CQT Lecture #2

CQT, Lecture#2:

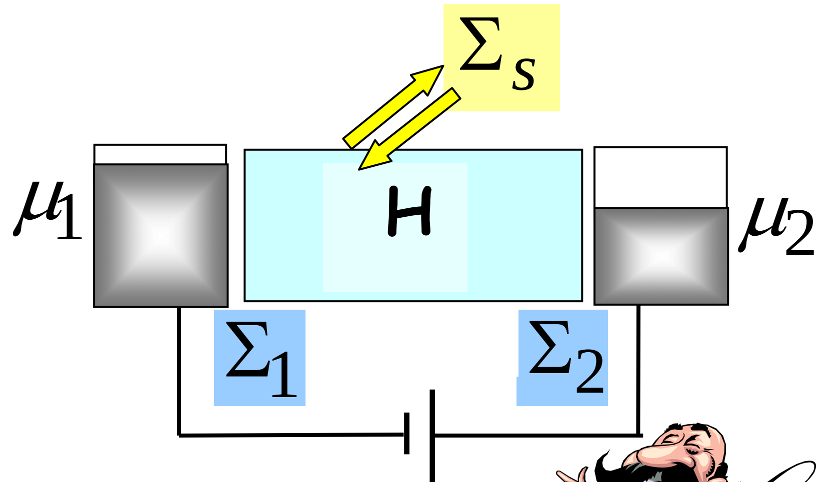
Electrical Resistance: A Simple Model

Objective:
To introduce a simple quantitative model for describing current flow in nanoscale structures and relate it to well-known large scale properties like Ohm's Law.

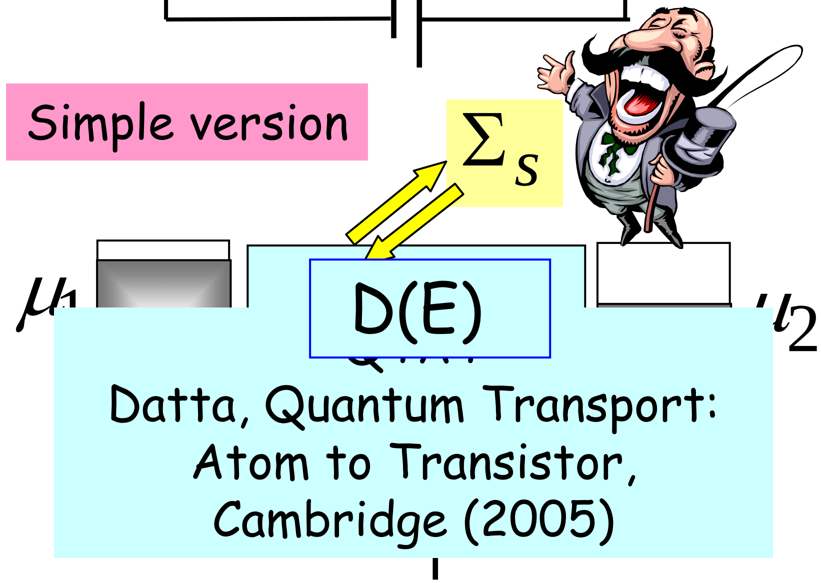
Model based on
Datta, Nanotechnology,15, S433 (2004).

Reference: QTAT, Chapter 1.

Unified Model for Quantum Transport Far from Equilibrium

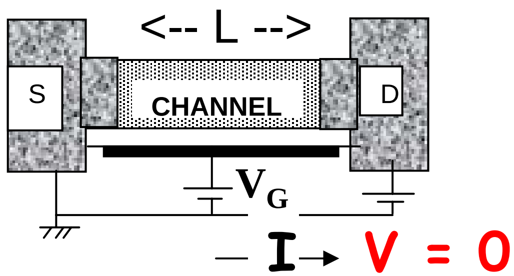


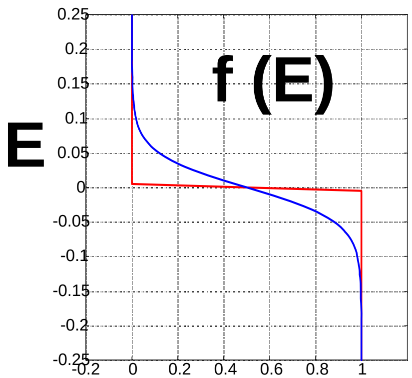
Simple version



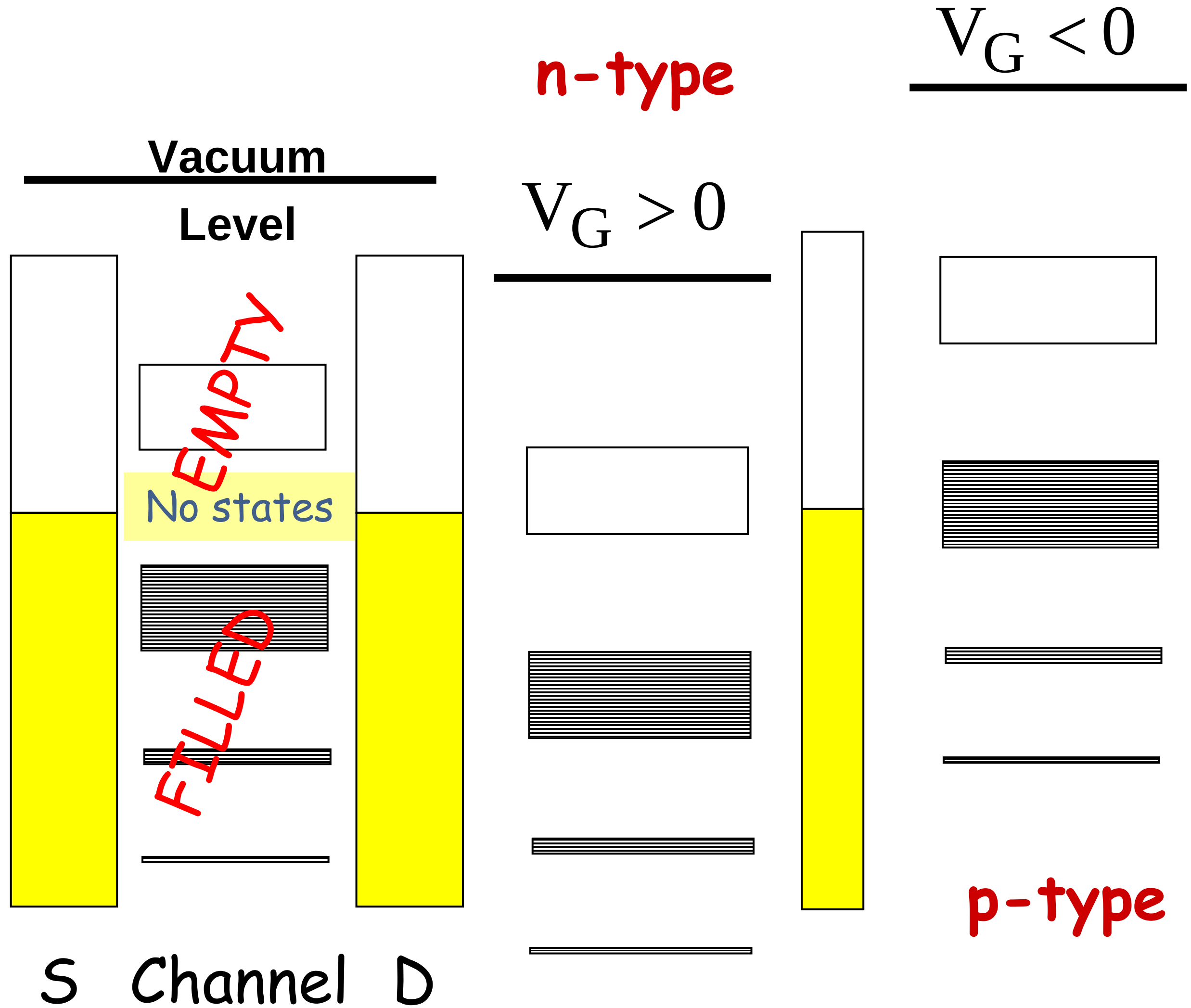
Datta, Quantum Transport: Atom to Transistor, Cambridge (2005)

Equilibrium Energy Level Diagram

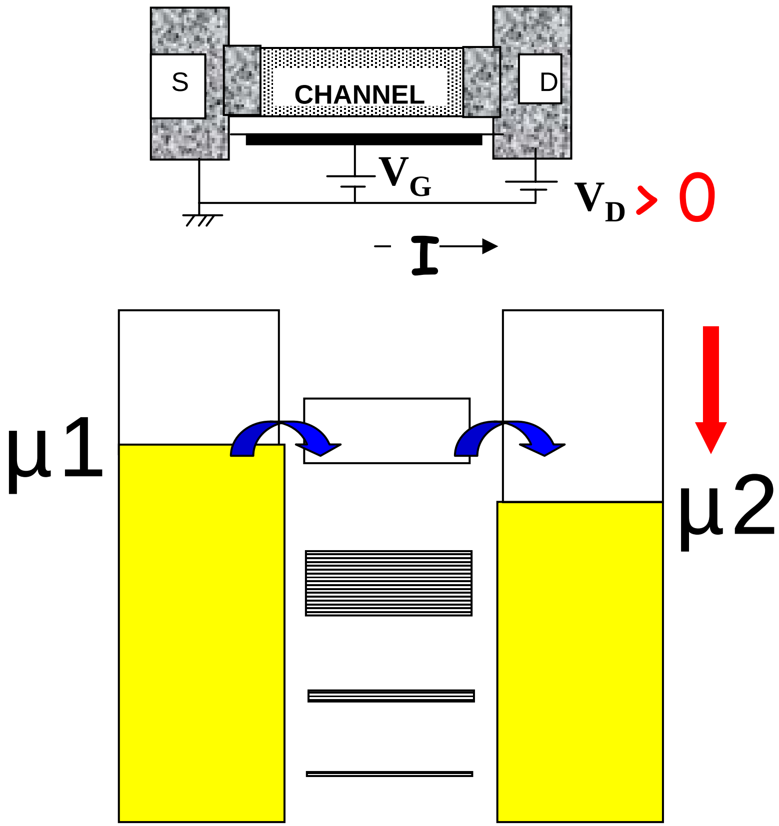


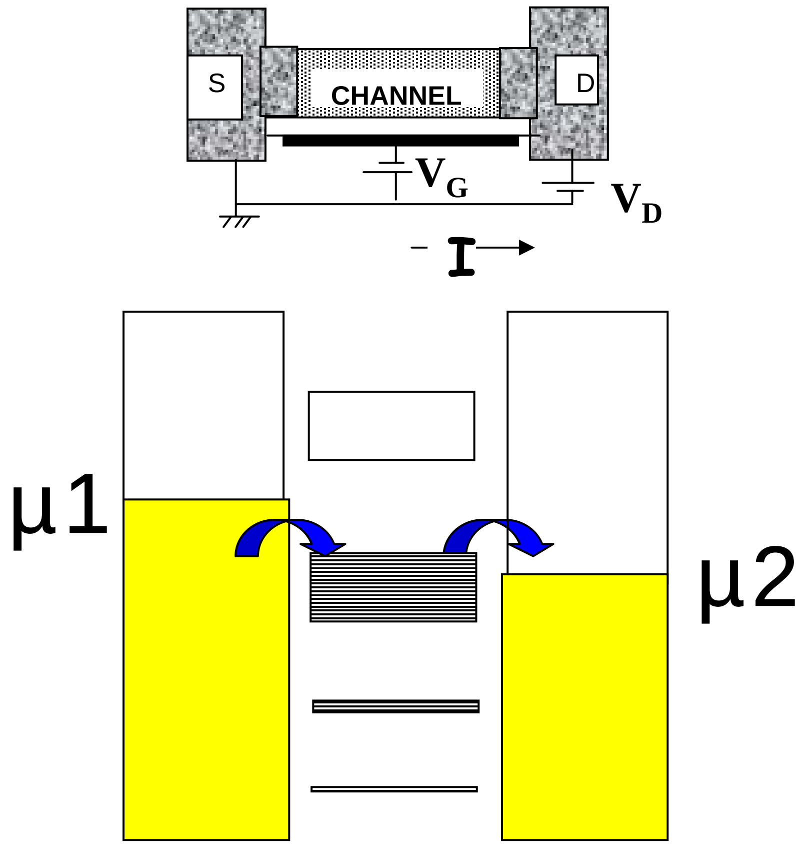


Fermi function



What makes electrons flow?

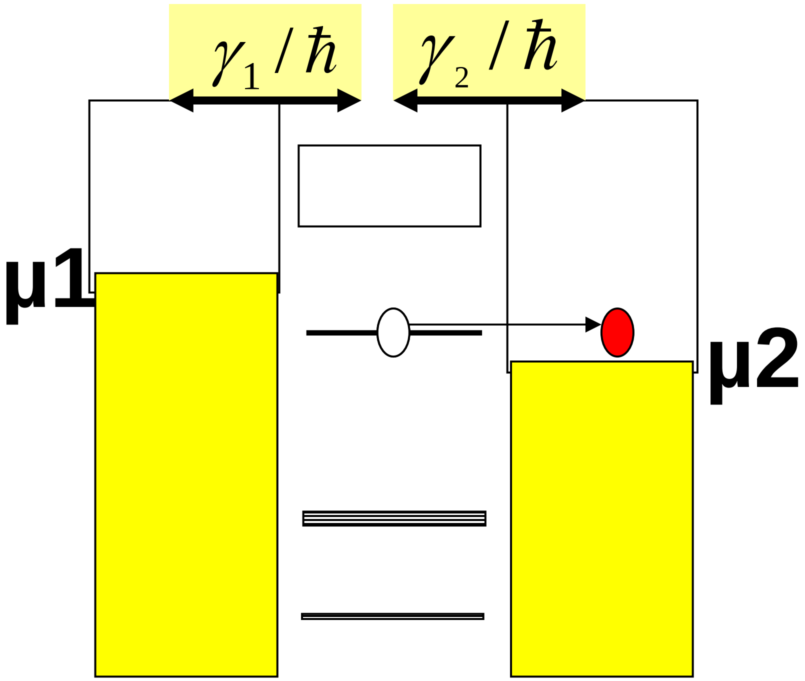


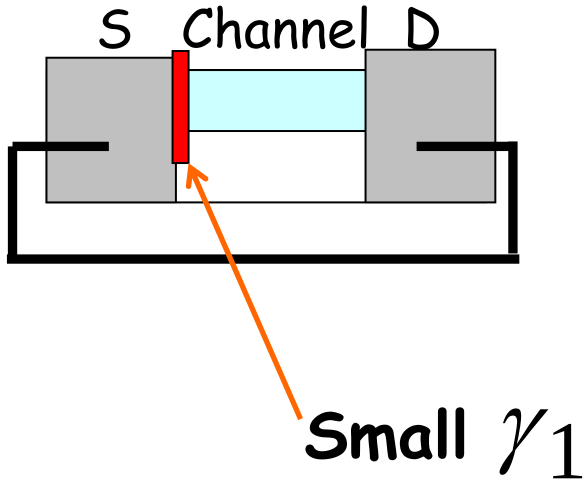


Escape rate

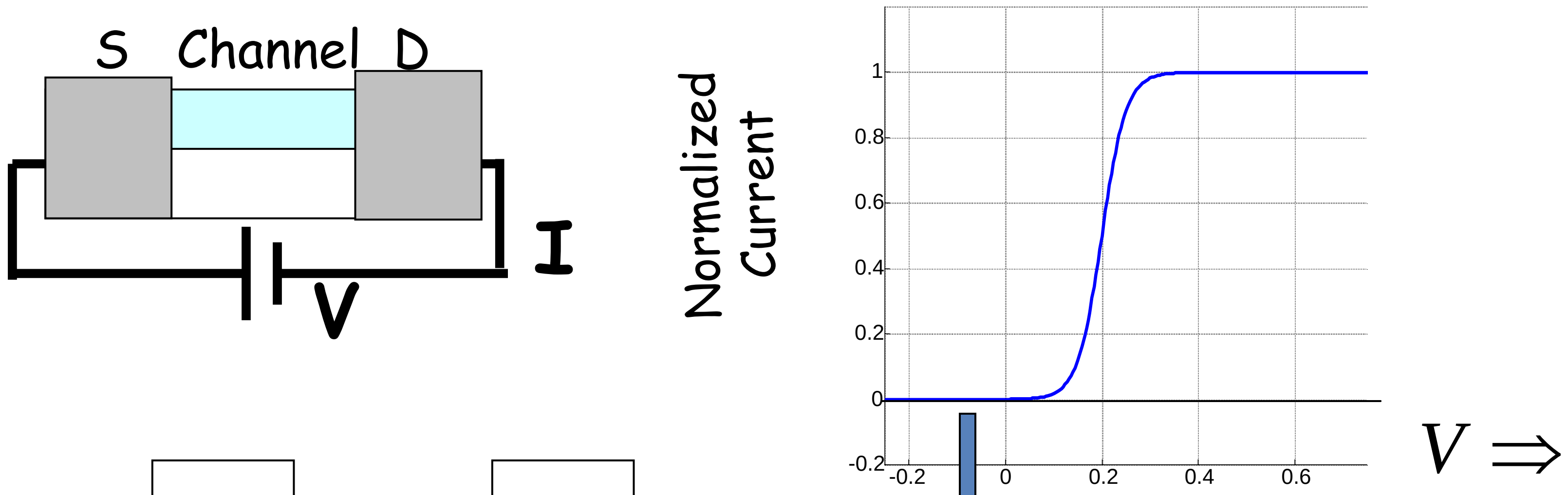
$$\gamma / \hbar : Escape\ Rate$$

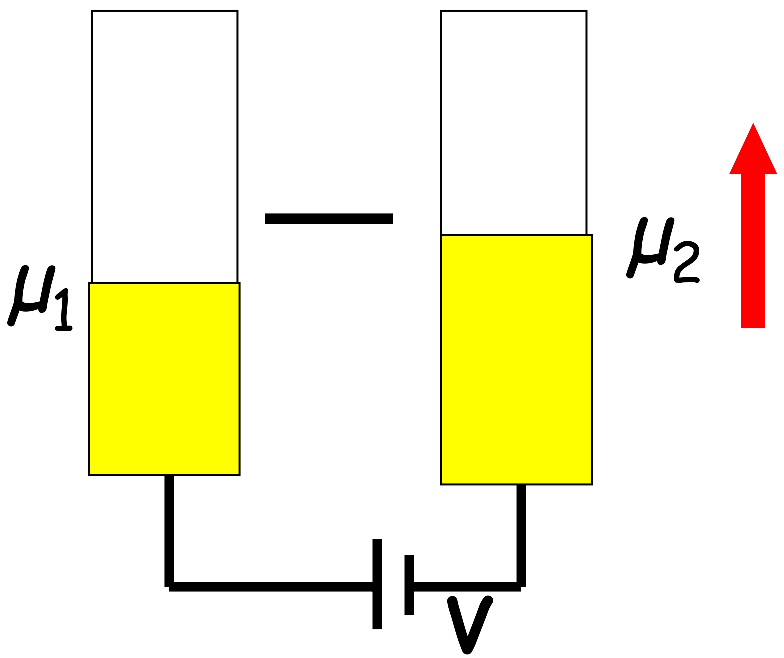
γ has dimensions of energy

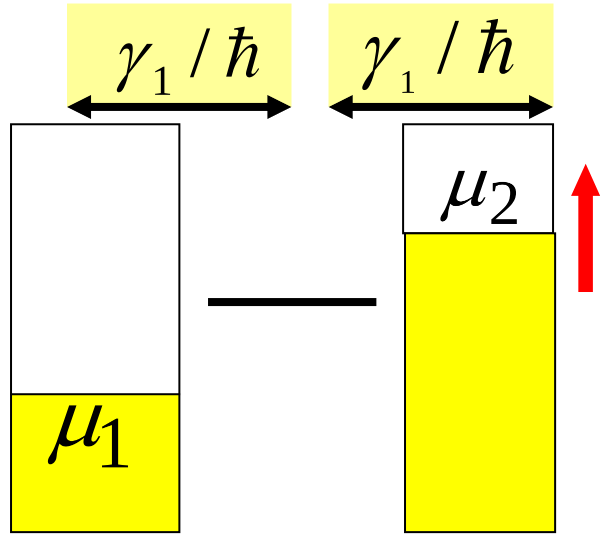




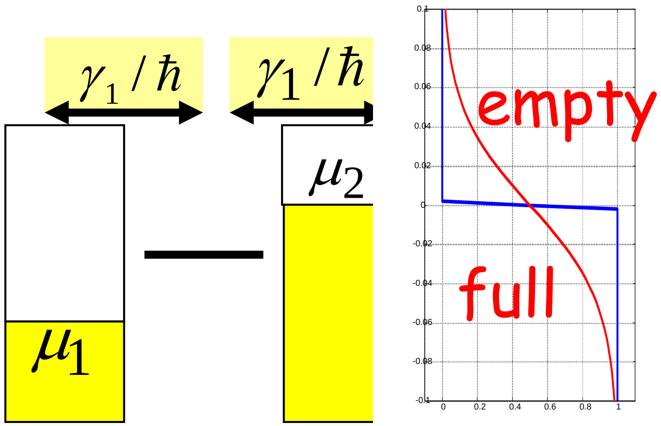
Current through a very small conductor

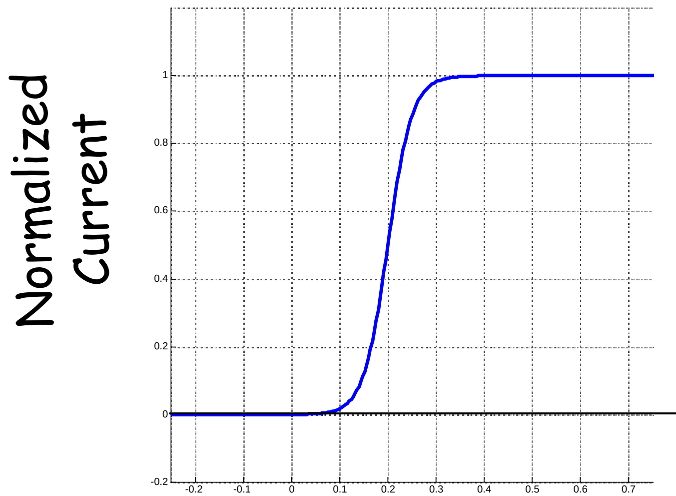






What is Conductance ?

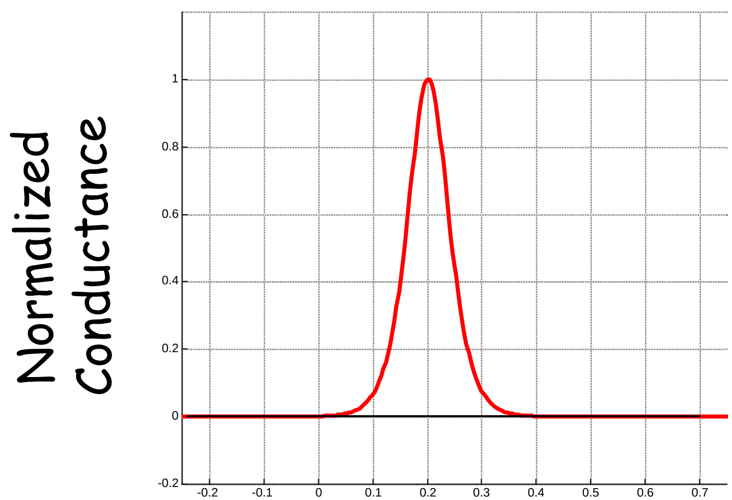




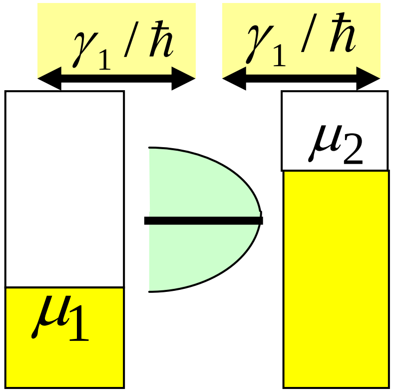
$\Rightarrow \dfrac{q\,\gamma_1}{2\,\hbar}$

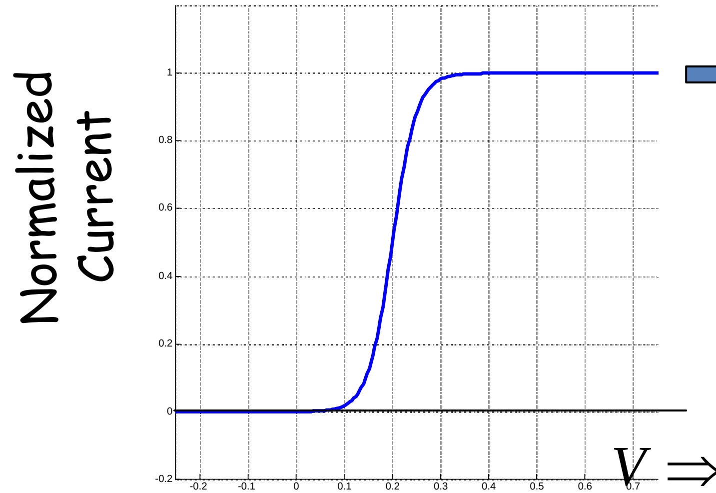
$V \Rightarrow$

$$\frac{dI}{dV} \sim \frac{q\gamma_1 / 2\hbar}{4kT / q}$$



Conductance quantum

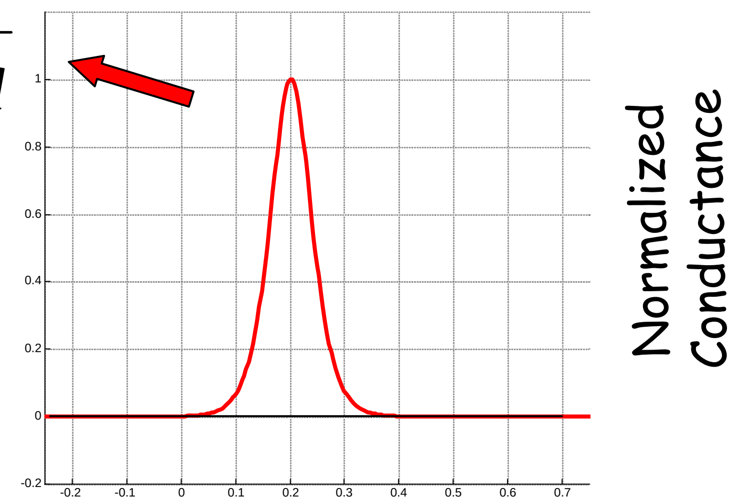




$$\frac{q\gamma_1}{2\hbar}$$

$$\frac{dI}{dV} \sim \frac{q\gamma_1/2\hbar}{(2\gamma_1 + \cancel{4kT})/q}$$

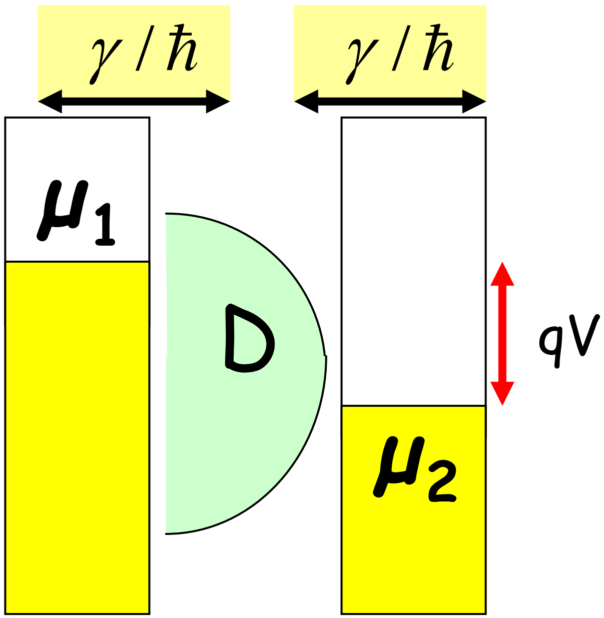
$$\sim \quad q^2/4\hbar \quad \text{if} \quad \gamma_1 >> kT$$



Conductance quantum

$$\sim \quad q^2/2\pi\hbar \quad \sim \quad 1/25.8\ K\Omega$$

Conductance: The bottom line

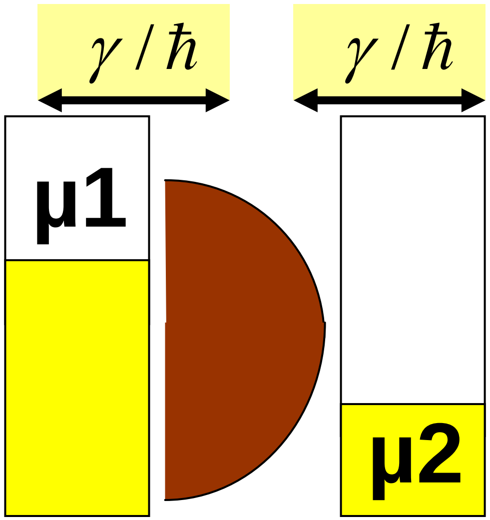


D: Density of states

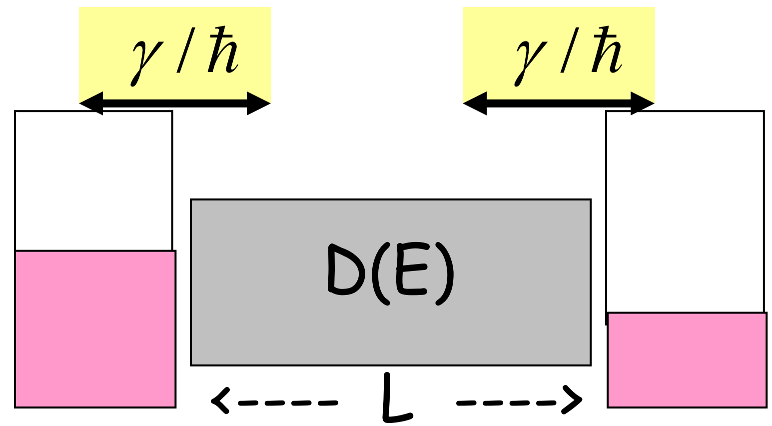
$$I \sim \underbrace{\frac{q\gamma}{2\hbar}}_{\text{Current per state}}$$

$$I/V = \underbrace{\frac{q^2}{2\pi\hbar}}_{\text{Conductance Quantum}} \underbrace{\langle \pi D \gamma \rangle}_{\text{Transmission}}$$

Bottom to Top: A Short-cut



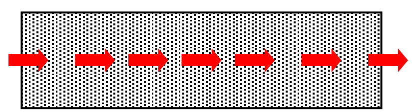
Cross-section A ; Length L



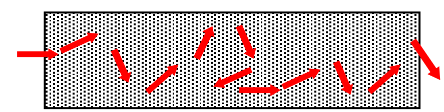
$$I/V = \frac{q^2}{2\pi\hbar}\langle \pi D \gamma \rangle$$

$$\underbrace{D(E)}_{/eV} = \underbrace{N_0(E)}_{/eV-nm^3}\ \underbrace{AL}_{nm^3}$$

Will show that



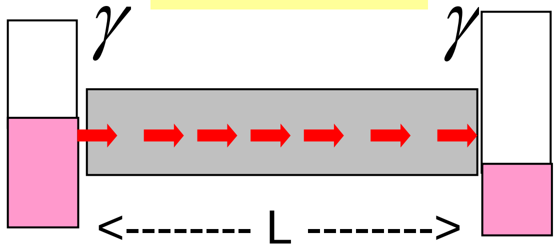
$$\gamma \sim \frac{\hbar v}{L} \rightarrow D\gamma \sim A$$

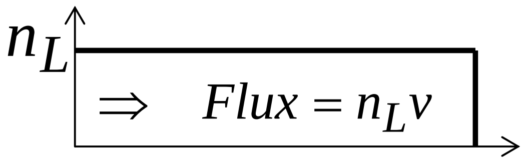


$$\gamma \sim \frac{2\hbar D}{L^2} \rightarrow D\gamma \sim A/L$$

Ballistic vs. Diffusive channels

Ballistic





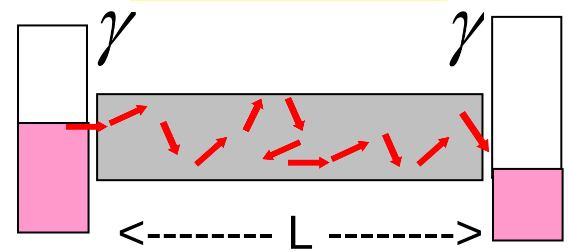
$$\Rightarrow \quad Flux = n_L v$$

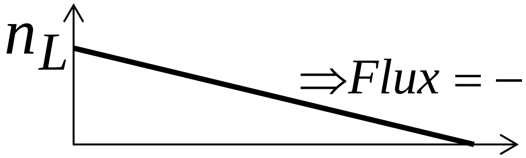
$$\frac{\gamma}{\hbar} = \frac{Flux}{Stored\ \ electrons}$$

$$= \frac{n_L v}{n_L L} = \frac{v}{L}$$

$$= \quad 1/Transit\ time$$

Diffusive





$$\Rightarrow Flux = -\tilde{D}\frac{\partial n_L}{\partial x}$$

$$\frac{\gamma}{\hbar} = \frac{Flux}{Stored\ \ electrons}$$

$$= \frac{\tilde{D} n_L / L}{n_L L / 2} = \frac{2\tilde{D}}{L^2}$$

$$= \quad 1/Transit\ time$$

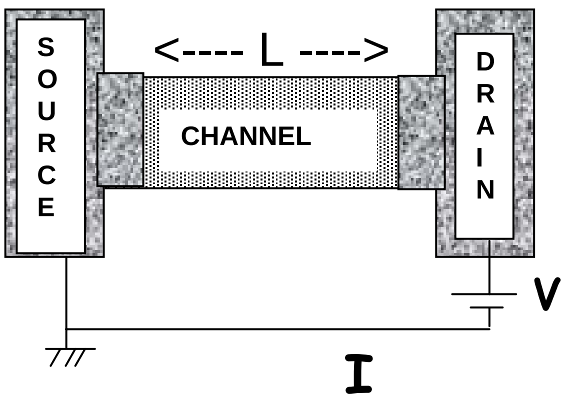
Drift-diffusion equations

Diffusive transport

$$\gamma = \frac{2\hbar\tilde{\mathcal{D}}}{L^2}$$

$$D = N_0 A L$$

$$\boxed{I/V = \frac{q^2}{2\hbar}\langle D\gamma\rangle}$$



$$I/V = (A/L)\;\; q^2 N_0 \tilde{\mathcal{D}}$$

conductivity

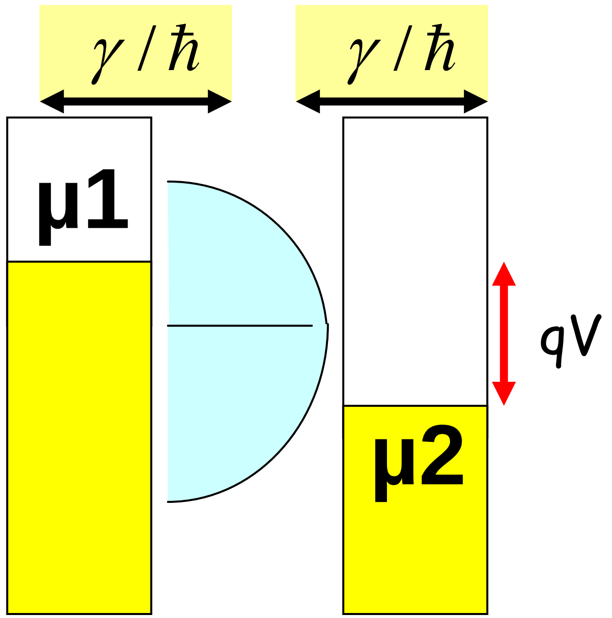
Non-degenerate

$$\rightarrow (A/L)\;\; q^2 \frac{n}{k_B T}\tilde{\mathcal{D}}$$

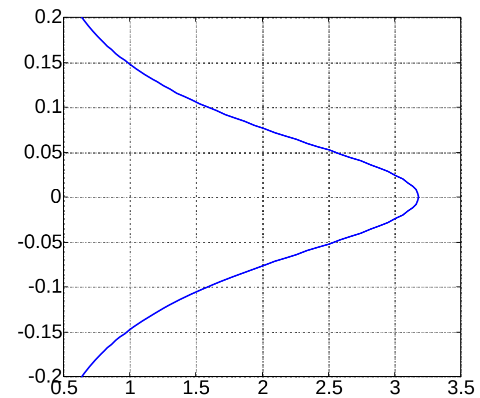
Einstein Relation : $\dfrac{\tilde{\mathcal{D}}}{\mu} = \dfrac{k_B T}{q}$

$$= (A/L)\;\; q\, n\, \mu$$

Transmission



One broadened level :



$$D(E) = \frac{\gamma/\pi}{(E-\varepsilon)^2 + \gamma^2}$$

$$D(E=\varepsilon) = 1/\pi\gamma$$

$$\langle \pi D \gamma \rangle = 1$$

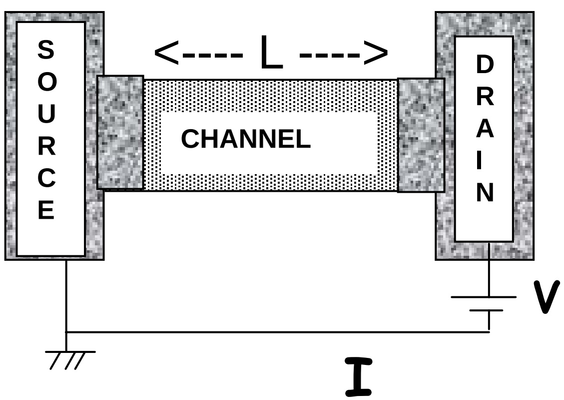
$$I/V = \underbrace{\frac{q^2}{2\pi\hbar}}_{\text{Conductance quantum}} \underbrace{\langle \pi D \gamma \rangle}_{\text{Transmission}}$$

Number of modes

Ballistic transport

$$\gamma = \hbar v_x / L$$

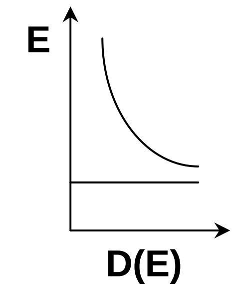
$$I/V = \underbrace{\frac{q^2}{2\pi\hbar}}_{\text{Conductance quantum}} \quad \underbrace{\langle \pi D \gamma \rangle}_{\text{Transmission}}$$



Electrons with effective mass 'm'

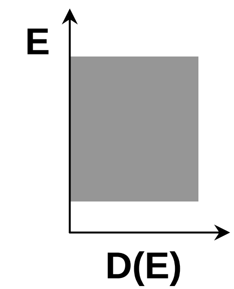
$$1D: \quad D = L(m/\pi\hbar v)$$

$$\langle \pi D \gamma \rangle = 1$$



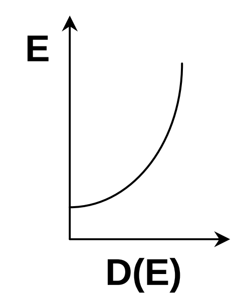
$$2D: \quad D = LW\, m/2\pi\hbar^2$$

$$\langle \pi D \gamma \rangle = W\, m v_x / 2\hbar \approx \frac{W}{\lambda/2}$$

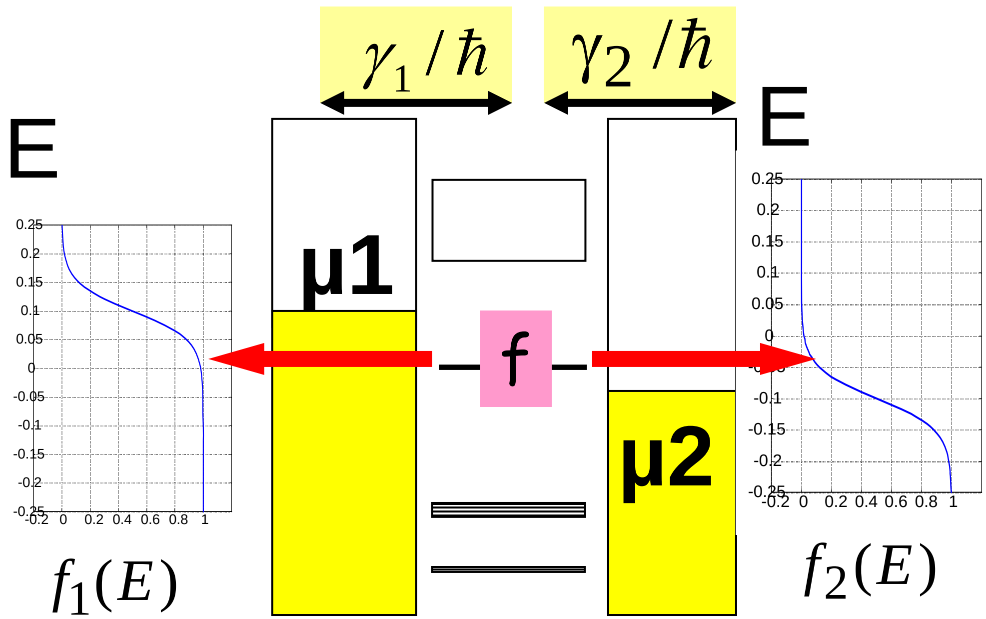


$$3D: \quad D = LAm^2 v / 3\pi^2 \hbar^3$$

$$\langle \pi D \gamma \rangle \approx \frac{A}{(\lambda/2)^2}$$



Current through one level



$$I_1 = q\frac{\gamma_1}{\hbar}[f_1 - f]$$

$$I_2 = q\frac{\gamma_2}{\hbar}[f - f_2]$$

$$\mathrm{I} = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}[f_1 - f_2]$$

Set $\gamma_2 = \gamma_1$:

1 - 0

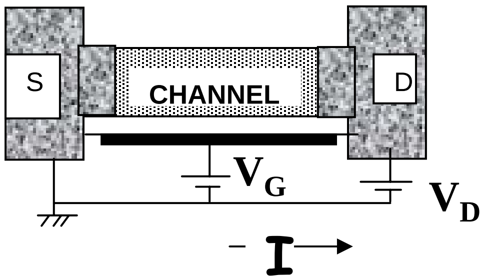
$$I \sim \frac{q}{\hbar}\frac{\gamma_1}{2}[\cancel{f_1} - \cancel{f_2}]$$

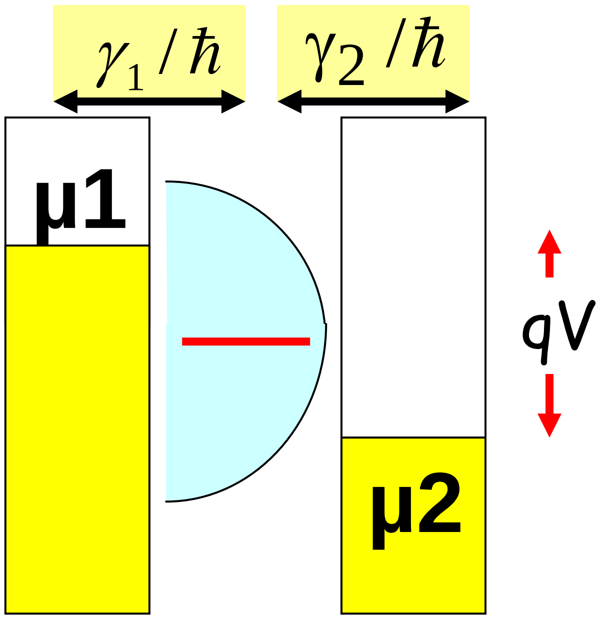
$$\boxed{I \sim \frac{q}{\hbar}\frac{\gamma_1}{2}}$$

Current with Broadening

$$n = \int dE \quad D(E) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

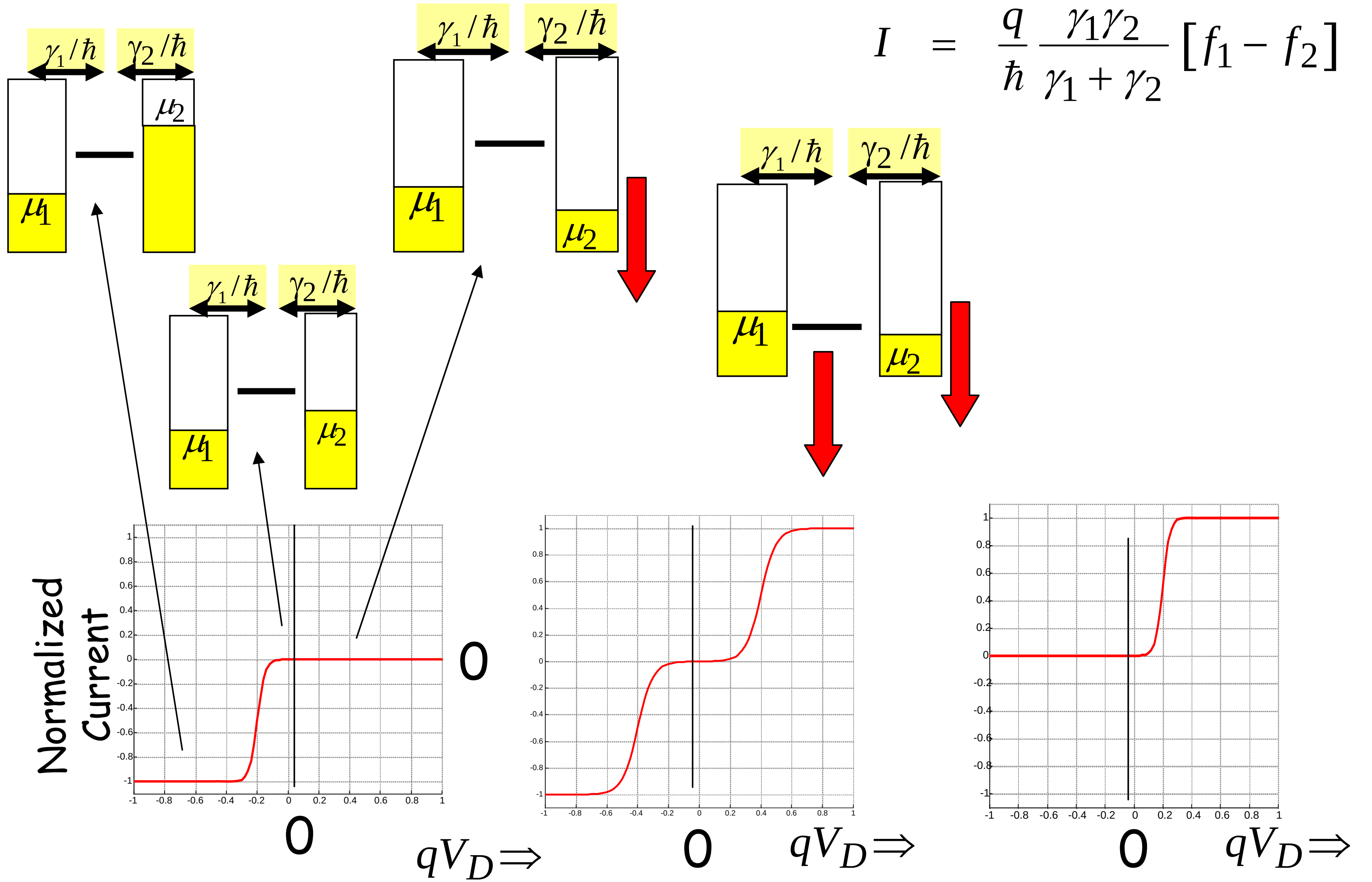
$$I = \frac{q}{\hbar} \int dE \quad D(E) \quad \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$



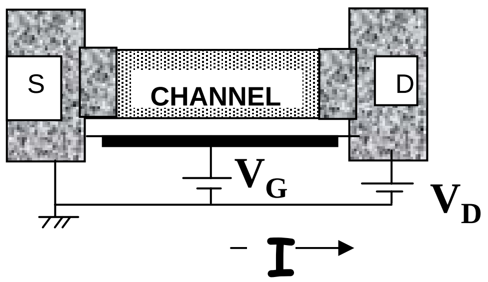


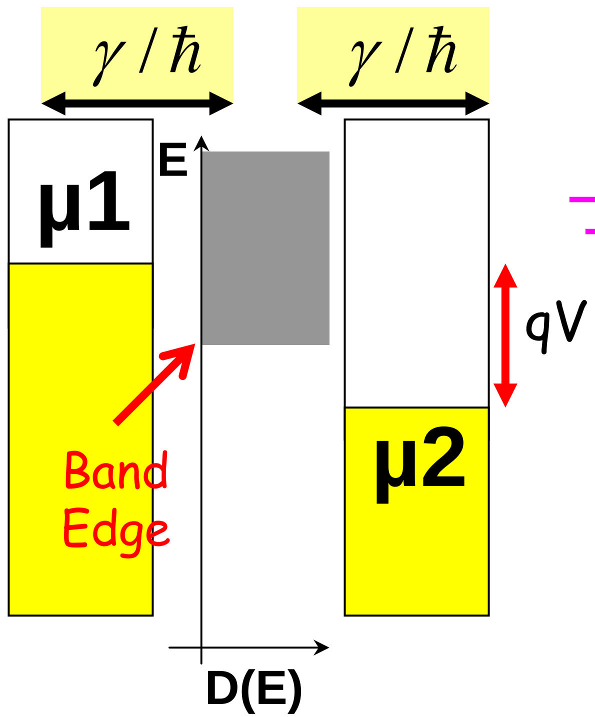
Importance of electrostatics

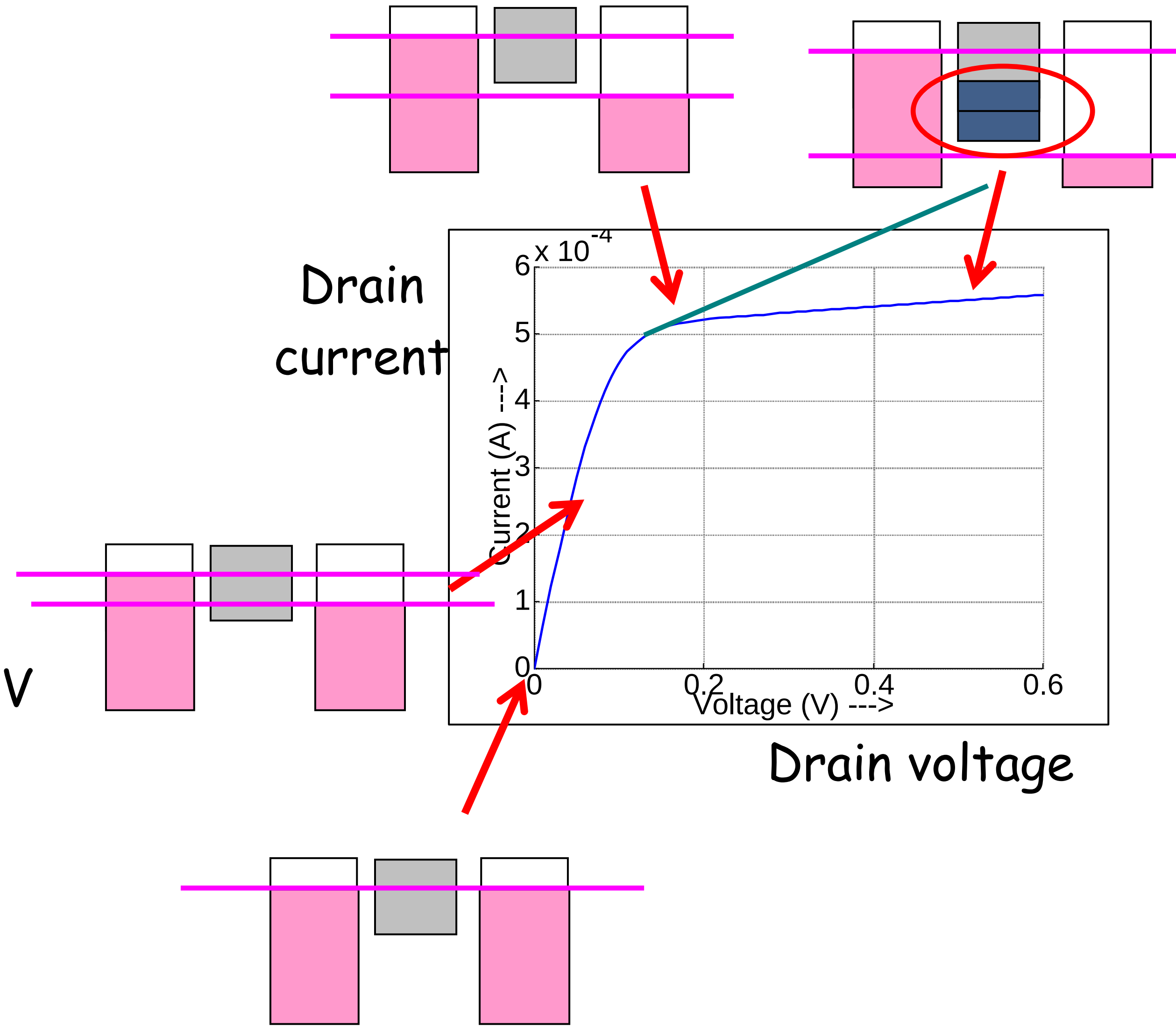
$$I = \frac{q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$



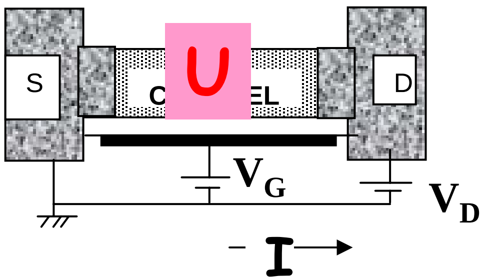
Why does the current in a transistor saturate ?

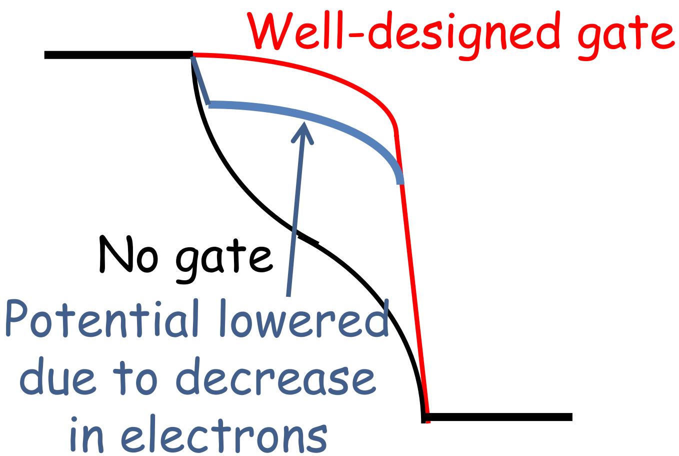






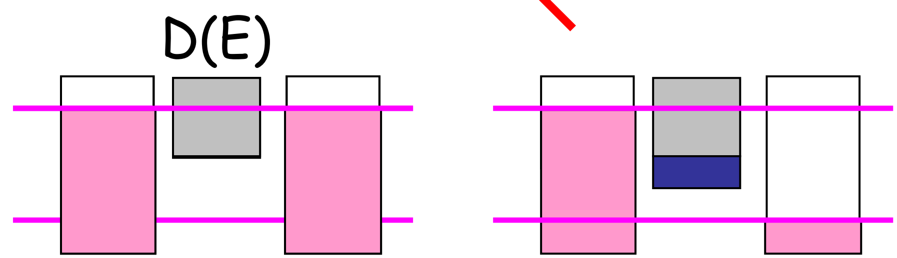
Self-consistent potential





$$n = D(E - \boxed{U})\left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}\right]$$

$$I = \frac{q}{\hbar} D(E - \boxed{U}) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}[f_1 - f_2]$$

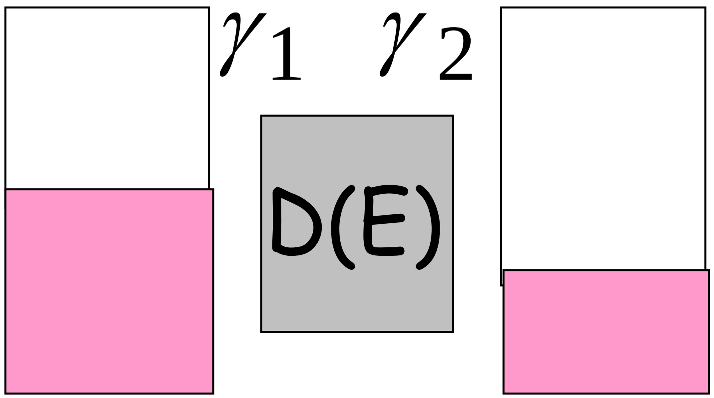


$$\nabla^2 U = 0$$

$$U = U_L + U_0(n - n_0)$$

$$\nabla^2 U \sim \textit{Charge Density}$$

Self-consistent field method

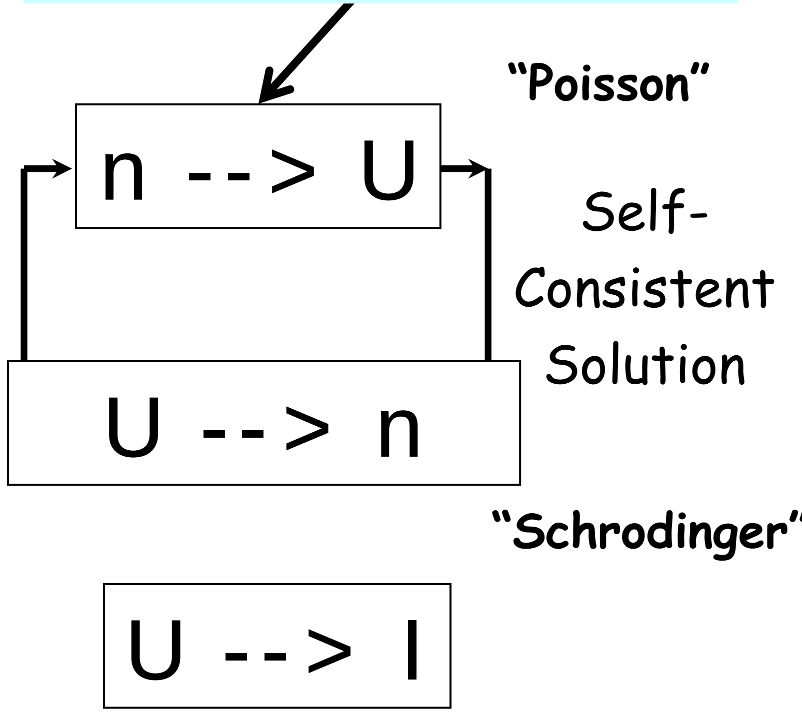


$$U = U_L + U_0 (n - n_0)$$

$$n = D(E-U)\left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}\right]$$

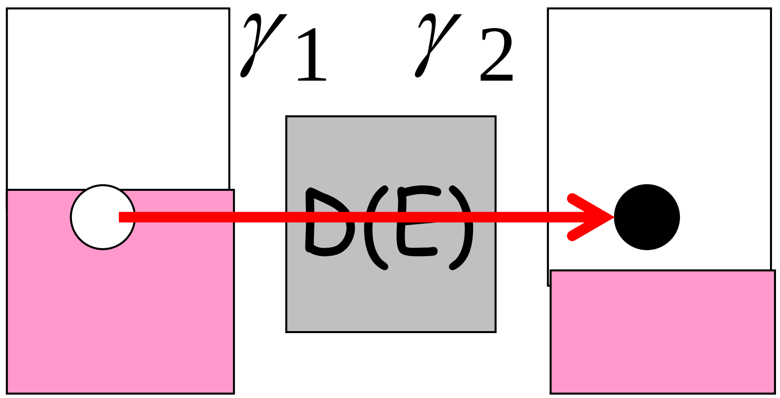
$$I = \frac{q}{\hbar} D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

Simplified treatment
of a very complicated problem



Nanowires / Nanotubes / Molecules

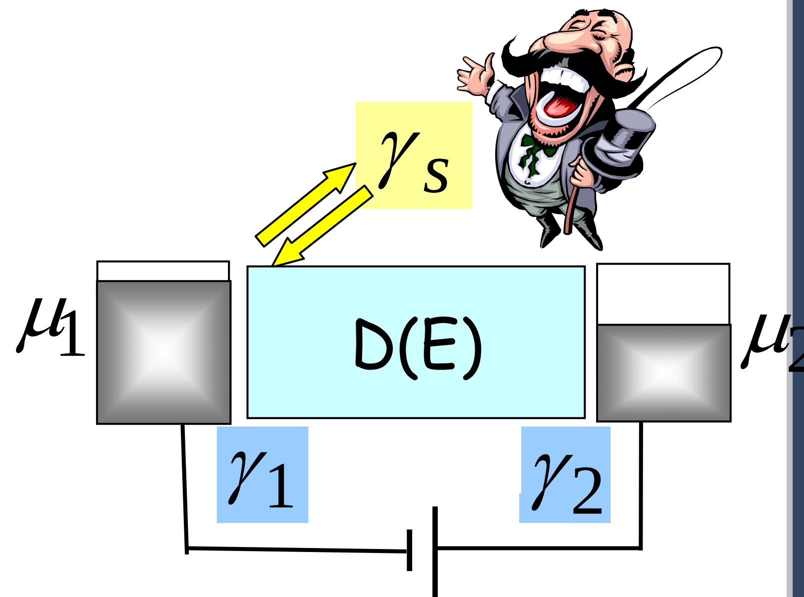
Modeling "demons"



$$U = U_L + U_0(n - n_0)$$

$$n = D(E-U)\left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}\right]$$

$$I = \frac{q}{\hbar} D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

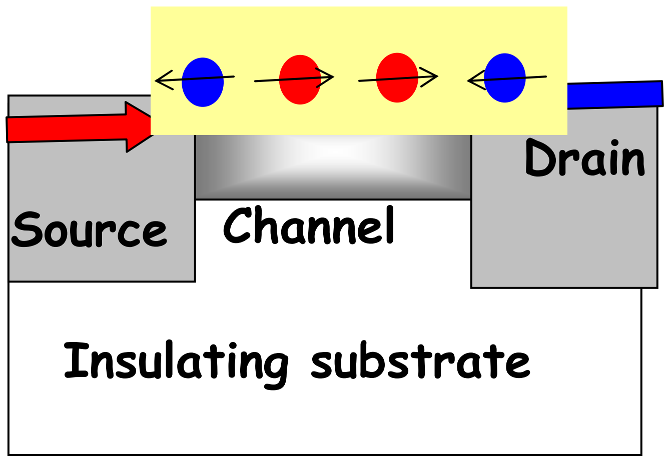


$$I_1 = q \frac{\gamma_1}{\hbar} D(E)[f_1 - f]$$

$$I_2 = q \frac{\gamma_2}{\hbar} D(E)[f - f_2]$$

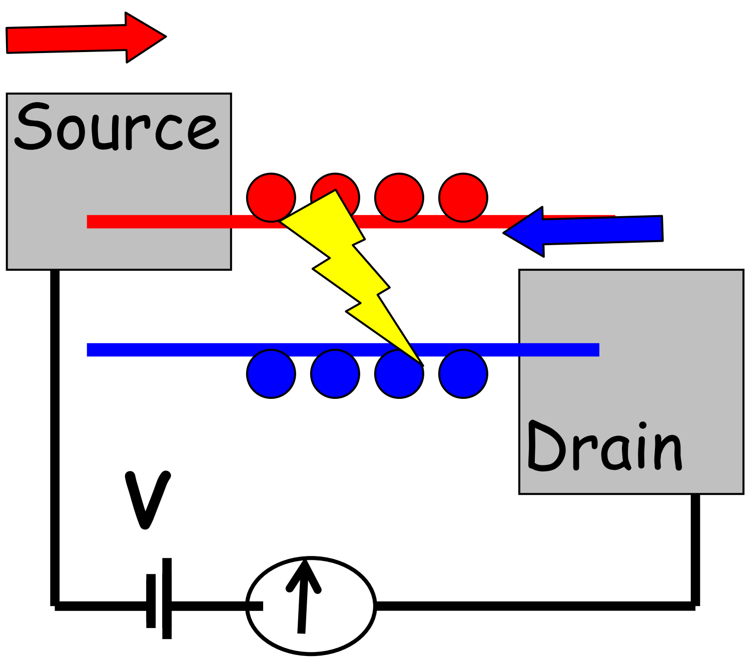
$$I_1 - I_2 = I_s$$

AP Spin Valves



$$I_R = q\frac{\gamma_1}{\hbar}D(E)\left[f_1 - f_R\right] = 0$$

$$I_B = q\frac{\gamma_2}{\hbar}D(E)\left[f_B - f_2\right] = 0$$



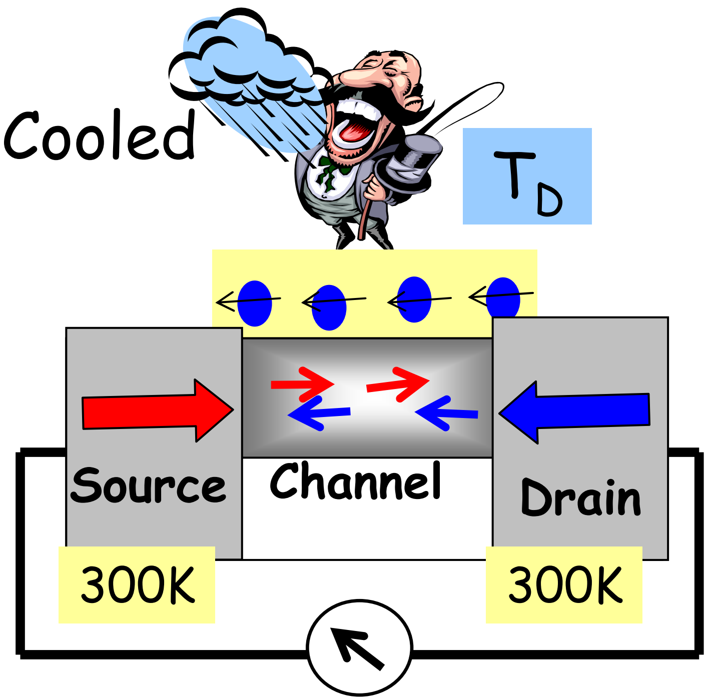
$$I = q\frac{\gamma_1}{\hbar}D(E)\left[f_1 - f_R\right]$$

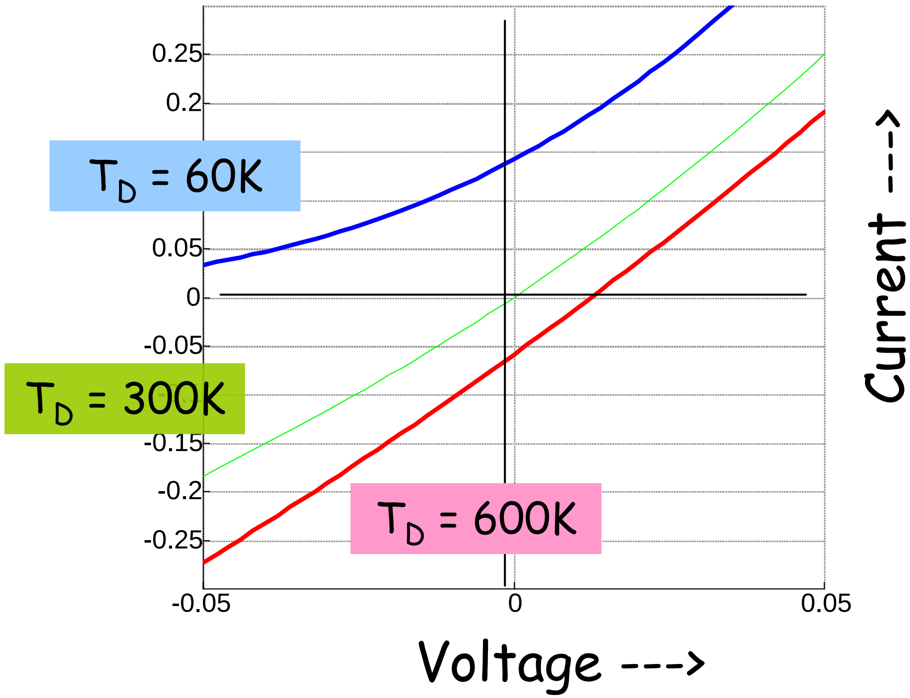
$$= q\frac{\gamma_2}{\hbar}D(E)\left[f_B - f_2\right]$$

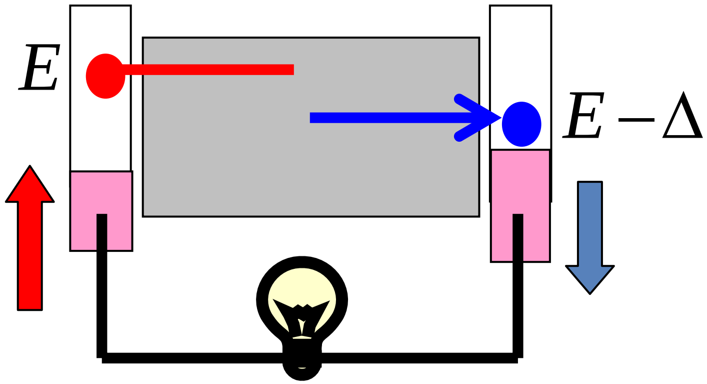
$$= q\frac{\gamma_s}{\hbar}D(E)\left[\begin{array}{l} f_R(1-f_B)B \\ \qquad - f_B(1-f_R)R \end{array}\right]$$

$$I \approx \quad q\frac{\gamma_s}{\hbar}D(E)\left[\begin{array}{l} f_1(1-f_2)B \\ \qquad - f_2(1-f_1)R \end{array}\right]$$

The cool demon as a heat engine

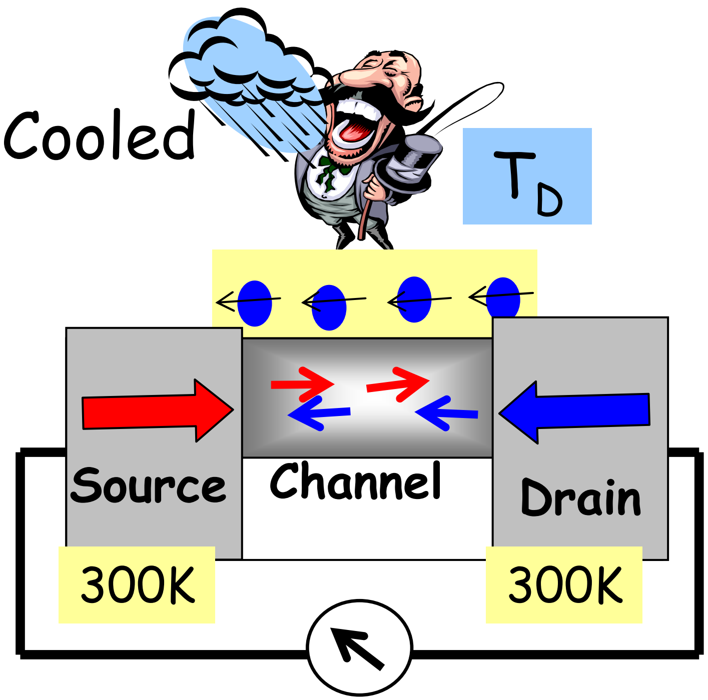


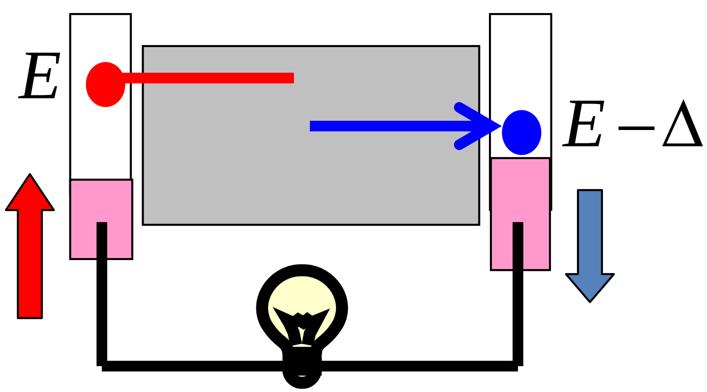




Q_1: heat from contacts
Q_2: heat to demon
Q_1 - Q_2 : useful work

Carnot's principle





$$I \approx q \frac{\gamma_S}{\hbar} D(E) \left[\begin{array}{l} f_1(1-f_2)B \\ \qquad - f_2(1-f_1)R \end{array} \right]$$

$$\frac{R}{B} < \frac{f_1(1-f_2)}{f_2(1-f_1)}$$

$$e^{-\Delta/kT_D} < \frac{e^{(E-\Delta-\mu_2)/kT}}{e^{(E-\mu_1)/kT}}$$

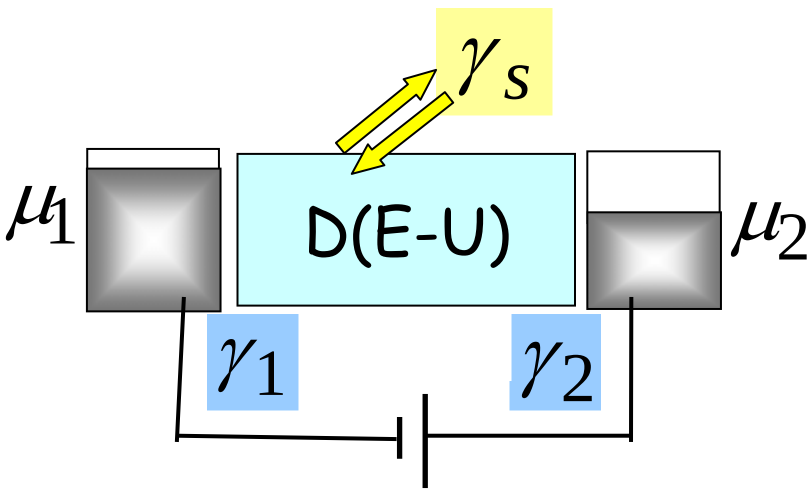
$$\frac{\Delta}{kT_D} > \frac{E-\mu_1}{kT} - \frac{E-\Delta-\mu_2}{kT}$$

$$\frac{Q_2}{kT_D} > \frac{Q_1}{kT}$$

Carnot's principle

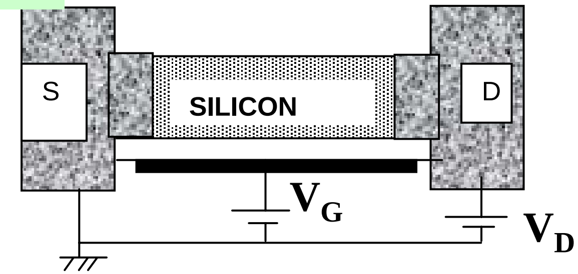
Summary

CQT, Lecture#2:
Electrical Resistance:
A Simple Model

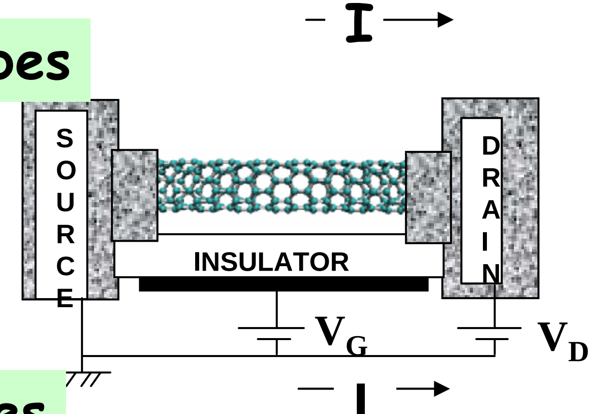


Simple quantitative model
For Transport
Far from Equilibrium

Nanowires



Nanotubes



Molecules