Stokes Flows

- Assume we’re considering a water flow
  - $\mu = 10^{-3} \text{ kg/(s m)}$, $\rho = 10^3 \text{ kg/m}^3$
- Length scale: $10 \mu\text{m} = 10^{-5} \text{ m}$
- Velocity scale: $10 \text{ mm/s} = 10^{-2} \text{ m/s}$
- Then: $Re = 10^{-1}$, so N-S eqn

\[
0 = -\nabla p' + \nabla^2 \mathbf{V}'
\]

becomes Poisson Eqn (Stokes Flow), simple, linear equation
Boundary Conditions

Generalized slip flow model for liquid or gas: \( \Delta u \bigg|_{wall} = u_{\text{fluid}} - u_{\text{wall}} = L_s \frac{\partial u}{\partial y} \bigg|_{wall} \)

Slip flow model strictly for ideal gases: \( u_{\text{gas}} - u_{\text{wall}} = \lambda \frac{2 - \sigma_v}{\sigma_v} \frac{\partial u}{\partial y} \bigg|_{wall} + \frac{3}{4} \frac{\mu}{\rho T_{\text{gas}}} \frac{\partial T}{\partial x} \bigg|_{wall} \)

Dimensionless Slip flow model for ideal gas:
\[
\frac{u_{\text{gas}}^* - u_{\text{wall}}^*}{Kn} = \frac{2 - \sigma_v}{\sigma_v} \frac{\partial u^*}{\partial y^*} \bigg|_{wall} + \frac{3}{2\pi} \frac{\gamma - 1}{\gamma} \frac{Kn^2}{Ec} \frac{\partial T^*}{\partial x^*} \bigg|_{wall}
\]
**Gas Accommodation Coefficients**

Accommodation coefficients are macro/molecular quantities that depend on:
- gas type
- surface material
- surface topology

<table>
<thead>
<tr>
<th>Gas</th>
<th>Metal</th>
<th>$\sigma_v$</th>
<th>$\sigma_T$</th>
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<tr>
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<tr>
<td>Ar</td>
<td>Silicon</td>
<td>0.80-0.90</td>
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<tr>
<td>N₂</td>
<td>Silicon</td>
<td>0.80-0.85</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Beskok, 2002

![Graph](image)

**FIGURE 2.3.** Tangential momentum accommodation coefficient $\sigma_v$ (TMAC) versus Knudsen number obtained from mass flowrate measurements for argon (left) and for nitrogen (right) (see details in (Arkilic, 1997)). (Courtesy of K. Breuer)
Continuum incompressible flow through long channels

No-slip

(White, 1991)

\[ u = \frac{-d\hat{p}}{dx} \left( \frac{r_0^2 - r^2}{4\mu} \right) \]

\[ Q_{\text{pipe}} = \frac{\pi r_0^4}{8\mu} \left( -\frac{d\hat{p}}{dx} \right) \]

\[ u(y, z) = \frac{16a^2}{\mu \pi^3} \left( -\frac{d\hat{p}}{dx} \right) \sum_{i=1,3,5,\ldots}^\infty (-1)^{(i-1)/2} \left[ 1 - \frac{\cosh(i\pi z/2a)}{\cosh(i\pi b/2a)} \right] \times \frac{\cos(i\pi y/2a)}{i^3} \]

\[ Q = \frac{4ba^3}{3\mu} \left( -\frac{d\hat{p}}{dx} \right) \frac{1 - 192a}{\pi^5b} \sum_{i=1,3,5,\ldots}^\infty \frac{\tanh(i\pi b/2a)}{i^5} \]

Micro/Nanoscale Physical Processes