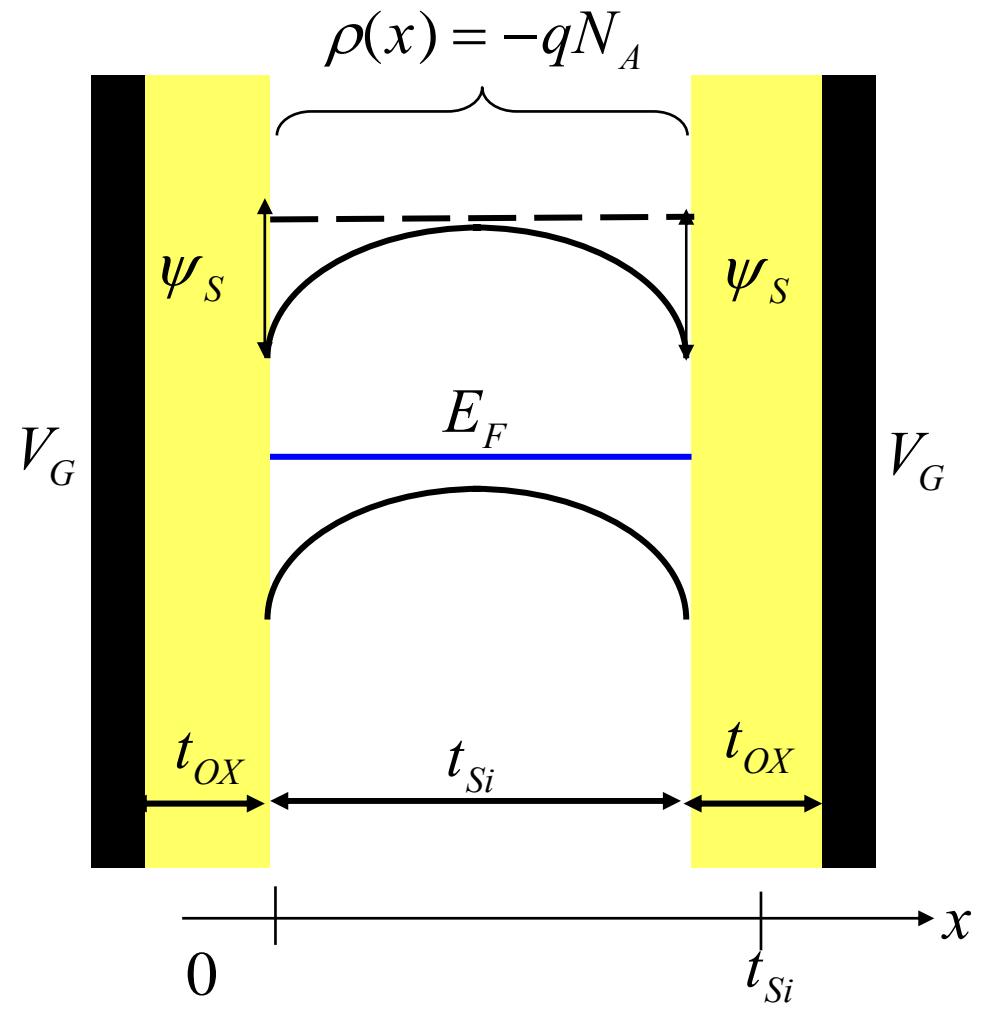
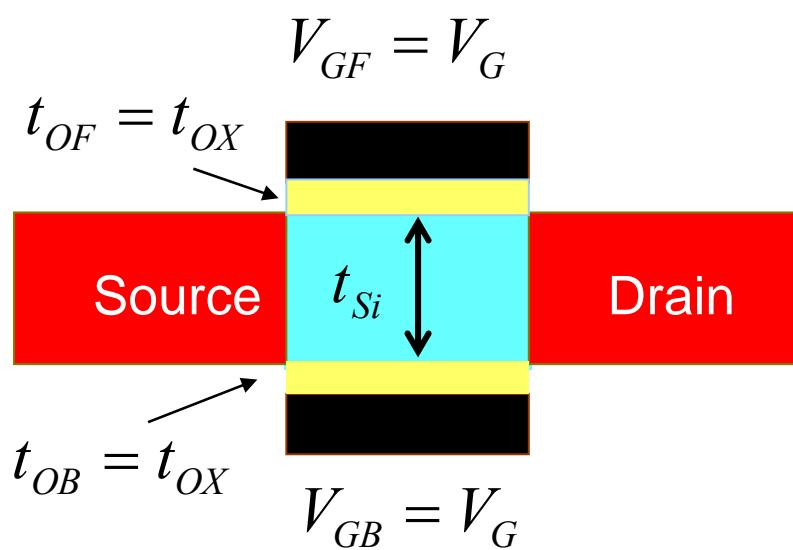
SOI summary

- 1) front and back gates are coupled electrostatically
- 2) front threshold voltage can be tuned by the back gate
- 3) for a thick BOX, the subthreshold swing is nearly ideal

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symmetrical double gate



DG electrostatics

$$V_{GF} = \phi_{msf} + \psi_{SF} - \frac{Q_{IF} + Q_B/2}{C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (\psi_{SF} - \psi_{SB}) \quad (7)$$

$$V_{GB} = \phi_{msb} + \psi_{SB} - \frac{Q_{IB} + Q_B/2}{C_{OB}} + \frac{C_{Si}}{C_{OB}} \times (\psi_{SB} - \psi_{SF}) \quad (8)$$

for double gate SOI:

$$V_G = \phi_{ms} + \psi_S - \frac{Q_I + Q_B/2}{C_{OX}}$$

DG subthreshold swing

$$V_G = \phi_{ms} + \psi_S - \frac{Q_I + Q_B / 2}{C_{OX}}$$

$$\frac{dV_G}{d\psi_S} = m = 1 \quad (\text{fully depleted, } Q_B \text{ independent of } \psi_S)$$

ideal subthreshold characteristics

DG above threshold

$$V_{GF} = \phi_{msf} + \psi_{SF} - \frac{Q_{IF} + Q_B/2}{C_{OF}} + \frac{C_{Si}}{C_{OF}} \times (\psi_{SF} - \psi_{SB}) \quad (7)$$

$$V_{GB} = \phi_{msb} + \psi_{SB} - \frac{Q_{IB} + Q_B/2}{C_{OB}} + \frac{C_{Si}}{C_{OB}} \times (\psi_{SB} - \psi_{SF}) \quad (8)$$

add these two equations:

$$2V_G = 2\phi_{ms} + 2\psi_S - \frac{Q_B}{C_{OX}} - \frac{Q_{IF} + Q_{IB}}{C_{OX}}$$

$$V_G = \phi_{ms} + \psi_S - \frac{Q_B}{2C_{OX}} - \frac{Q_I}{2C_{OX}}$$

DG above threshold (ii)

$$V_G = \phi_{msf} + \psi_S - \frac{Q_B}{2C_{OX}} - \frac{Q_I}{2C_{OX}}$$

$$V_T = \phi_{msf} + 2\psi_B - \frac{Q_B}{2C_{OX}}$$

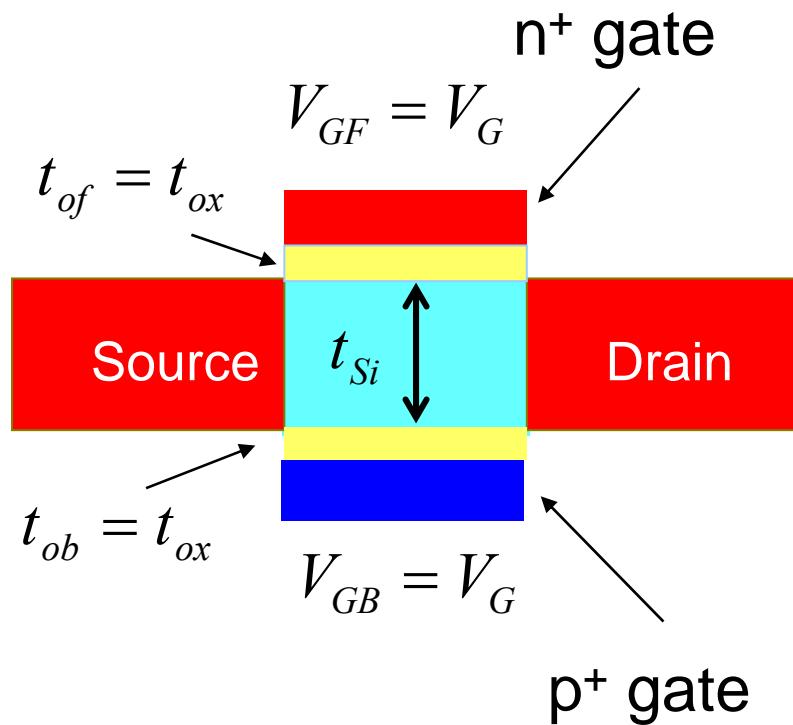
for $V_G > V_T$:

$$V_G - V_T = -\frac{Q_I}{2C_{OX}}$$

$$Q_I = -2C_{OX} (V_G - V_T)$$

twice as much charge \Rightarrow twice as much current

asymmetric gates



$$V_{GF} = V_{GB} = V_G$$

$$t_{OF} = t_{OB} = t_{OX}$$

$$\phi_{msf} \neq \phi_{msb}$$

how should

$$Q_I = -2C_{OX}(V_G - V_T)$$

be modified?

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