# ME 517: Micro- and Nanoscale Processes

# Lecture 11: Atomic Force Microscopy - Cantilever I

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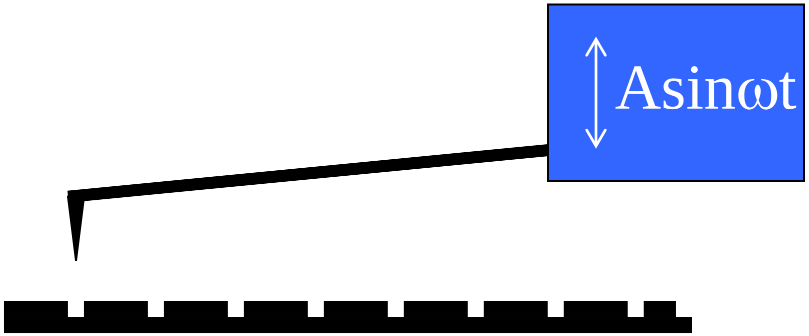
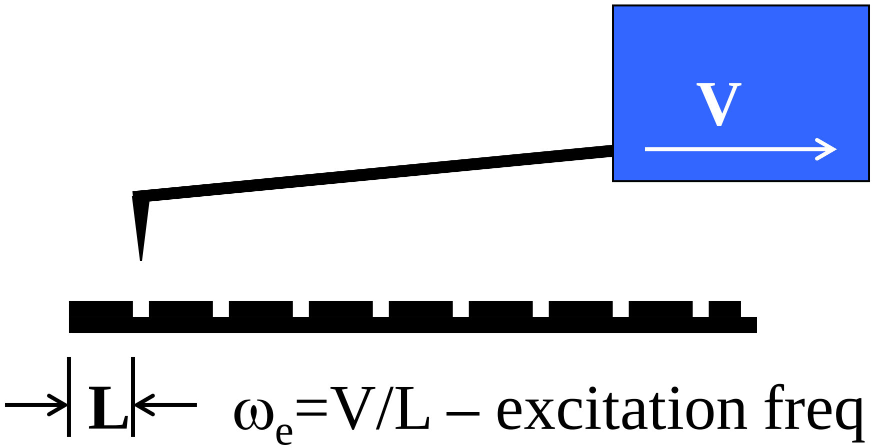
# AFM Dynamic Problems

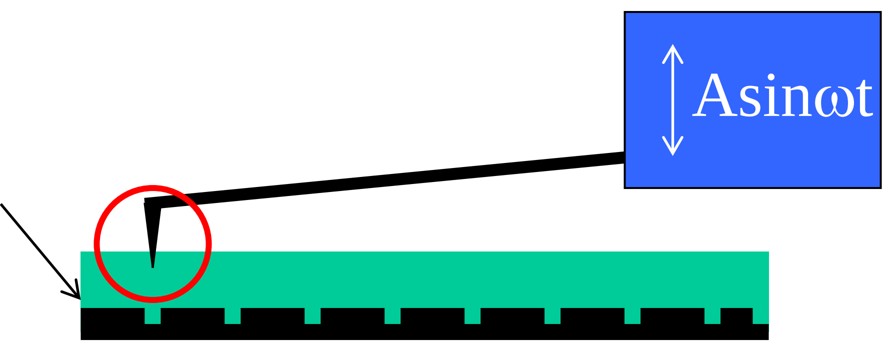
## Tapping Mode

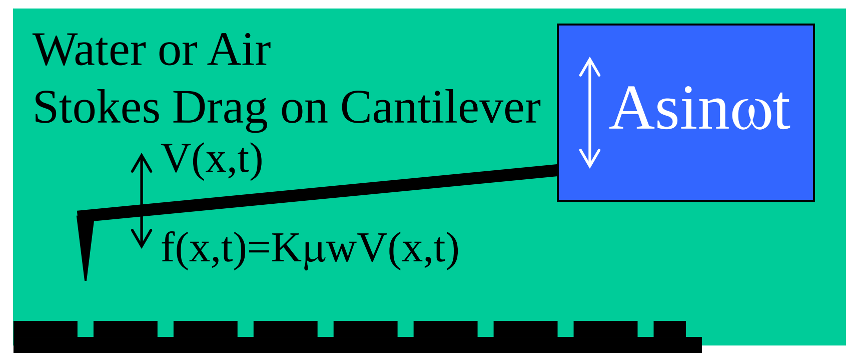



## Contact Mode




$\omega_e = V/L$ – excitation freq




Model boundary condition
as damped free end

# *Force Resolution of the AFM*

Hooke's Law

$$F = -K x$$

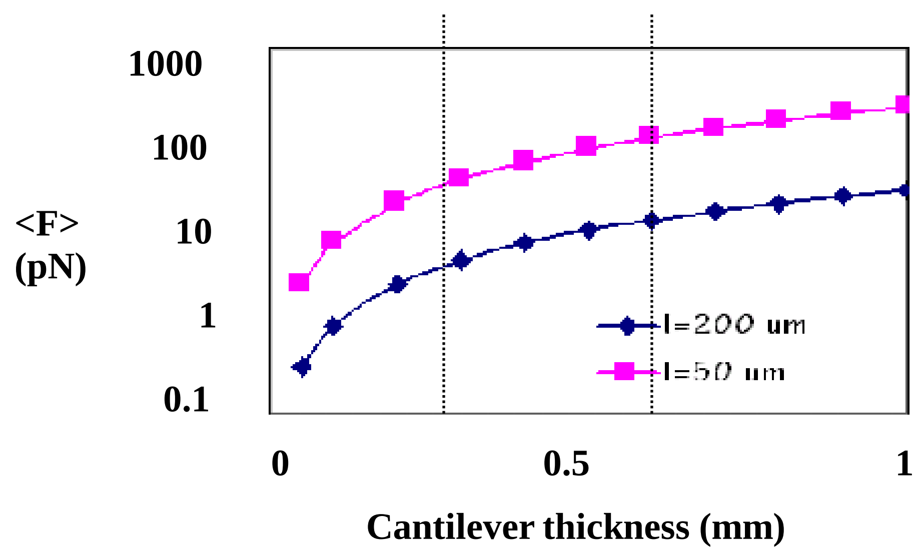
Spring constant dependence on the dimensions of the cantilevers

$$K = \frac{E t^3 w}{4 l^3}$$

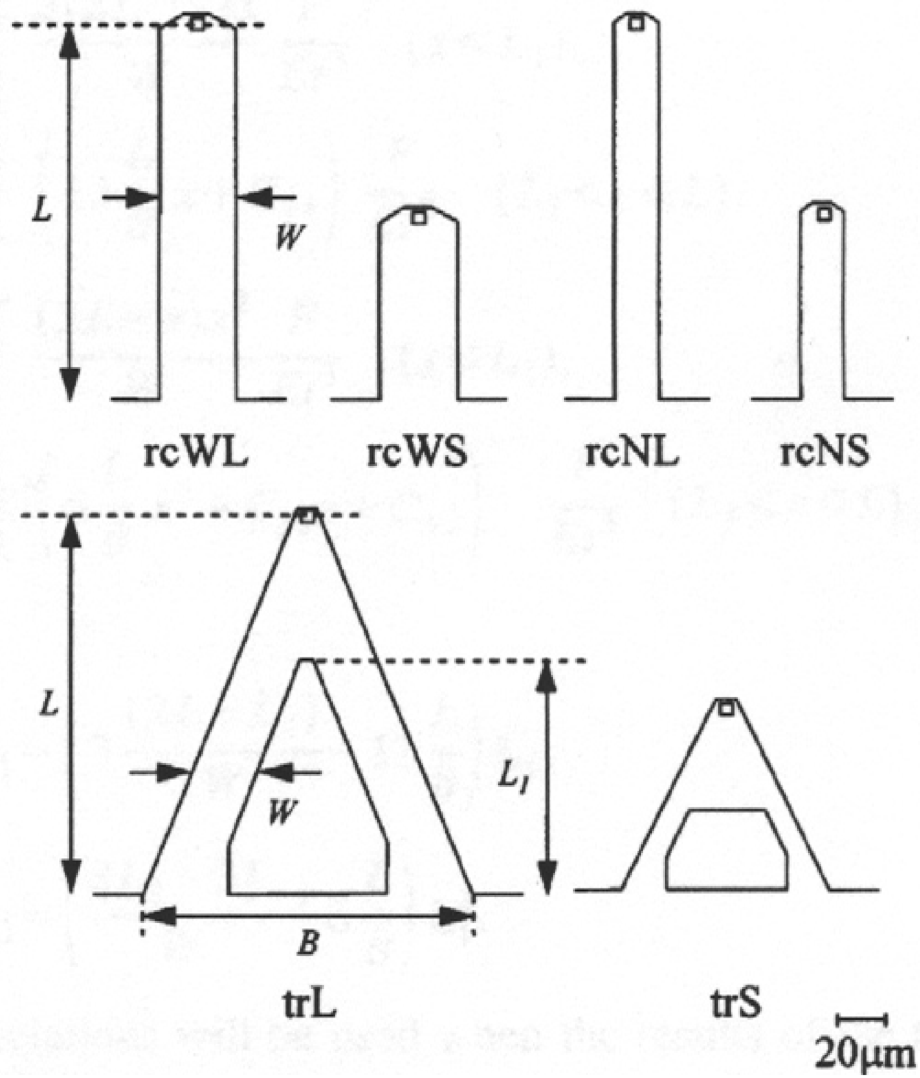
**E – elastic modulus**
**t – thickness**
**w – width**
**l -length**

Force resolution limitation

$$\langle F \rangle = \sqrt{4 K k_b T}$$

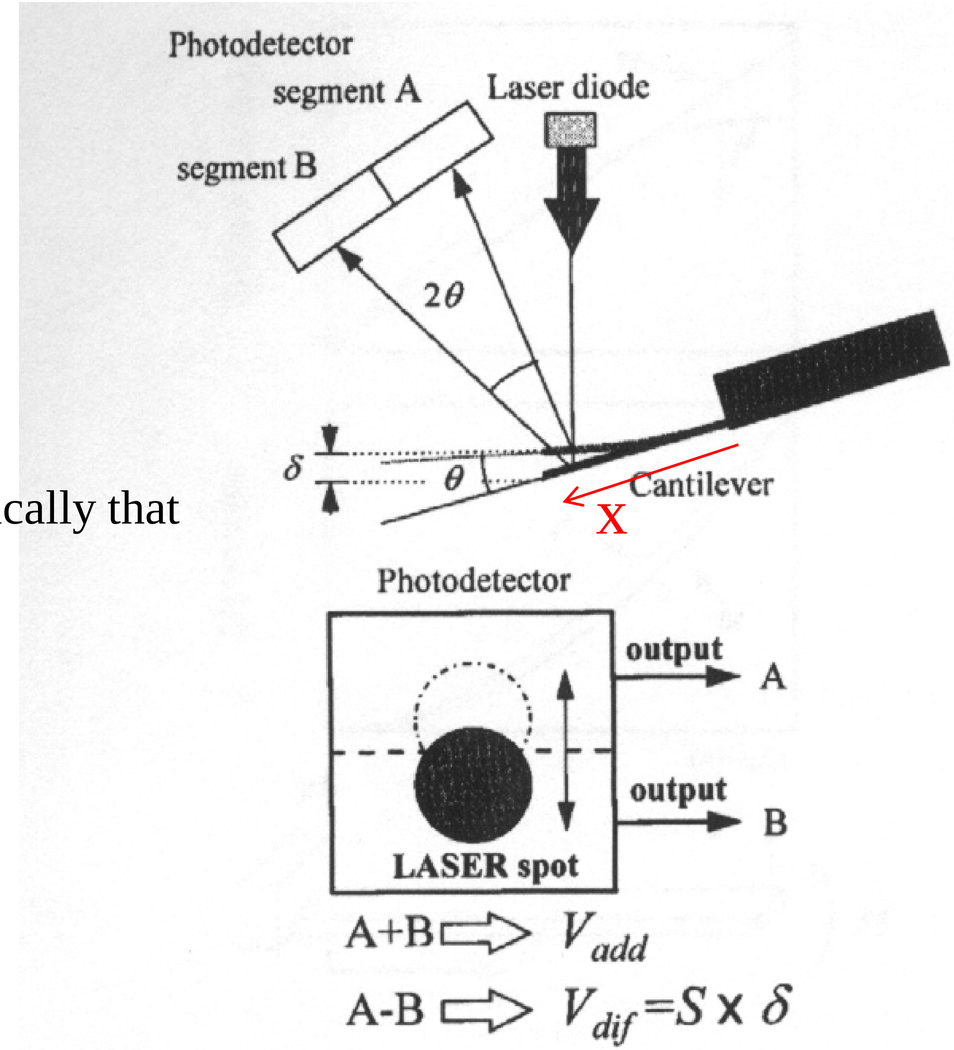


# Typical Cantilever Geometries




T. Miyani and M. Fujihira, “Calibration of surface stress measurements with atomic force microscopy,” J. Appl. Phys., Vol. 81 (11), p. 7099, 1997.

# Cantilever Readout Config




Can show geometrically that for small angles,

$$\frac{d\delta}{dx} = \theta$$

# Calculation of Moment of Inertia

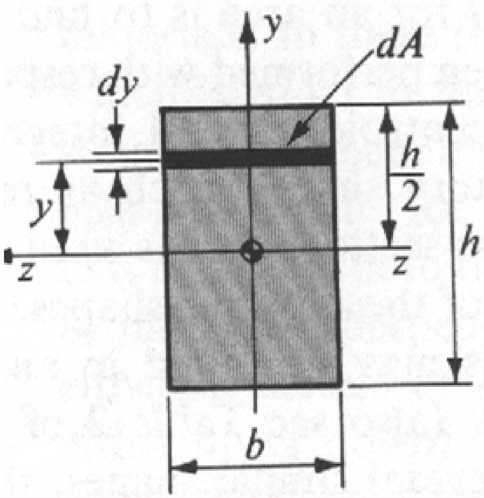



**Fig. 5-6**

**EXAMPLE 5-1**

Find the moment of inertia around the horizontal axis passing through the centroid for the rectangular area shown in Fig. 5-6.

**SOLUTION**

The centroid of this section lies at the intersection of the two axes of symmetry. Here it is convenient to take $dA$ as $b\,dy$. Hence

$$I_{zz} = I_o = \int_A y^2\,dA = \int_{-h/2}^{+h/2} y^2 b\,dy = b\left|\frac{y^3}{3}\right|_{-h/2}^{+h/2} = \frac{bh^3}{12}. \qquad (5\text{-}3)$$

Similarly

$$I_{yy} = \frac{b^3h}{12}$$

**Mechanics of Materials,** Popov, 1976

# Cantilever Deflection Fundamentals

• Bernoulli-Euler Beam Flexure Formula

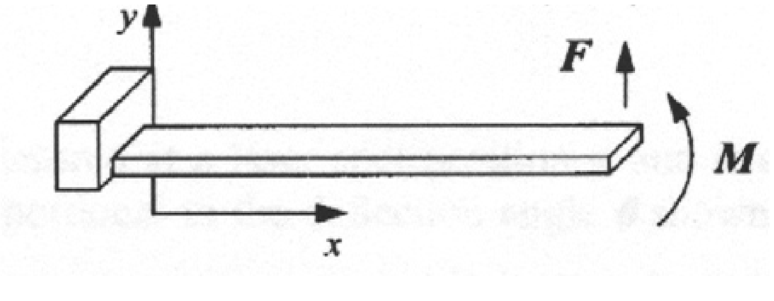
δ =deflection distance, x=position, M=applied moment, F=applied force, E=Young's modulus, I=moment of inertia

$$\frac{d^2\delta}{dx^2} = \frac{M(x)}{EI} \quad \text{Or equivalently} \quad \frac{d^4 v}{dx^4} = \frac{F(x)}{EI}$$

• Subject to boundary conditions:

–Fixed left side (x=0): $\delta=0$, $d\delta/dx=0$

–Free right side (x=L): $\delta$ =?, $d^2\delta/dx^2=0$, $d^3\delta/dx^3=0$



# Cantilever Fundamentals Cont'd.

- For beams with constant cross sections and compositions, these equations integrate to:

$$EI\delta(x) = \int_0^x dx \int_0^x F(x)\,dx + C_1 x + C_2$$

$$EI\delta(x) = \int_0^x dx \int_0^x dx \int_0^x dx \int_0^x F(x)\,dx + C_1 x^3 + C_2 x^2 + C_3 x + C_4$$