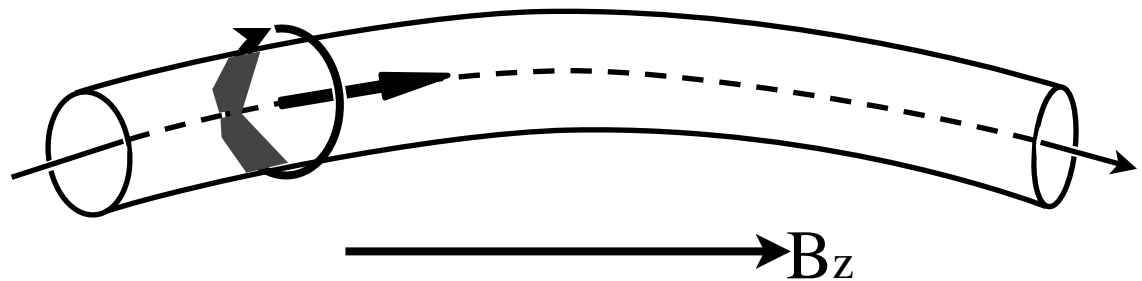


Coupling of spin and nanomechanical motion in carbon nanotubes

Mark Rudner

Niels Bohr Institute

16 July 2014



In collaboration with:

Emmanuel Rashba (*Harvard*)

Andras Palyi (*Eotvos*), Philipp Struck (*Konstanz*), Karsten Flensberg (*Copenhagen*), Guido Burkard (*Konstanz*)

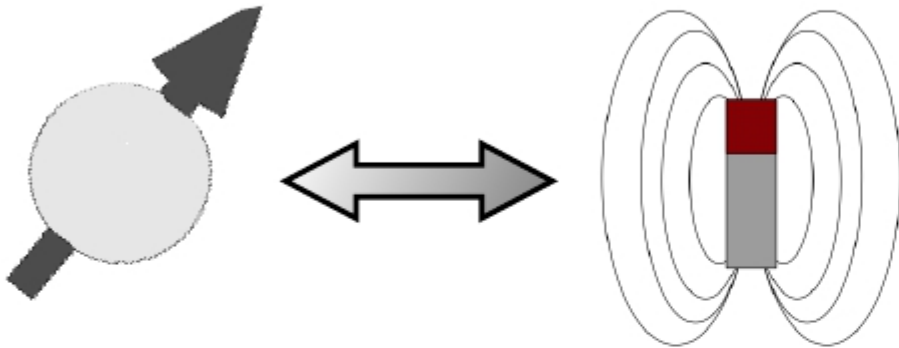
For details see:

MR and E. I. Rashba, Phys. Rev. B 81, 125426 (2010).

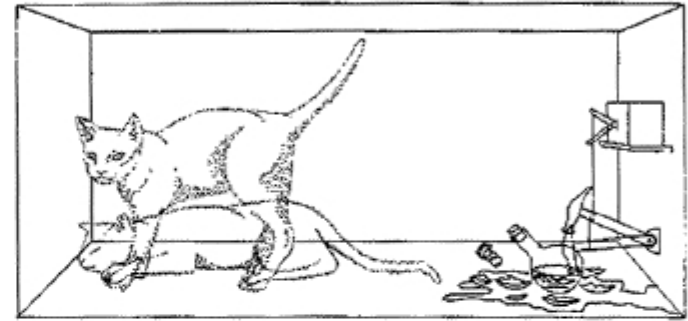
A. Palyi, P. R. Struck, MR, K. Flensberg, and G. Burkard, Phys. Rev. Lett. 108, 206811 (2012).

Understand, control, and exploit electronic properties beyond charge

Electron and nuclear spin

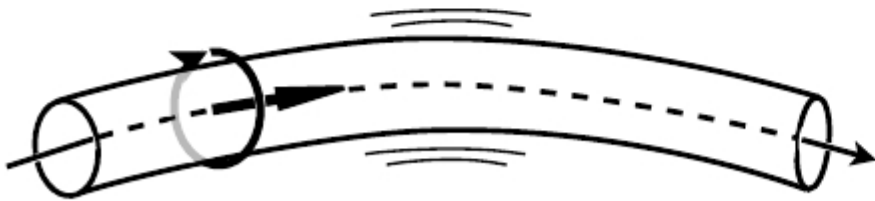


Quantum coherence/Entanglement

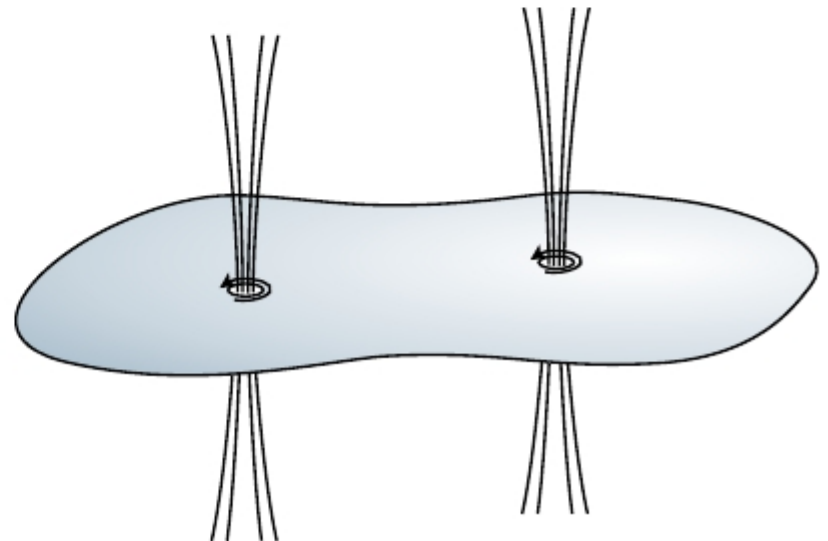


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Nanomechanical motion (NEMS)



Exotic collective modes

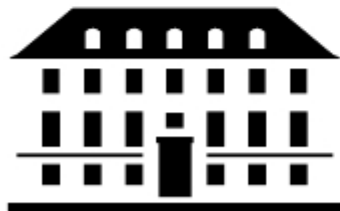


Vibrant research community in Copenhagen

Copenhagen University



Center for
Quantum
Devices



The Niels Bohr
International Academy

Microsoft®
Research

Outline

I. Nanotube basics, spin-orbit coupling and phonons

II. Electron spin relaxation in CNT quantum dots

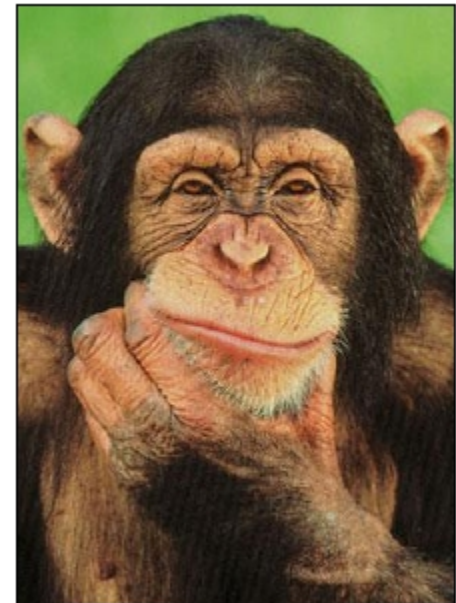
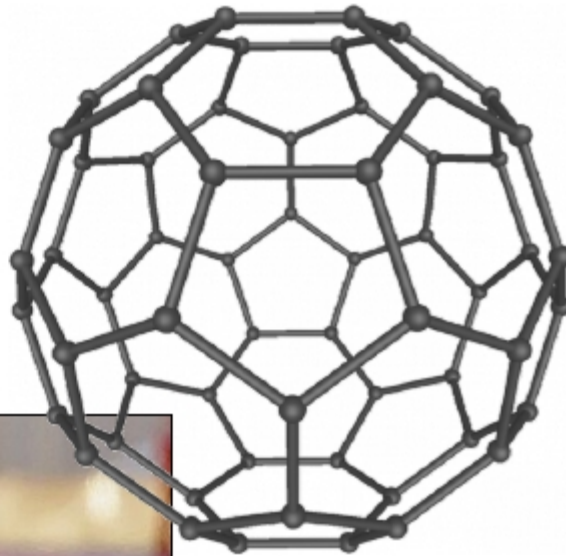
III. Coupling to phonon “cavity” mode in suspended CNT

Experimental Inspiration:

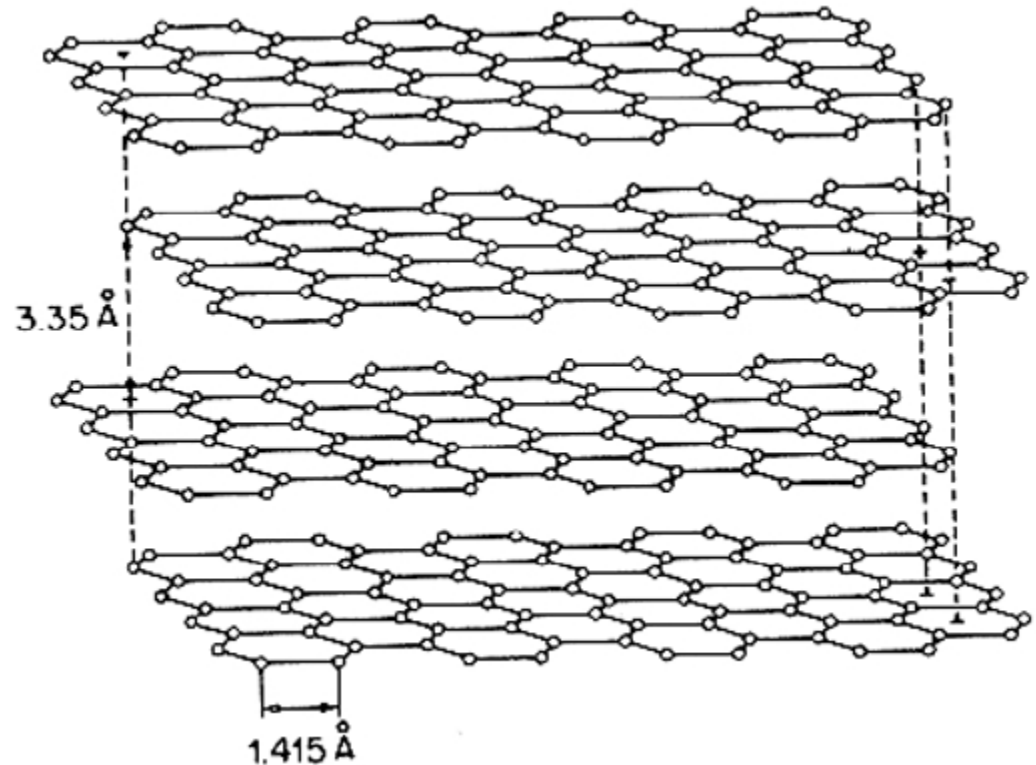
H. O. H. Churchill, et al., Phys. Rev. Lett. 102, 166802 (2009).

G. A. Steele, et al., Science 325, 1103 (2009).

In nature, carbon comes in many forms

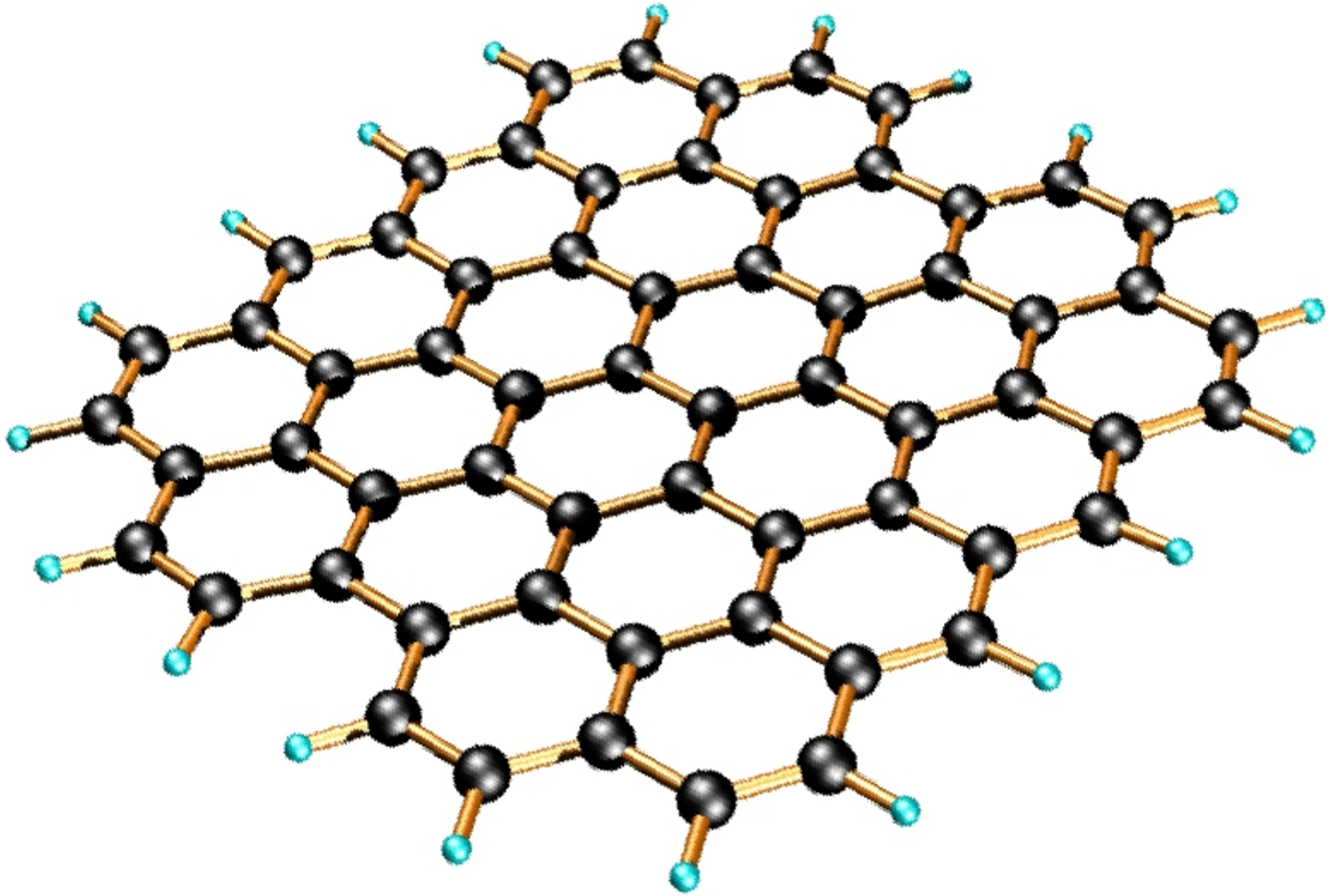


Graphite: stacked 2D sheets of carbon

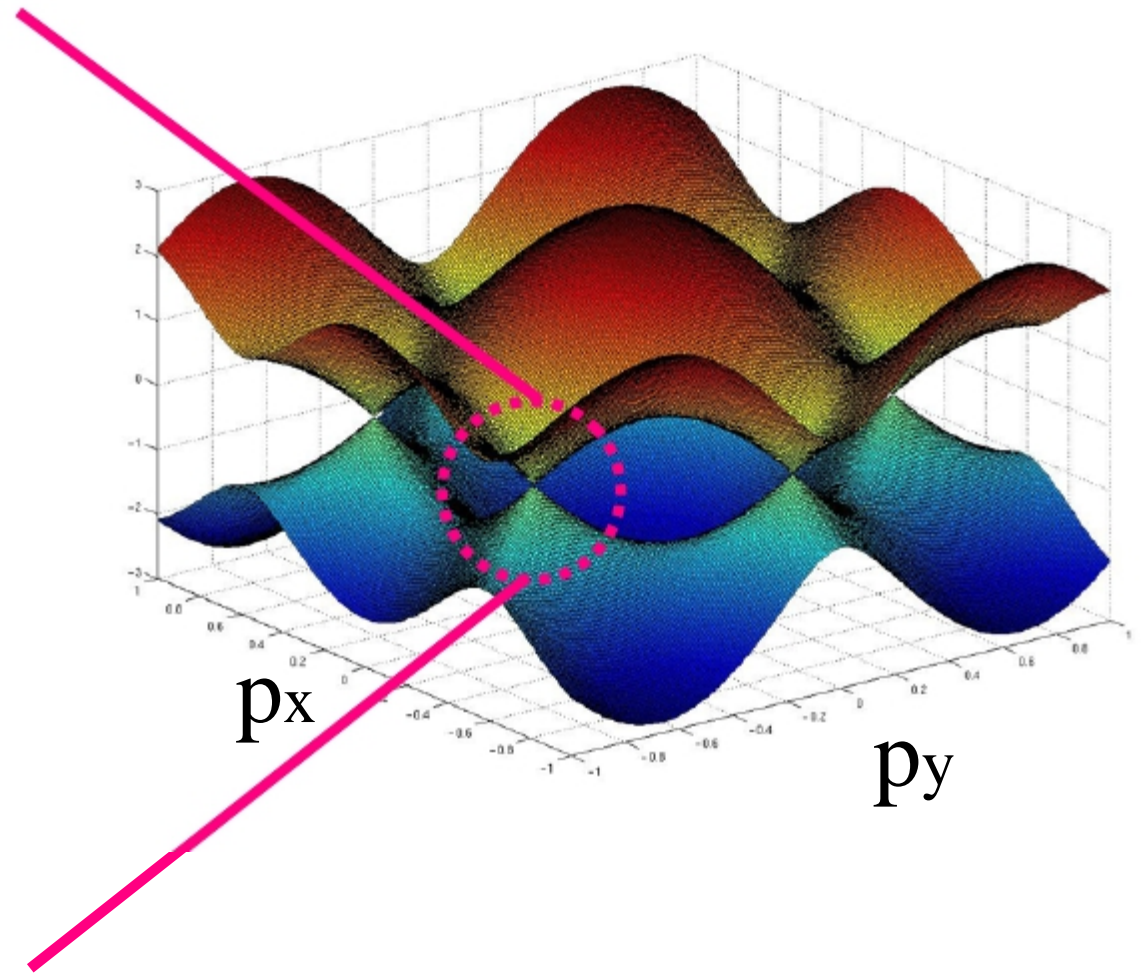
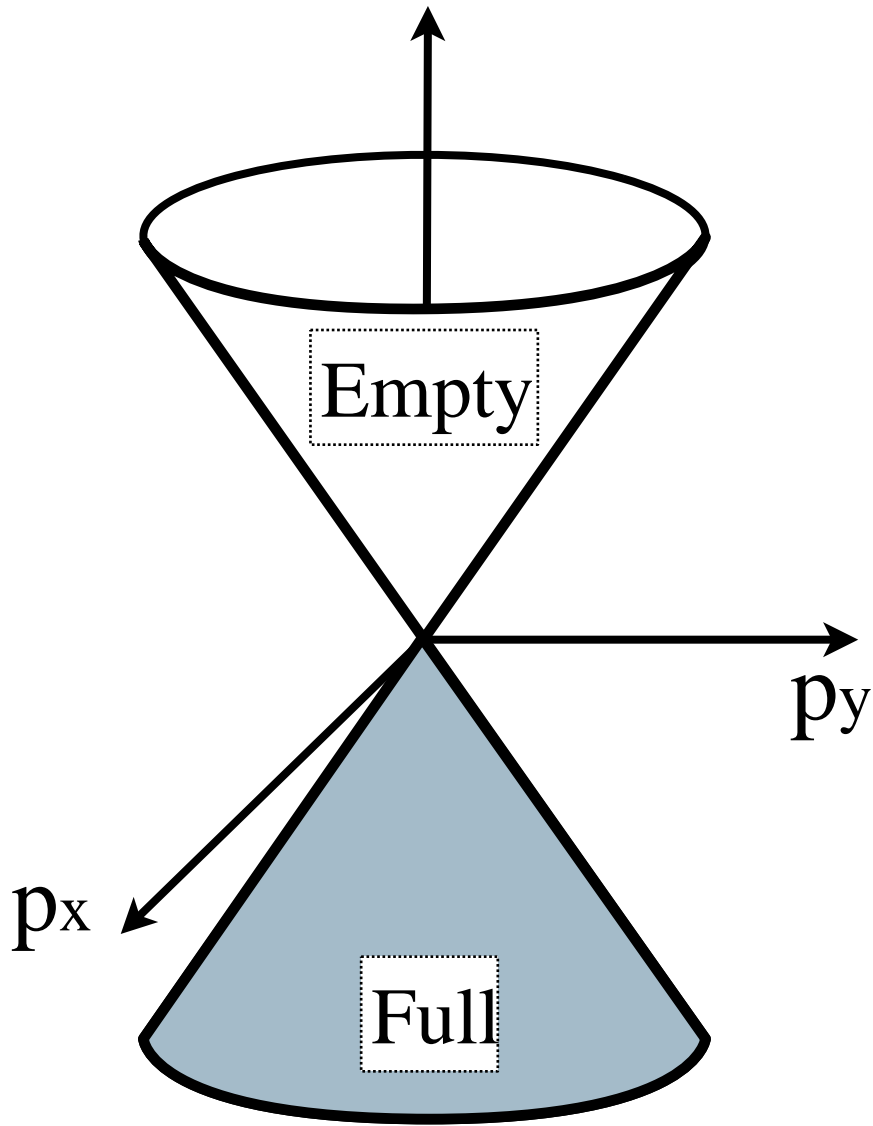


- * Strong in-plane bonds, weak interaction between planes

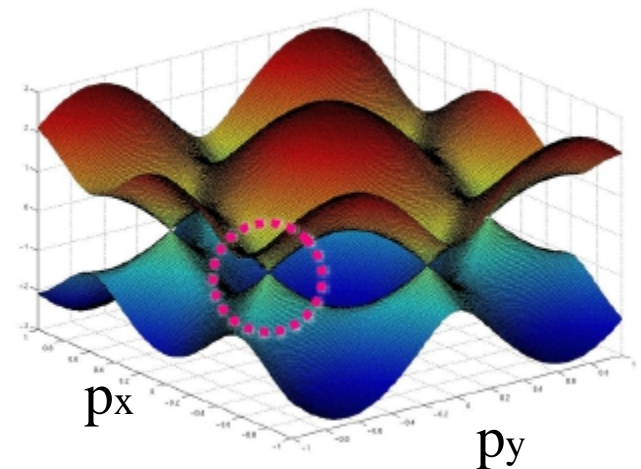
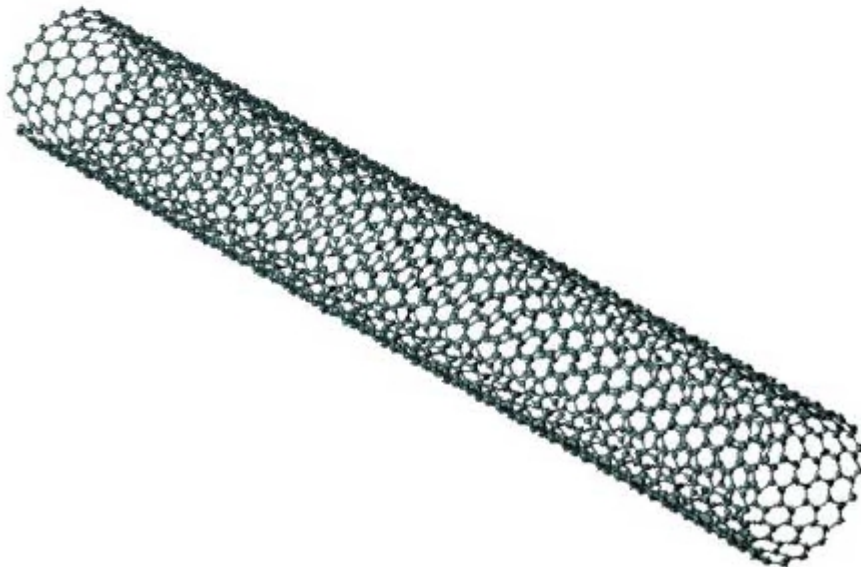
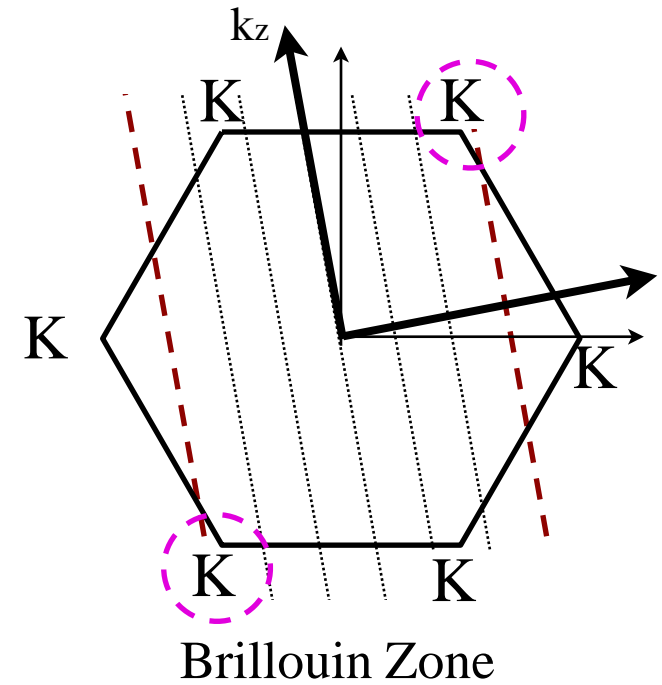
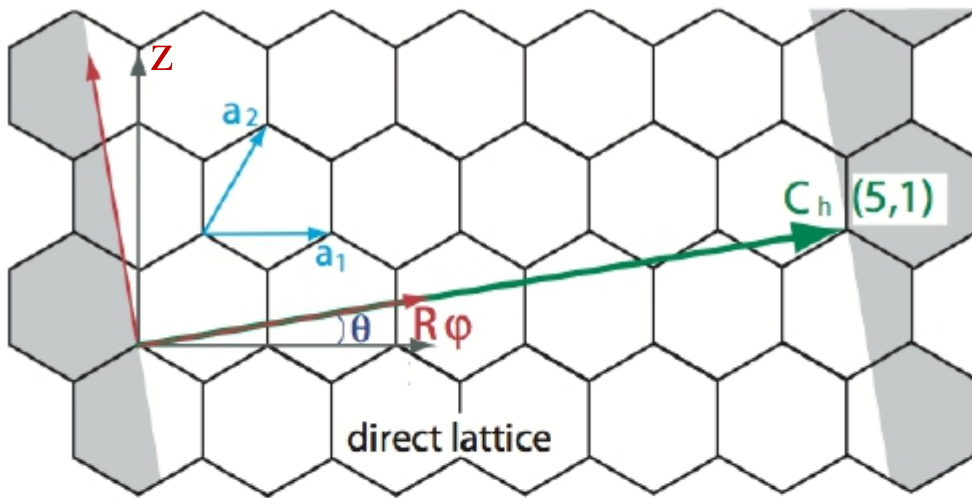
Graphene: a single atomic plane of carbon



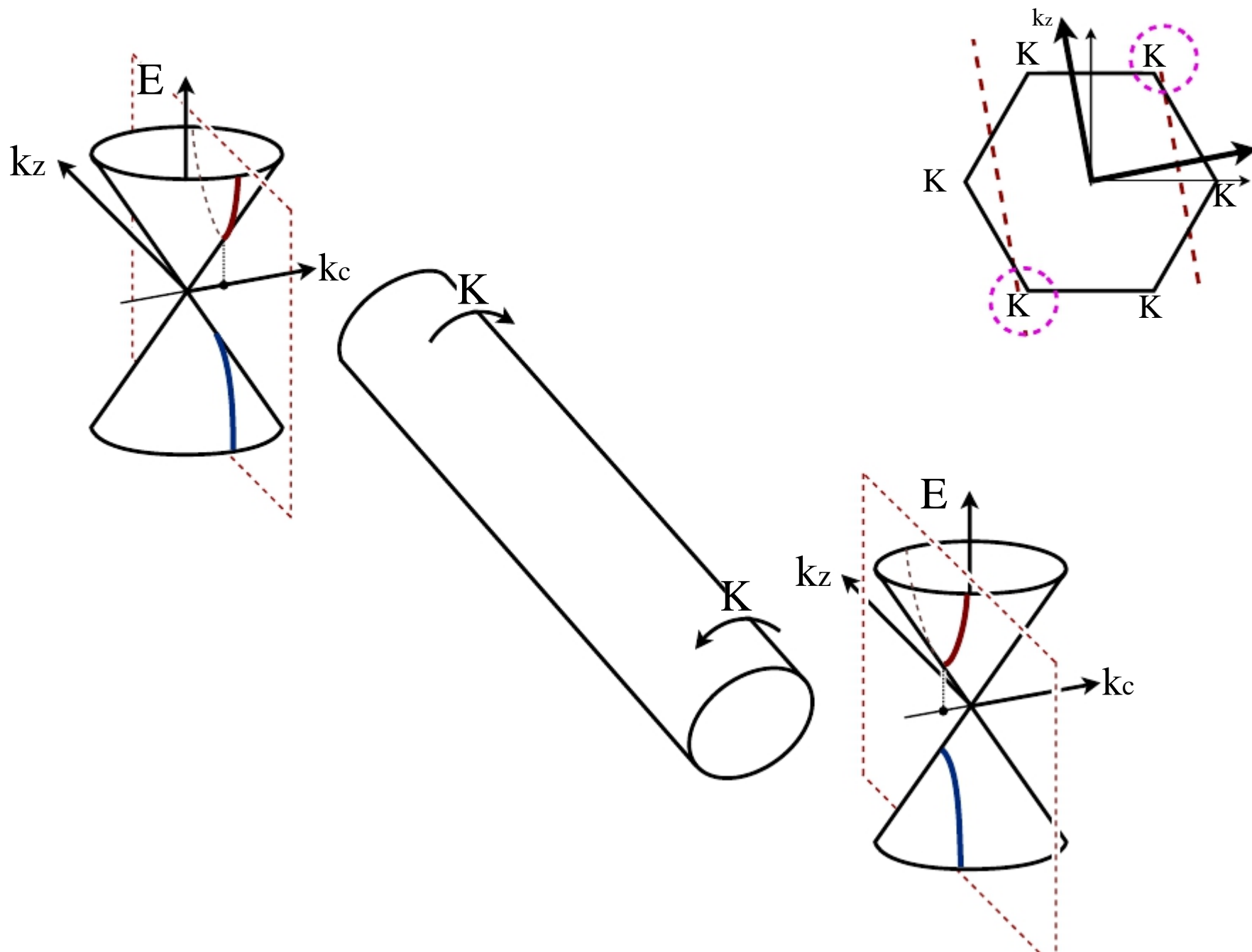
Conduction and valence bands touch at Fermi Level



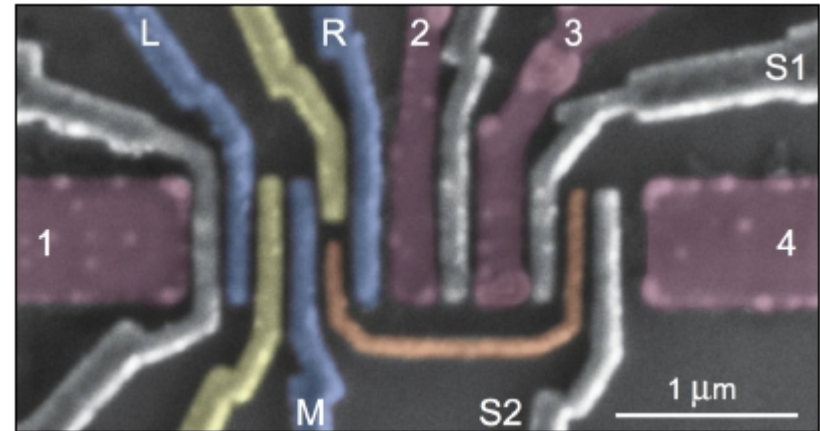
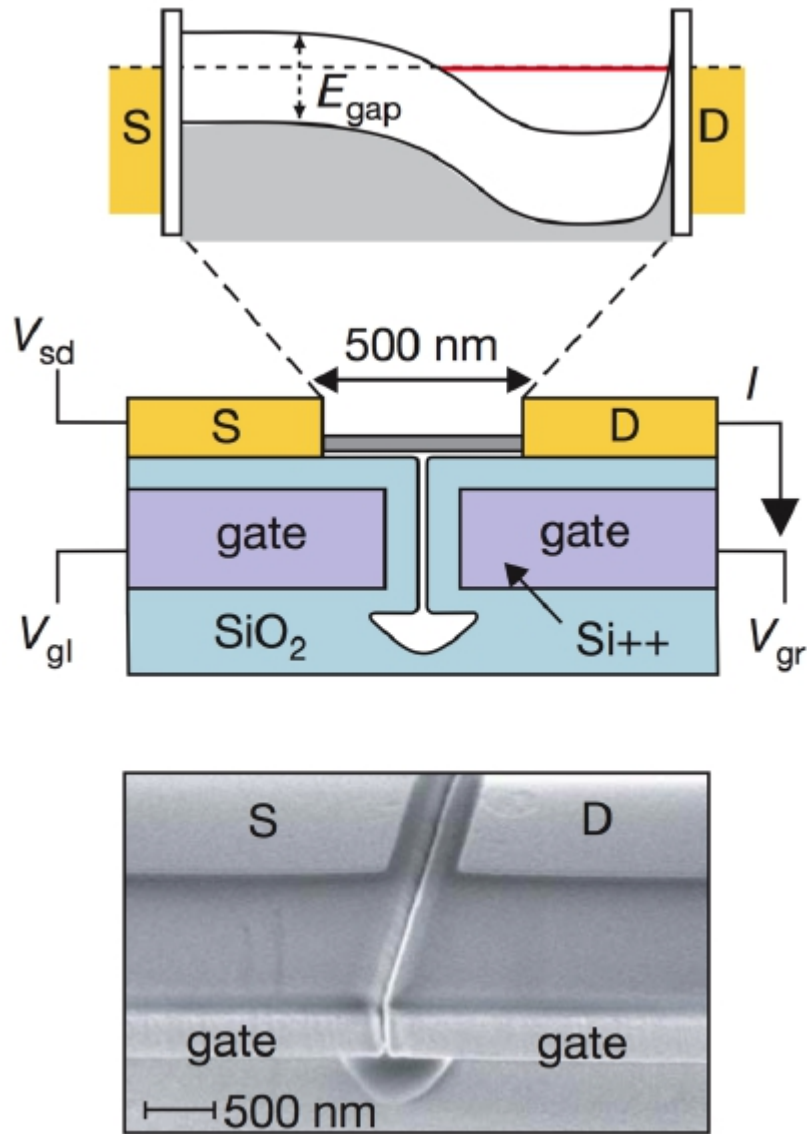
Nanotube formed by “rolling up” graphene



States near K and K carry opposite circulating currents



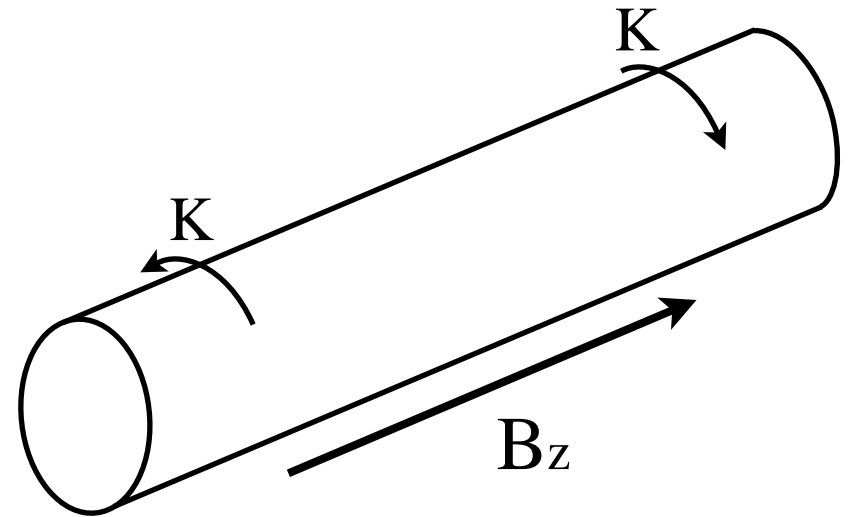
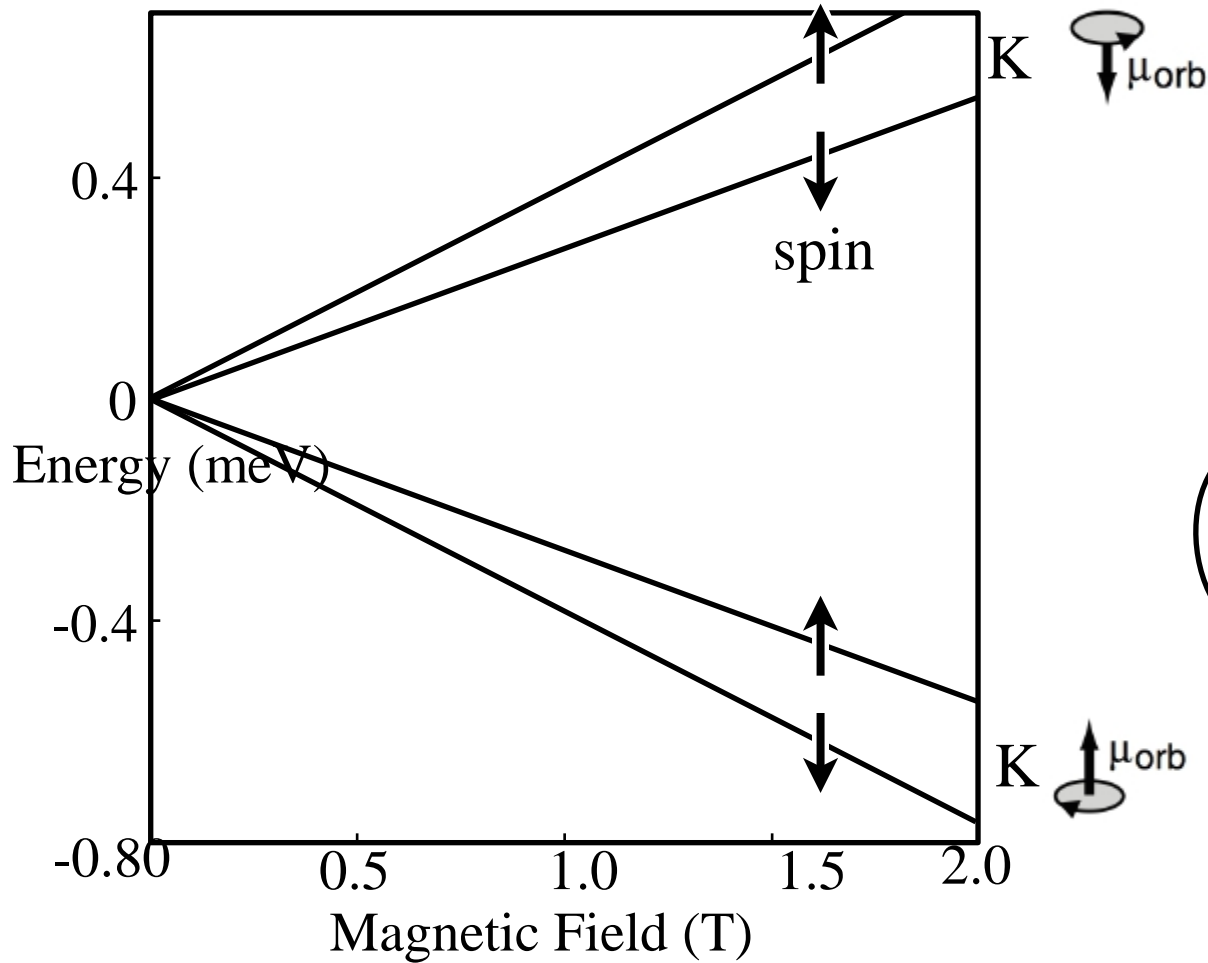
Electrons confined to quantum dots via electrostatic gates



H.O.H.Churchill, et al., Phys.Rev.Lett. **102**, 166802 (2009)

Degeneracy lifted in applied field by orbital, spin moments

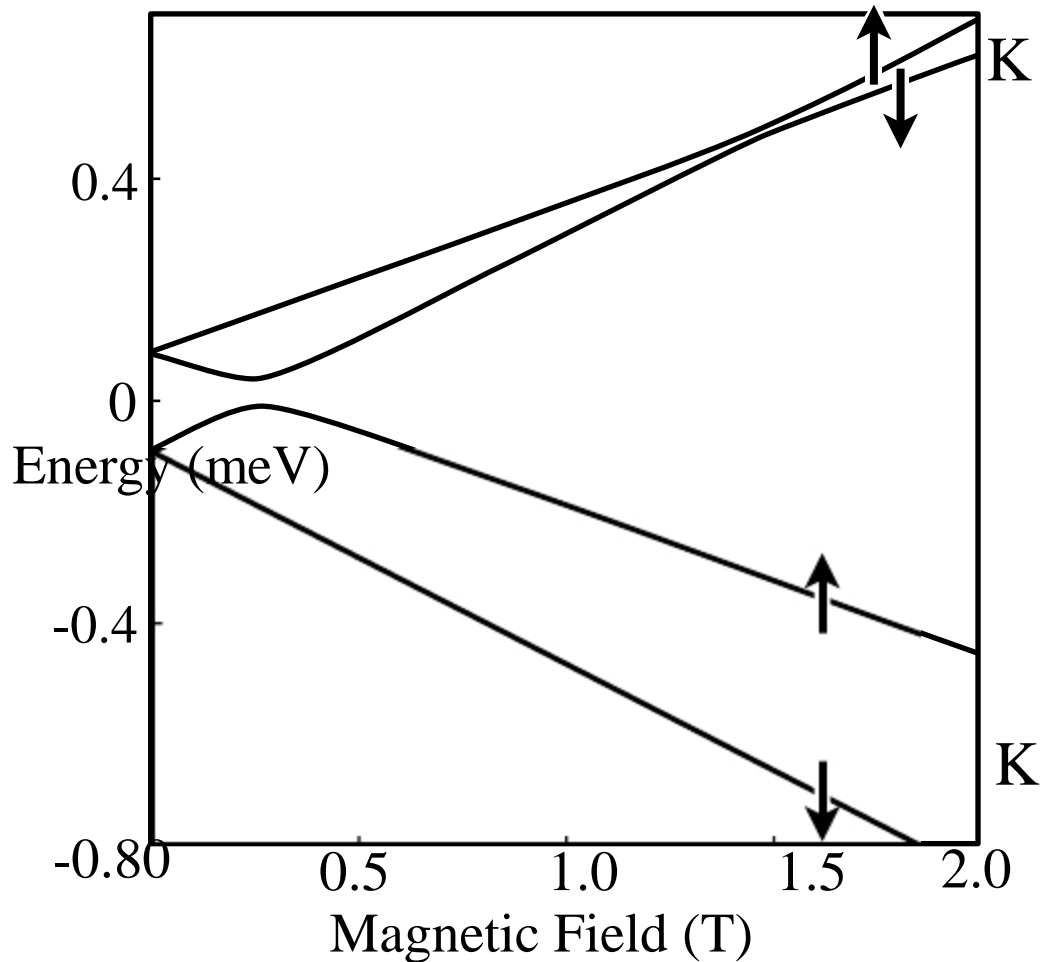
Single localized electron spectrum



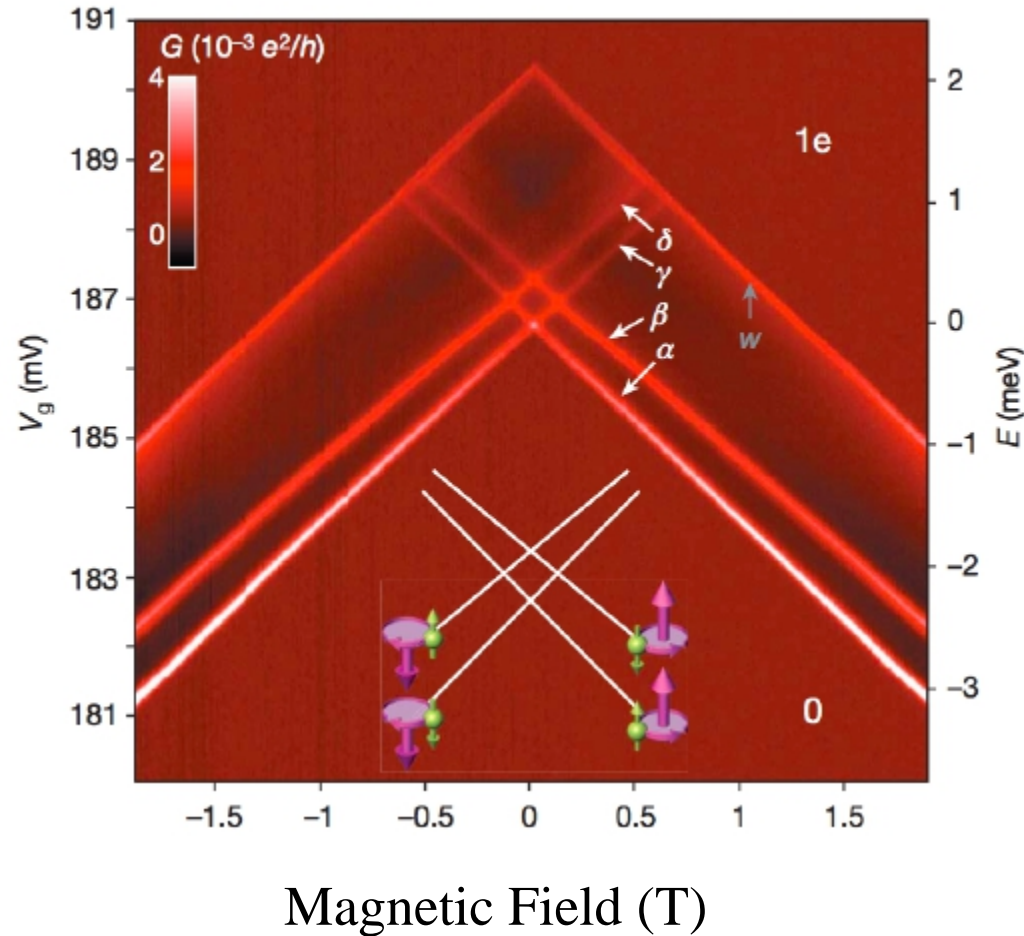
$$\mu_{orb} = \frac{eL}{2m_e} \quad 10\mu_B$$

Degeneracy partially lifted at $B = 0$ by spin-orbit coupling

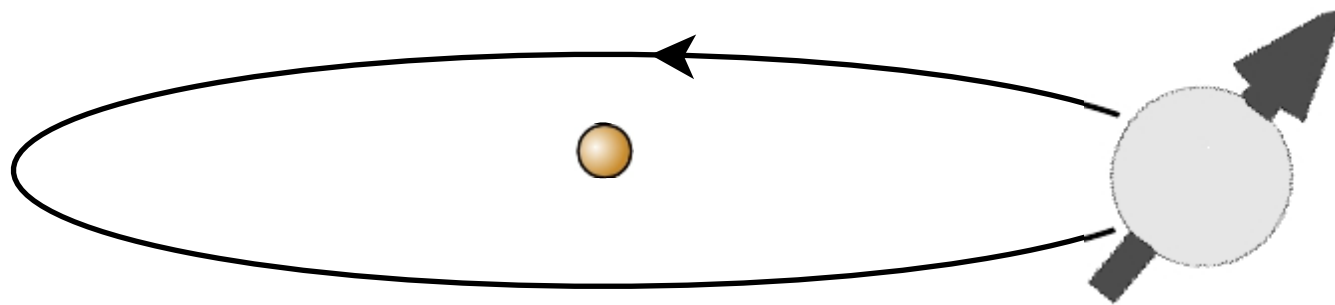
H.O.H.Churchill, et al., Phys.Rev.Lett. **102**, 166802 (2009)



F.Kuemmeth, S.Irani, D.C.Ralph, and P.L.McEuen, Nature **452**, 448 (2008)



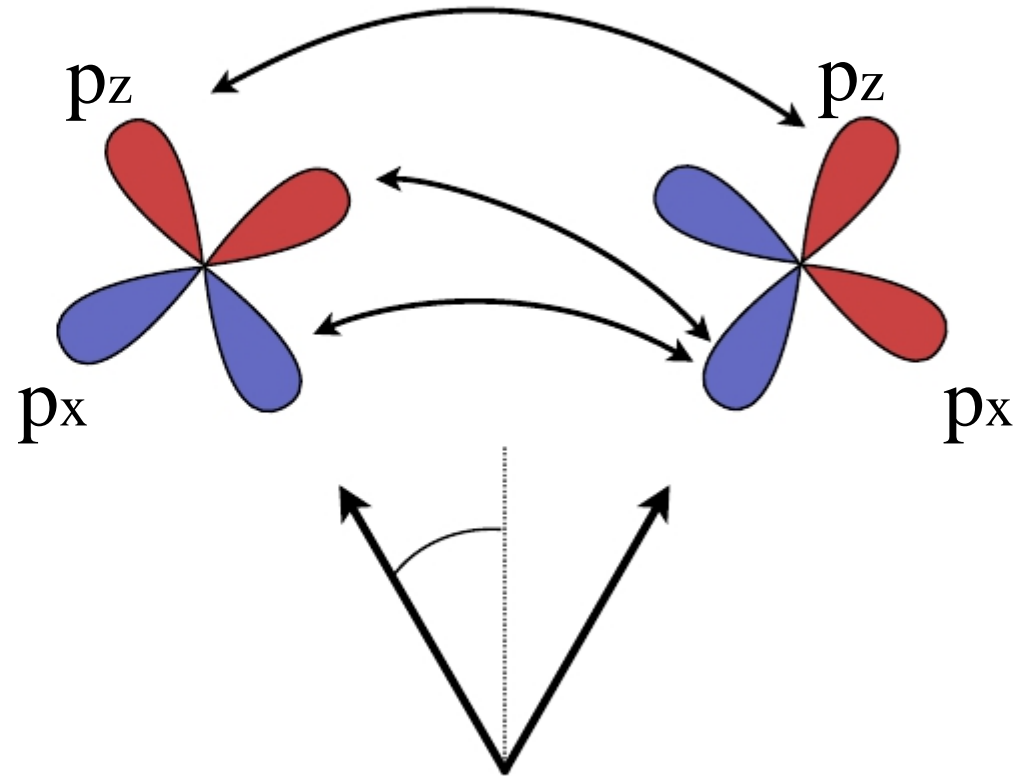
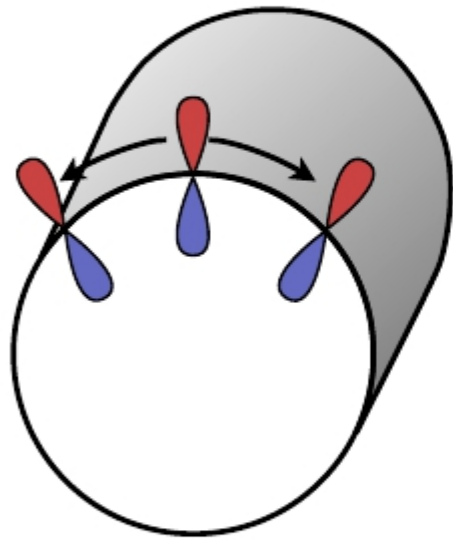
Relativity: orbiting electron 'sees' magnetic field from nucleus



$$\text{ESO} \leftarrow \dots \text{L} \cdot \text{S}$$

orbital angular momentum spin

(Atomic spin orbit) + (nanotube curvature) = sizable effect



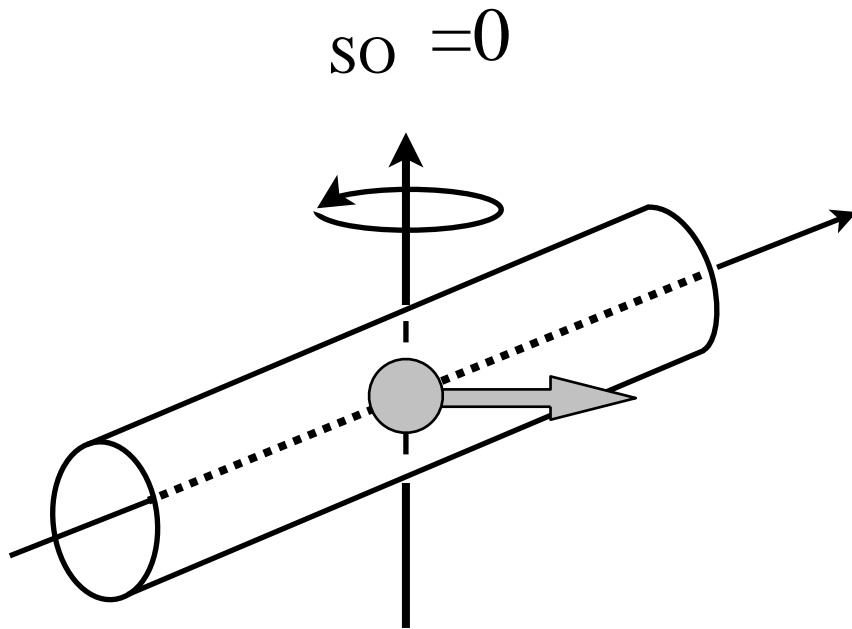
T.Ando,J.Phys.Soc.Japan **69**,1757 (2000)

D.Huertas-Hernando,F.Guinea,and A.Brataas,Phys.Rev.B **74**,155426 (2006)

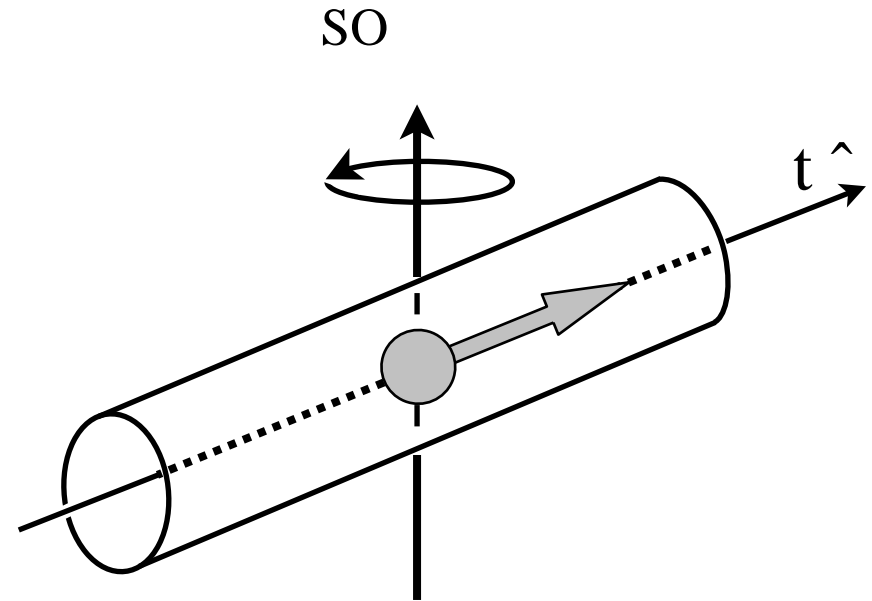
F.Kuemmeth,S.Ilani,D.C.Ralph,and P.L.McEuen,Nature **452**,448 (2008)

W.Izumida,K.Sato,and R.Saito,J.Phys.Soc.Japan **78**,074707 (2009)

Spin-orbit coupling “locks” spin to nanotube axis



Spin Stationary

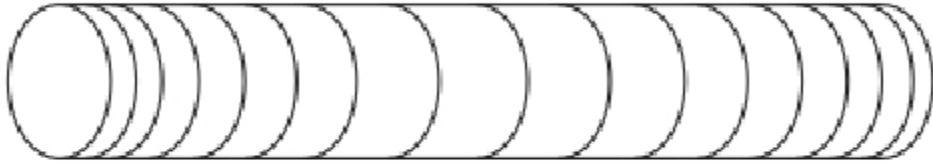


Spin Follows Tube

$$H_{SO} = \frac{SO}{2} \sigma(\hat{t} \cdot \mathbf{s})$$

+/- 1 for K/K' valley

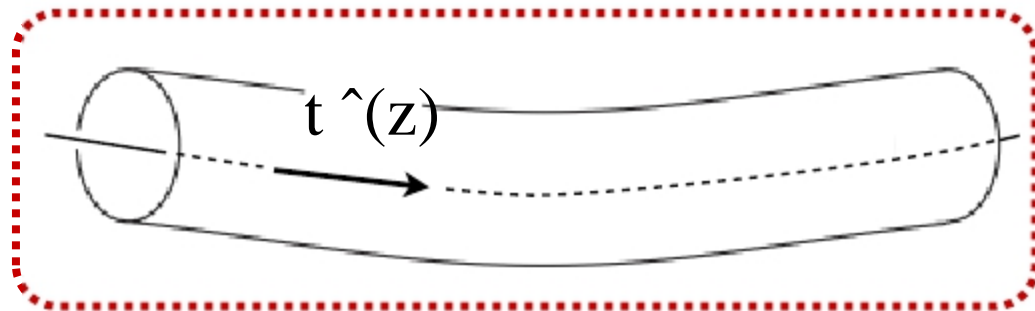
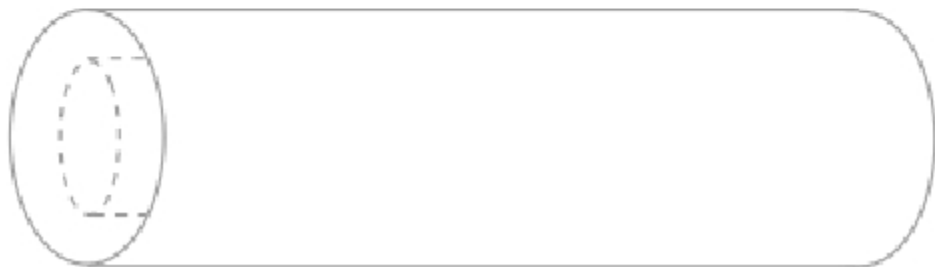
Nanotube supports many vibrational modes



E.Mariani and F.von Oppen,Phys.Rev.B **80**,155411 (2009)

G.D.Mahan,Phys.Rev.B **65**,235402 (2002); H.Suzuura andT.Ando,Phys.Rev.B **65**,235412 (2002)

In flexural mode, local tube axis direction fluctuates



$$q = q^2$$

E.Mariani and F.von Oppen,Phys.Rev.B **80**,155411 (2009)

G.D.Mahan,Phys.Rev.B **65**,235402 (2002); H.Suzuura andT.Ando,Phys.Rev.B **65**,235412 (2002)

Outline

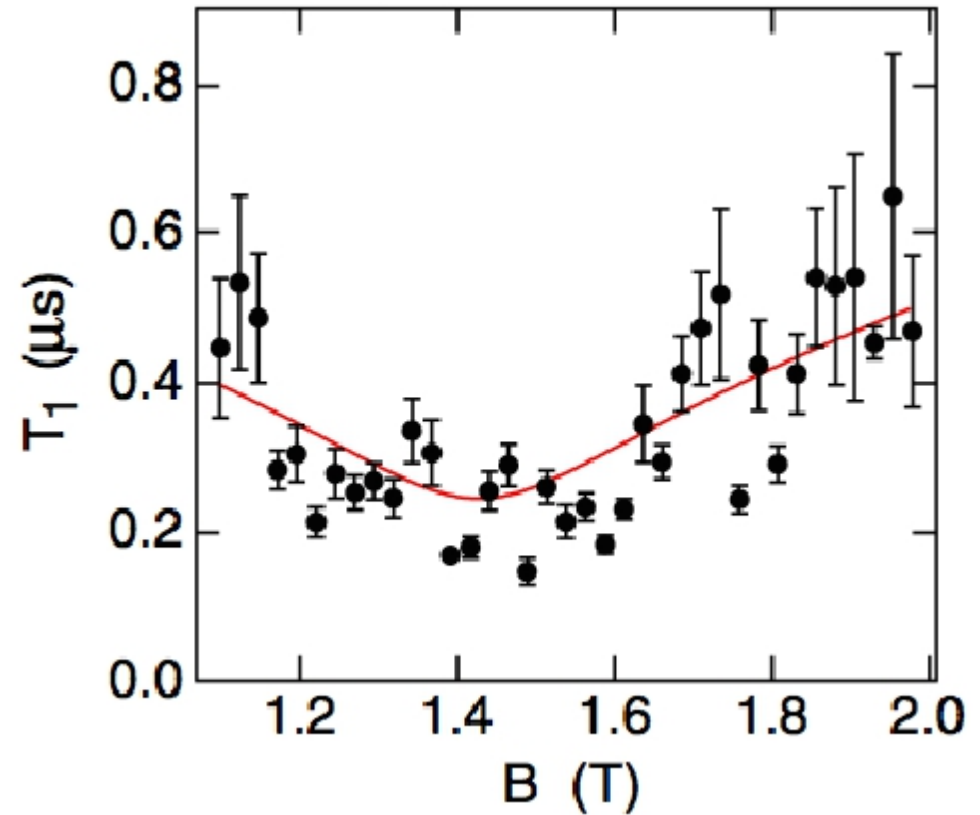
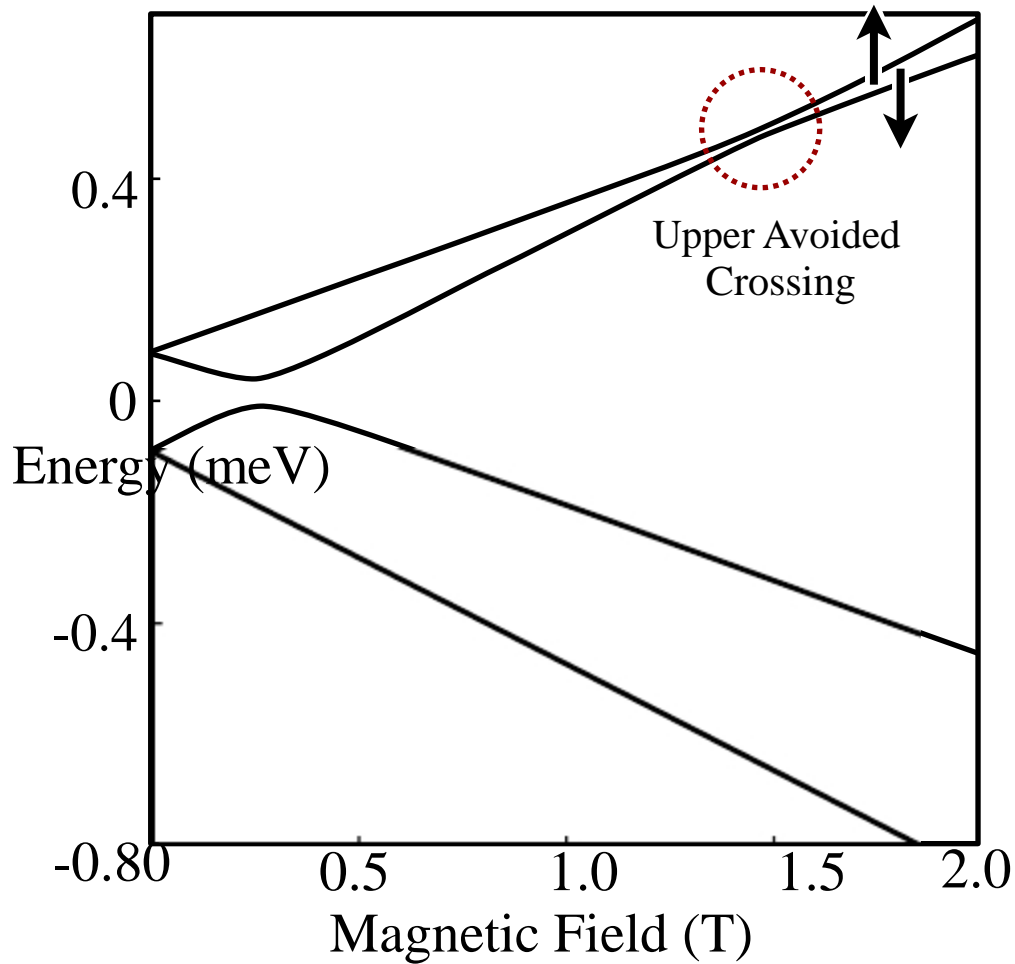
~~I. Nanotubes basics, spin-orbit coupling and phonons~~

II. Electron spin relaxation in CNT quantum dots

~~III. Coupling to phonon “cavity” mode in suspended CNT~~

Observation: minimum in spin lifetime T_1 near narrow avoided level crossing

H.O.H.Churchill, et al., Phys.Rev.Lett. **102**, 166802 (2009)



Through spin-orbit interaction, spin couples to local direction of tube axis (deflection)

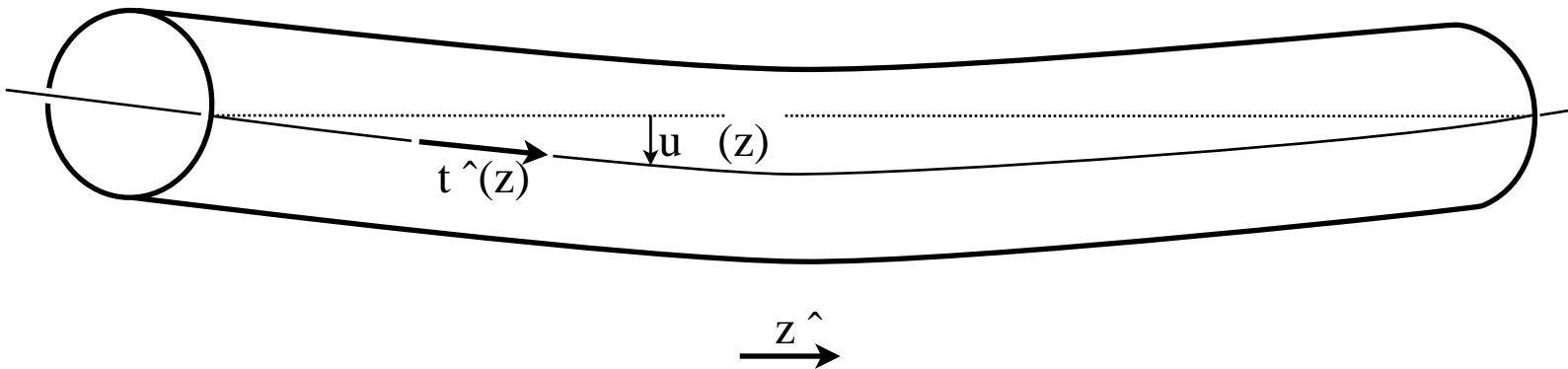
$$\hat{t}(z) = \hat{z} + du(z)/dz$$

Zero-order (unperturbed) Hamiltonian:

$$H_0 = \frac{SO}{2} 3S_z \quad \mu_{orb} 3B_z + \mu_B(s \cdot B)$$

Spin-deflection (phonon) coupling:

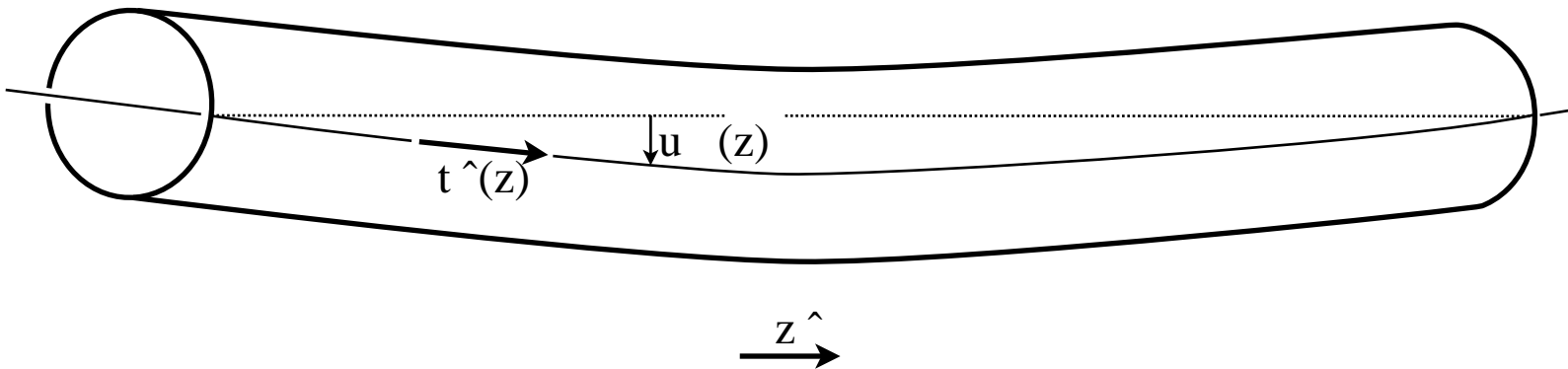
$$H_{s \text{ ph}} = \frac{SO}{2} \frac{du}{dz} \cdot s \quad \mu_{orb} \frac{du}{dz} \cdot B$$



Spin couples to phonons through deflections

$$u(z) = \frac{1}{2 L_z} \sum_q \hat{X}_q (a_q + a_q^\dagger) e^{iqz}; \quad q = \frac{2\pi}{L_z} n$$

L_z nanotube mass per unit length
 L_z nanotube length
 \hat{X}_q phonon polarization vector
 a_q^\dagger phonon creation operator



Calculate spin relaxation rate from Fermi's Golden Rule

$$W_{fi} = \frac{2}{2\pi} \overline{|\langle \psi_f | H_{s-ph} | \psi_i \rangle|^2} \frac{L_z}{2} \frac{dq}{d\epsilon}$$

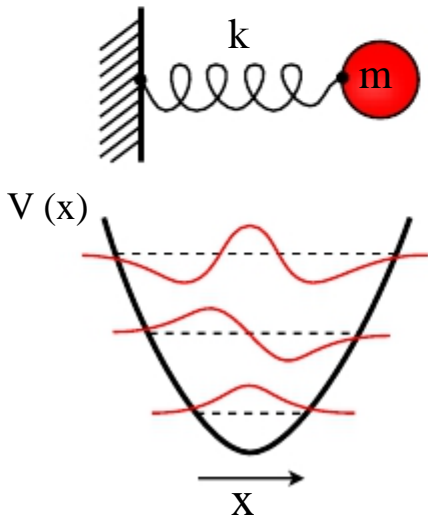
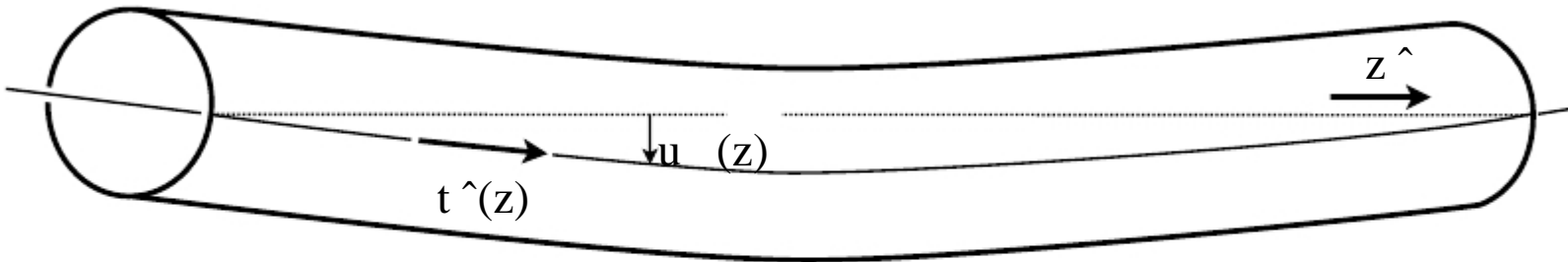
Overbar indicates average over thermal distribution of phonons

L_z	nanotube length
$ \psi_i\rangle$	electron initial state
$ \psi_f\rangle$	electron final state

Vibration amplitudes grow large for long wavelengths

Normal modes: $u = A e^{iqz}$, $\frac{du}{dz} = qA$

$\frac{du}{dz}$ ← deflection brings down one factor of q



Vibration amplitude (rough estimate):

$$\frac{1}{2} m \langle x^2 \rangle = \frac{1}{2} \frac{\hbar^2}{m \omega} \left(n + \frac{1}{2} \right)$$

<u>Low Temperature</u>	<u>High Temperature</u>
$qA = q \cdot \frac{\hbar^2}{M q}$ const	$qA = q \cdot \frac{k_B T}{M q^2} = \frac{1}{q}$

$$q = q^2$$

Relaxation rate shows divergence at small energy transfer

$$W_{fi} = \frac{2}{2} \frac{|\langle f | H_s | i \rangle|^2}{2} \frac{L_z}{2} \frac{dq}{dz} \quad q \rightarrow 0$$

1) At low temperature, $\frac{du}{dz} \rightarrow \text{const}$; at high temperature, $\frac{du}{dz} \rightarrow 1/2$

2) Van Hove singularity in density of states contributes factor of $1/2$ to the rate

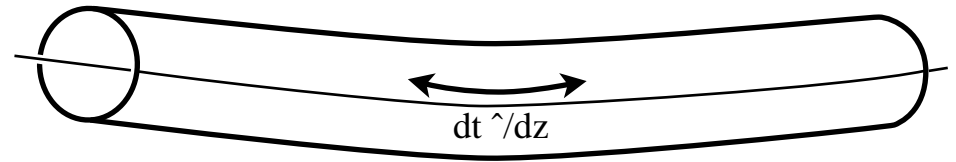
<u>Low Temperature</u>	<u>High Temperature</u>
$1/T_1$ $1/2$	$1/T_1$ $3/2$

Compare with “usual” deformation potential mechanism

Deflection q



Deformation q_2



Rate for deformation potential mechanism suppressed by relative factor q_2

<u>Low Temperature</u>	
$1/T_1$	$1/2$

<u>High Temperature</u>	
$1/T_1$	$3/2$

<u>Low Temperature</u>	
$1/T_1$	$1/2$

<u>High Temperature</u>	
$1/T_1$	$1/2$

Outline

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~~II. Electron spin relaxation in CNT quantum dots~~

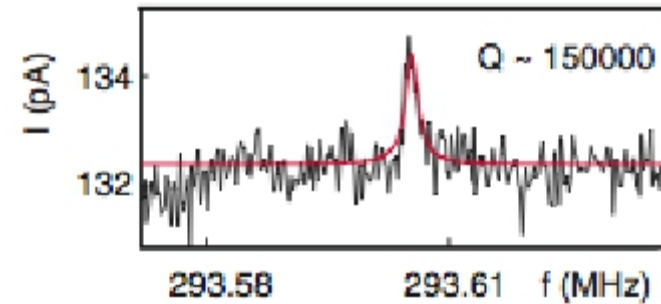
III. Coupling to phonon “cavity” mode in suspended CNT

Ultraclean suspended CNTs act as “phonon cavity”

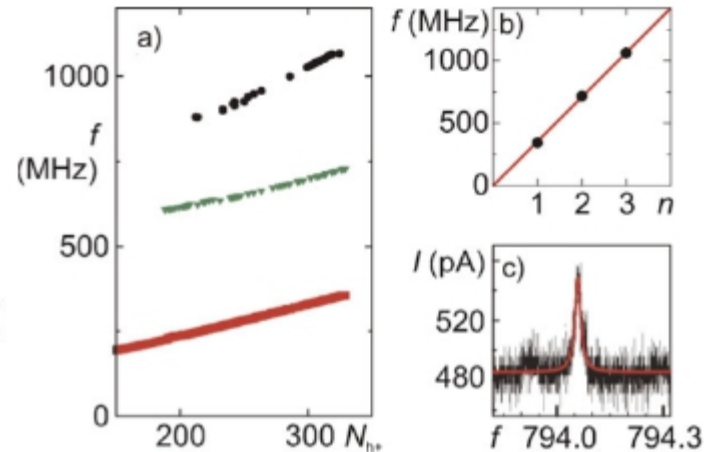
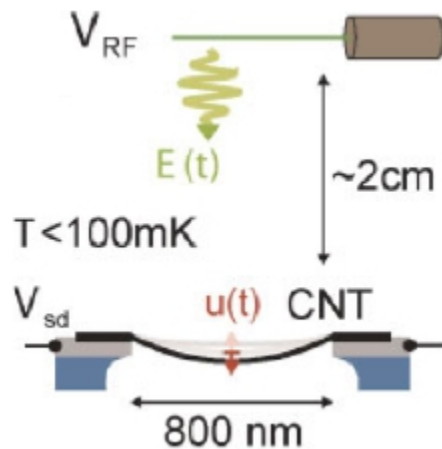
Strong Coupling Between Single-Electron Tunneling and Nanomechanical Motion

G. A. Steele,* A. K. Hüttel,† B. Witkamp, M. Poot, H. B. Meerwaldt, L. P. Kouwenhoven, H. S. J. van der Zant

G.A.Steele et al., Science **325**,1103 (2009)



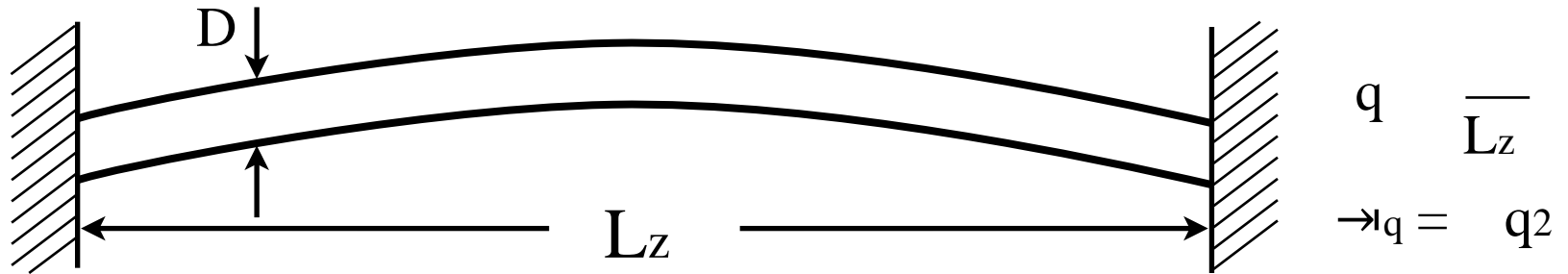
A quality factor exceeding 10^5



A.K. Hüttel, et al., Phys. Status Solidi B **247**, 2974 (2010).

see also: B. Lassagne et al., Science **325**,1107 (2009)

Outlook: strong (coherent) coupling regime within reach



Coupling strength:

$$H \rightarrow g(\hat{a} + \hat{a}^\dagger), \quad g = \frac{\hbar \omega}{2} \frac{D}{2M} \approx 10 \text{ neV}$$

Zeeman energy, oscillator frequency:

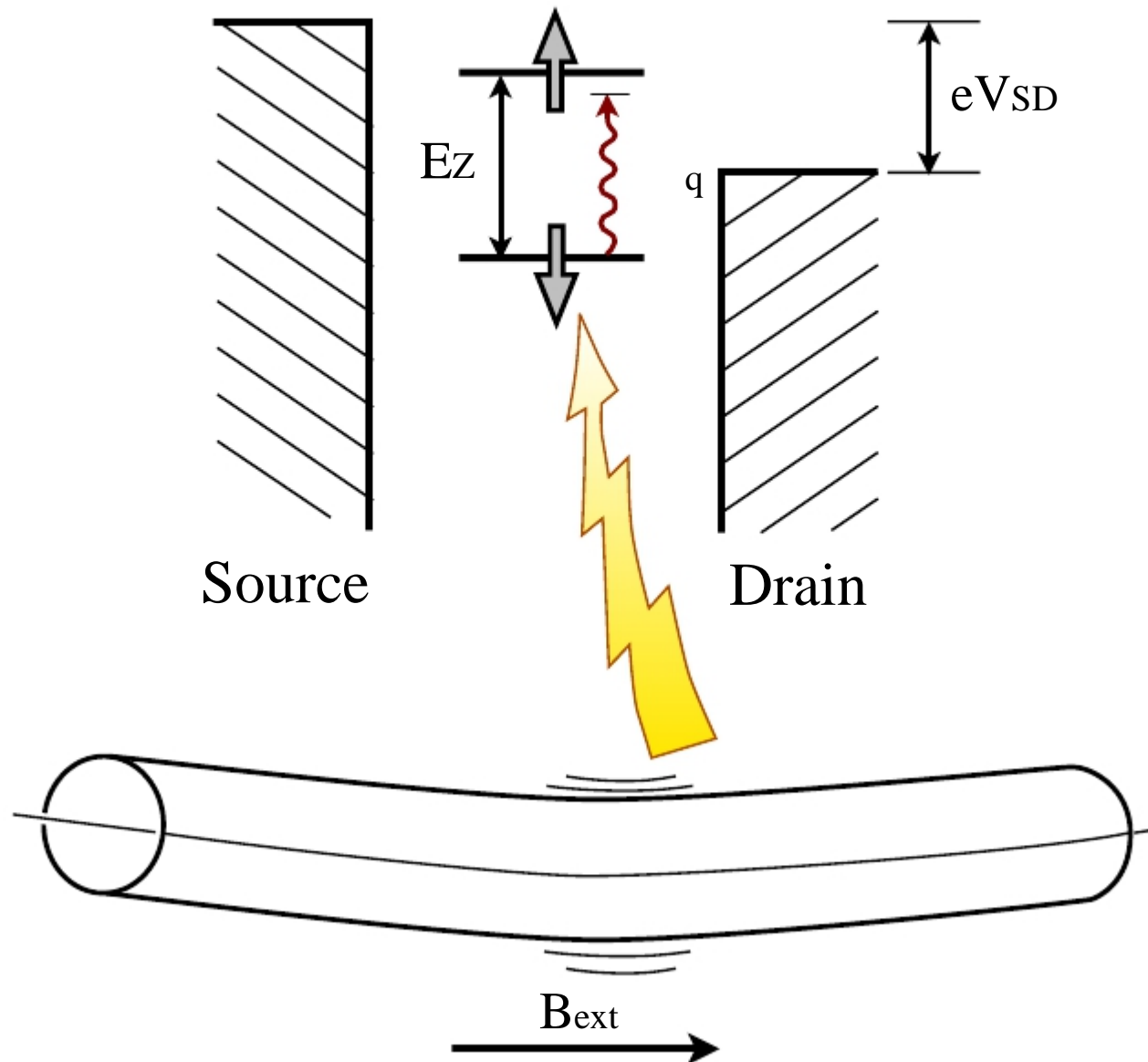
$$E_Z = \hbar q \approx k_B T = 10 \mu\text{eV}$$

Coupling parameter:

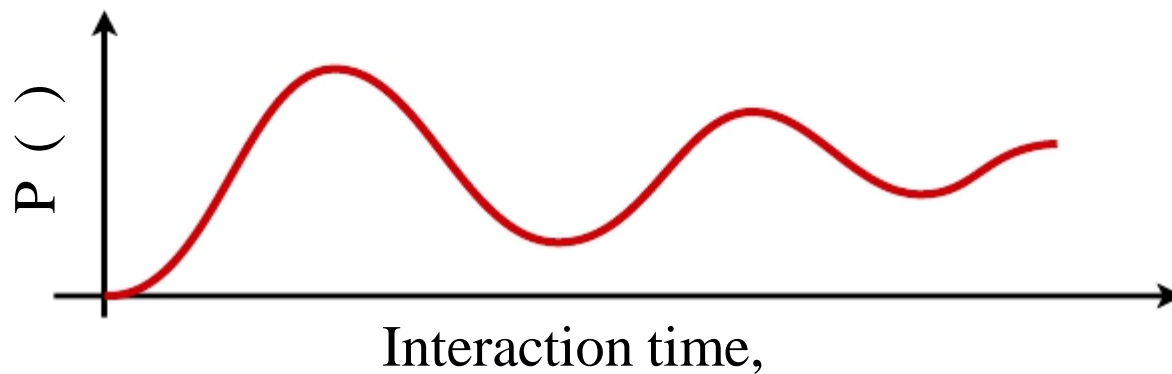
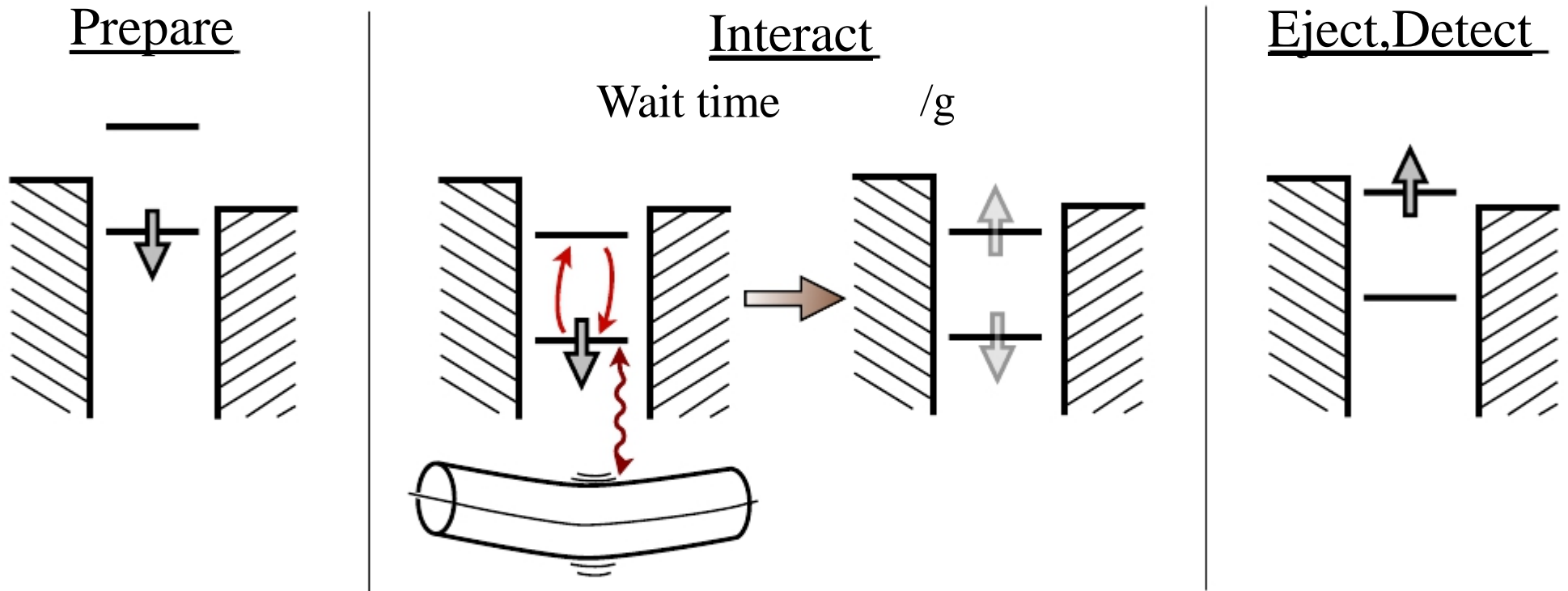
$$g \approx \frac{Q}{q} \approx 10 \text{ -- } 100$$

$$\begin{aligned}
 M &= D L_z \rightarrow s \\
 s &= 10^{-24} \text{ kg/nm}^2 \\
 &= 10^4 \mu\text{eV nm}^2 \\
 Q &= 10^5
 \end{aligned}$$

Coupling leads to enhancement of transport near resonance



Pulsed gate measurements may reveal coherent oscillations



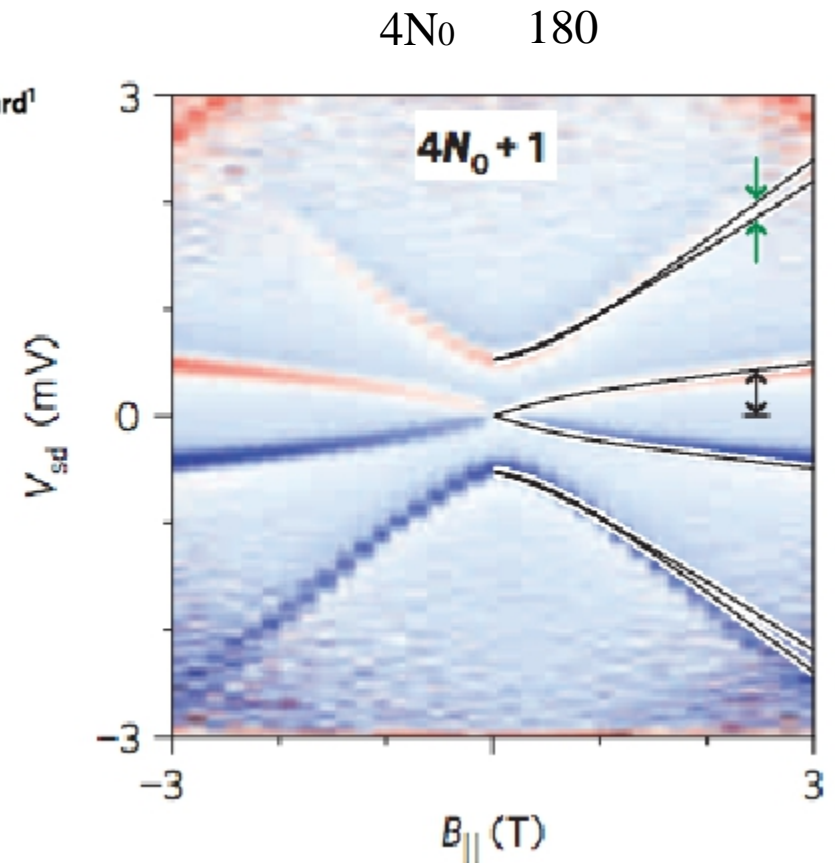
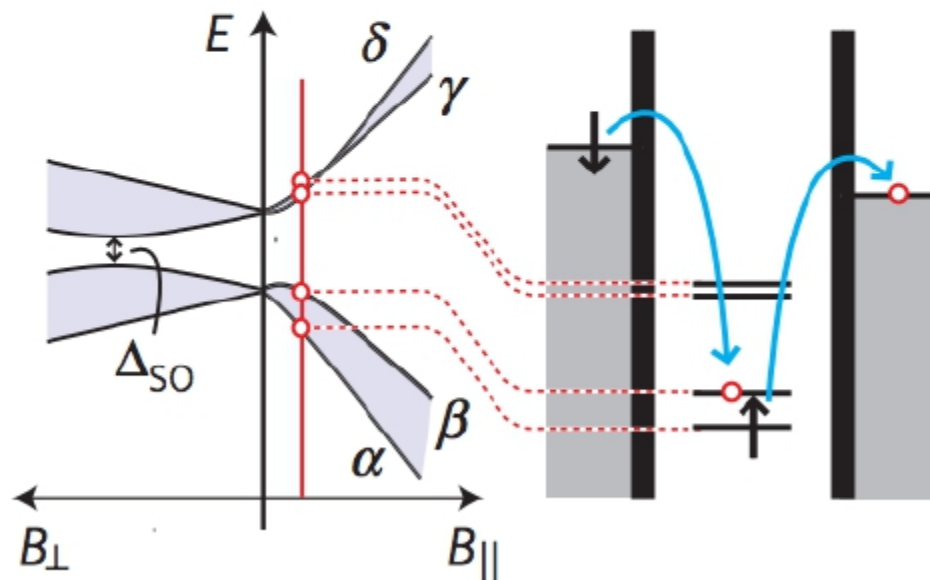
Sense motion of CNT via coupling to microwave cavity?

Concern: typical vibration amplitudes in 1-10 pm range

Possible solution: use large electron number to get sizable dipole moment

Gate-dependent spin-orbit coupling in multielectron carbon nanotubes

T. S. Jespersen^{1*†}, K. Grove-Rasmussen^{1,2*†}, J. Paaske¹, K. Muraki², T. Fujisawa³, J. Nygård¹ and K. Flensberg¹ **NATURE PHYSICS** | VOL 7 | APRIL 2011 |



Quantum dots in suspended CNTs might make good qubits

- 😊 CNTs can be grown nearly nuclear spin free
- 😊 Phonon cavity eliminates bath density of states at qubit energy
- 😊 Coupling to external antenna for electrical control, readout
- 😐? Measurement sensitivity?
- 😐? Multi-qubit coupling?
- 😞 Difficult to fabricate

