Rarefied Micro/Nanoflows: a brief tour

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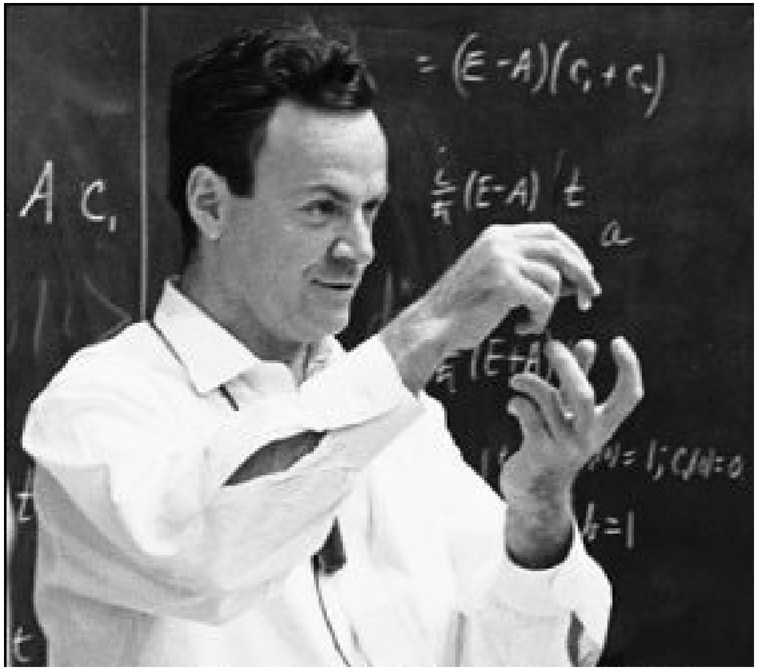
All Science in 1 Sentence



Richard Feynmann,
***Six Easy Pieces*, Ch. 1**

I believe it is the atomic hypothesis that ***All things are made of atoms-little particles that that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.”***

Boltzmann Kinetic Equation

Kinetic Theory of Gases

Molecular Hypothesis

Classical Conservation Laws

Statistics

$$f = f(t, \vec{x}, \vec{u})$$

Ludwig Botlzmann, 1844-1906

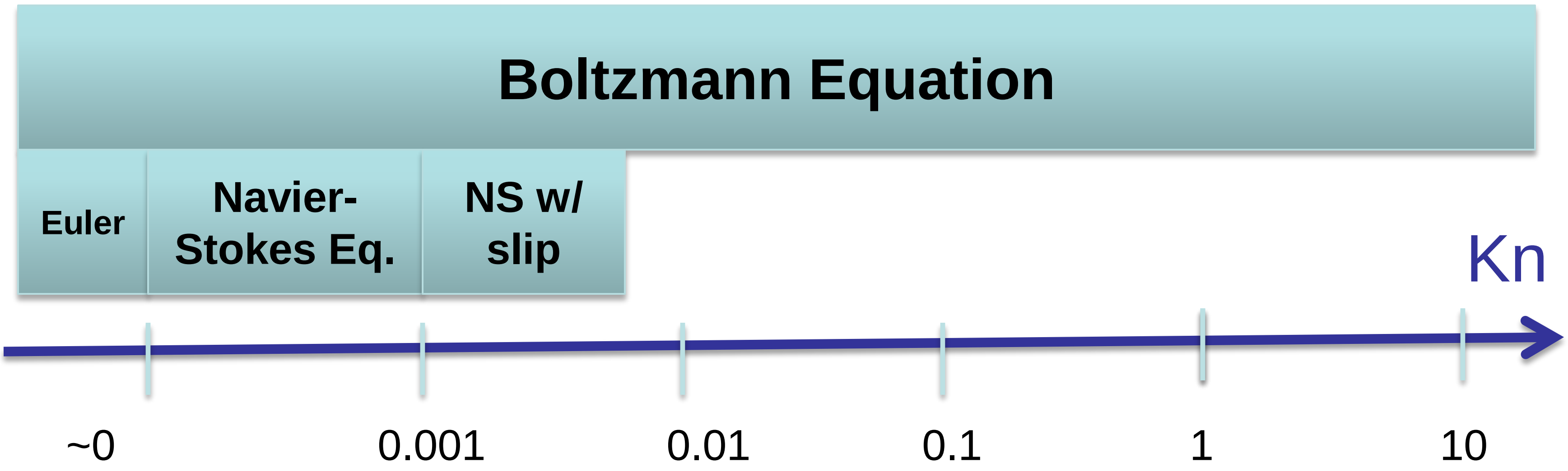
$$\frac{\partial (nf)}{\partial t} + \vec{u} \cdot \nabla_x (nf) + \frac{\vec{F}}{m} \cdot \nabla_v (nf) = n^2 \int \int_0^{4\pi} (f^* f_1^* - f f_1) \, | \vec{u}_r | \frac{d\sigma_T}{d\Omega} d\Omega d\vec{u}_1$$

Molecular Interaction Law

Purdue course **AAE590 Molecular Gas Dynamics** (Fall 2014; offered online)

Flow Rarefaction Regimes

Knudsen number: $Kn = \frac{\lambda}{L}$



Boltzmann Equation

Euler

Navier-Stokes Eq.

NS w/ slip

Kn

~0

0.001

0.01

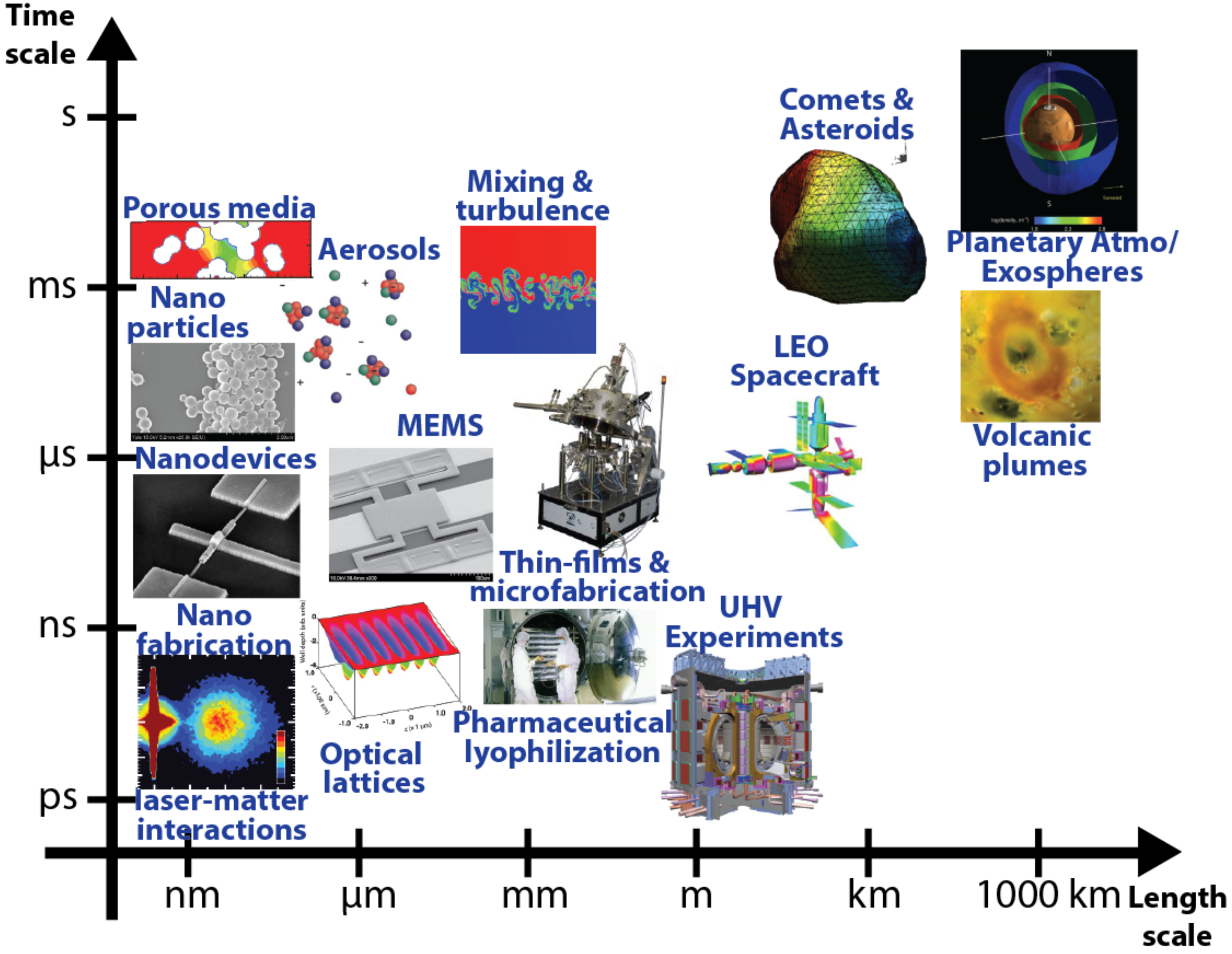
0.1

1

10

Mean free path calculator **Gas Dynamics Toolbox**:
http://web.ics.purdue.edu/~alexeenk/GDT/

Applications of Rarefied Gas Dynamics

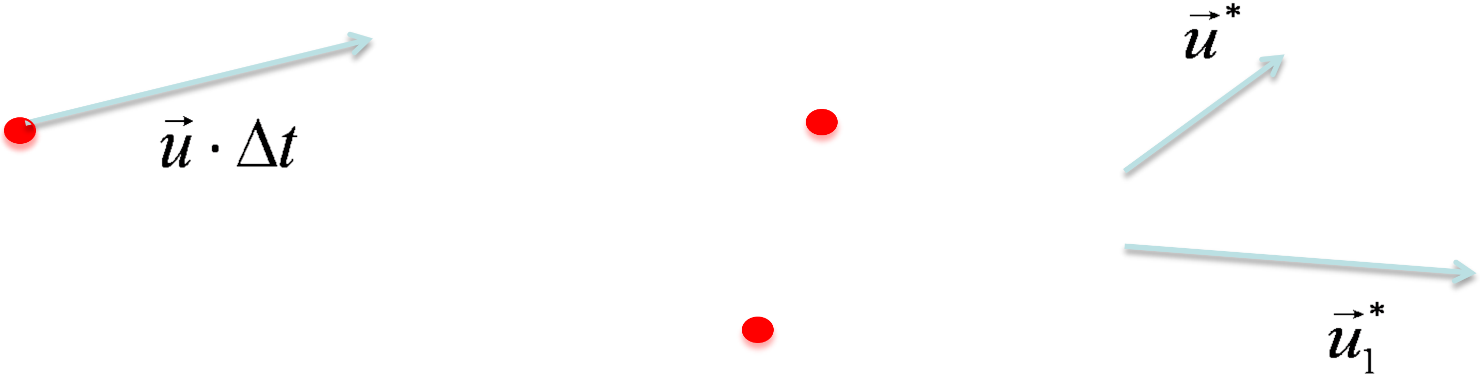


Stochastic Simulation Method: DSMC

Direct Simulation Monte Carlo (DSMC) is a stochastic method to obtain solution of the Boltzmann equation governing the dilute gas flows in molecular regime:

$$\frac{\partial(nf)}{\partial t}+\vec{u}\cdot\nabla_x(nf)=n^2\int\int_0^{4\pi}(f^* f_1^* - ff_1)\,|\,\vec{u}_r\,|\,\frac{d\sigma_T}{d\Omega}d\Omega d\vec{u}_1$$

Free flight *Binary molecular collisions*



<u>Input</u>: geometry; boundary conditions (thermal, evaporation/condensation); molecular interaction model !!

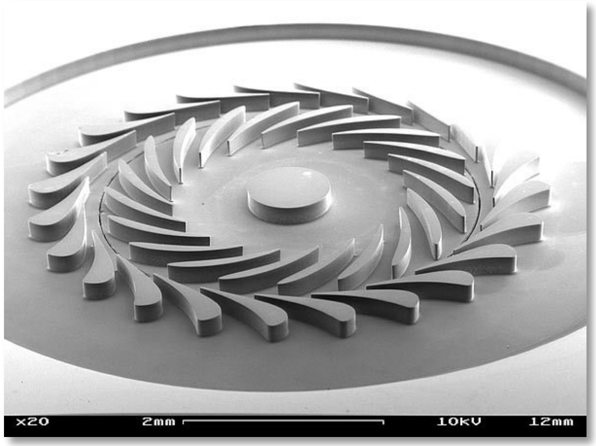
Deterministic Approach: ES-BGK

Unsteady Boltzmann model kinetic equation for velocity distribution function:

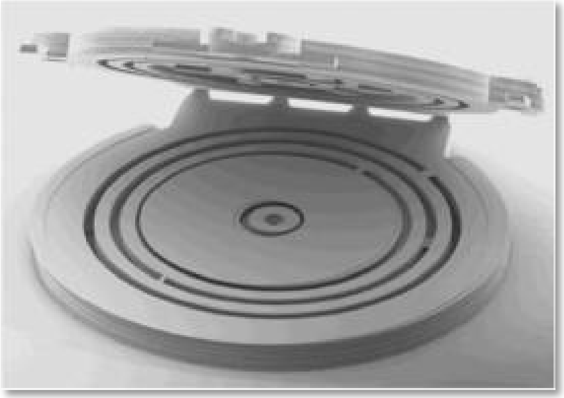
$$\frac{\partial f}{\partial t} + \vec{u} \cdot \nabla_x f = \frac{1}{\tau}(f_0 - f)$$

- ESBGK collision operator
  - H-theorem proved by Andries et al, 2000.
- discrete-ordinate method in velocity space with high-order quadratures
- FVM in physical space with 3rd-order WENO fluxes.
- 2nd and 3rd -order time integration with Runge-Kutta TVD schemes
- domain decomposition in physical space

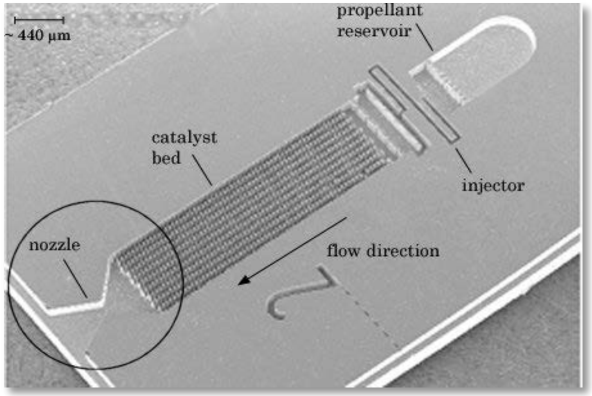
Gas-Based Microdevices



MIT micro-turbine



USC micro-burner



NASA GSFC/UVM H2O2 Monopropellant Micro-Thruster

Microscale Challenges:

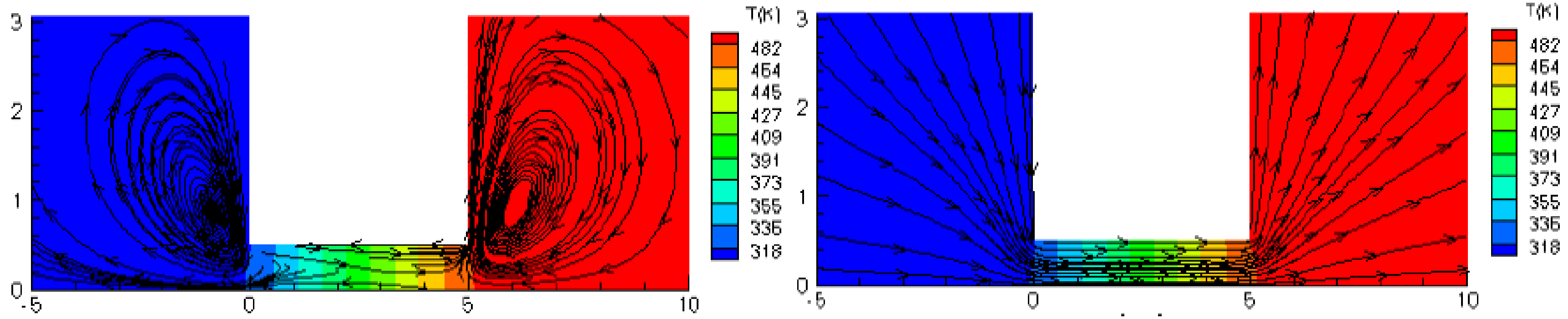
- Gas-phase extinction limit, min Re~40. Catalyst needed
- Amplified heat transfer losses
- Increased viscous losses: Isp drops for Re<200.

Rarefied flow analysis provides methods for design to overcome these challenges

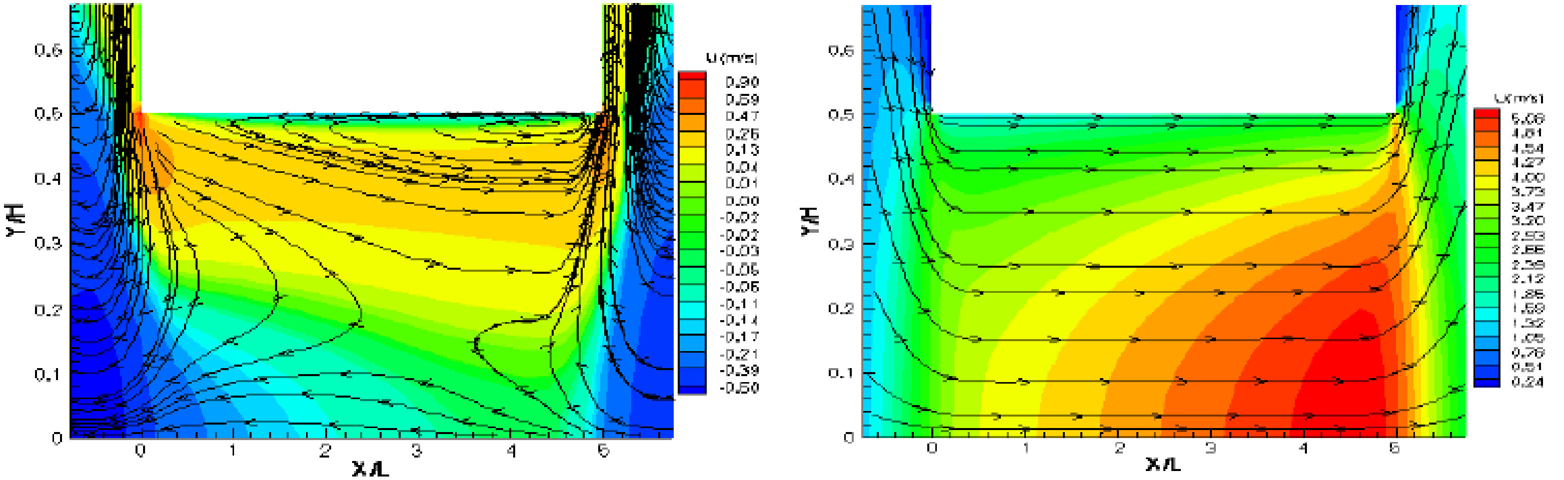
Exploiting Rarefied Microflows

L/H= 5, Kn= 0.2, T_1=300K,T_2=500K, linear wall temperature profile.

Closed Flow: **Open Flow:**



•Temperature contours and streamlines for thermal creep flow in a 2D channel

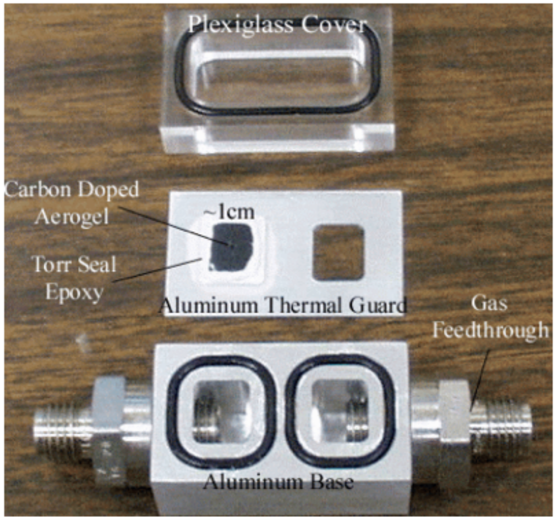


•X-component of velocity contours and streamlines (zoom) in a 2D channel

Knudsen Compressor

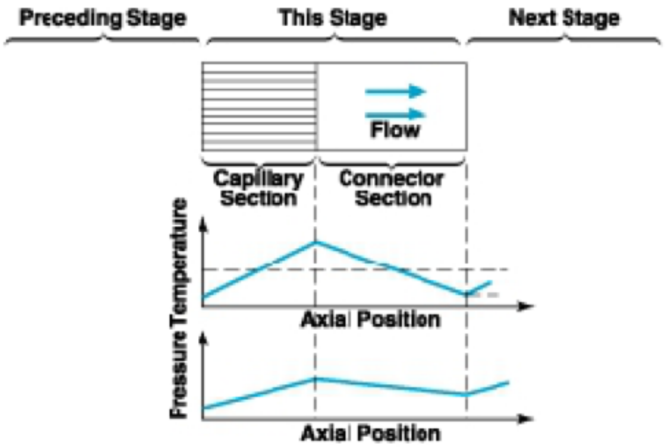
A Knudsen compressor is a cascade of multiple, individually heated compressor stages that exploit the pumping effect of thermal transpiration.

Phil Muntz et al, USC



15-stage: radiant
20 MW/cm^2, 30cm^{2}x2cm
760 → 640 torr

- **Aerogel pore diameter 20-50 nm**
- **Multistage**
- **Resistant and radiant heating.**



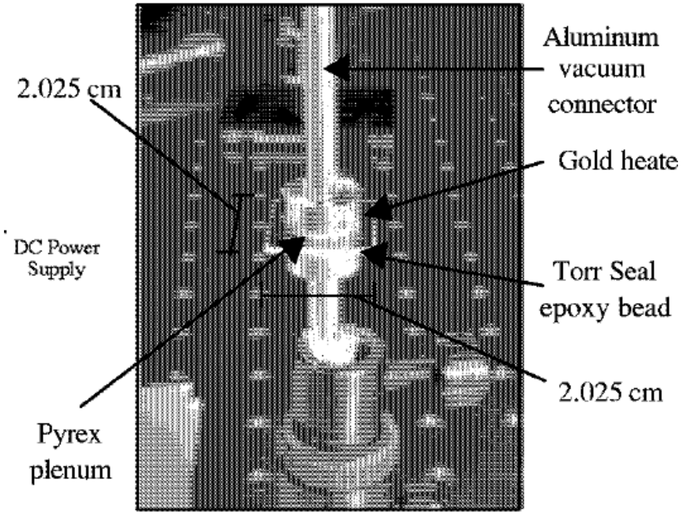
Schematic diagram of gas flow

Exploiting Rarefied Microflows

Vargo, Muntz , Shiflett, Tang, “Knudsen Compressor as a Micro/Meso-scale Vacuum Pump”, *JVST A*, 1999:

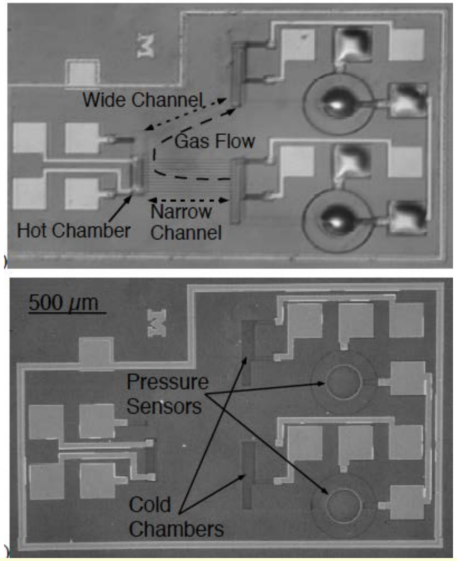
a cascade of multiple, individually heated compressor stages that exploit the pumping effect of rarefied thermal transpiration.

Vargo and Muntz (USC), 2001



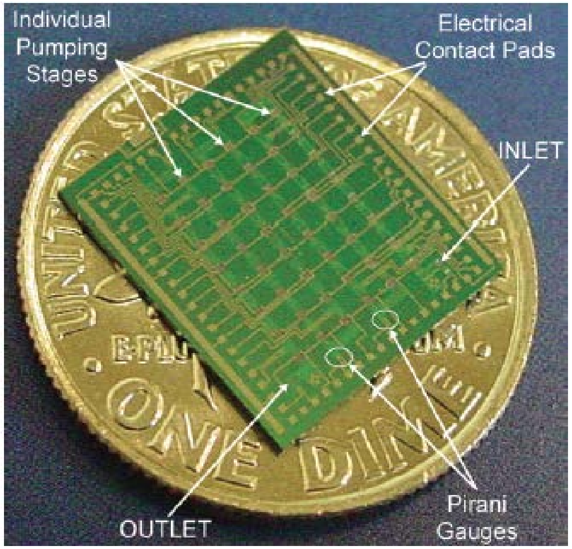
1-stage:
1.5 W, 2 cmx2cm
760 → 750 torr

McNamara&Gianchandani (UMich), 2003



1-stage:
80 mW, 2 x 2 mm^2
760 → 350 torr

Gupta, Ann, Gianchandani (UMich), 2012



48-stage:
1350 mW, 1cm^2
760 → 50 or 250 → 5 torr

Emerging Applications of RGD

- Knudsen force actuation/sensing

  Rarefied thermally-driven flow + thermoelectrics

- Microplasmas

  Rarefied flows + Field-emission cold plasmas

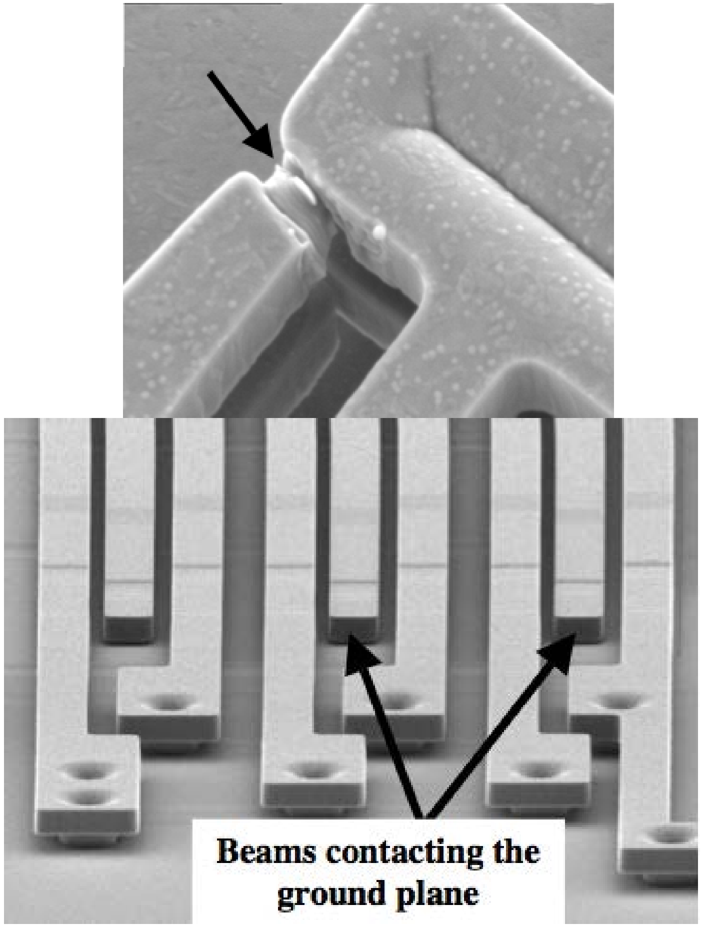
Actuation Mechanisms in MEMS

- Electrostatic Force:

$$F_e = \frac{1}{2}\varepsilon_0 A \frac{V^2}{g^2}$$

ε_0: permittivity
A: normal area
V: voltage
g: gap size

  - + Large force output at small geometry scales.
  - - Electrostatic discharge at high E-fields (MV/cm)
  - - Stiction due to the non-linear force that increases quadratically with distance.
- Piezoelectric Materials
- Pneumatic Actuation
- Fluidic Thermal Actuation = Knudse force



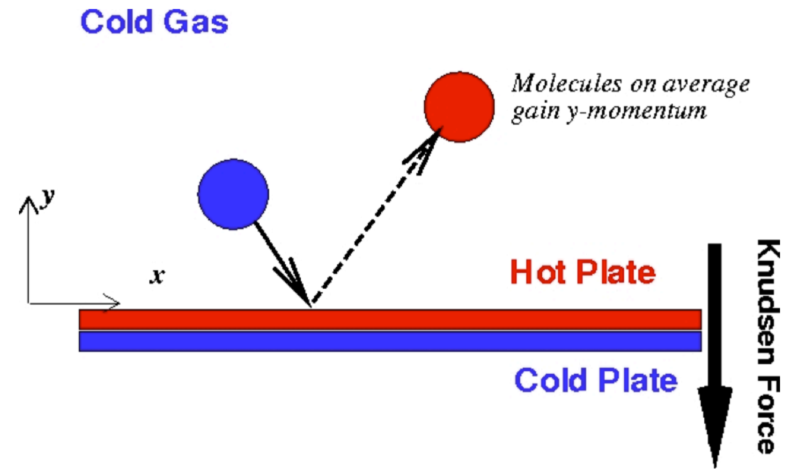
J. A.Walraven, *Failure Mechanisms in MEMS*, International Test Conference, 2003

Thermal Knudsen Forces

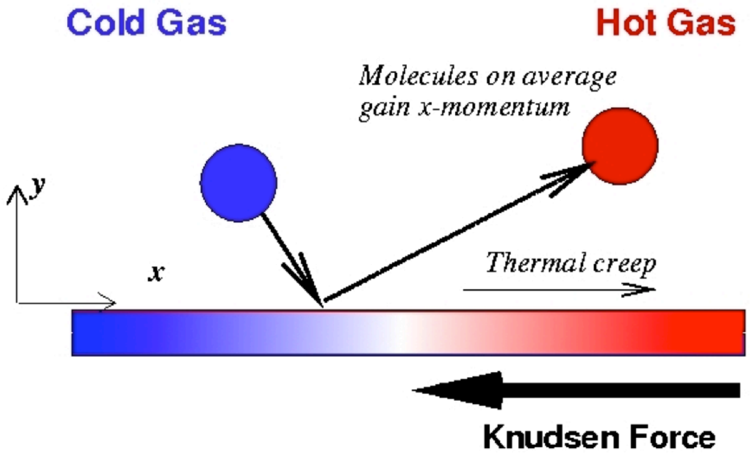
Crookes' Radiometer
(Sir William Crookes, 1874)

- Initially misattributed to light pressure
- Reynolds (1876) explained that gas reflecting from dark vanes is hotter and thus faster
- Maxwell (1879) introduced the notion of thermal creep around the edges
- Einstein (1925) suggested that thermal force was localized near edge of the vane and the force proportional to perimeter of vane.
- Observed also in
  - Vacuum microbalance gauges
  - Thermal transpiration and creep through porous media
  - Thermophoresis on small unequally heated particles
- Direct measurements at **microscale** using laser heating by Passian *et al* (2003, 2004).

Knudsen Forces



Crookes Radiometer: Transverse

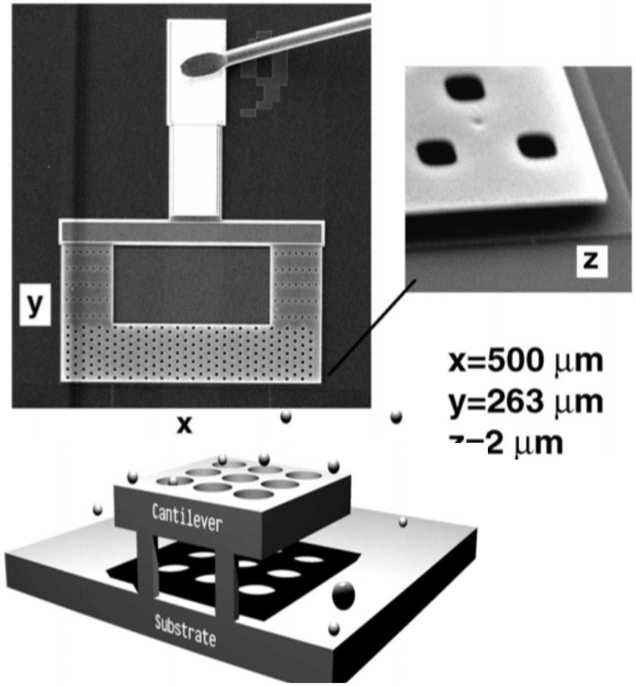


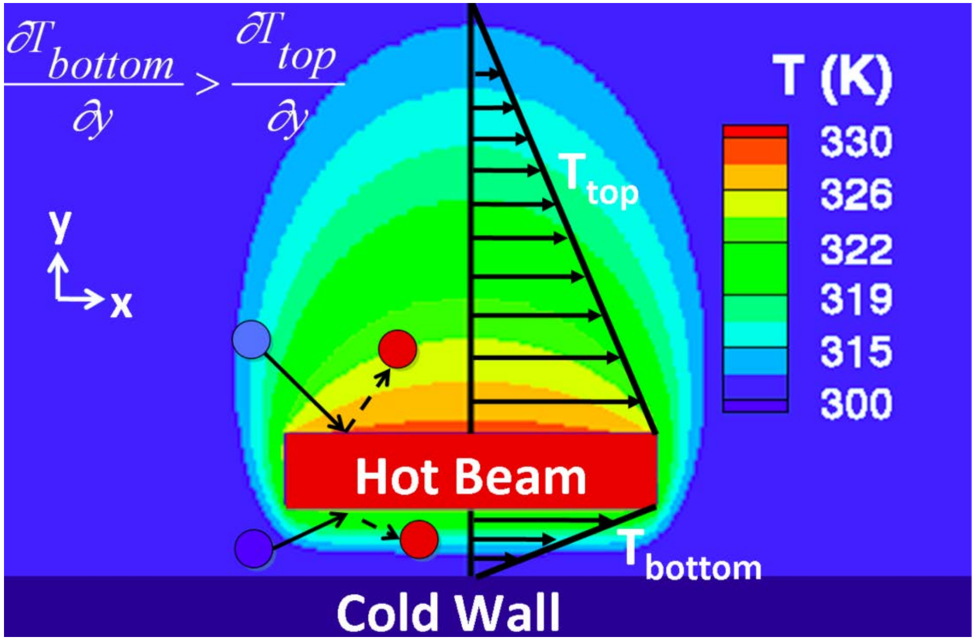
Transpiration: Longitudinal

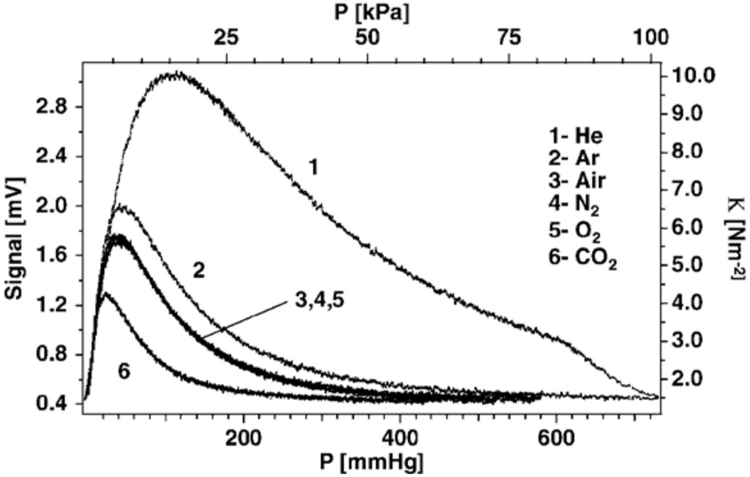
$$U_{creep} \sim \lambda \cdot \frac{dT}{dx} = Kn \cdot \Delta T$$

Kn force actuation

- Consequence of a thermal non-equilibrium between gas and solid
- Can be generated by resistive heating as well as optically Experimental data Passian et al, *PRL*, 2003 measurements using heated AFM probes







Knudsen Force: Modeling Approach

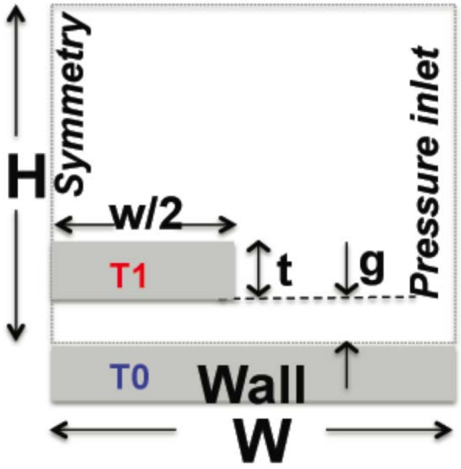
2D quasi-stationary Boltzmann-ESBGK equation:

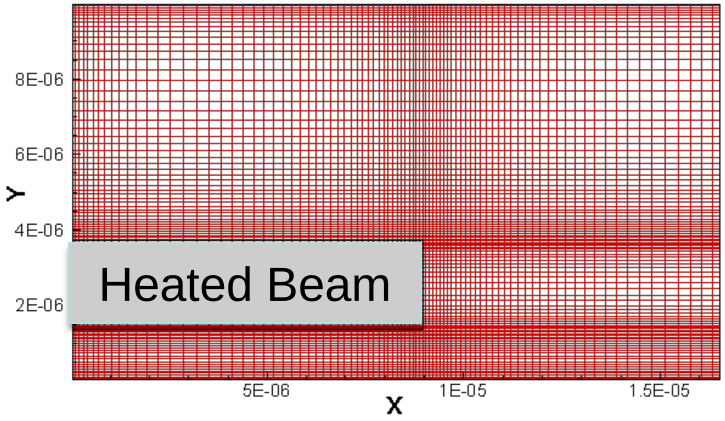
$$\boxed{u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}=\frac{f_0-f}{\tau}}$$

$1/\tau$ – collision frequency

f_0 – Gaussian d.f.

- 2^{nd}-order upwind FVM scheme in physical space
- High order discrete ordinates in velocity space
- Typical Mesh: N=150x150x16x64 = 23 M; Memory: ≈700MB;
- Simulations for planar geometry with equivalent front-to-side area ratio



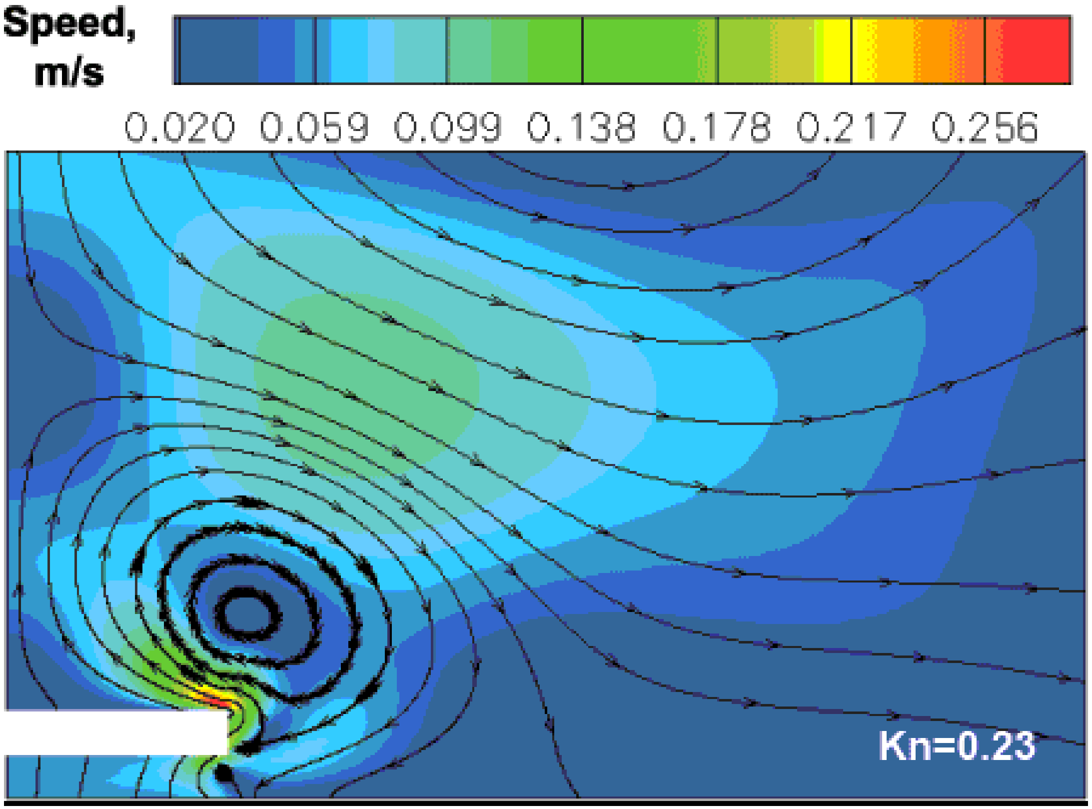


Heated Beam

Non-uniform grid

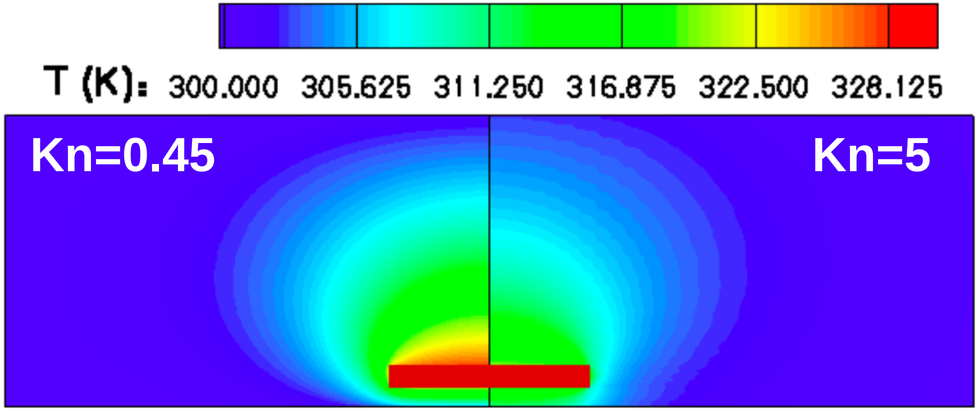
Passian et al data: Simulation Results

- Velocity Contours and Streamlines:

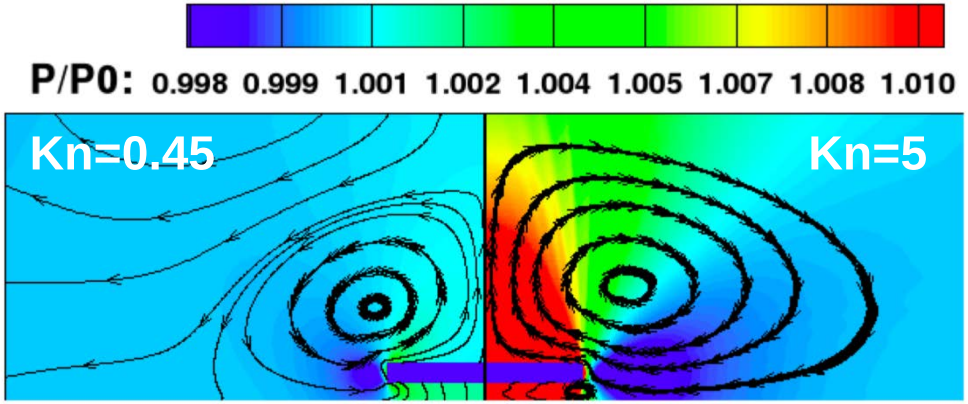


Passian et al data: Simulation Results

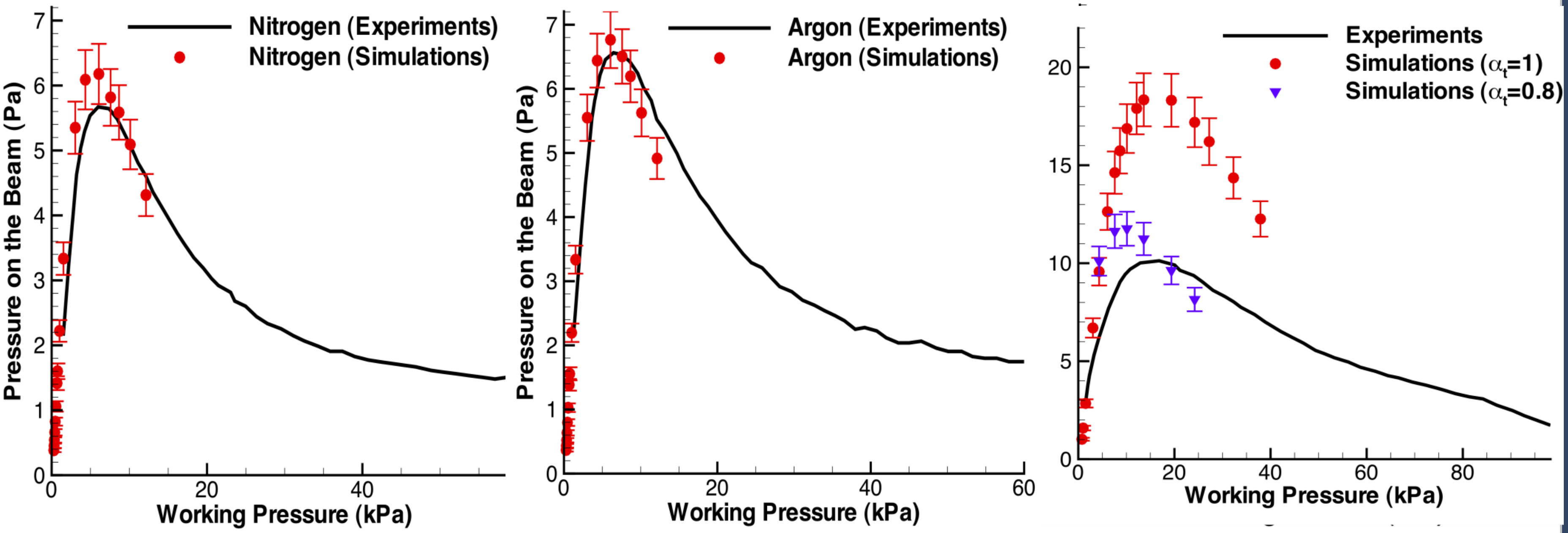
- Temperature Contours:



- Non-dimensional Pressure and Streamlines:



Experimental Validation



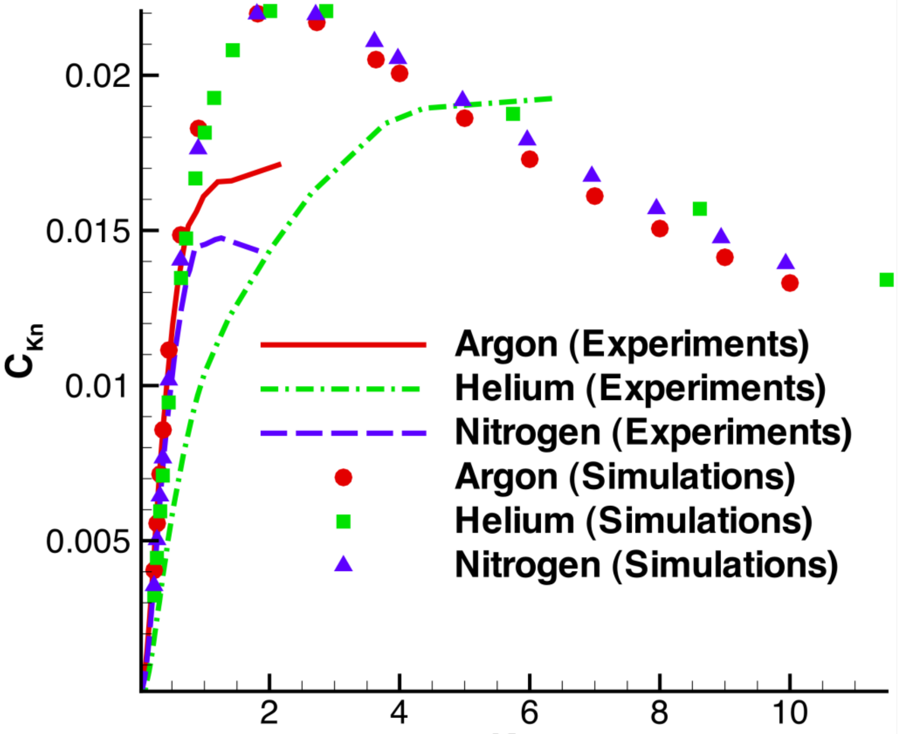
- Argon and Nitrogen simulations agree with experiments within 10%.
- Deviation for Helium is about 80% at the maximum of Knudsen force.

Nabeth, Chigullapalli, Alexeenko, *PRE*, 2011

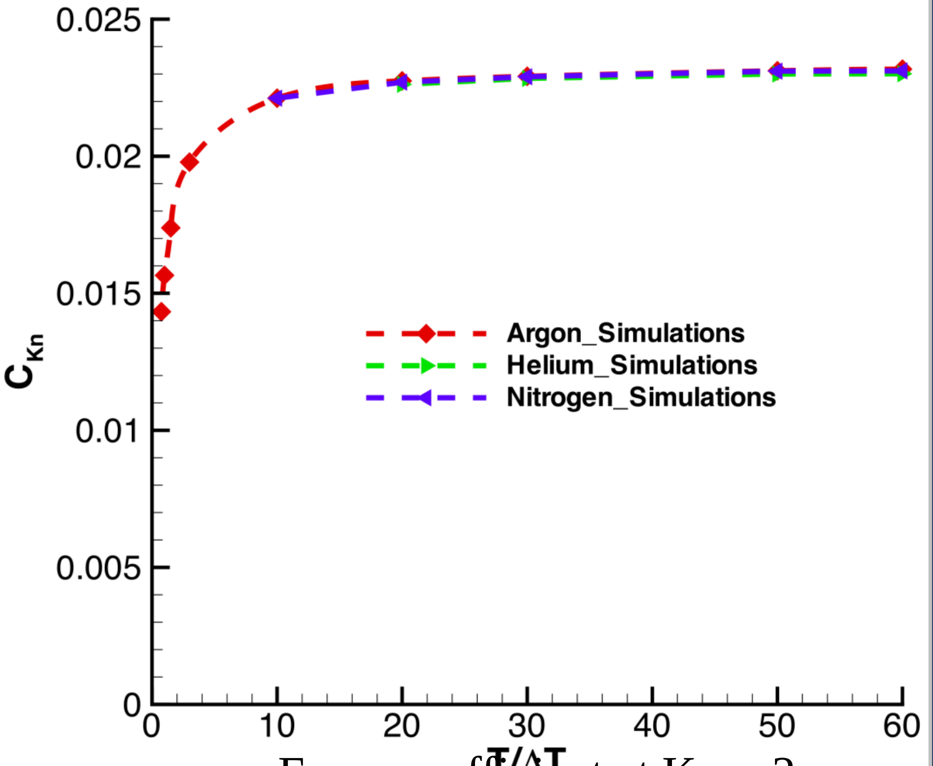
Compact Model for Kn Force

Closed-form model for non-dimensional Knudsen thermal force coefficient for uniformly heated beam:

$$C_{Kn}(Kn,\frac{T}{\Delta T}) = \frac{F'}{w\rho R\Delta T} = \frac{1+D(\frac{T}{\Delta T})^{d} + E(\frac{T}{\Delta T})^{e}}{AKn^{a}+BKn^{b}+CKn^{c}}$$



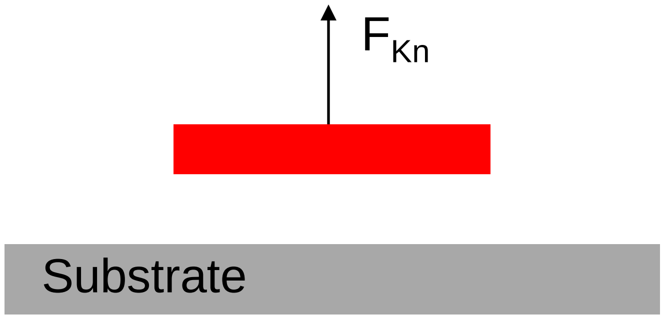
Force coefficient at T/ΔT = 10



Force coefficient at Kn = 2

Force Enhancement and Reversal

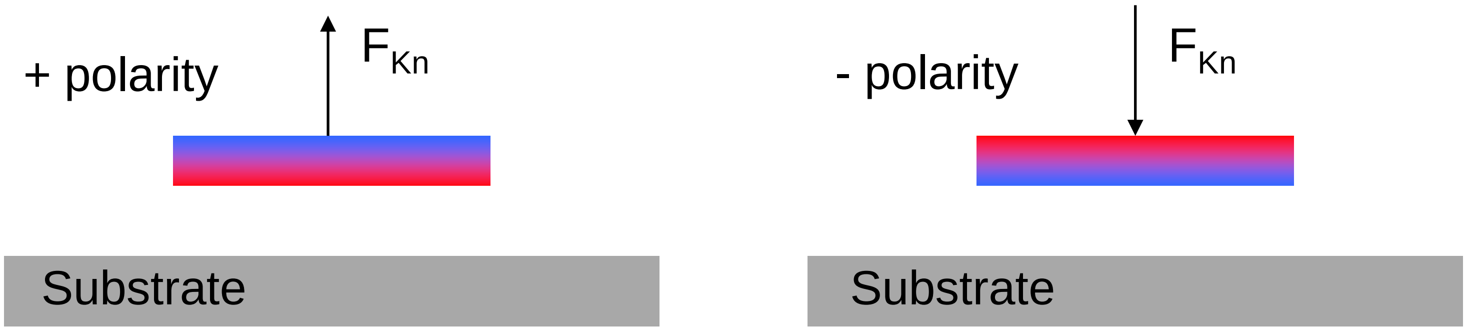
- Uniformly heated beam (Passian et al, 2003)



F_{Kn}

Substrate

- Thermoelectric heating for bi-directional actuation (this work):



+ polarity

F_{Kn}

Substrate

- polarity

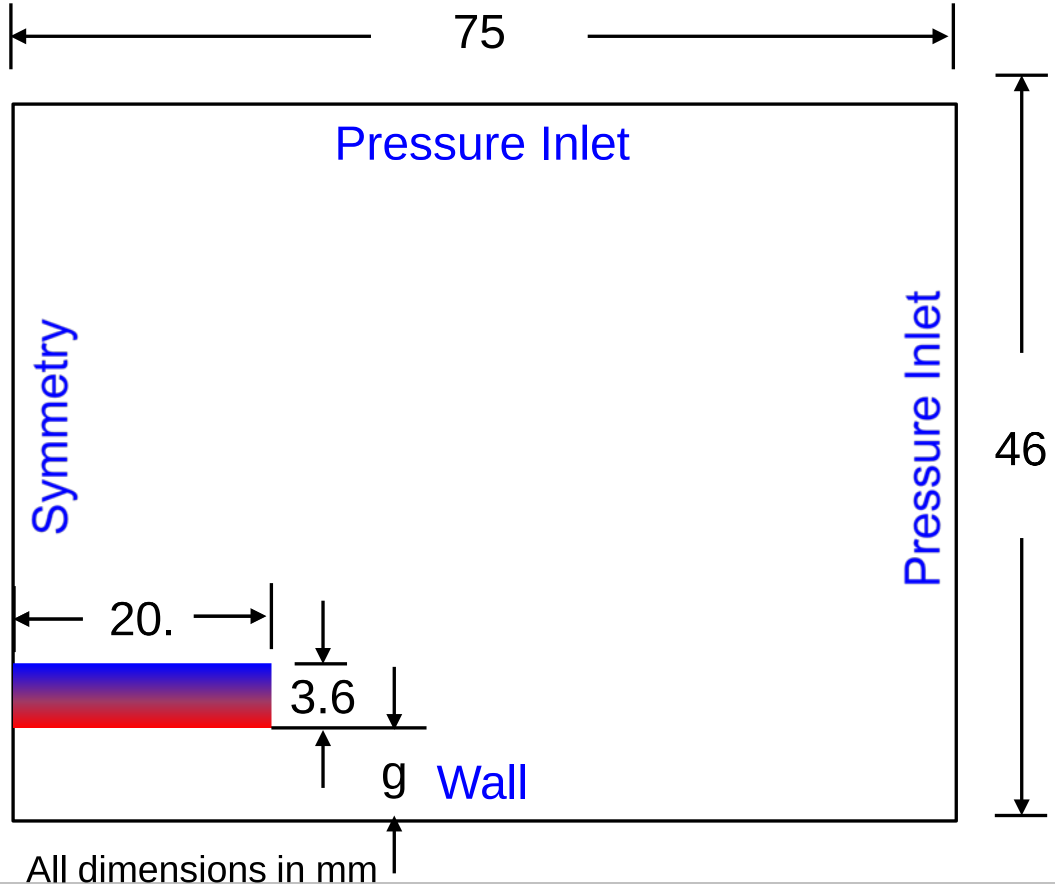
F_{Kn}

Substrate

Modeling the Knudsen Force

Simulate Knudsen force on suspended heating element using ES-BGK model

- 2D-2V finite volume solver with second-order upwind fluxes
- 8th-order Gauss-Hermite quadrature in velocity space

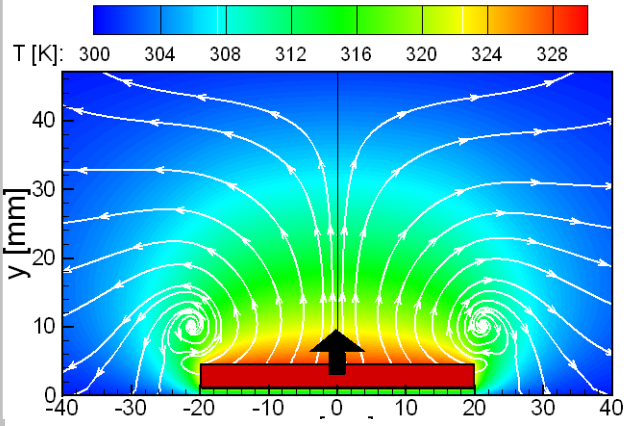


Parameter	Value
P_{ref}	298 – 12,522 Pa
$\lvert \Delta T_b \rvert$	25 K
$T_{b,mean}$	326 K
$T_{substrate}$	298 K
$\Delta T_{b,s}$	26 K

Richardson Extrapolation:
160x160 mesh for force within 2%

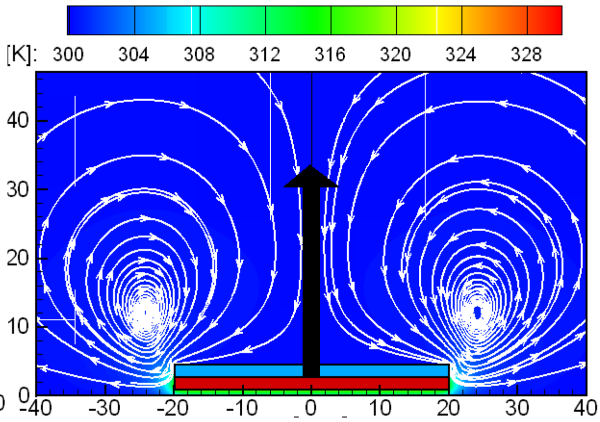
Simulation Results

Uniform Heating



0.105% of p_0

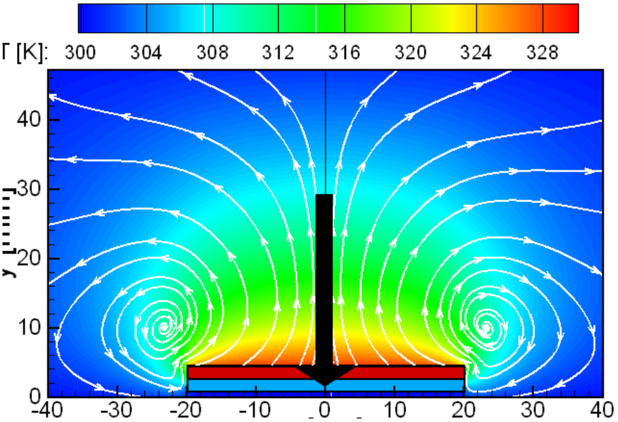
Thermoelectric: Bottom-Up



0.938% of p_0

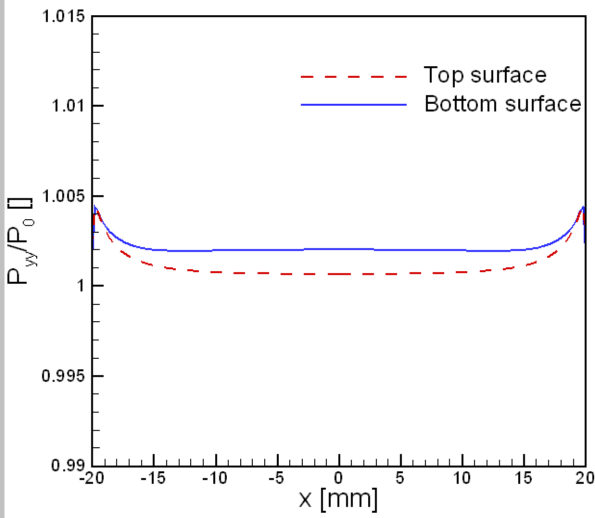
X8.9 enhancement

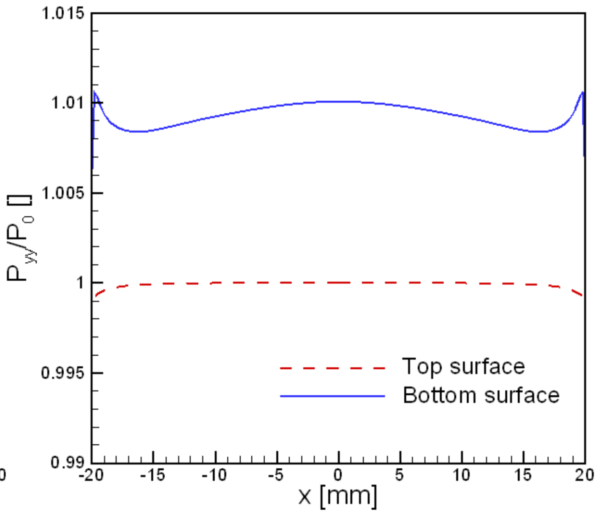
Thermoelectric: Top-Down

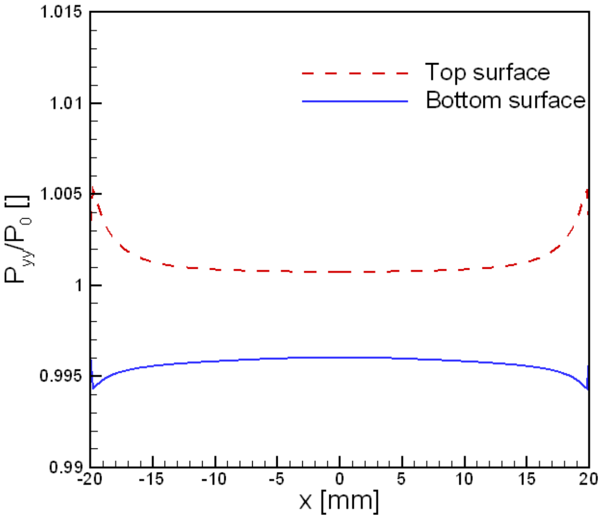


$-0.557\ \%$ of p_0

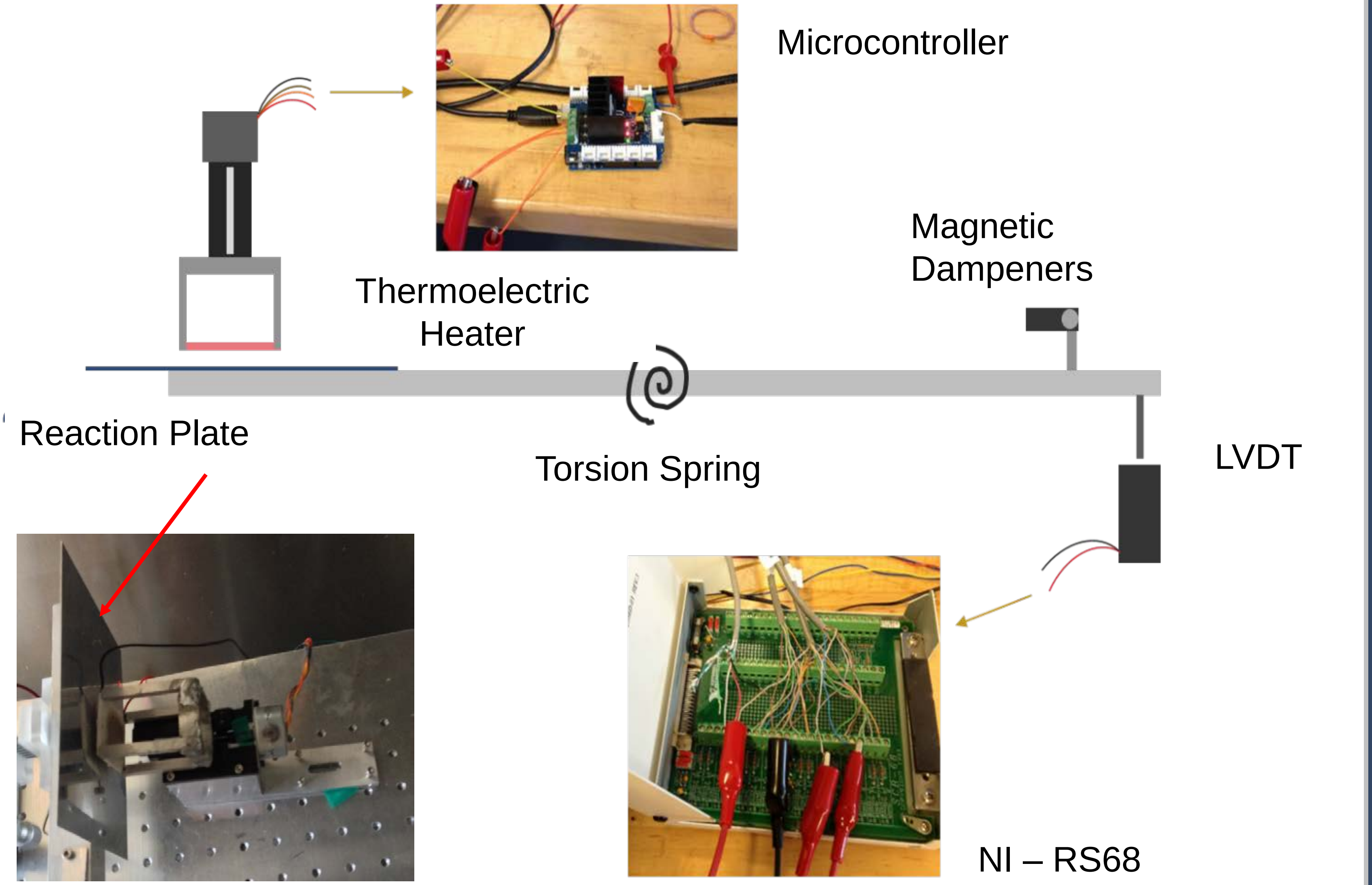
X5.3 enhancement & reversal







Experimental Setup



Microcontroller

Magnetic Dampeners

Thermoelectric Heater

Reaction Plate

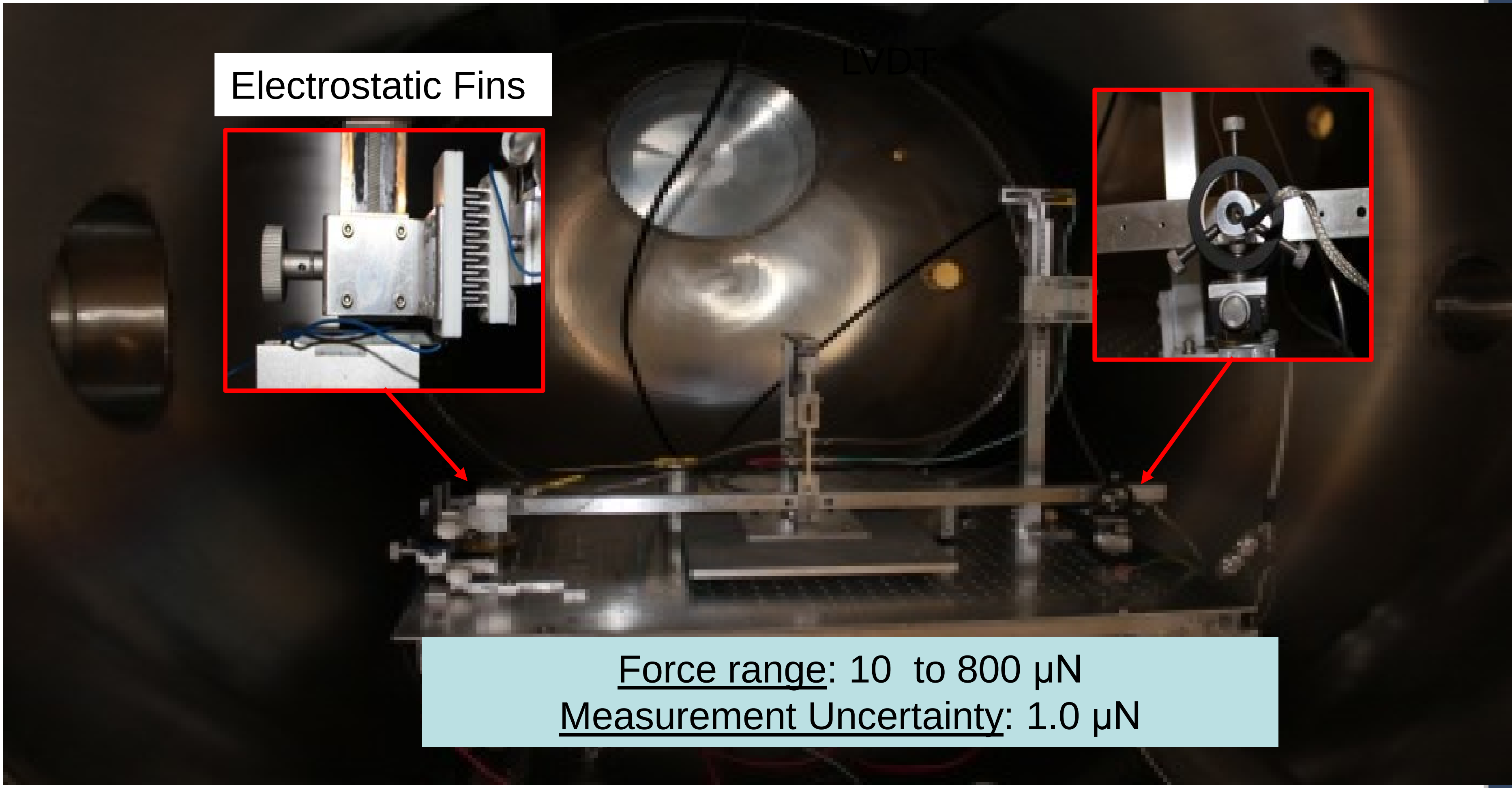
Torsion Spring

LVDT

NI – RS68

Experimental Setup

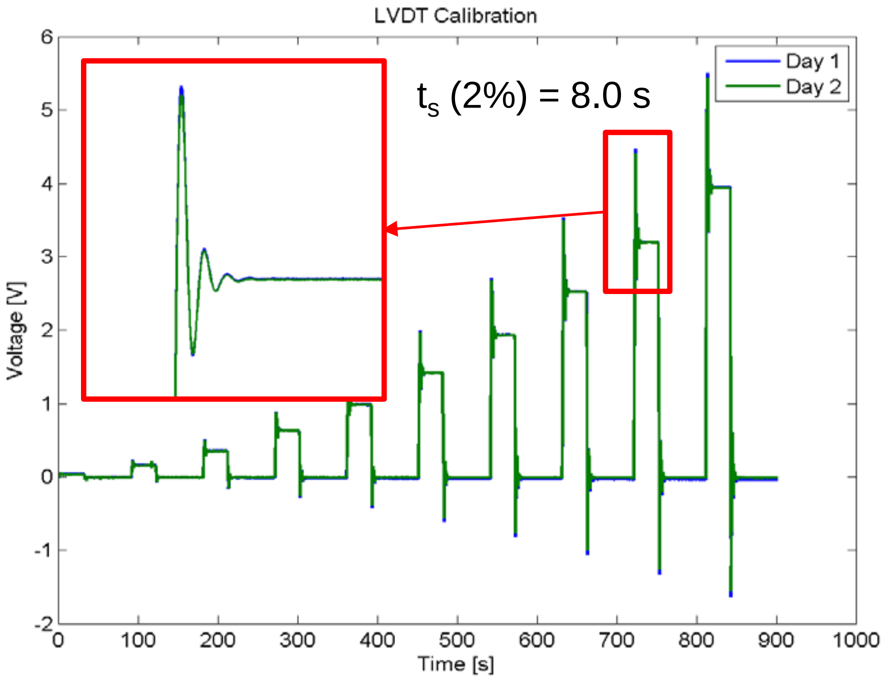
Purdue LEAP MicroNewton Thrust Stand



<u>Force range</u>: 10 to 800 μN

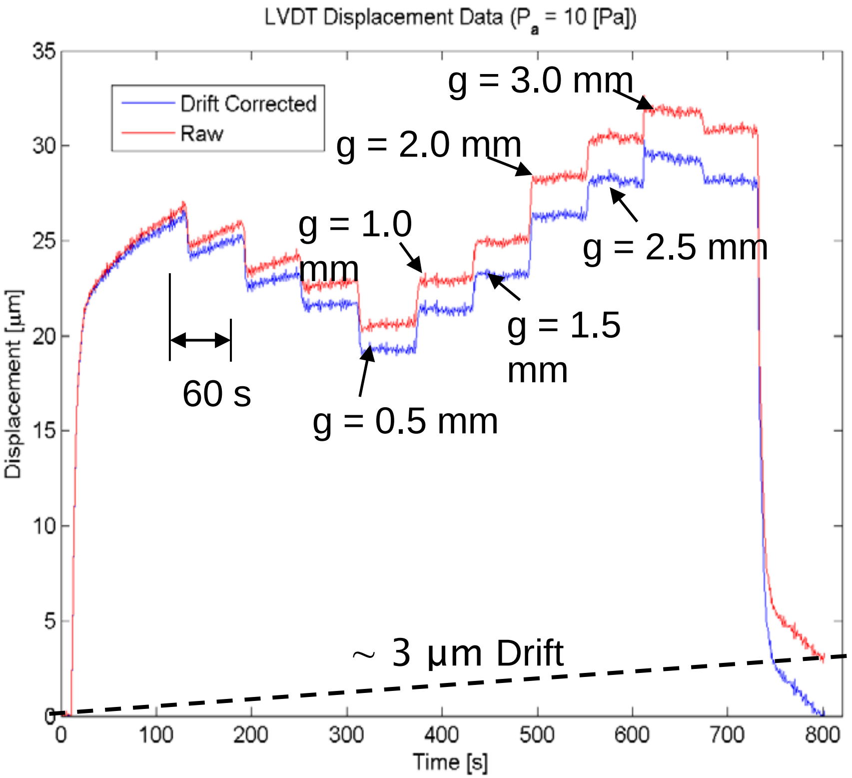
<u>Measurement Uncertainty</u>: 1.0 μN

Measurement Technique

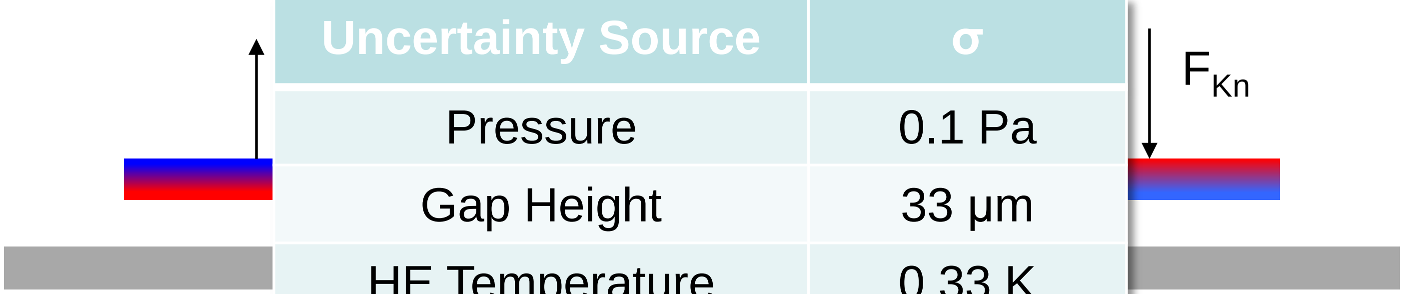


- Calibration performed at the beginning of each test day
- Sweep 0 - 100 V, 30 seconds high, 60 seconds low
    - Evaluate force from LVDT voltage

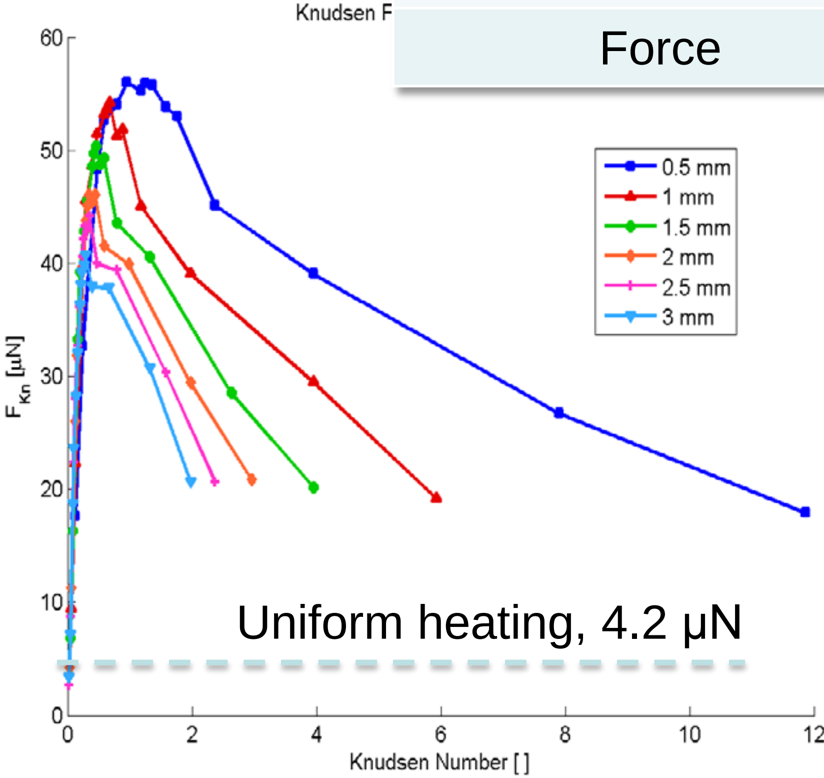
- Measure displacement of reaction plate
- Sweep 0.5 – 3.0 mm gap size at fixed pressure
    - Hold for 60 seconds to ensure steady state
    - Apply linear drift correction



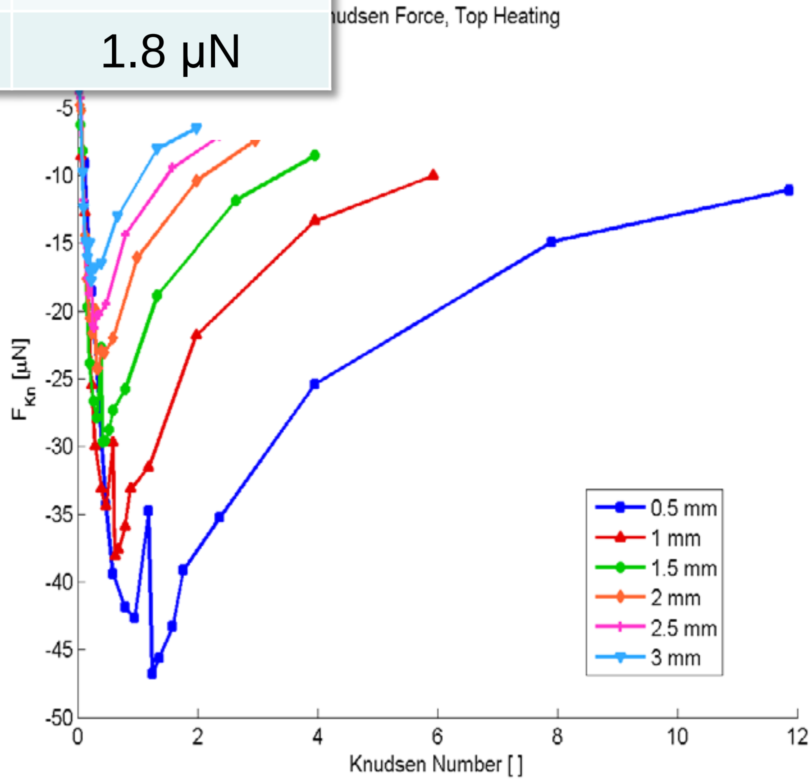
Themoelectric Kn force:Measurements



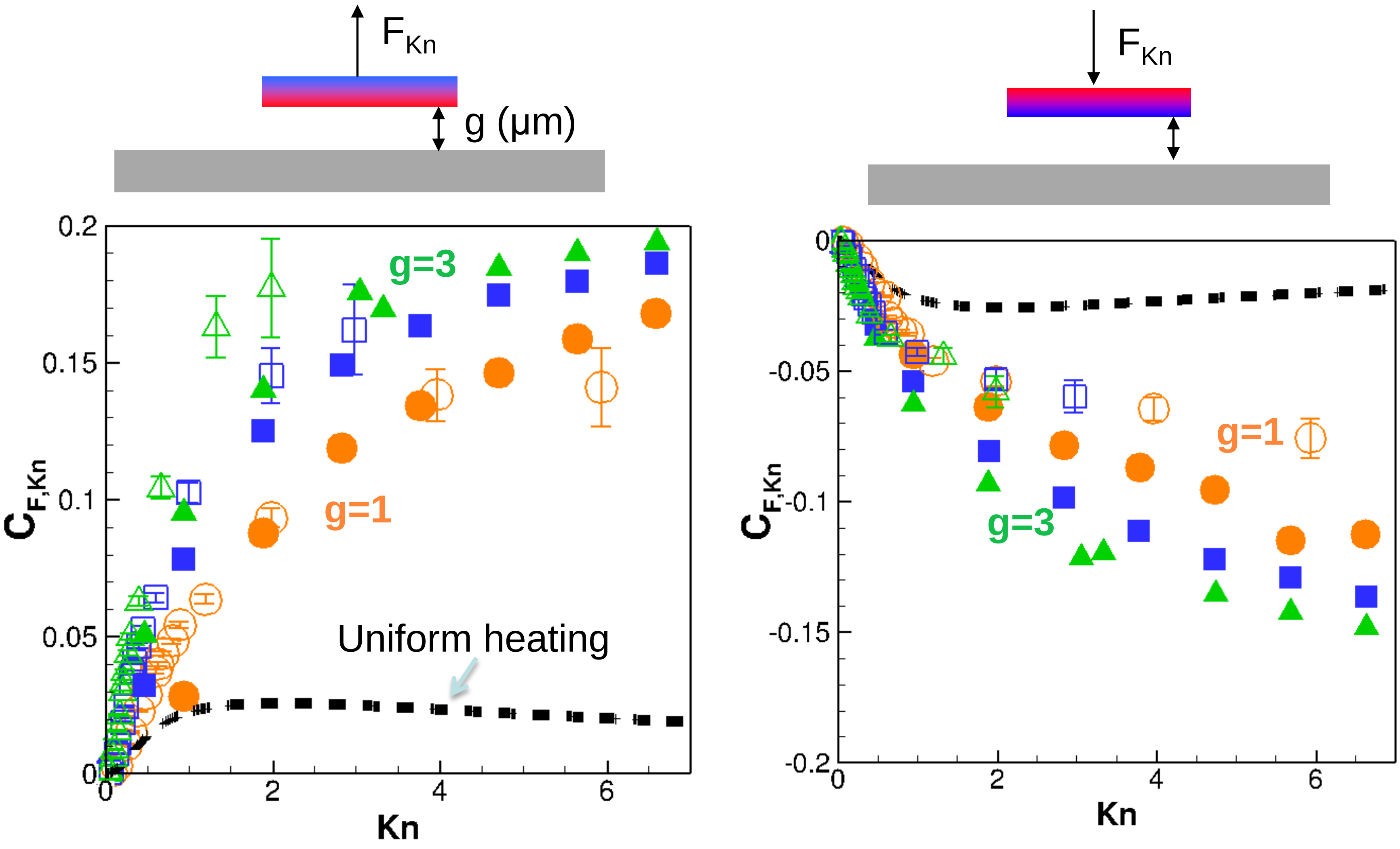
Uncertainty Source	σ
Pressure	0.1 Pa
Gap Height	33 μm
HE Temperature	0.33 K
Substrate Temperature	4.0 K
Force	1.8 μN



Uniform heating, 4.2 μN



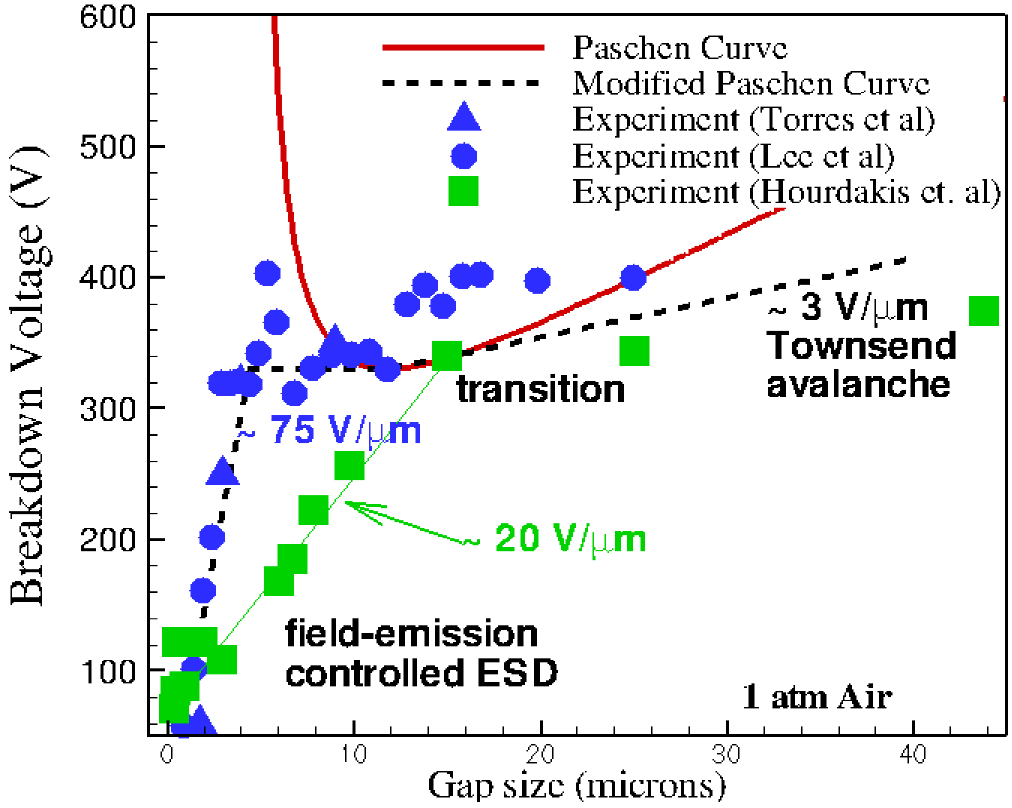
Comparison of Modeling and Experiment



Maximum 36% error in $C_{F,Kn}$ between simulation and experiment

- Difference likely due to 3D effects in experimental measurements
- Errors likely stem from model inputs related to the heating element

Experiments: DC breakdown in microgaps



Fowler-Nordheim field emission:

$$j_{fe} = \frac{A\beta^2 E^2}{\phi t^2(y)} \exp\left(\frac{-B\phi^{3/2} v(y)}{\beta E}\right)$$

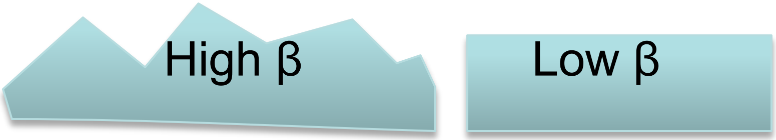
$$A = \frac{e}{2\pi h} = 6.2\text{E-6}\ A/eV$$

e – electron charge
h – Planck's constant

$$B = \frac{4\kappa}{3} = 6.85\text{E7} \qquad \kappa = \sqrt{\frac{8\pi^2 m}{h^2}}$$

Field enhancement factor β = 1.5 – 100.
Depends on geometry and material.

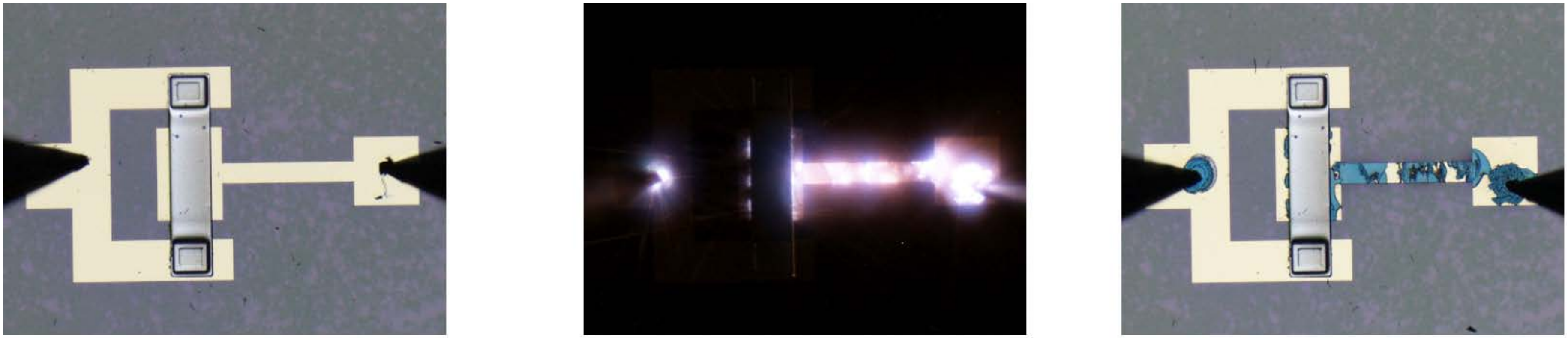
Field-emission become dominant for E>100 MV/m or 100 V/µm



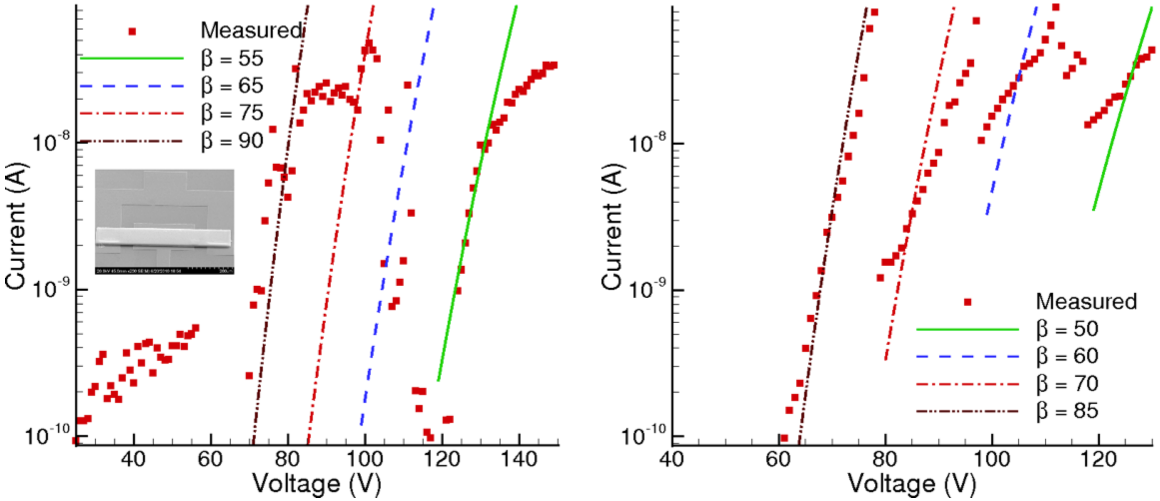
MEMS microdischarges measurements

Example: Nickel RF MEMS capacitive switch

200 V DC



Garg et al, “Direct Measurement of Field Emission Current in Electrostatic MEMS”, MEMS 2011



I-V measurements and PIC/MCC model*:*

Venkattraman, Garg, Peroulis, Alexeenko *APL*, 2012