ME 517: Micro- and Nanoscale Processes

Lecture 20: Microfluidics - Stokes Flows

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Stokes Flows

- Assume we're considering a water flow
  - $\mu=10^{-3}$ kg/(s m), $\rho=10^{3}$ kg/m^{3}
- Length scale: 10 μm=10^{-5} m
- Velocity scale: 10 mm/s=10^{-2} m/s
- Then: Re=10^{-1}, so N-S eqn

$$0=-\nabla p' + \nabla^2 \vec{V}'$$

$$0 = -\nabla p + \mu \nabla^2 \vec{V}$$

- becomes Poisson Eqn (Stokes Flow), simple, linear equation

Boundary Conditions

Generalized slip flow model for ***liquid or gas***: Navier 1836

$$\Delta u\big|_{wall} = u_{fluid} - u_{wall} = L_s \frac{\partial u}{\partial y}\bigg|_{wall}$$

TW

Slip flow model ***strictly for ideal gases***: Maxwell ~1886

$$u_{gas} - u_{wall} = \lambda \frac{2-\sigma_v}{\sigma_v} \frac{\partial u}{\partial y}\bigg|_{wall} + \frac{3}{4} \frac{\mu}{\rho T_{gas}} \frac{\partial T}{\partial x}\bigg|_{wall}$$

Dimensionless Slip flow model for ideal gas:

$$u^*_{gas} - u^*_{wall} = Kn \frac{2-\sigma_v}{\sigma_v} \frac{\partial u^*}{\partial y^*}\bigg|_{wall} + \frac{3}{2\pi} \frac{\gamma - 1}{\gamma} \frac{Kn^2 \,\mathrm{Re}}{Ec} \frac{\partial T^*}{\partial x^*}\bigg|_{wall}$$

Gas Accommodation Coefficients

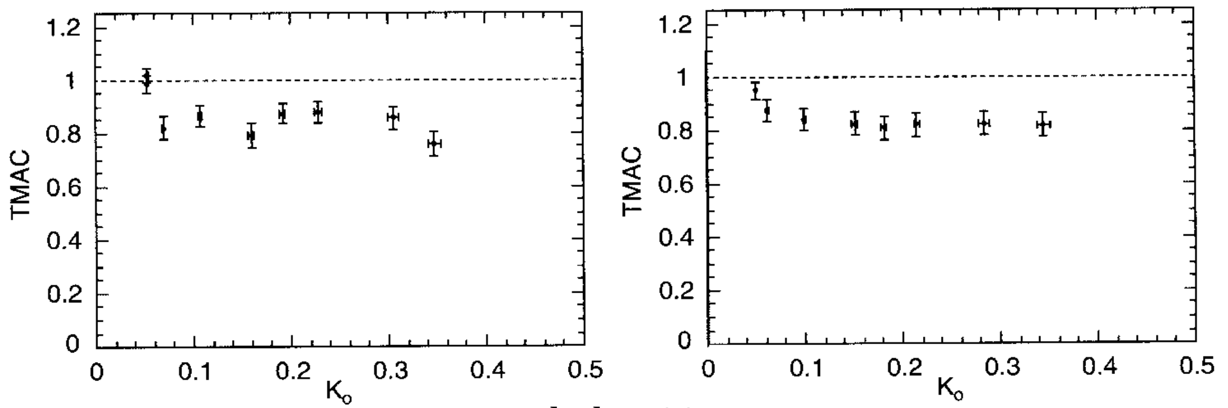
Accommodation coefficients are macro/molecular quantities that depend on

- gas type
- surface material
- surface topology

Gas	*Metal*	σ_V	σ_T
Air	Aluminum	0.87-0.97	0.87-0.97
Air	Iron	0.87-0.96	0.87-0.93
Air	Bronze	0.88-0.95	n/a
Ar	Silicon	0.80-0.90	n/a
N_2	Silicon	0.80-0.85	n/a

$$\sigma_v = \frac{\tau_i - \tau_r}{\tau_i - \tau_w}$$

$$\sigma_T = \frac{dE_i - dE_r}{dE_i - dE_w}$$

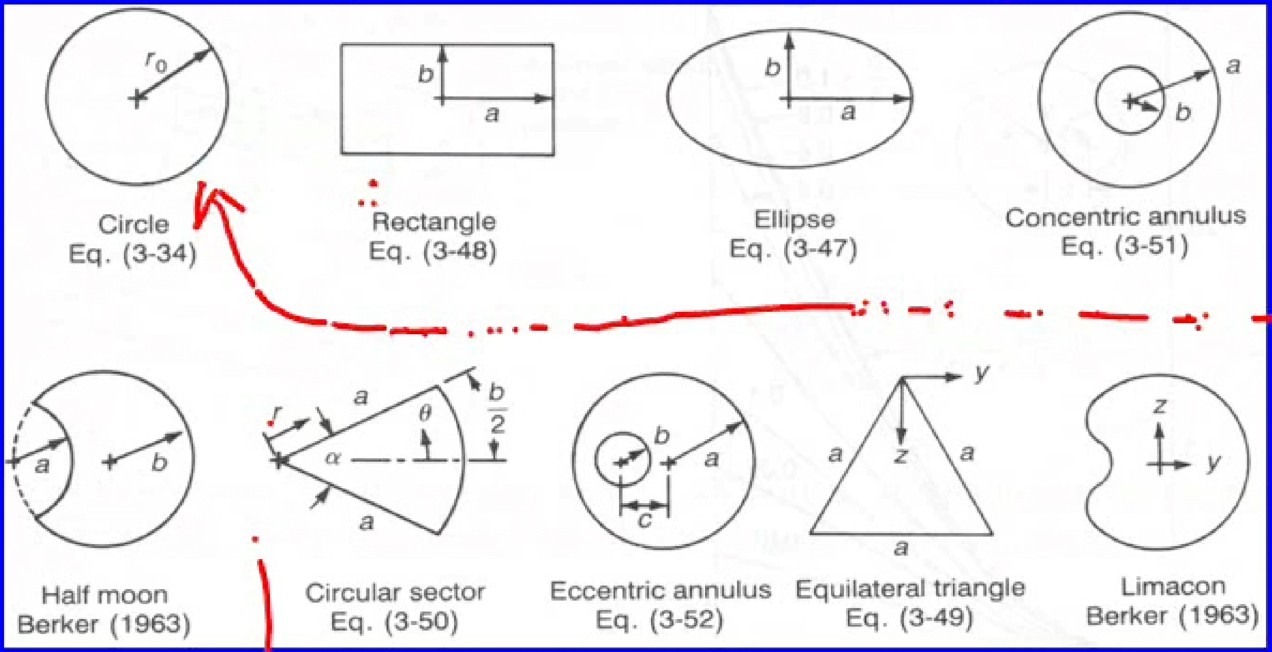


Beskok, 2002

FIGURE 2.3. Tangential momentum accommodation coefficient σ_v (TMAC) versus Knudsen number obtained from mass flowrate measurements for argon (left) and for nitrogen (right) (see details in (Arkilic, 1997)). (Courtesy of K. Breuer)

Continuum incompressible flow through long channels (White, 1991)

No - slip



$$u = \frac{-d\hat{p}/dx}{4\mu}\left(r_0^2 - r^2\right)$$

$$Q_{\text{pipe}} = \frac{\pi r_0^4}{8\mu}\left(-\frac{d\hat{p}}{dx}\right)$$

$$u(y, z) = \frac{16a^2}{\mu\pi^3}\left(-\frac{d\hat{p}}{dx}\right)\sum_{i=1,3,5,\ldots}^{\infty}(-1)^{(i-1)/2}\left[1 - \frac{\cosh(i\pi z/2a)}{\cosh(i\pi b/2a)}\right] \times \frac{\cos(i\pi y/2a)}{i^3}$$

rect

$$Q = \frac{4ba^3}{3\mu}\left(-\frac{d\hat{p}}{dx}\right)\left[1 - \frac{192a}{\pi^5 b}\sum_{i=1,3,5,\ldots}^{\infty}\frac{\tanh(i\pi b/2a)}{i^5}\right]$$

$$u(y, z) = \frac{-d\hat{p}/dx}{2\sqrt{3}\,a\mu}\left(z - \frac{1}{2}a\sqrt{3}\right)(3y^2 - z^2)$$

$$Q = \frac{a^4\sqrt{3}}{320\mu}\left(-\frac{d\hat{p}}{dx}\right)$$

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