

# ME 517: Micro- and Nanoscale Processes

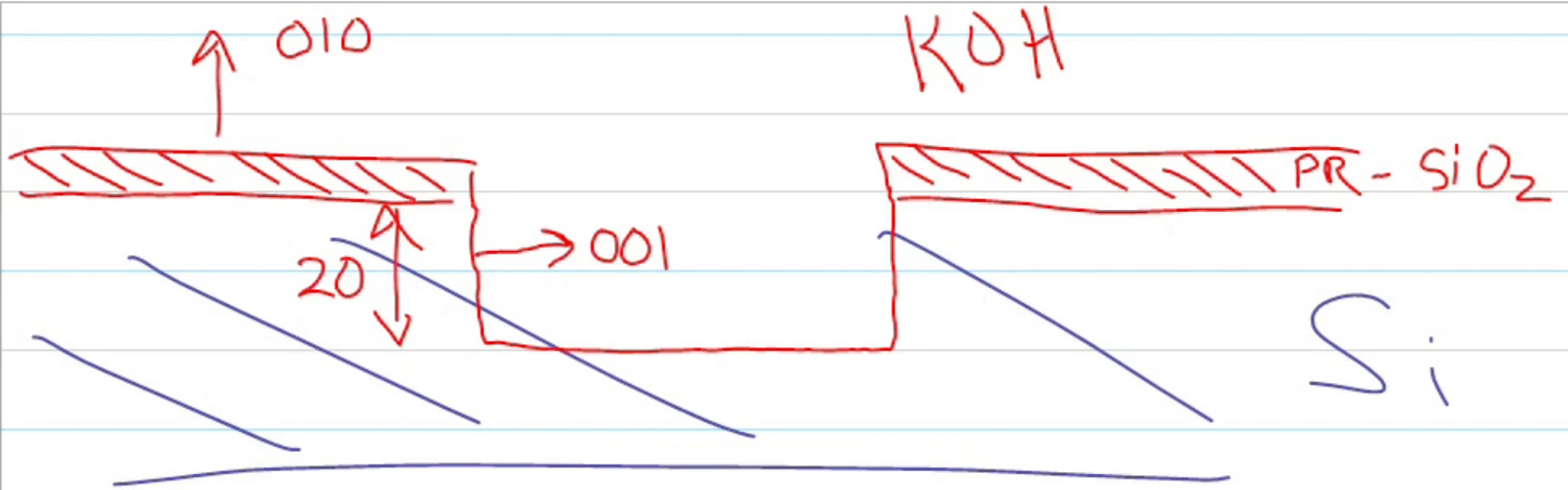
## Lecture 15: Electrostatics III

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$\leftarrow 100 \rightarrow$

Title



# Electrostatic Cantilever

Mechanical Transducers

279

← Kovacs

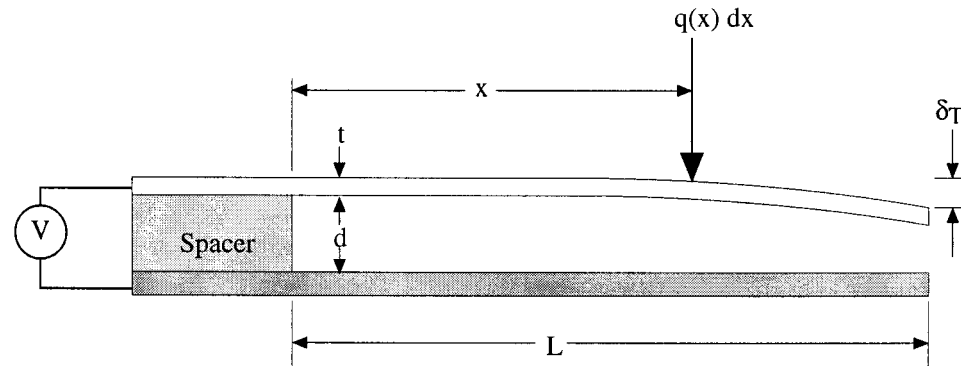
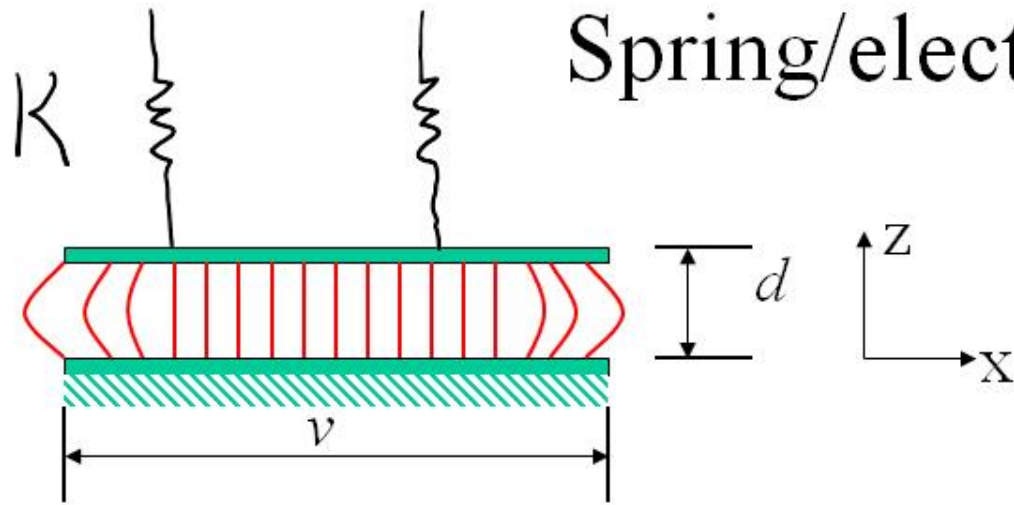


Illustration of an electrostatically deflected cantilever structure showing variable definitions for analysis. After Petersen (1978a).

- cantilever suspended above substrate forming parallel plate capacitor *in a vacuum*
- Analysis requires Euler-Bernoulli Beam Flexure Formula and electrostatics

Petersen, K.E., “Dynamic Micromechanics on Silicon: Techniques and Devices,” IEEE Trans. On Electron Devices, vol. ED-25, no. 10, Oct. 1978, p. 1241-1250.

# Spring/electrode model

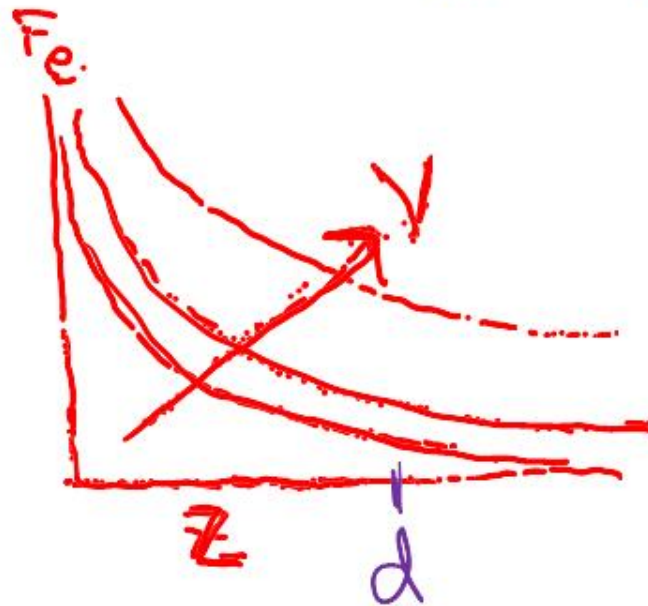
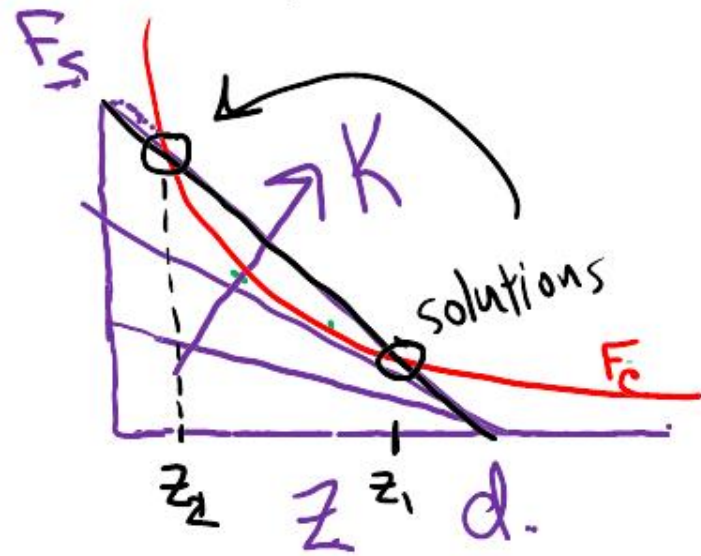


$$F_s = K(d - z)$$

$$F_e = \frac{1}{2} \epsilon_0 \epsilon_r \frac{VW}{z^2} V^2$$

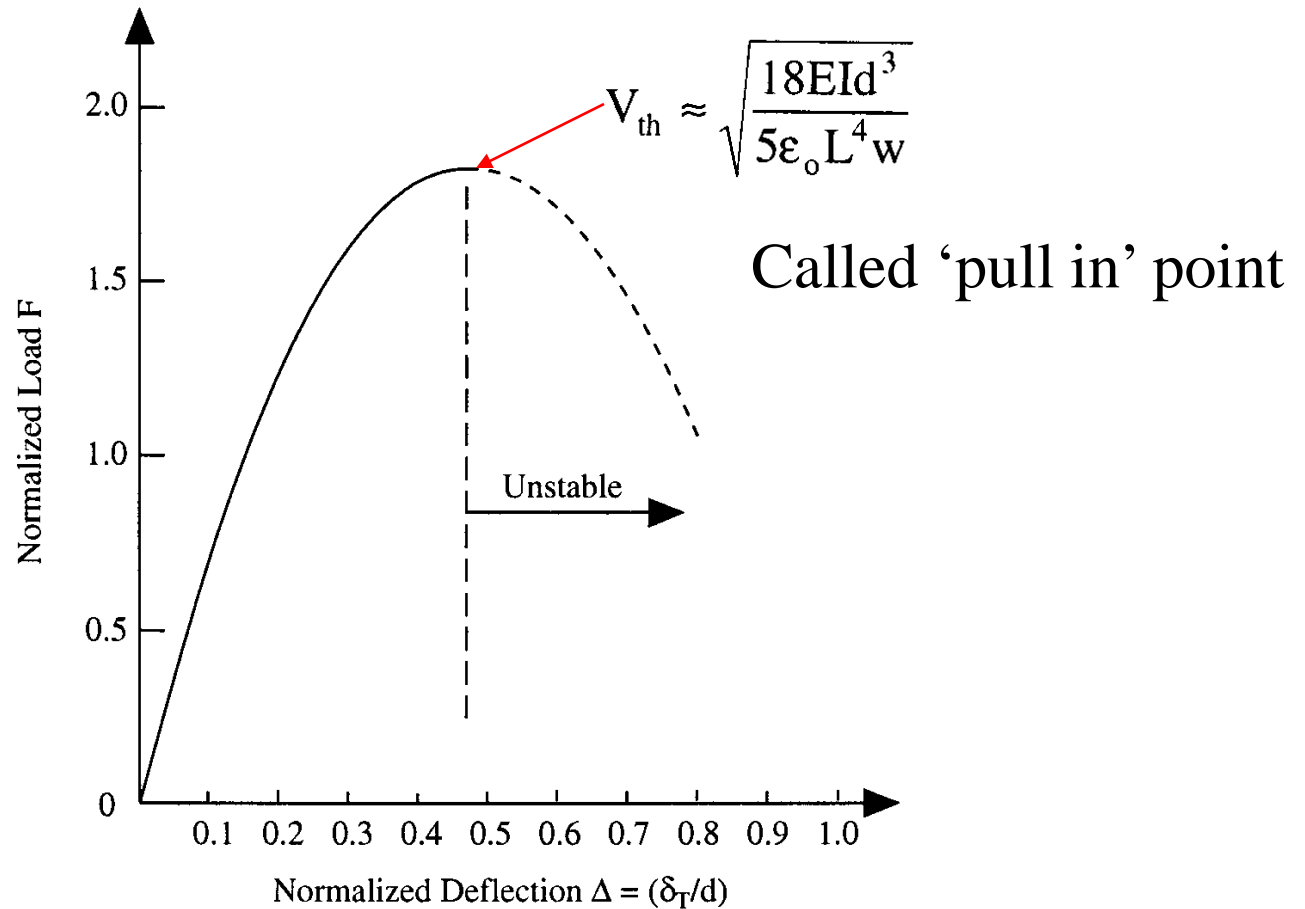
$$F_s = F_e$$

$$K(d - z) = \frac{1}{2} \epsilon_0 \epsilon_r \frac{VW}{z^2} V^2$$



"snap in"  
"pull in"

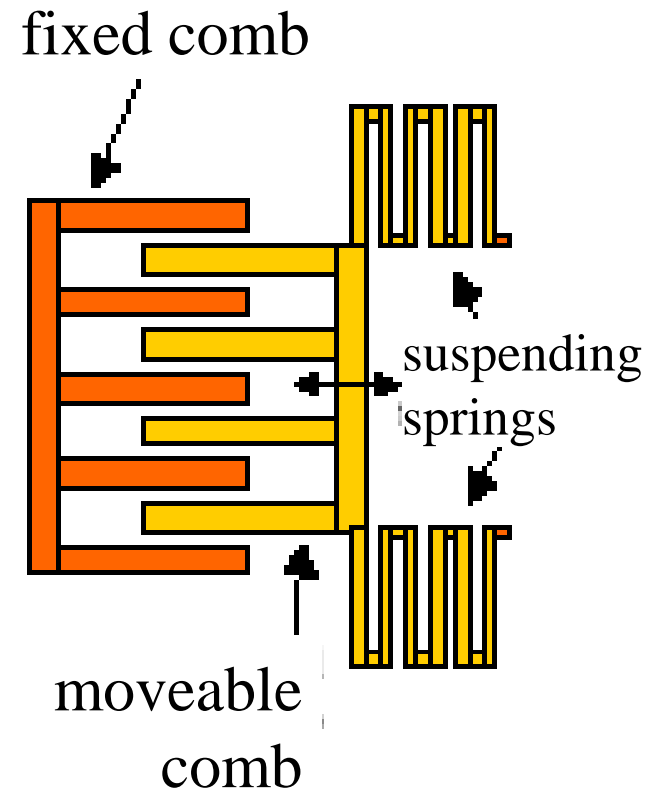
# Electrostatic Cantilever



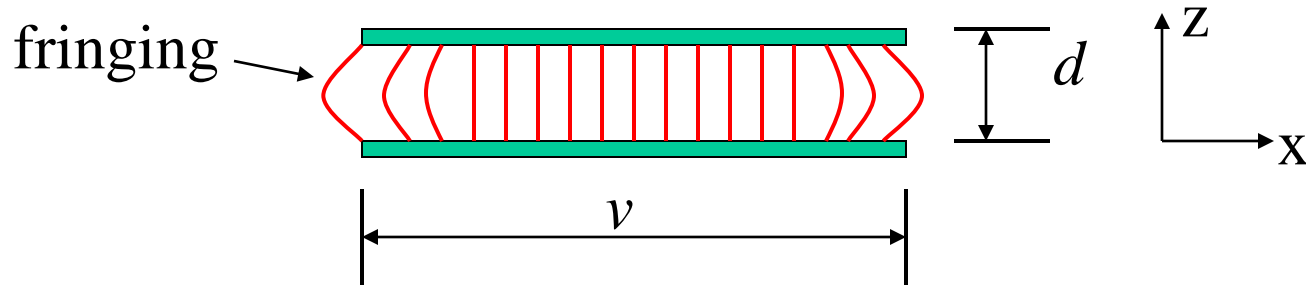
*Plot showing the normalized load,  $F$ , versus the normalized deflection,  $\Delta$ , for an electrostatically deflected cantilever. Adapted from Petersen (1978a). Beyond a certain threshold deflection, spontaneous collapse of the cantilever occurs.*

# Electrostatic Comb Drives

- Interlacing comb fingers create large effective capacitor area (Tang, Nguyen, Howe, 1989)
- Salient features:
  - linear relationship between capacitance and displacement
  - interlacing fingers provide much larger surface area than parallel plate capacitor
  - electrostatic actuation—low power consumption because no current



# Capacitor

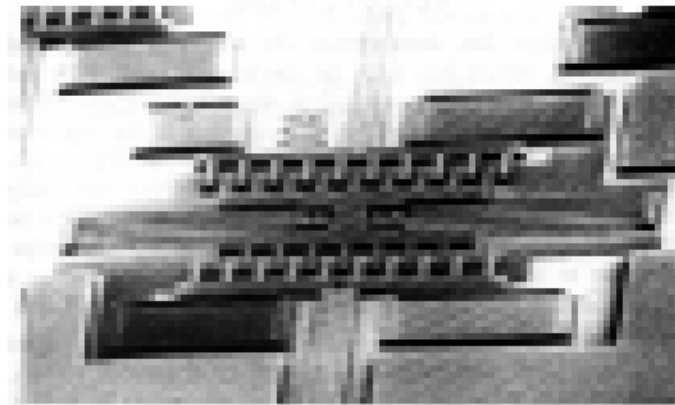
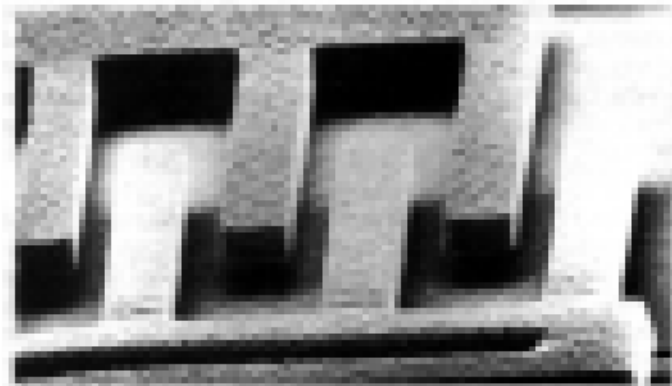
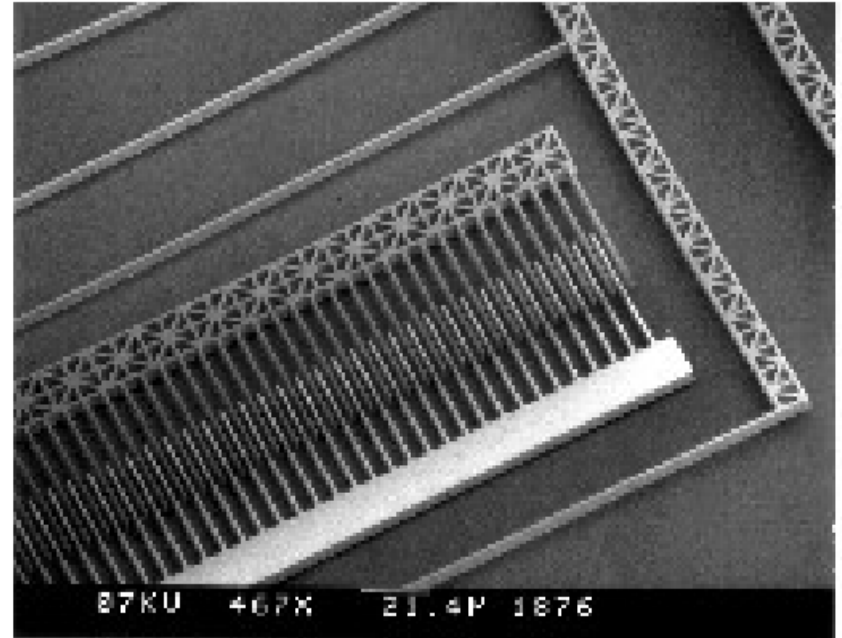


- Simplest geometry—parallel plate above
- Assuming that  $d$  is much smaller than  $v$  or  $w$ , fringing will not play a major role—assume that the field is uniform.

$$\begin{aligned} W_e &= \frac{1}{2} \int_V \rho V dv = \frac{1}{2} \int_V \epsilon_0 \epsilon_r \left( \frac{V}{d} \right)^2 dv \\ &= \frac{1}{2} \epsilon_0 \epsilon_r \left( \frac{V}{d} \right)^2 v w d = \frac{1}{2} \epsilon_0 \epsilon_r \frac{v w}{d} V^2 \end{aligned}$$

# Comb Drive

- How are these structures produced?





# Comb Drive

Capacitance approximately

$$C = \epsilon_0 \epsilon_r A/d$$
$$= 2n \epsilon_0 \epsilon_r lh/d$$

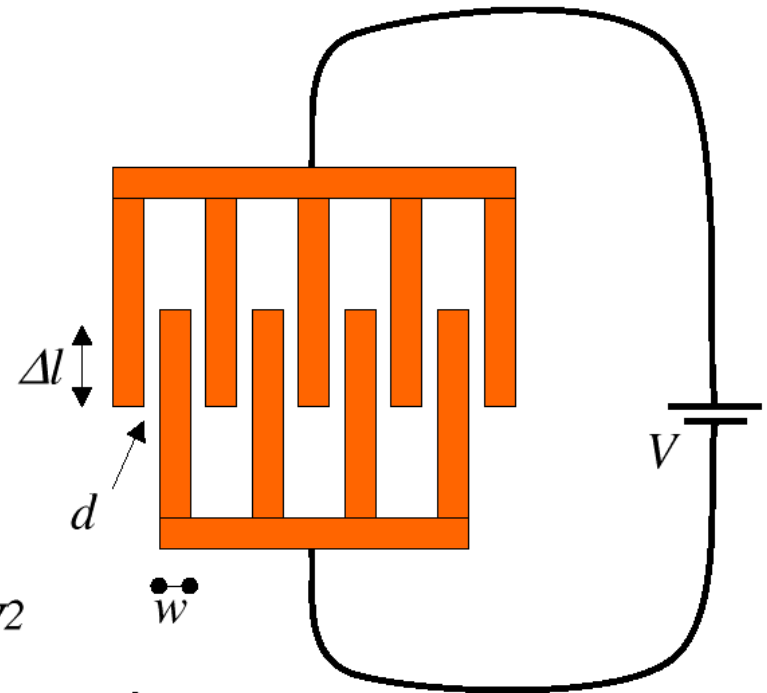
Change in capacitance when moving by  $\Delta x$ :

$$\Delta C = \epsilon_0 \epsilon_r \Delta A / d$$
$$= 2n \epsilon_0 \epsilon_r \Delta l h/d$$

Electrostatic force:

$$F_{el} = 1/2 V^2 dC/dx = n \epsilon_0 \epsilon_r h/d V^2$$

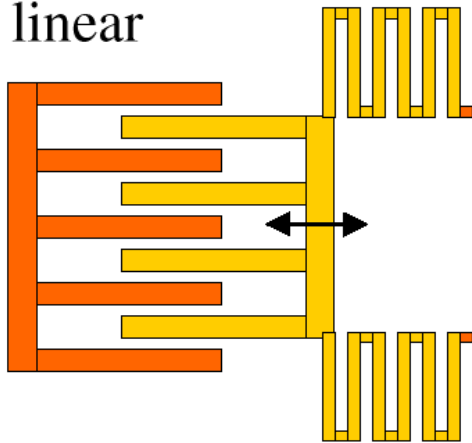
Note:  $F_{el}$  independent of  $\Delta l$  over wide range (fringing field), and quadratic in  $V$ .



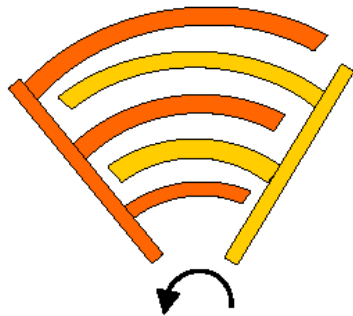
# Comb Drive Design

## Combs

linear

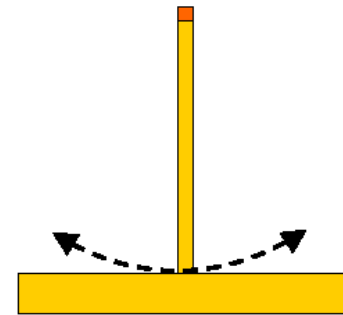


rotational

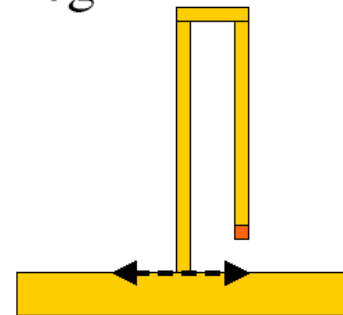


## Suspensions

cantilever / bridge



crab leg

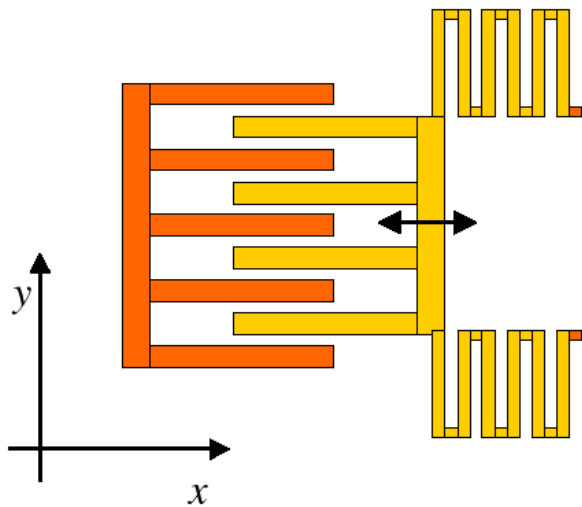


# Comb Drive Failure Modes

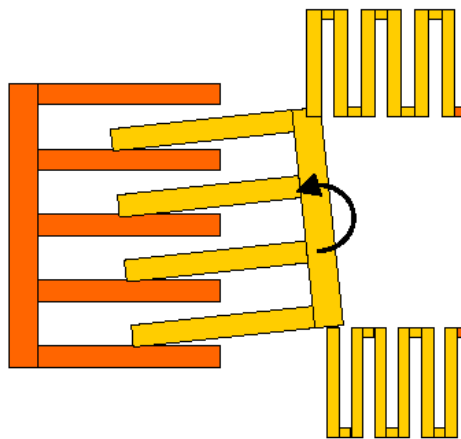
Comb drives require low stiffness in  $x$  direction but high stiffness in  $y, z$  direction as well as rotations.

Note: comb fingers are in unstable equilibrium w.r.t.  $y$  direction.

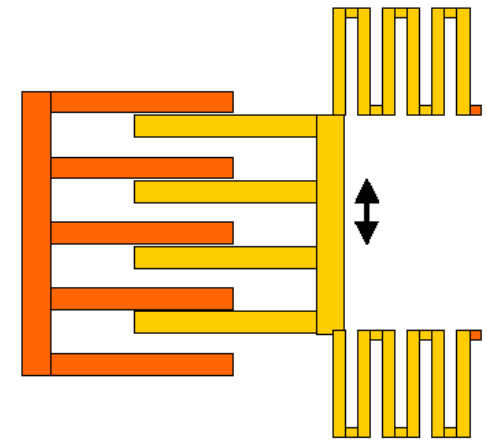
good



bad



bad



# The RF-MEMS Switch:

- If you apply a DC voltage so that both ends of the switch are charged differently you create an electrostatic force. This force causes an attraction between the two areas.

