A Primer on the Virtual Source Model for Nanoscale MOSFETs

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the MIT VS Model

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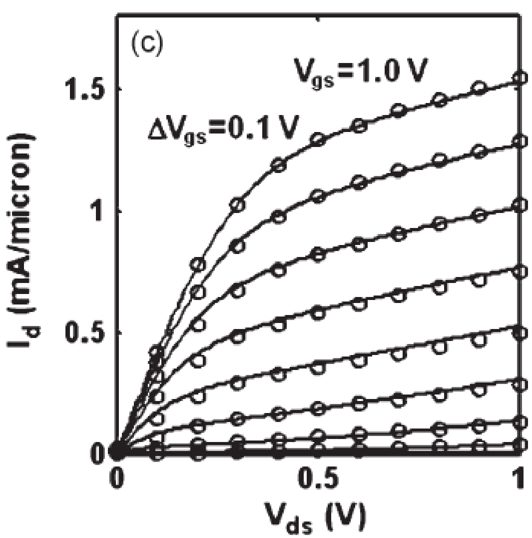
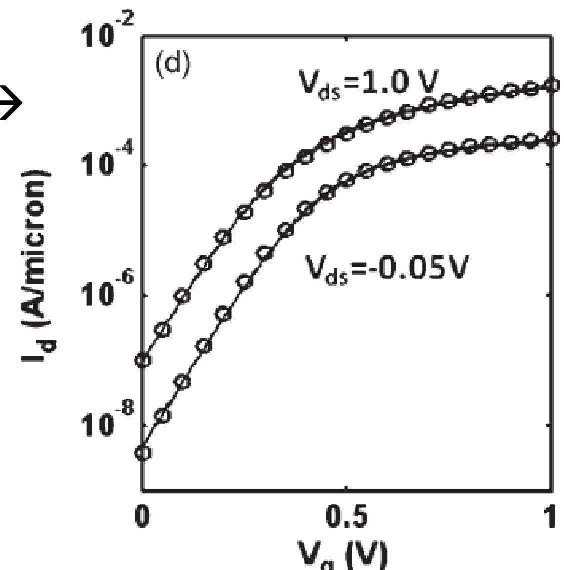
A Simple Semiempirical Short-Channel MOSFET Current–Voltage Model Continuous Across All Regions of Operation and Employing Only Physical Parameters

Ali Khakifirooz, *Member, IEEE*, Osama M. Nayfeh, *Member, IEEE*, and Dimitri Antoniadis, *Fellow, IEEE*



← 32 nm technology →

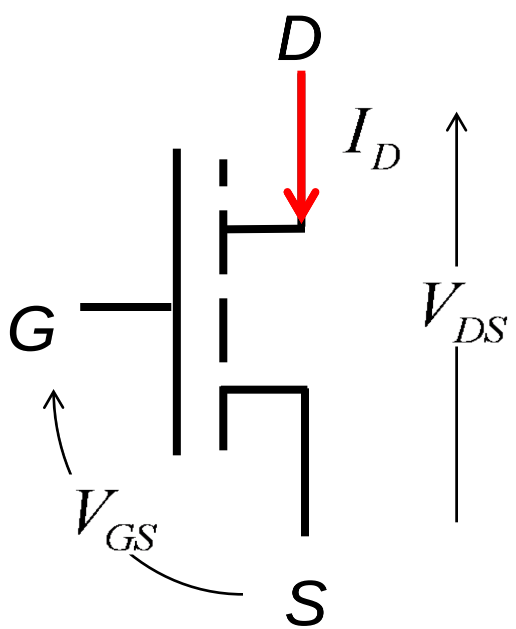


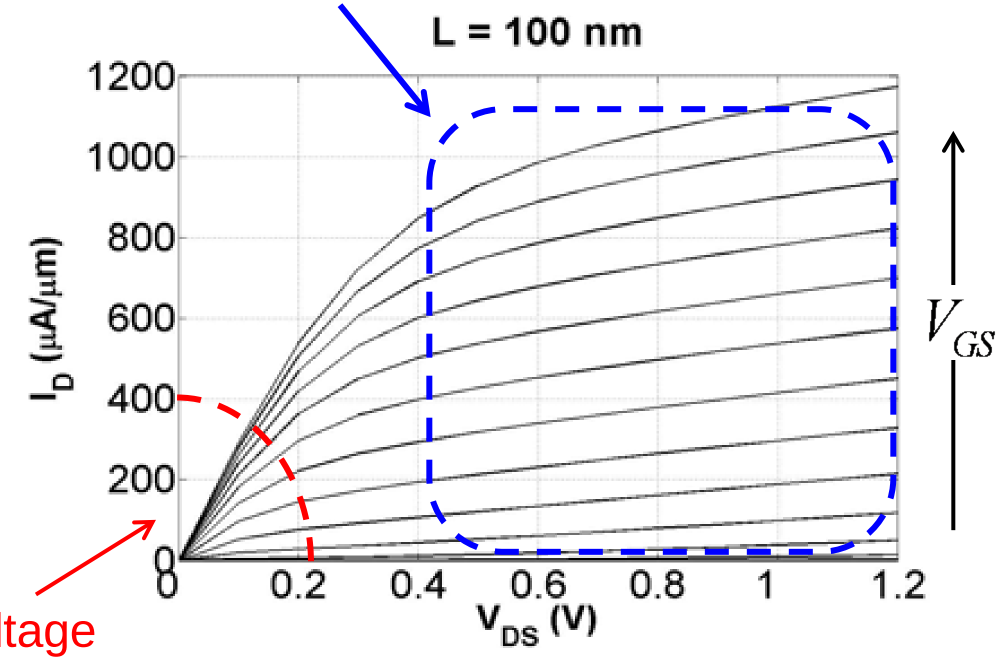
outline

1) Traditional MOSFET theory
2) VS model: above threshold
3) VS model: subthreshold
4) VS model: quasi-ballistic and ballistic
5) Summary

MOSFET IV characteristic

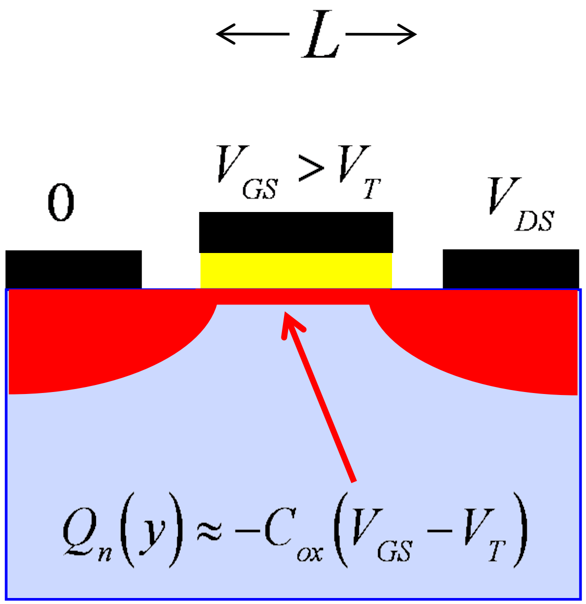
circuit symbol

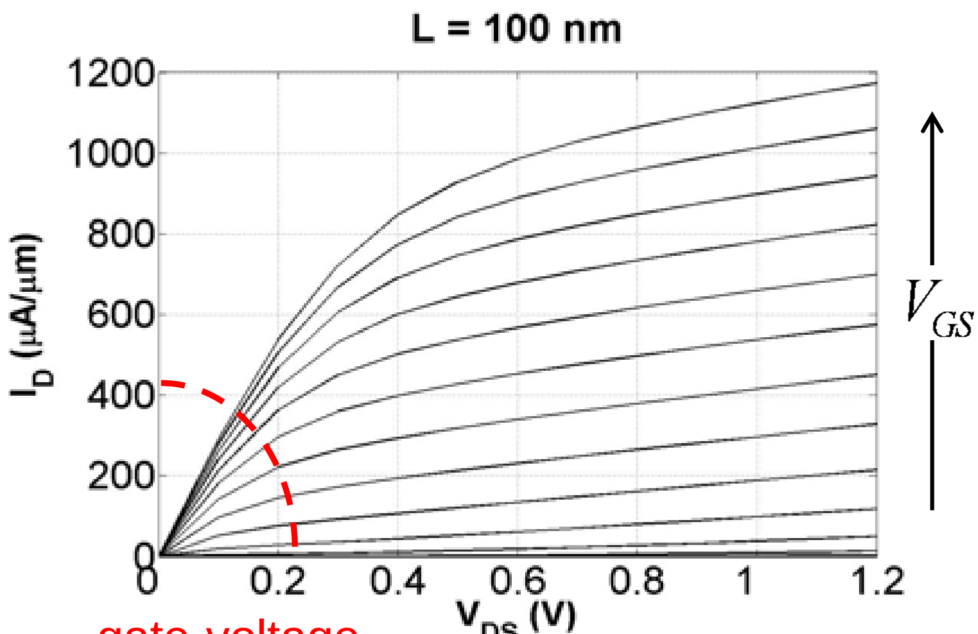




(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

MOSFET IV: low V_{DS}





gate-voltage controlled resistor

$$I_D = -W\,Q_n(y)\langle \upsilon_y(y)\rangle$$

$$Q_n = -C_{ox}\left(V_{GS} - V_T\right)$$

$$\langle \upsilon_y(y)\rangle = -\mu_n \mathcal{E}_y$$

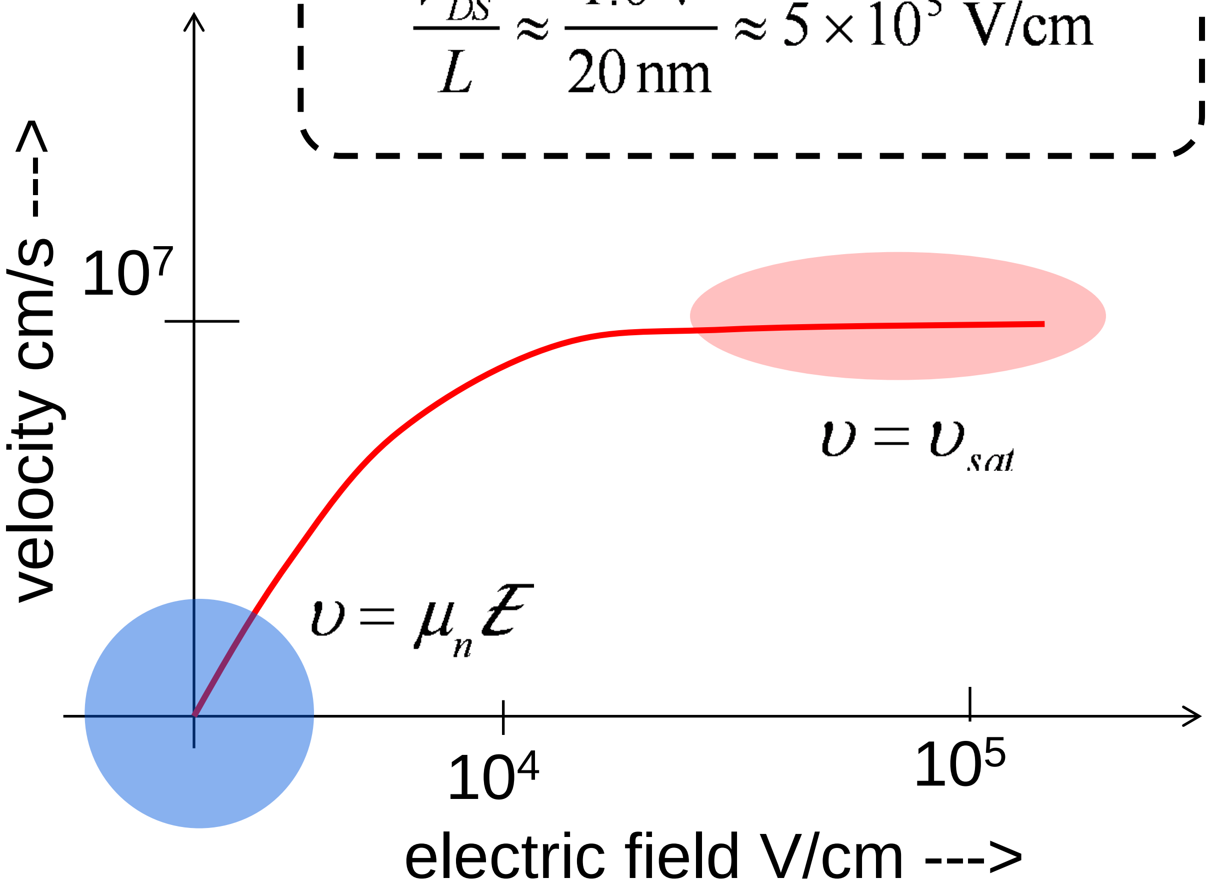
$$\mathcal{E}_y = -V_{DS}/L$$

$$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

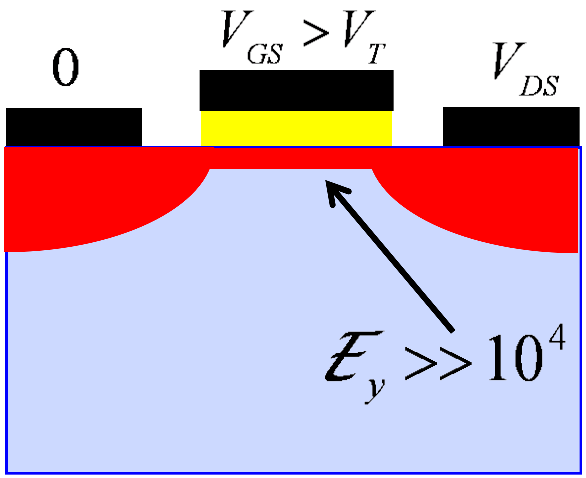
✓

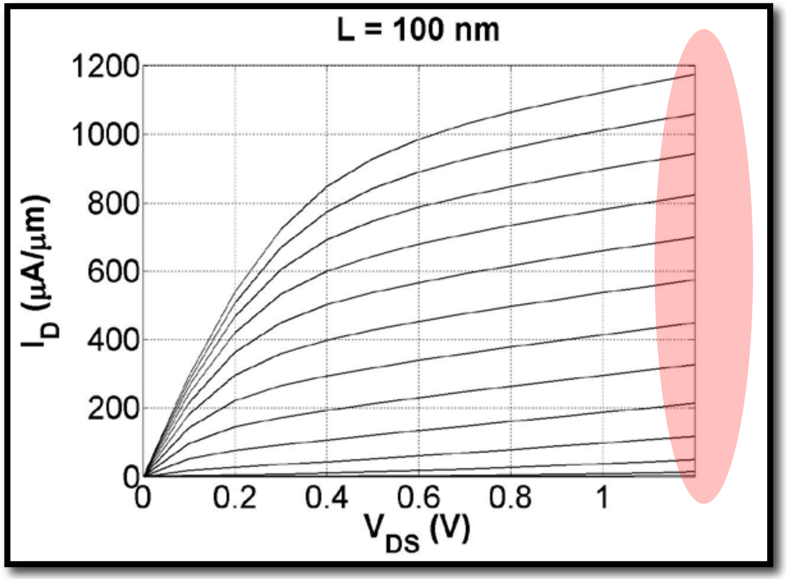
High V_{DS}: velocity saturation

$$\frac{V_{DS}}{L} \approx \frac{1.0\,\text{V}}{20\,\text{nm}} \approx 5\times 10^{5}\ \text{V/cm}$$



MOSFET IV: velocity saturation





(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$I_D = -W\,Q_n(y)\langle \upsilon_y(y)\rangle$$

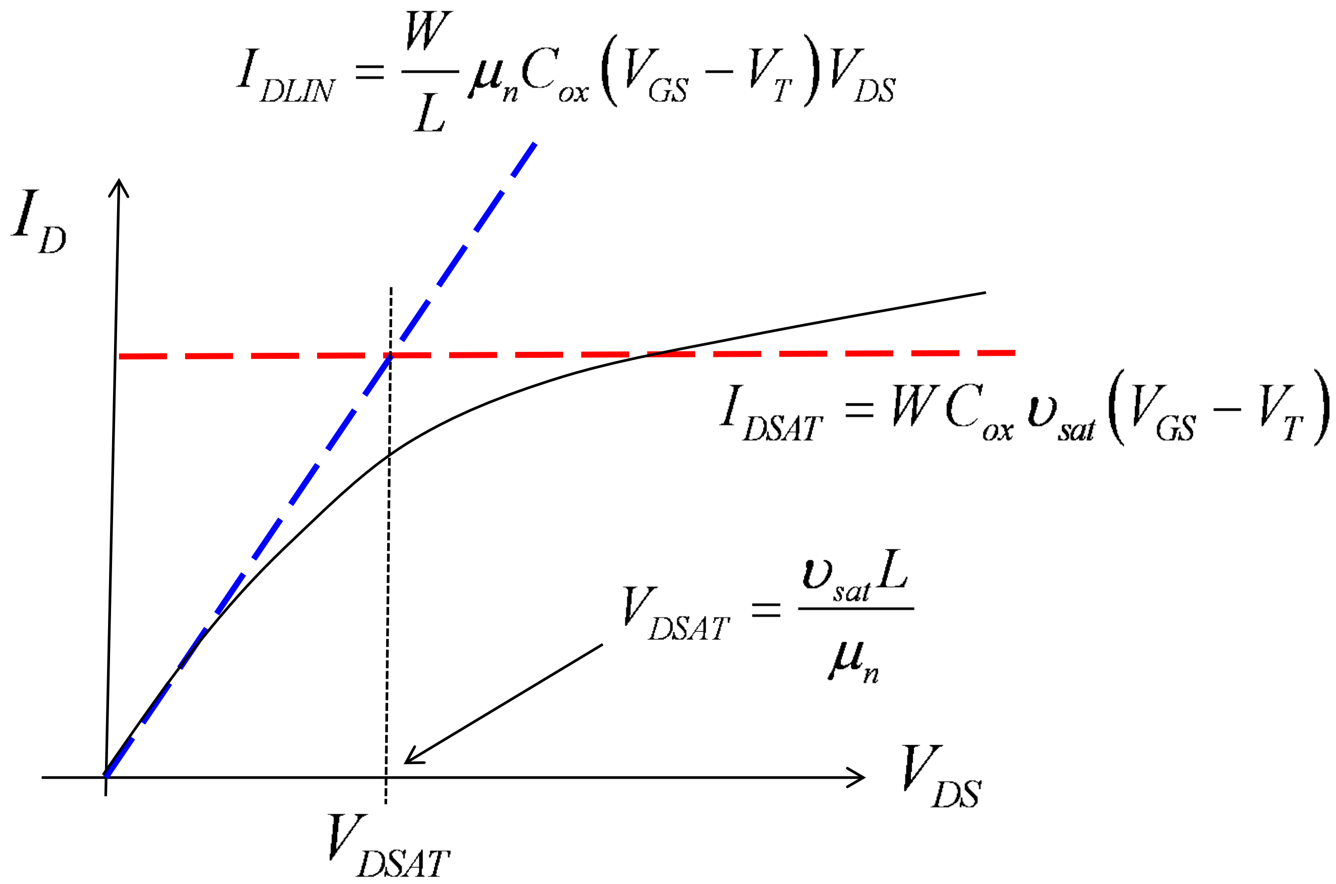
$$Q_n = -C_{ox}\left(V_{GS} - V_T\right)$$

$$\langle \upsilon_y \rangle = \upsilon_{sat}$$

$$I_D = W\,C_{ox}\,\upsilon_{sat}\left(V_{GS} - V_T\right)$$

✓

MOSFET: IV (re-cap)



We have developed a 2-piece approximation to the MOSFET IV characteristic.

piecewise model for $I_D(V_{GS}, V_{DS})$

$$I_D/W = -Q_n(V_{GS})\langle \upsilon(V_{DS})\rangle$$

$$V_{GS} \geq V_T : \quad Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$$

$$V_{GS} < V_T : \quad Q_n(V_{GS}) = 0$$

$$V_{DS} \leq V_{DSAT} : \langle \upsilon(V_{DS})\rangle = \left(\mu_n \frac{V_{DS}}{L}\right)$$

$$V_{DS} > V_{DSAT} : \langle \upsilon(V_{DS})\rangle = \upsilon_{sat}$$

If we can make the average velocity go smoothly from the low V_{DS} to high V_{DS} limits, then we will have a smooth model for $I_D(V_{GS}, V_{DS})$ – above threshold.

From low V_{DS} to high V_{DS}

$$\frac{1}{\langle \upsilon(V_{DS}) \rangle} = \frac{1}{\mu_n V_{DS}/L} + \frac{1}{\upsilon_{sat}} \rightarrow$$

$$\langle \upsilon(V_{DS}) \rangle = \left[\frac{V_{DS}/V_{DSAT}}{1 + V_{DS}/V_{DSAT}} \right] \upsilon_{sat}$$

$$\langle \upsilon(V_{DS}) \rangle = F_{SAT}(V_{DS}) \upsilon_{sat}$$

$$F_{SAT}(V_{DS}) = \frac{V_{DS}/V_{DSAT}}{\left[1 + (V_{DS}/V_{DSAT})^{\beta} \right]^{1/\beta}}$$

The extra parameter, β, is empirically adjusted to fit the IV characteristic. Typically, $\beta \approx 1.4 - 1.8$ for both N-MOSFETs and for P-MOSFETs.

saturating function: $F_{SAT}(V_D)$

$$\langle \upsilon(V_{DS}) \rangle = F_{SAT}(V_{DS})\,\upsilon_{sat}$$

$$F_{SAT}(V_{DS}) = \frac{V_{DS}/V_{DSAT}}{\left[1+\left(V_{DS}/V_{DSAT}\right)^{\beta}\right]^{1/\beta}}$$

Although this is just an empirical method to produce smooth curve that properly goes between the small and large V_D limits, it works very well in practice, which suggests that it captures something important about MOSFETs.

level 0 model

1) $I_D/W = -Q_n(V_G)\langle\upsilon(V_{DS})\rangle$

2) $V_{GS} \le V_T: \quad Q_n(V_{GS}) = 0$

$V_{GS} > V_T: \quad Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$

3) $\langle\upsilon(V_{DS})\rangle = F_{SAT}(V_{DS})\upsilon_{sat}$

4) $F_{SAT}(V_{DS}) = \dfrac{V_{DS}/V_{DSAT}}{\left[1+(V_{DS}/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{sat}L}{\mu_n}$

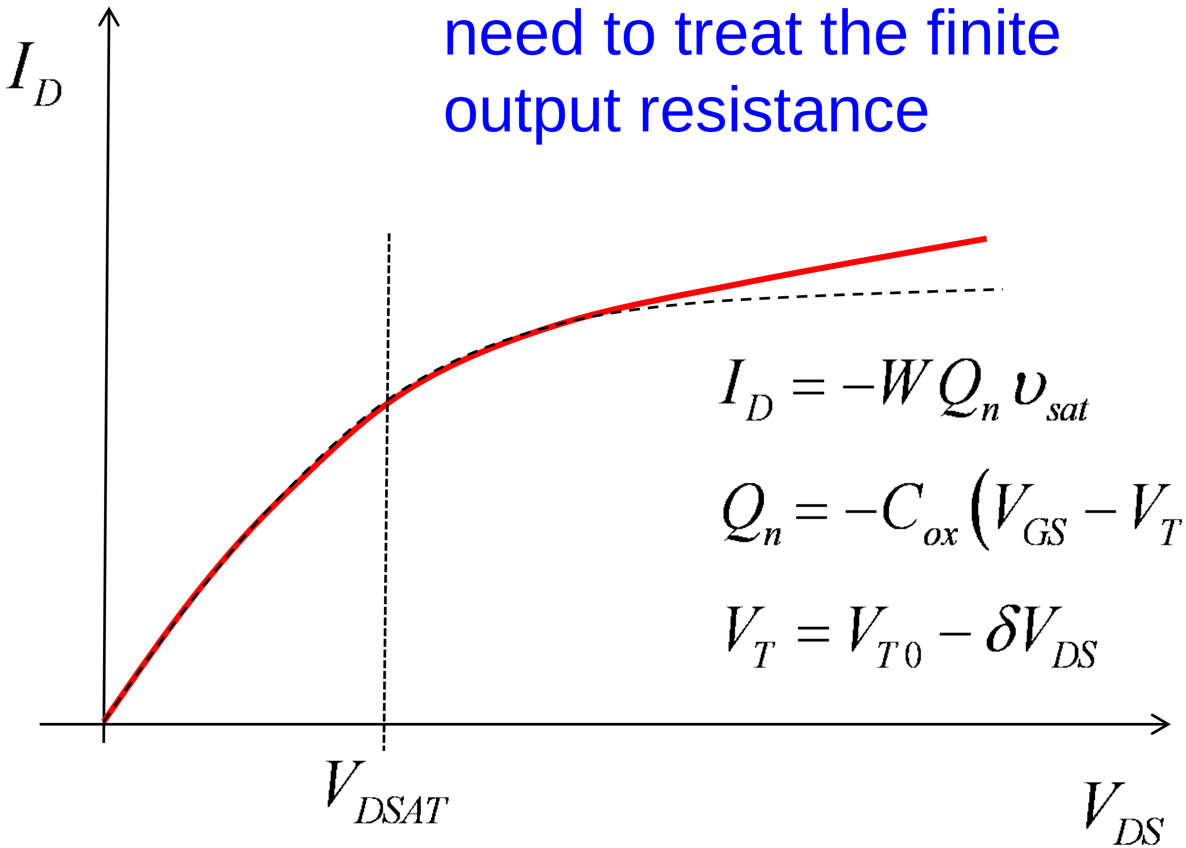
With this simple model, we can compute reasonable MOSFET IV characteristics, and the model can be extended step by step to make it more and more realistic.

There are only 5 device-specific parameters in this model:

$$C_{ox}, V_T, \upsilon_{sat}, \mu_n, L$$

$$+ \quad \beta$$

output resistance



need to treat the finite output resistance

$$I_D = -W Q_n \upsilon_{sat}$$

$$Q_n = -C_{ox}\left(V_{GS} - V_T\right)$$

$$V_T = V_{T0} - \delta V_{DS}$$

level 0' model

1) $I_D/W = -Q_n(V_G)\langle \upsilon(V_{DS})\rangle$

2) $V_{GS} \le V_T: \quad Q_n(V_{GS}) = 0$

$V_{GS} > V_T: \quad Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$

$$V_T = V_{T0} - \delta V_{DS}$$

3) $\langle \upsilon(V_{DS})\rangle = F_{SAT}(V_{DS})\upsilon_{sat}$

4) $F_{SAT}(V_{DS}) = \dfrac{V_{DS}/V_{DSAT}}{\left[1+(V_{DS}/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{sat}L}{\mu_n}$

There are only 6 device-specific parameters in this model:

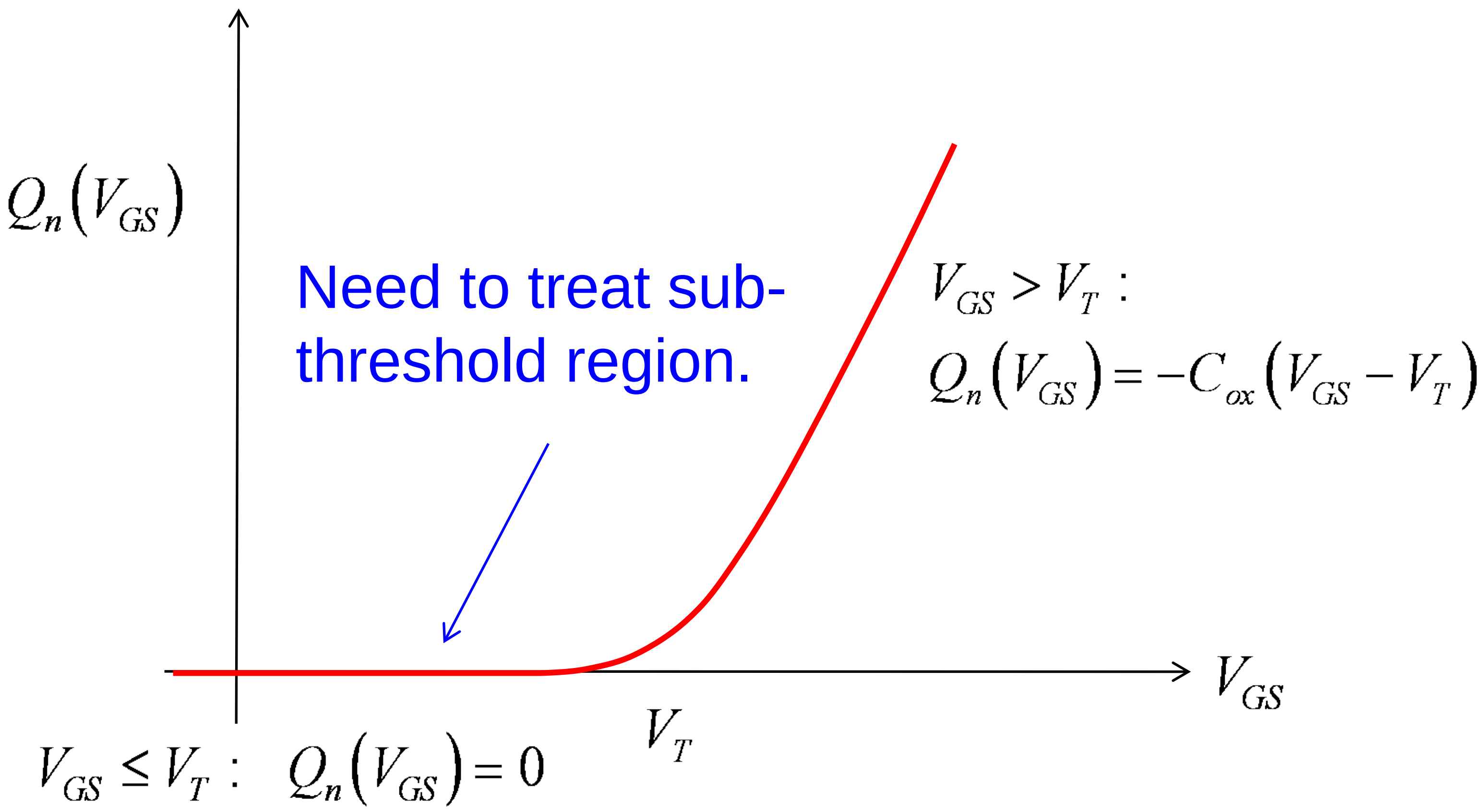
$$C_{ox}, V_T, \delta, \upsilon_{sat}, \mu_n, L$$

$$+ \quad \beta$$

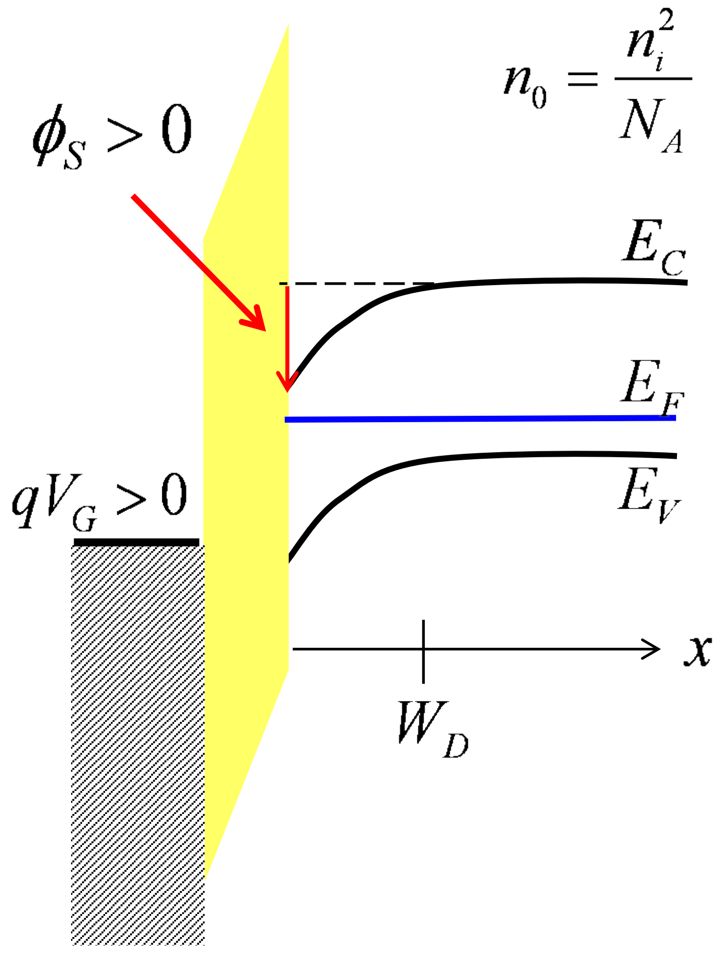
outline

1) Traditional MOSFET theory
2) VS model: above threshold
3) **VS model: subthreshold**
4) VS model: quasi-ballistic and ballistic
5) Summary

below threshold

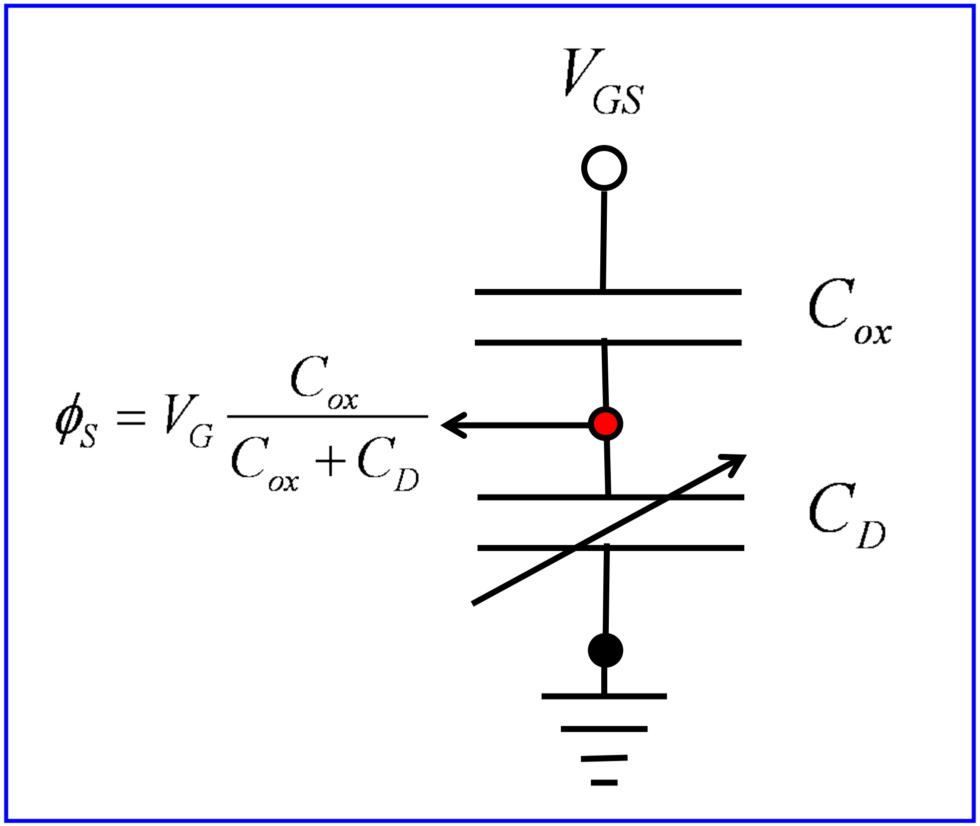


subthreshold (surface potential)



$$Q_n(\phi_S) \approx -q\frac{n_i^2}{N_A}e^{q\phi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

subthreshold (gate voltage)



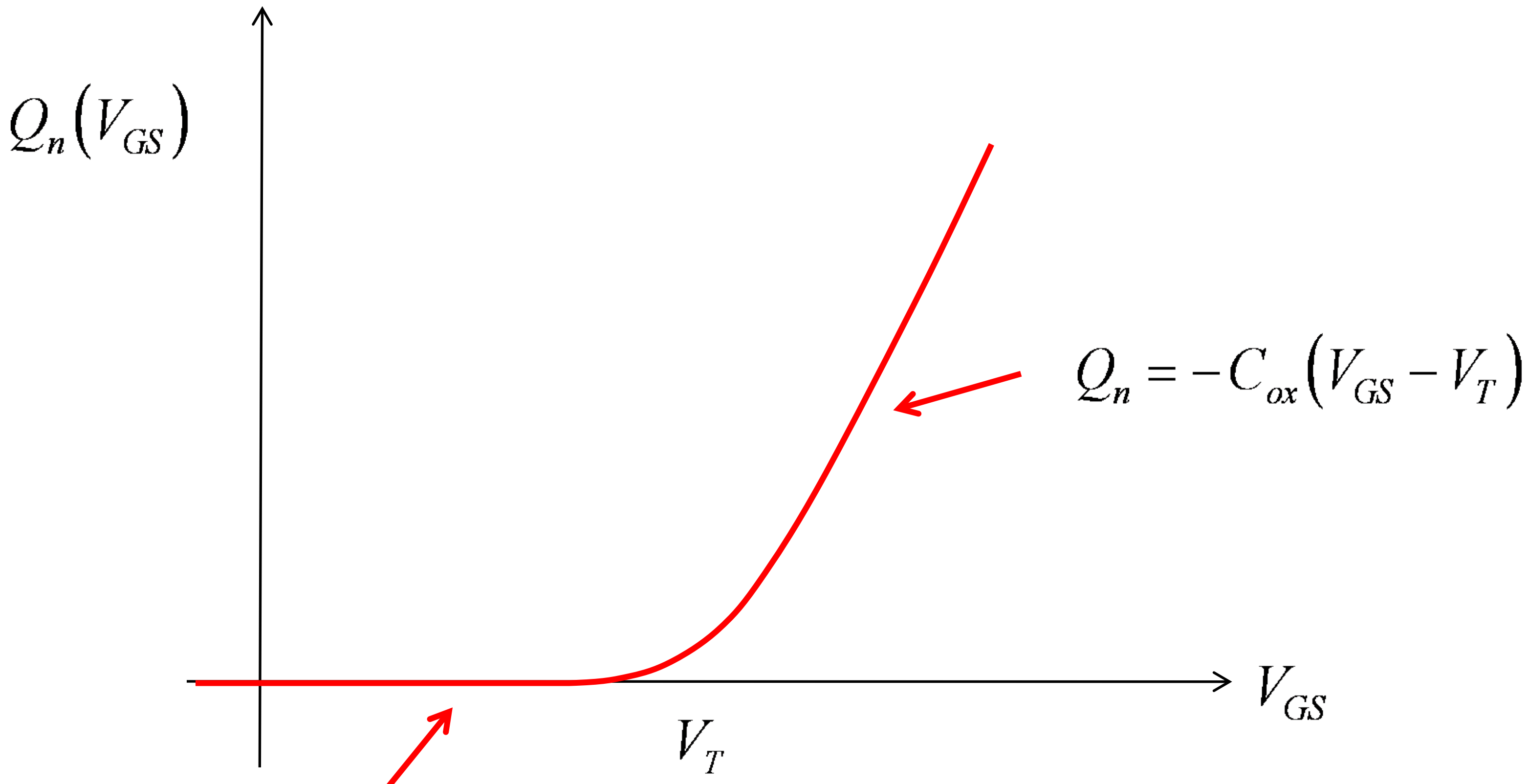
$$\phi_S = \frac{V_{GS}}{m} \qquad m = 1 + C_D / C_{ox} \geq 1$$

$$Q_n(\phi_S) = -q \frac{n_i^2}{N_A} e^{q\phi_S / k_B T} \left(\frac{k_B T / q}{\mathcal{E}_S} \right)$$

$$Q_n(V_{GS}) = -q \frac{n_i^2}{N_A} e^{q V_{GS} / m k_B T} \left(\frac{k_B T / q}{\mathcal{E}_S} \right)$$

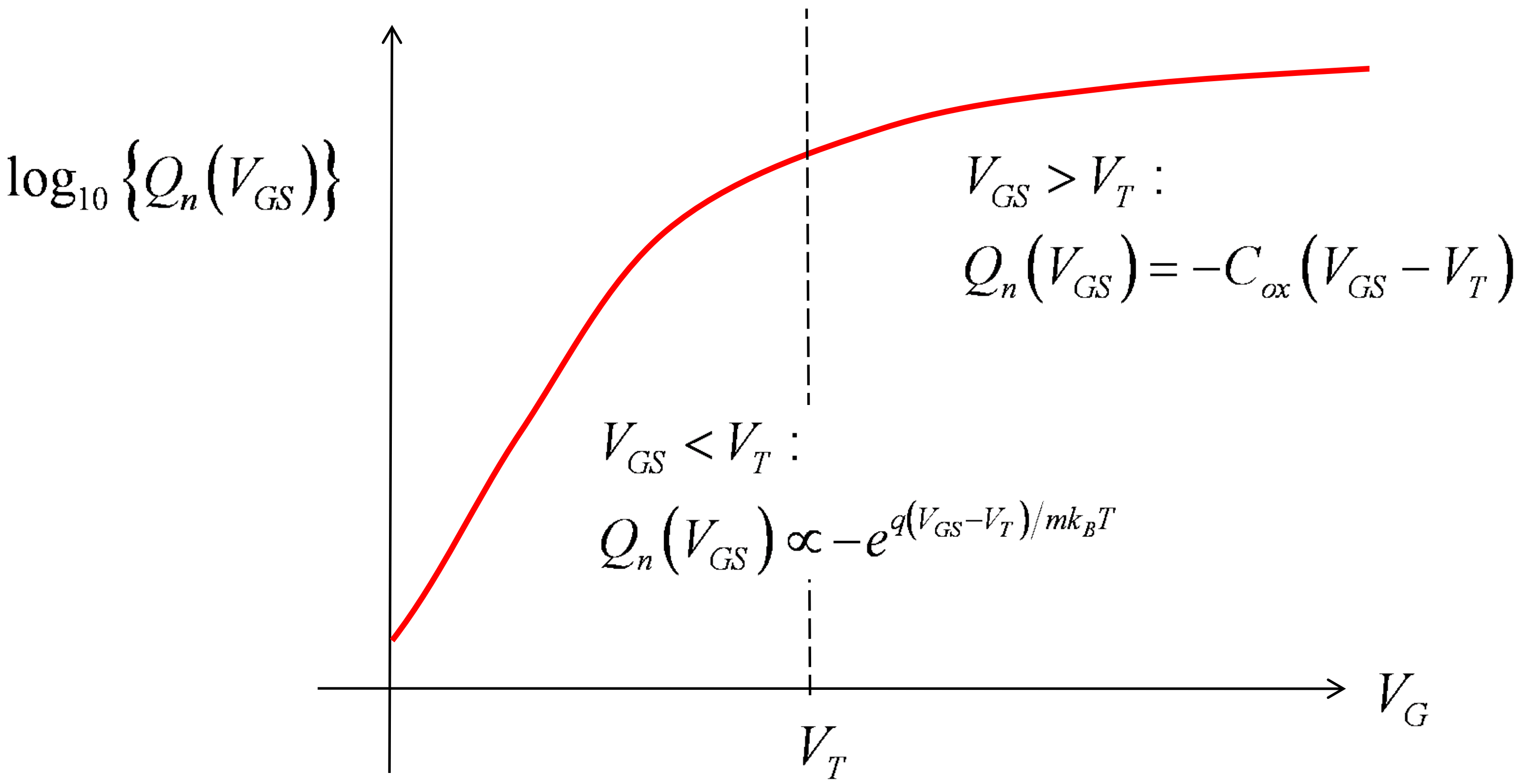
See “ECE 612 Lundstrom”

charge vs. gate voltage



$$Q_n(V_{GS}) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q(V_{GS}-V_T)/mk_B T}$$

subthreshold characteristics



subthreshold (summary)

$V_{GS} << V_T$:

$$Q_n(V_{GS}) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q(V_{GS}-V_T)/mk_B T}$$

$V_{GS} >> V_T$:

$$Q_n(V_{GS}) = -C_{ox}(V_{GS}-V_T)$$

Is there a single expression that works both below and above threshold?

Yes – a numerical one – the “surface potential model”
Yes – an empirical one – with some fitting parameters

empirical treatment

$$Q_n(V_{GS}) = -C_{ox} m (k_B T/q) \ln\left(1 + e^{q(V_{GS}-V_T)/mk_BT}\right)$$

$V_{GS} << V_T$:

$$\ln(1+x) \approx x$$

$$Q_n(V_{GS}) \approx -C_{ox} m (k_B T/q) e^{q(V_{GS}-V_T)/mk_BT}$$

$$Q_n(V_{GS}) = -(m-1) C_{ox} (k_B T/q) e^{q(V_{GS}-V_T)/mk_BT}$$

correct answer

G. T. Wright, "Threshold modelling of MOSFETs for CAD of CMOS VLSI," *Electron Lett.*, **21**, pp. 223–224, Mar. 1985.

empirical treatment

$$Q_n(V_{GS}) = -C_{ox} m(k_B T/q) \ln\left(1 + e^{q(V_{GS}-V_T)/mk_B T}\right)$$

$V_{GS} > V_T$:

$$\ln(1+x) \approx \ln(x)$$

$$Q_n(V_{GS}) \approx -C_{ox}(V_{GS} - V_T)$$

$$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$$

correct answer

G. T. Wright, “Threshold modelling of MOSFETs for CAD of CMOS VLSI,” *Electron Lett.*, **21**, pp. 223–224, Mar. 1985.

Level 1 model

1) $I_D/W = -Q_n(V_{GS})\langle\upsilon(V_{DS})\rangle$

2) $Q_n(V_{GS}) = -C_{ox}m(k_BT/q)\ln\left(1+e^{q(V_{GS}-V_T)/mk_BT}\right)$

$V_T = V_{T0} - \delta V_{DS}$

3) $\langle\upsilon(V_{DS})\rangle = F_{SAT}(V_{DS})\upsilon_{sat}$

4) $F_{SAT}(V_{DS}) = \dfrac{V_{DS}/V_{DSAT}}{\left[1+(V_{DS}/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{sat}L}{\mu_n}$

There are only 7 device-specific parameters in this model:

$$C_{ox}, V_T, \delta, m, \upsilon_{sat}, \mu_n, L$$

$$+ \quad \alpha, \beta$$

Appendix 2: MVS empirical charge model

expression used in the MIT VS Model

$$Q_n(V_{GS}) = -C_{inv} m (k_B T/q) \ln\left(1 + e^{q\left(V_{GS} - V_T + \alpha (k_B T/q) F_f\right)/m k_B T}\right)$$

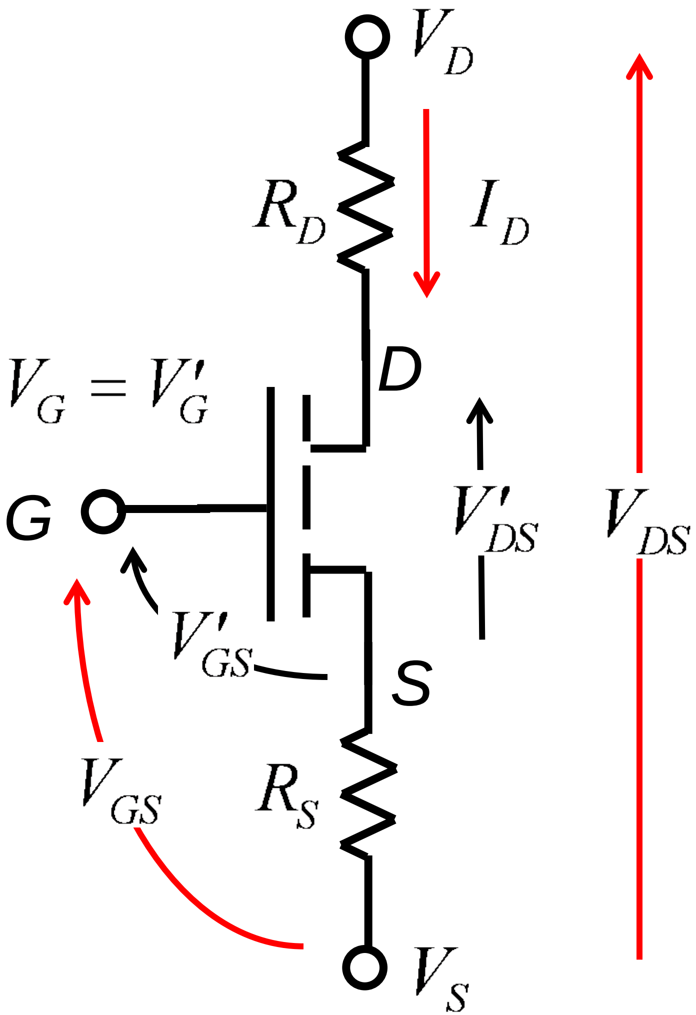
$$V_T = V_{T0} - \delta V_{DS}$$

DIBL

different V_T's in strong and weak inversion

Ali Khakifirooz, Osama M. Nayfeh, and Dimitri Antoniadis, "A Simple Semi-empirical Short-Channel MOSFET Current–Voltage Model Continuous Across All Regions of Operation and Employing Only Physical Parameters," *IEEE Trans. Electron Devices*, **56**, pp. 1674-1680, 2009.

intrinsic vs. extrinsic voltages



$$V'_G = V_G$$

$$V'_D = V_D - I_D\left(V'_G, V'_S, V'_D\right) R_D$$

$$V'_S = V_S + I_D\left(V'_G, V'_S, V'_D\right) R_S$$

Level 1' model

1) $I_D/W = -Q_n(V'_{GS})\langle\upsilon(V'_{DS})\rangle$

2) $Q_n(V'_{GS}) = -C_{ox}m(k_BT/q)\ln\left(1+e^{q(V'_{GS}-V_T)/mk_BT}\right)$

$V_T = V_{T0} - \delta V'_{DS}$

3) $\langle\upsilon(V'_{DS})\rangle = F_{SAT}(V'_{DS})\upsilon_{sat}$

4) $F_{SAT}(V'_{DS}) = \dfrac{V'_{DS}/V_{DSAT}}{\left[1+(V'_{DS}/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{sat}L}{\mu_n}$

There are only 8 device-specific parameters in this model:

$$C_{ox}, V_T, \delta, m, \upsilon_{sat}, \mu_n, L,$$

$$R_{SD} = R_S + R_D$$

$$+ \quad \alpha, \beta$$

outline

1) Traditional MOSFET theory
2) VS model: above threshold
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5) Summary

MOSFETs

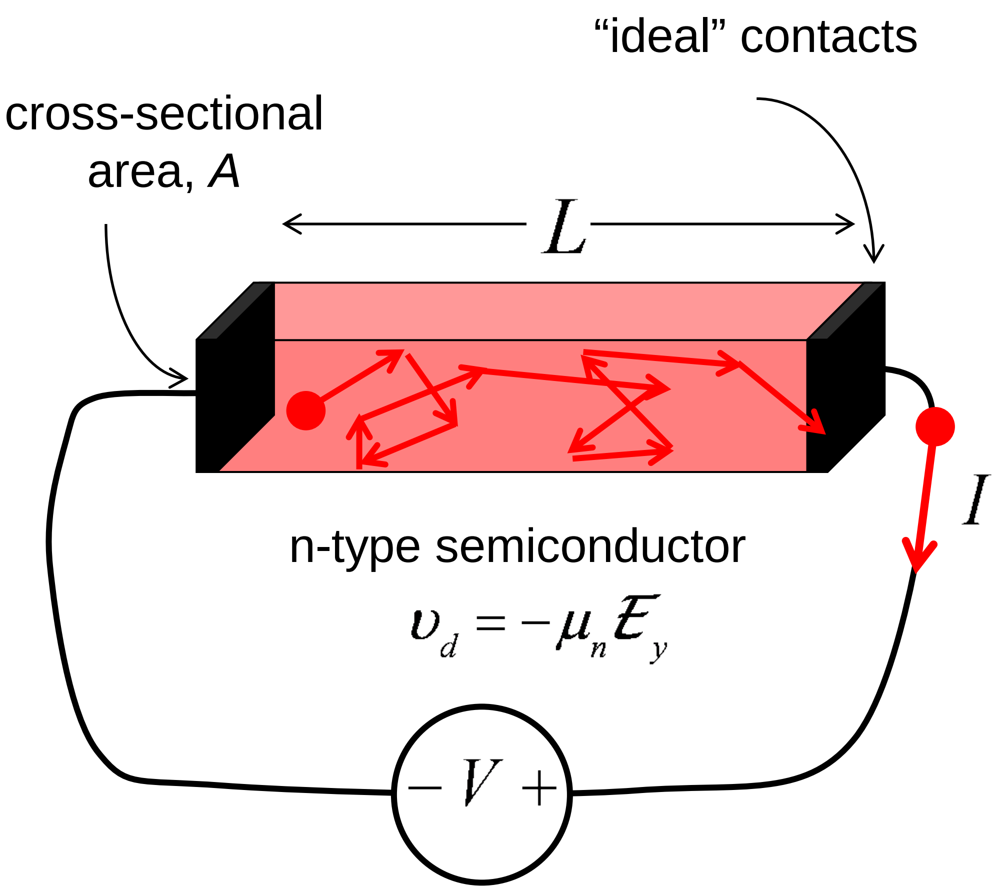
$$I_D/W = -Q_n\left(V_{GS}\right)\left\langle \upsilon\left(V_{DS}\right)\right\rangle$$

electrostatics ***transport***

$$\left\langle \upsilon_y \right\rangle = -\mu_n \mathcal{E}_y$$

$$\left\langle \upsilon_y \right\rangle = \upsilon_{sat}$$

mobility



Mobility is a concept that describes long channels (many MFP's long).

mobility

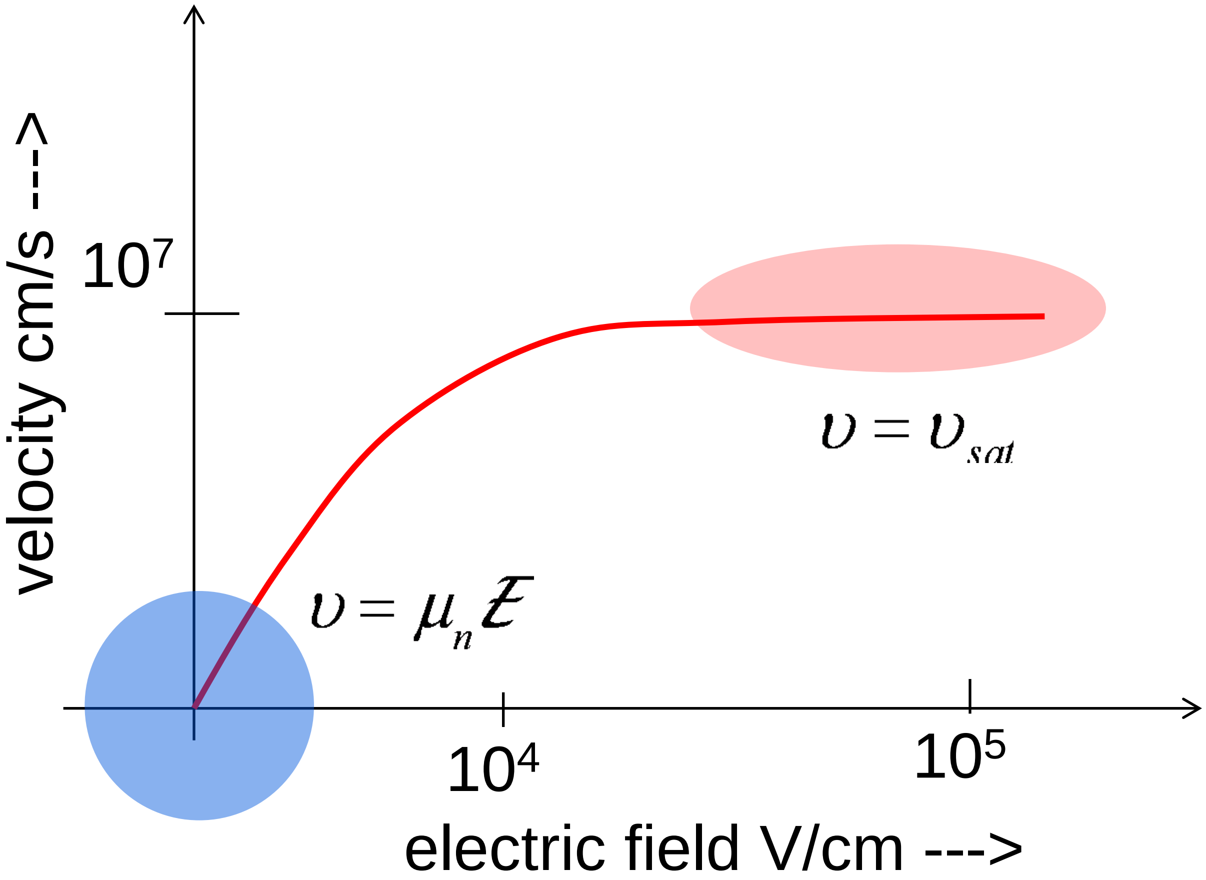
$$D_n = \frac{\upsilon_T \lambda}{2} \ \text{cm}^2/\text{s}$$

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

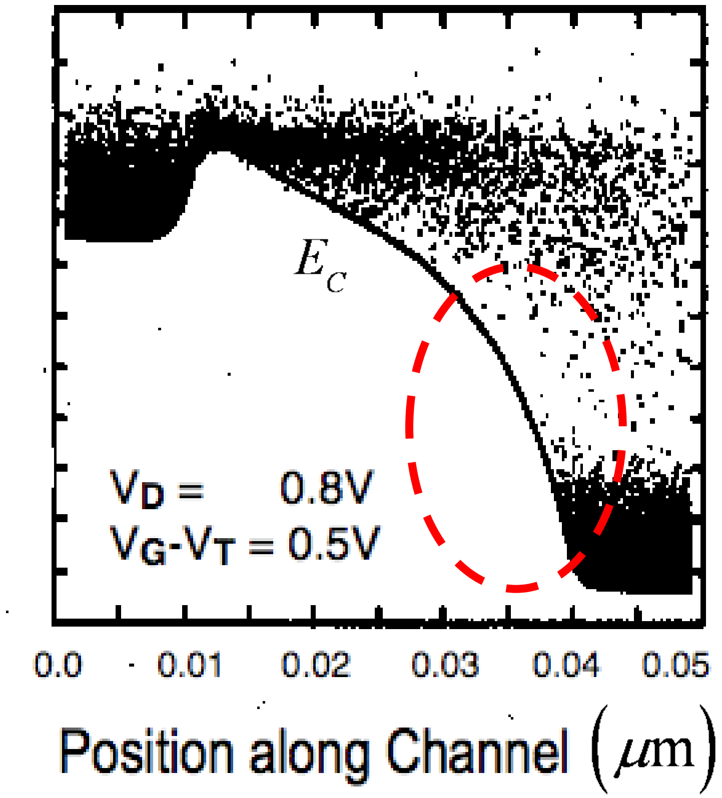
$$L >> \lambda$$

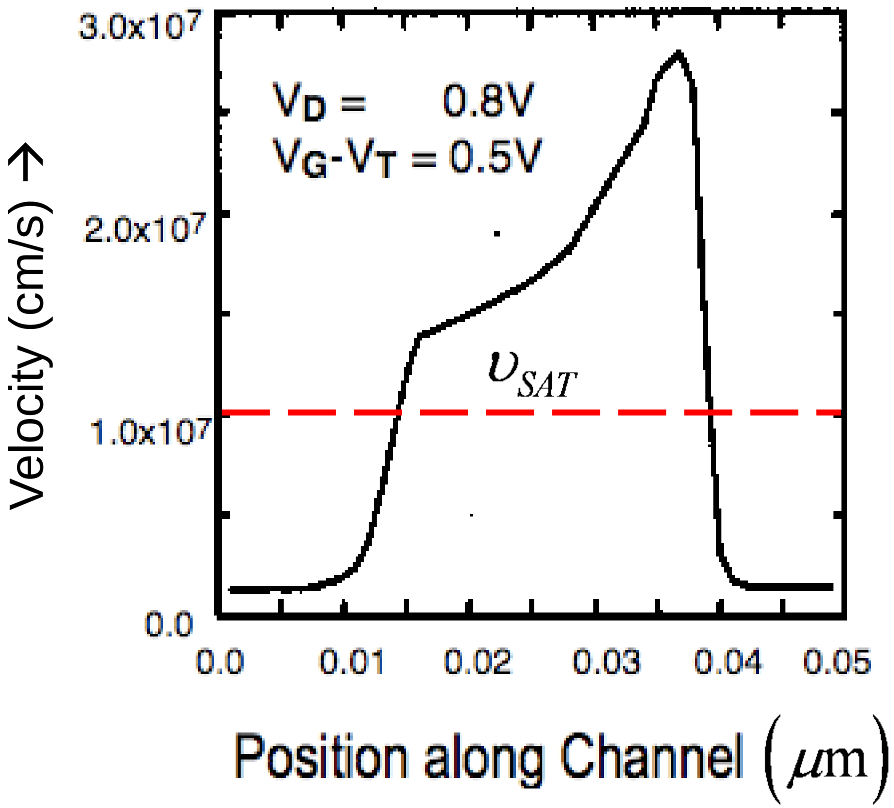
$$\mu_n = \frac{\upsilon_T \lambda}{2(k_B T/q)} \ \text{cm}^2/\text{V-s}$$

velocity saturation in bulk semiconductors



“velocity overshoot”





D. Frank, S. Laux, and M. Fischetti, Int. Electron Dev. Mtg., Dec., 1992.

The MVS model

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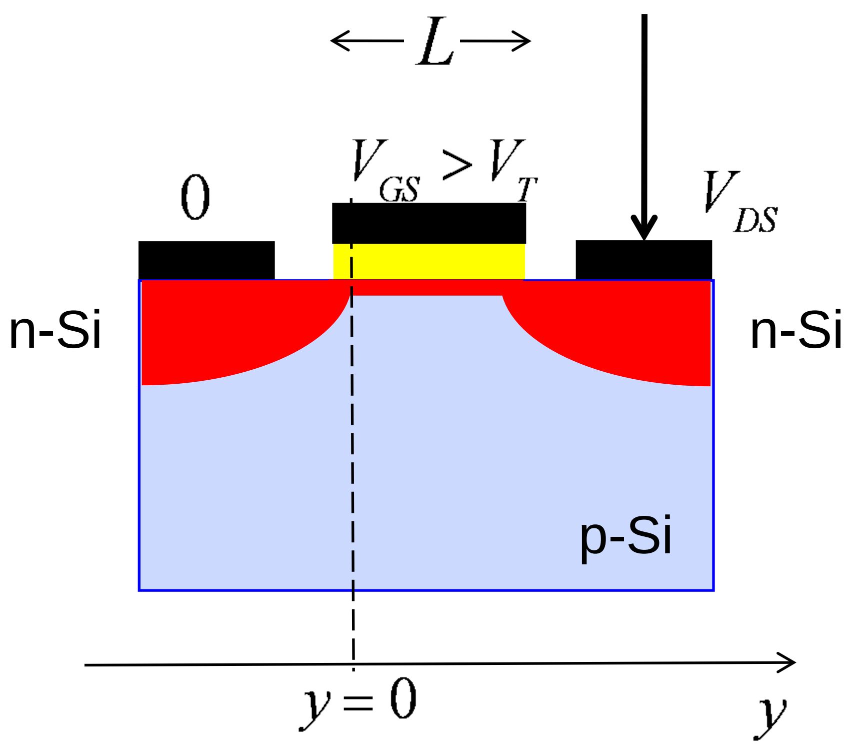
$$\frac{1}{\mu_n} \rightarrow \frac{1}{\mu_{app}}$$ “apparent mobility”

$$\upsilon_{sat} \rightarrow \upsilon_{inj}$$ “injection velocity”

M. S. Lundstrom and D. A. Antoniadis, “Compact Models and the Physics of Nanoscale FETs,” *IEEE Trans. Electron Dev.*, **99**, pp. 225-233, 2014.

apparent mobility



$$\frac{1}{\lambda_{app}} = \frac{1}{\lambda} + \frac{1}{L}$$

$$\mu_n = \frac{\upsilon_T \lambda}{2(k_B T/q)} \text{ cm}^2/\text{V-s}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B}$$

$$\mu_B = \frac{\upsilon_T L}{2(k_B T/q)} \text{ cm}^2/\text{V-s}$$

"ballistic mobility"

The MFP cannot be longer than the channel length.

ballistic limit (low V_{DS})

$$I_D = \frac{W}{L}\mu_{app}C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B}$$

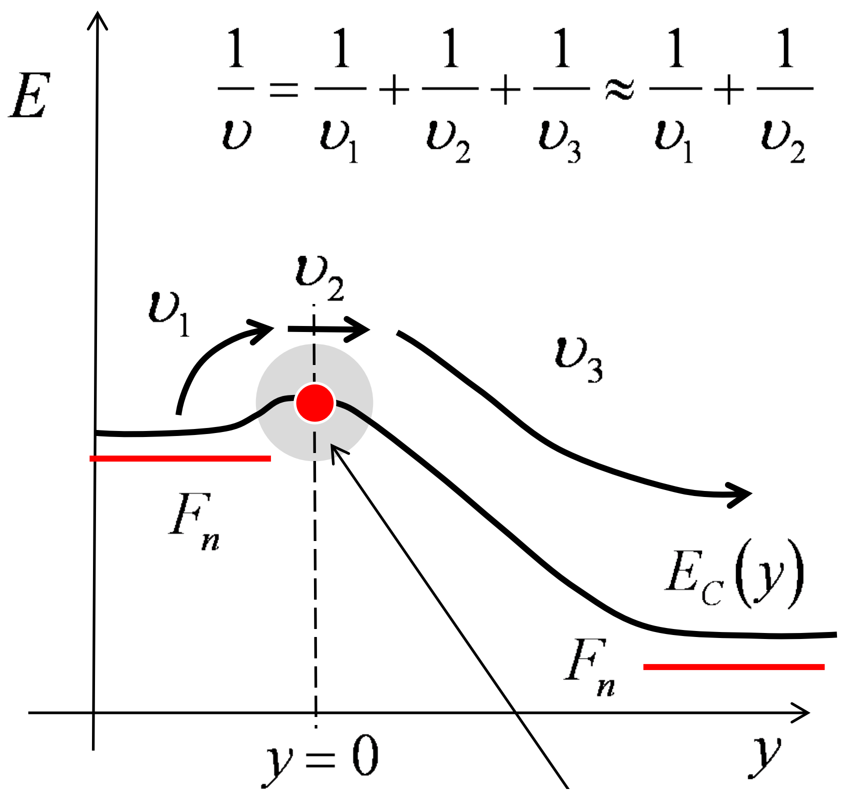
$$\mu_B = \frac{\upsilon_T L}{2\left(k_B T/q\right)} \text{ cm}^2/\text{V-s}$$

$$I_D = WC_{ox}\frac{\upsilon_T}{2\left(k_B T/q\right)}\left(V_{GS} - V_T\right)V_{DS}$$

ballistic MOSFET

injection velocity

$$\frac{1}{\upsilon} = \frac{1}{\upsilon_1} + \frac{1}{\upsilon_2} + \frac{1}{\upsilon_3} \approx \frac{1}{\upsilon_1} + \frac{1}{\upsilon_2}$$



$$Q_n \approx -C_{ox}\left(V_{GS} - V_T\right) \quad \mathrm{C/cm^2}$$

$$\frac{1}{\upsilon_{inj}} = \frac{1}{\upsilon_T} + \frac{1}{D_n/\ell}$$

ballistic limit (high V_{DS})

$$I_D = WC_{ox}\upsilon_{inj}\left(V_{GS} - V_T\right)$$

$$\frac{1}{\upsilon_{inj}} = \frac{1}{\upsilon_T} + \frac{1}{D_n/\ell}$$

$$I_D = WC_{ox}\upsilon_T\left(V_{GS} - V_T\right)$$

ballistic MOSFET

The VS nanotransistor model

1) $I_D/W = -Q_n(V'_{GS})\langle \upsilon(V'_{DS})\rangle$

2) $Q_n(V'_{GS}) = -C_{ox} m (k_B T/q)\ln\left(1 + e^{q(V'_{GS}-V_T)/mk_B T}\right)$

$V_T = V_{T0} - \delta V'_{DS}$

3) $\langle \upsilon(V'_{DS})\rangle = F_{SAT}(V'_{DS})\upsilon_{sat}$

4) $F_{SAT}(V'_{DS}) = \dfrac{V'_{DS}/V_{DSAT}}{\left[1+\left(V'_{DS}/V_{DSAT}\right)^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{inj} L}{\mu_{app}}$

There are only 8 device-specific parameters in this model:

$C_{ox}, V_T, \delta, m, \upsilon_{inj}, \mu_{app}, L,$

$R_{SD} = R_S + R_D$

$+ \quad \alpha, \beta$

The MVS model

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$$\frac{1}{\mu_{eff}} \rightarrow \frac{1}{\mu_{app}} = \frac{1}{\mu_{eff}} + \frac{1}{\mu_B}$$

$$\upsilon_{sat} \rightarrow \upsilon_{inj} = \left[\frac{1}{\upsilon_T} + \frac{1}{D_n/\ell}\right]^{-1}$$

conclusions

1) Traditional MOSFET models correctly describe the shape of the IV characteristics of nanoscale MOSFETs – because they are still “barrier controlled devices.”

2) To get the magnitude of the current right, the mobility and saturation velocity need to be replaced by the ***apparent mobility*** and the ***injection velocity***.

3) These two parameters are not “fudge factors” – they have clear physical meaning.