ME 517: Micro- and Nanoscale Processes

Lecture 13: Electrostatics I

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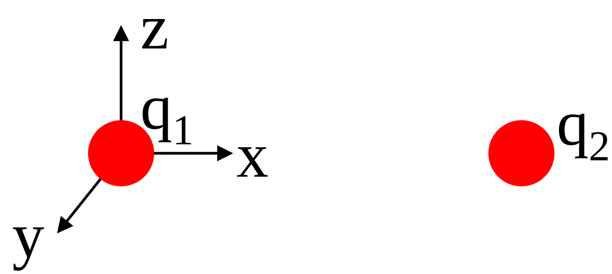
Electrostatics



Coulomb's law—force between two point charges is given by:

$$F_{elect} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{x^2}$$

where q_1 and q_2 are the two charges measured in coulombs (C), x is the distance separating them, ε_0 is the permittivity of free space, ε_r is the relative permittivity of the medium between the charges.

$\varepsilon_0 = 8.85\text{x}10^{-12}\ C^2/N\text{-}m^2$

$\varepsilon_r = 1$ for a vacuum, $\simeq 1$ for air, > 1 for other materials

Electrostatics

Gauss's law—the electric flux Φ_e through a surface bounding a charge distribution is given by

$$\Phi_e = \oint_\Sigma \vec{E} \bullet d\vec{S} = \frac{1}{\varepsilon_0 \varepsilon_r} \oint_\Sigma \vec{D} \bullet d\vec{S}$$

$$= \frac{1}{\varepsilon_0 \varepsilon_r} \int_V \rho dv = \frac{\text{net charge within } \Sigma}{\varepsilon_0 \varepsilon_r}$$

where dS is a local surface normal, Σ is the bounding area, E is the electric field, ρ is the charge density, and D is the electric flux density.

It is helpful to construct a potential field such that

$$\vec{E}\left(space\right) = -\nabla W\left(space\right)$$

where W is the electrostatic potential measured in J/C (or volts)

Electrostatics

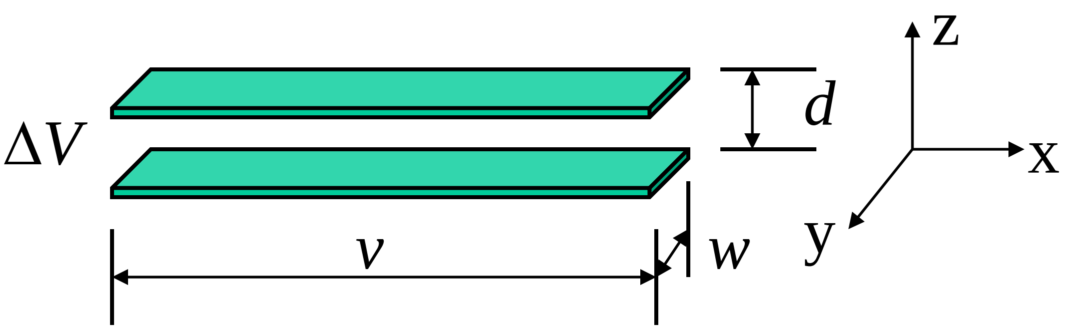
Define the potential energy W_e stored in an electrostatic field as:

$$W_e = \frac{1}{2}\int_V \rho V dv = \frac{1}{2}\int_V \varepsilon_0 \varepsilon_r \left|\vec{E}\right|^2 dv$$

where W_e is measured in Joules.

Physically W_e represents the total amount of work that would be required to place the charge within the electrostatic field.

Capacitor



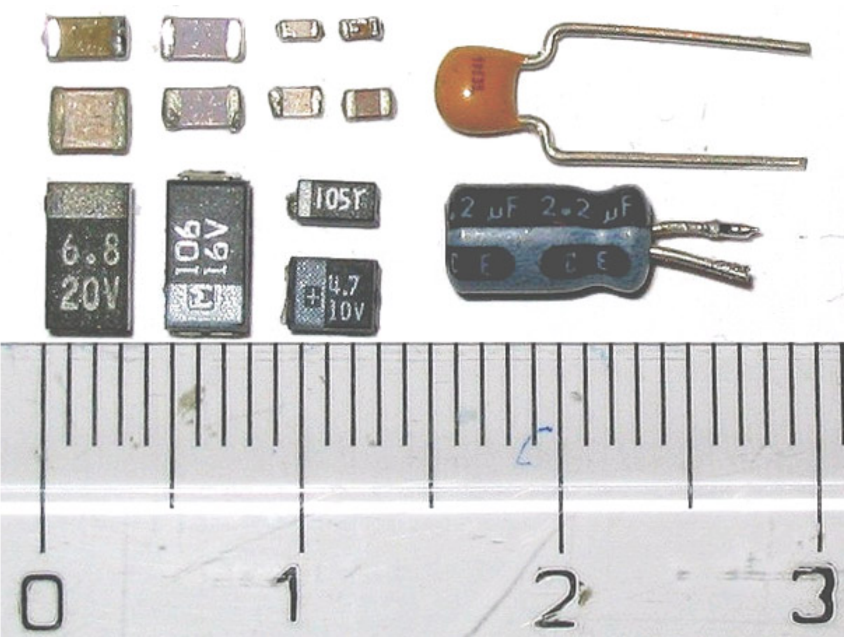
We call the above geometry a capacitor and define its capacitance as

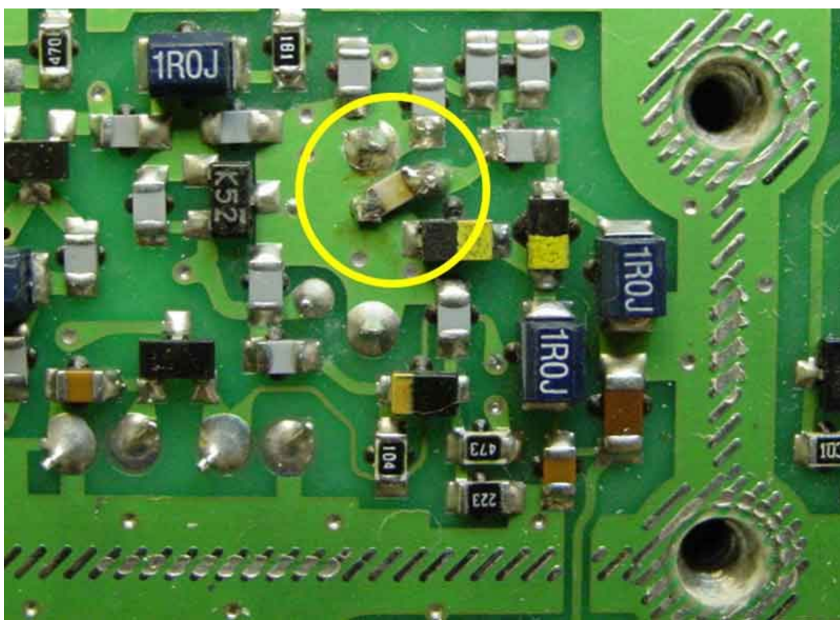
$$C = \frac{Q}{\Delta V}$$

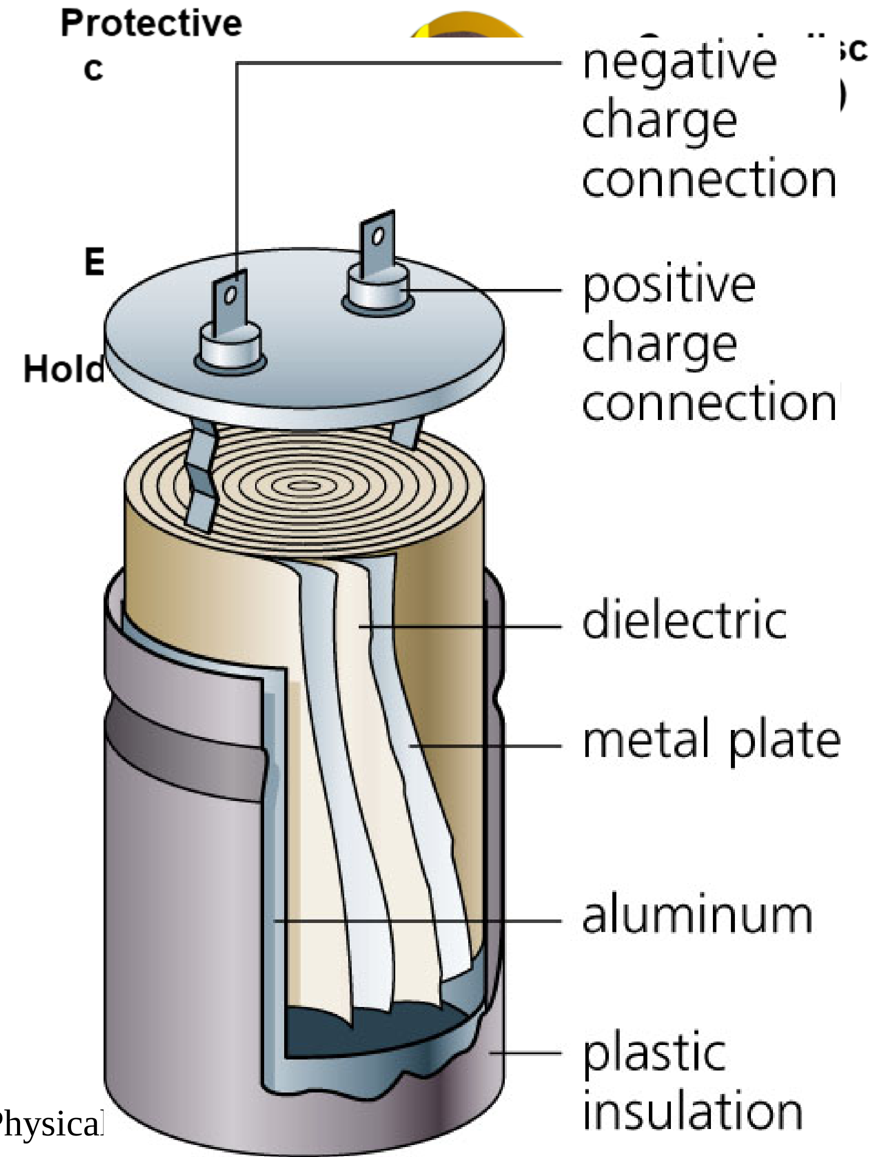
where C is measured in C/V, a unit defined as the farad (F).

Source: wikipedia

Typical Capacitors

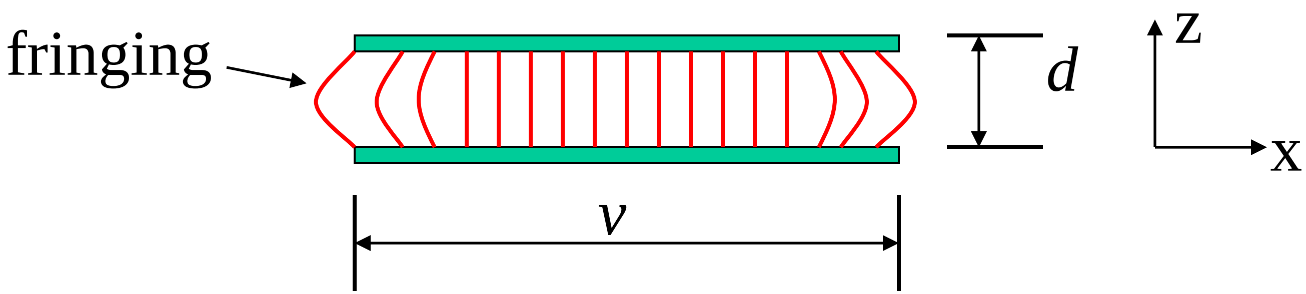






Physical

Capacitor

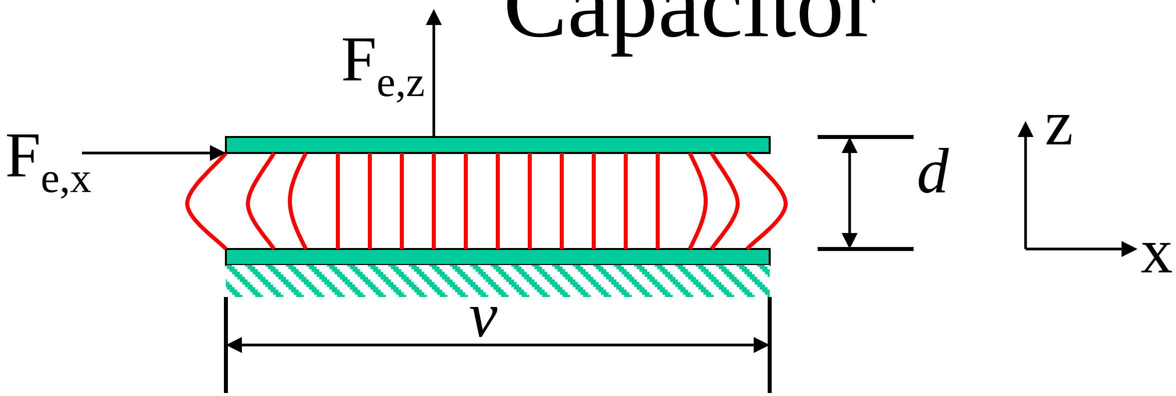


- Simplest geometry—parallel plate above
- Assuming that *d* is much smaller than *v* or *w*, fringing will not play a major role—assume that the field is uniform.

$$W_e = \frac{1}{2}\int_V \rho V dv = \frac{1}{2}\int_V \varepsilon_0 \varepsilon_r \left(\frac{V}{d}\right)^2 dv$$

$$= \frac{1}{2}\varepsilon_0 \varepsilon_r \left(\frac{V}{d}\right)^2 vwd = \frac{1}{2}\varepsilon_0 \varepsilon_r \frac{vw}{d} V^2$$

Capacitor



Calculate force exerted on free plate by fixed plate

Force is the gradient of potential so

$$F_{e,z} = \frac{\partial W_e}{\partial z} = \frac{\partial\left(\frac{1}{2}\varepsilon_0\varepsilon_r\frac{vw}{d}V^2\right)}{\partial z} = -\frac{1}{2}\varepsilon_0\varepsilon_r\frac{vw}{d^2}V^2$$

$$F_{e,x} = \frac{\partial W_e}{\partial x} = \frac{\partial\left(\frac{1}{2}\varepsilon_0\varepsilon_r\frac{(v-x)w}{d}V^2\right)}{\partial x} = -\frac{1}{2}\varepsilon_0\varepsilon_r\frac{w}{d}V^2$$