ME 517: Micro- and Nanoscale Processes

Lecture 14: Electrostatics II

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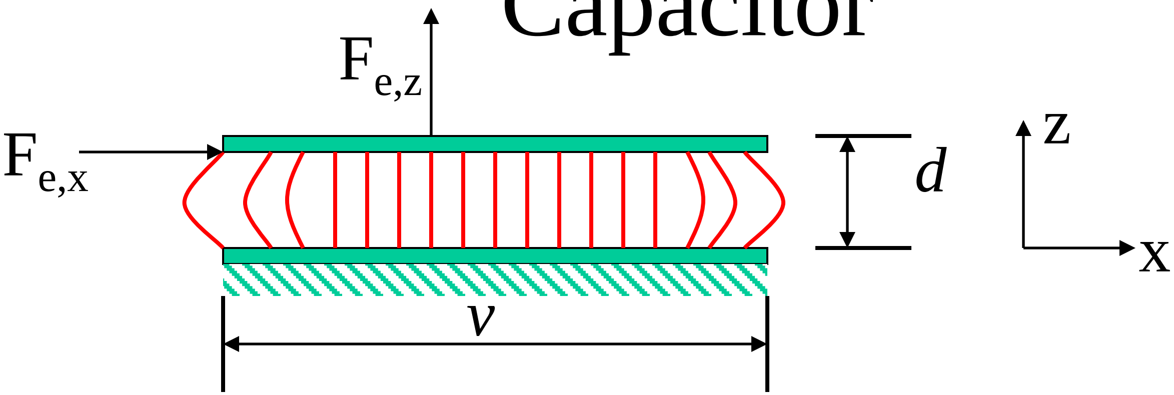
Capacitor



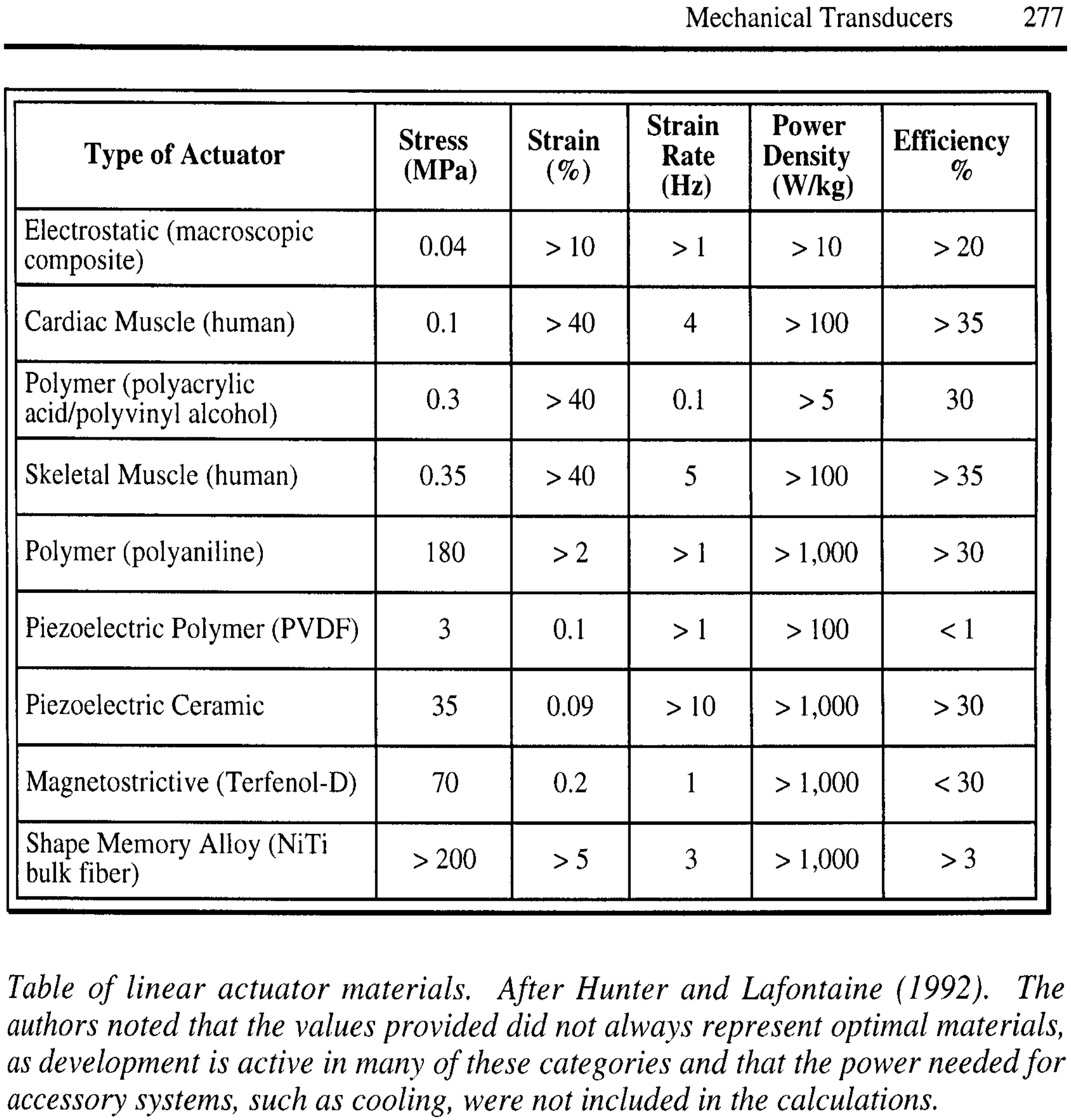
Calculate force exerted on free plate by fixed plate

Force is the gradient of potential so

$$F_{e,z} = \frac{\partial W_e}{\partial z} = \frac{\partial\left(\frac{1}{2}\varepsilon_0\varepsilon_r\frac{vw}{d}V^2\right)}{\partial z} = -\frac{1}{2}\varepsilon_0\varepsilon_r\frac{vw}{d^2}V^2$$

$$F_{e,x} = \frac{\partial W_e}{\partial x} = \frac{\partial\left(\frac{1}{2}\varepsilon_0\varepsilon_r\frac{(v-x)w}{d}V^2\right)}{\partial x} = -\frac{1}{2}\varepsilon_0\varepsilon_r\frac{w}{d}V^2$$

Comparison of Actuators



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Type of Actuator	Stress (MPa)	Strain (%)	Strain Rate (Hz)	Power Density (W/kg)	Efficiency %
Electrostatic (macroscopic composite)	0.04	> 10	> 1	> 10	> 20
Cardiac Muscle (human)	0.1	> 40	4	> 100	> 35
Polymer (polyacrylic acid/polyvinyl alcohol)	0.3	> 40	0.1	> 5	30
Skeletal Muscle (human)	0.35	> 40	5	> 100	> 35
Polymer (polyaniline)	180	> 2	> 1	> 1,000	> 30
Piezoelectric Polymer (PVDF)	3	0.1	> 1	> 100	< 1
Piezoelectric Ceramic	35	0.09	> 10	> 1,000	> 30
Magnetostrictive (Terfenol-D)	70	0.2	1	> 1,000	< 30
Shape Memory Alloy (NiTi bulk fiber)	> 200	> 5	3	> 1,000	> 3

Table of linear actuator materials. After Hunter and Lafontaine (1992). The authors noted that the values provided did not always represent optimal materials, as development is active in many of these categories and that the power needed for accessory systems, such as cooling, were not included in the calculations.

Electrostatics Scaling

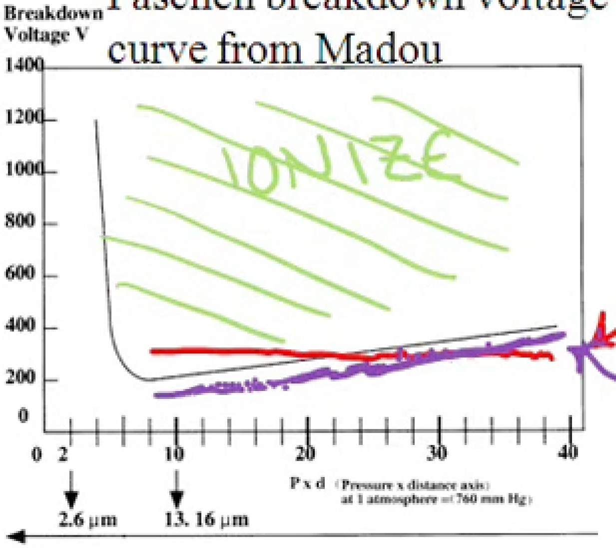
Energy Storage

$$W_e = \frac{1}{2}\varepsilon_0\varepsilon_r\frac{vw}{d}V^2$$

Force Generation

$$F_{i,e} = \frac{\partial W_e}{\partial x_i} = -\varepsilon_0\varepsilon_r\frac{w}{d}V^2$$

Paschen breakdown voltage curve from Madou



Two modes of operation:

- Constant voltage: $V \propto L^0$
- Maximum voltage: $V \propto L^1$

- Constant Voltage: $W_e \propto L^1$, $F \propto L^0$
- Maximum Voltage: $W_e \propto L^3$, $F \propto L^2$
- Even bigger gains possible with very small gaps

Electrostatic Cantilever

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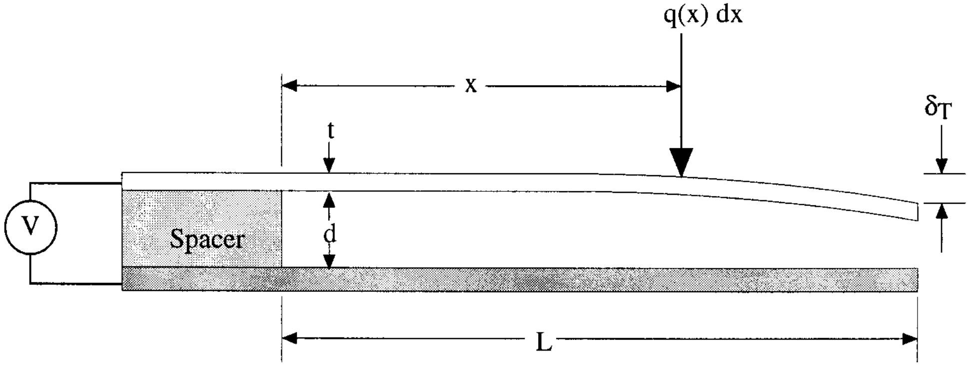
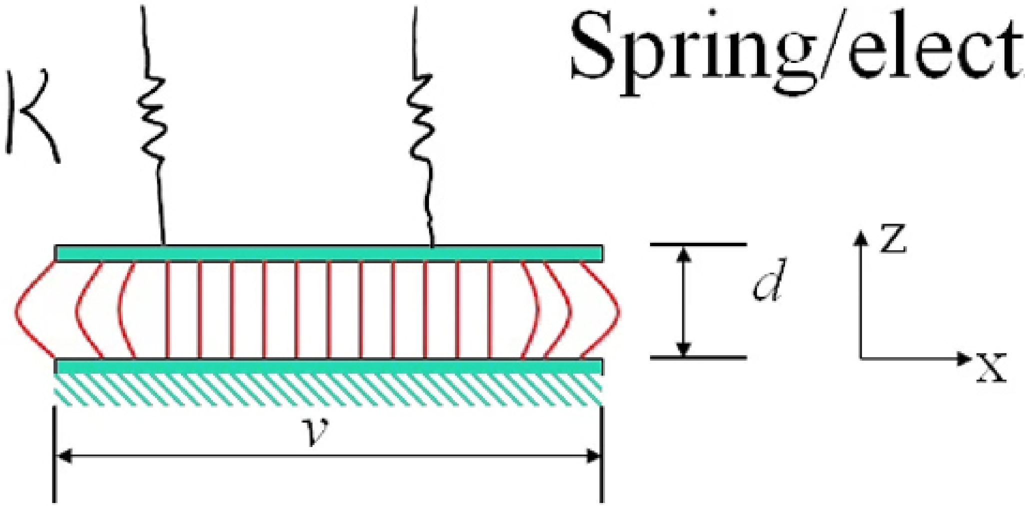


Illustration of an electrostatically deflected cantilever structure showing variable definitions for analysis. After Petersen (1978a).

- cantilever suspended above substrate forming parallel plate capacitor *in a vacuum*
- Analysis requires Euler-Bernoulli Beam Flexure Formula and electrostatics

Petersen, K.E., “Dynamic Micromechanics on Silicon: Techniques and Devices,” IEEE Trans. On Electron Devices, vol. ED-25, no. 10, Oct. 1978, p. 1241-1250.

Spring/electrode model



$F_s = K(d-z)$

$F_e = \frac{1}{2}\varepsilon_0\varepsilon_r\frac{vW}{z^2}V^2$

$F_s = F_e$

$K(d-z) = \frac{1}{2}\varepsilon_0\varepsilon_r\frac{vW}{z^2}V^2$