The R3 Model: Homework Solutions

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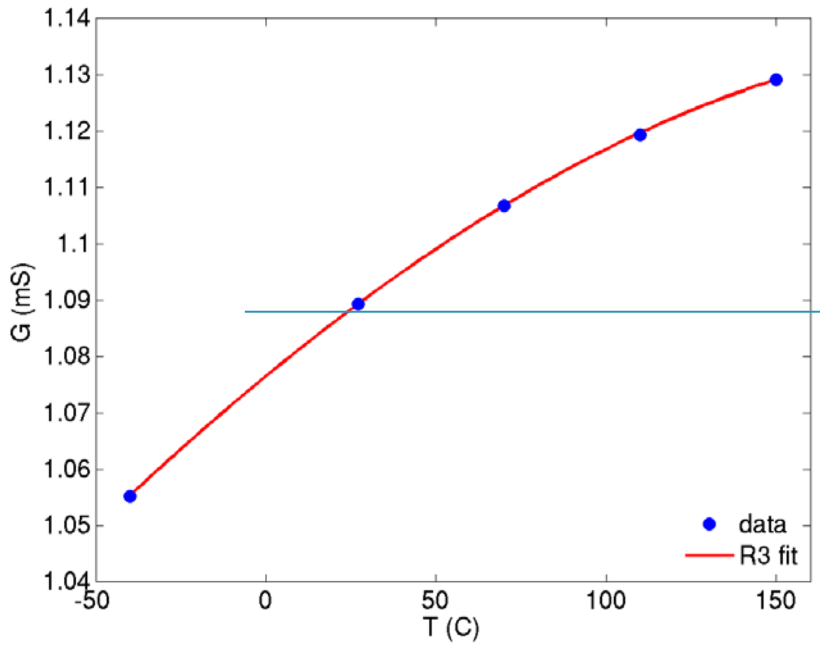
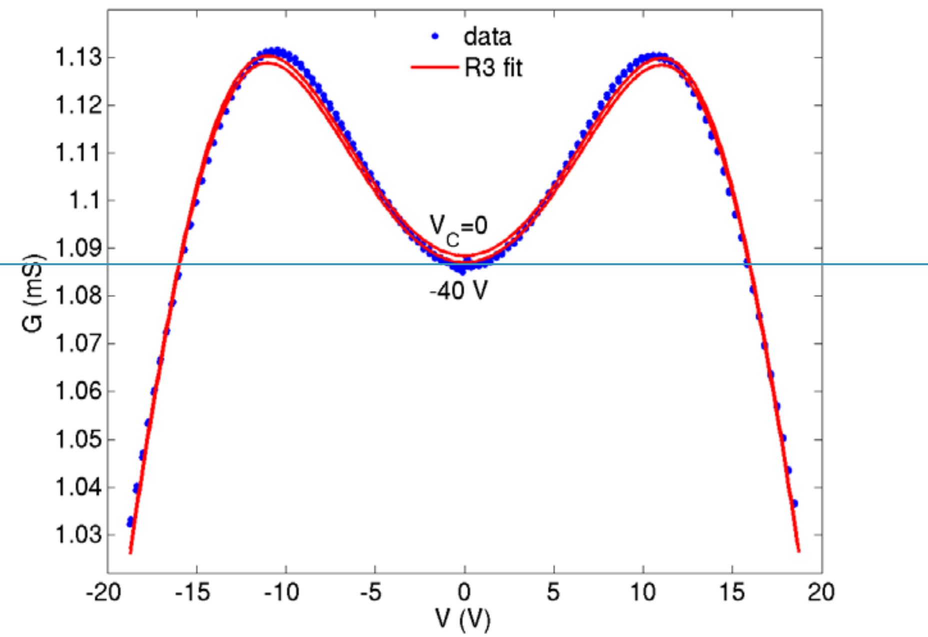
Homework (8)

- From plots below, what is approx. max. resistor temperature?
  - hint: there's a reason the $G(T)$ plot is on the left





$$G_0 = \lim_{V \to 0} G(V, V_c = 0)$$

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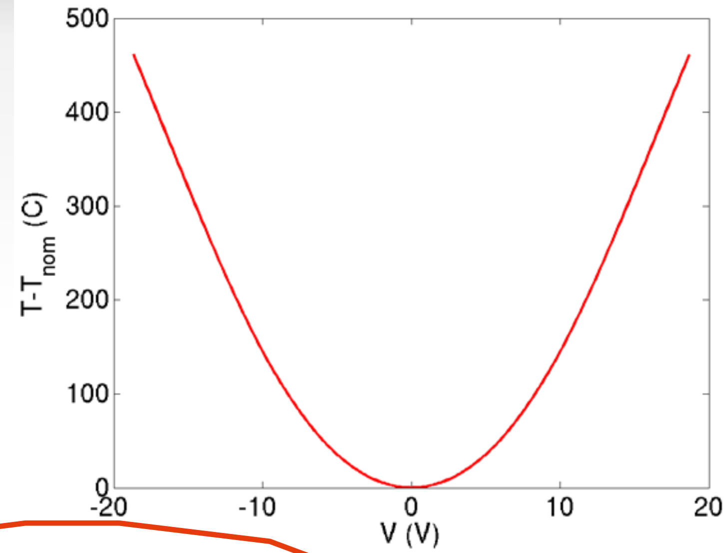
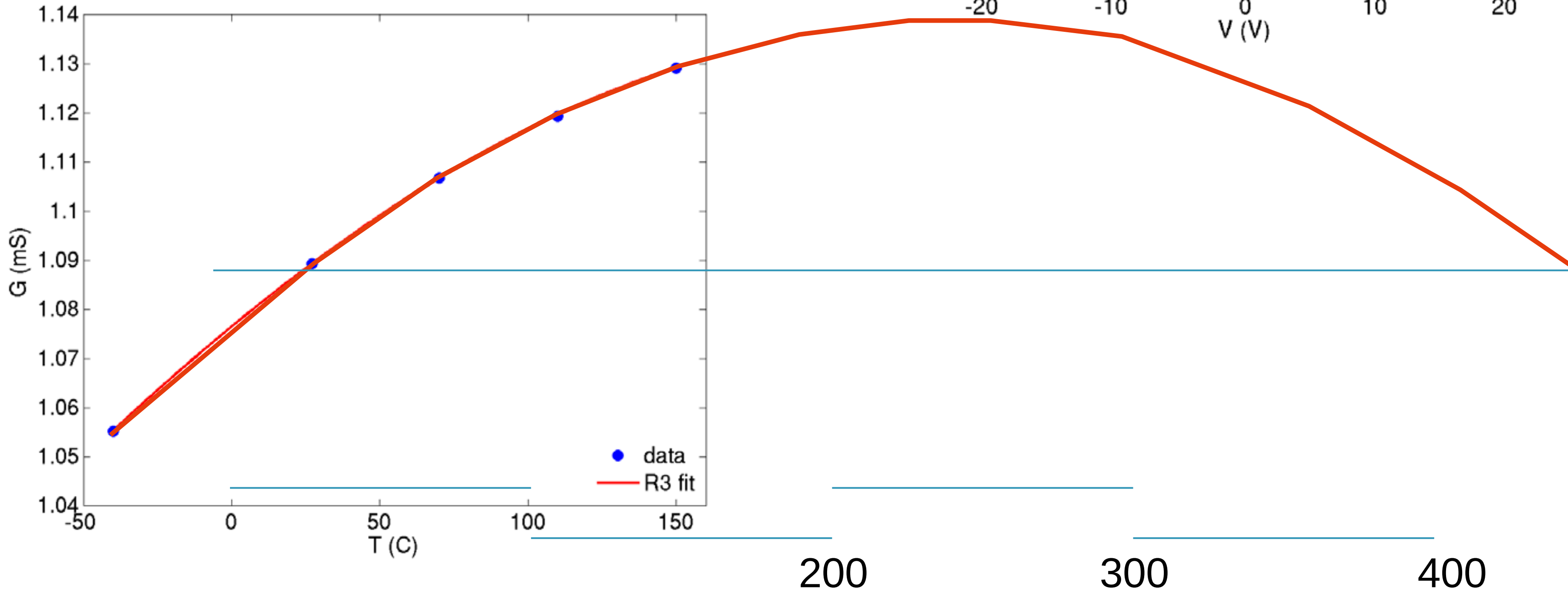
Solution (8)

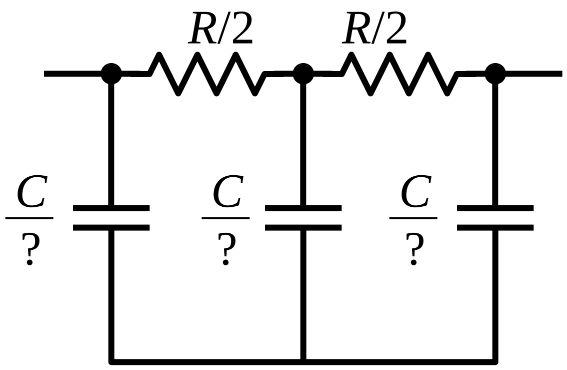
- Extrapolate $G(T)$ to where it reduces below $G(27)$





Homework (7)

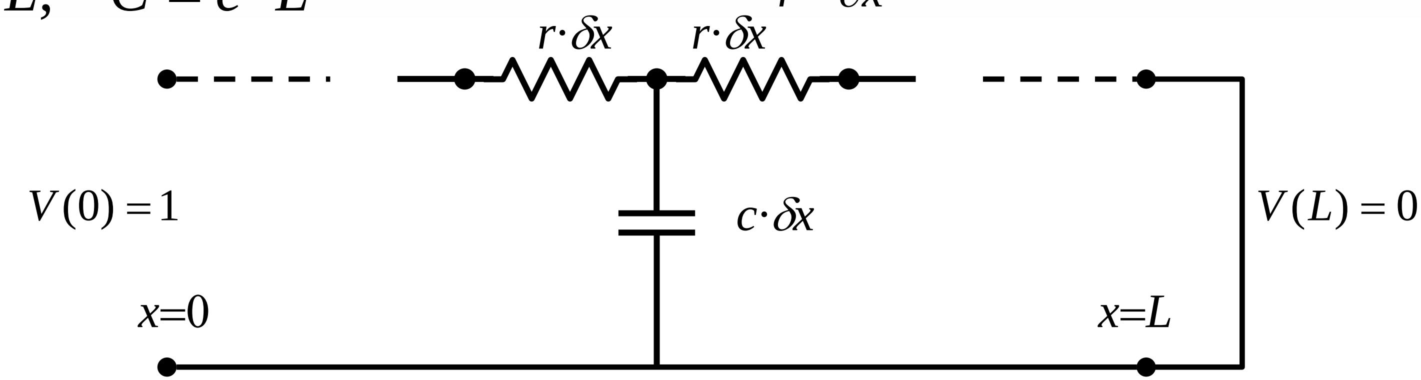
- Consider a resistor split into two sections, to increase the frequency range over which modeling is accurate. How should the capacitance be split?
  - 1/3, 1/3, 1/3
  - 1/4, 1/2, 1/4
  - or is there a more accurate way?
  - hint: compare the input admittance of a distributed RC model to the two-R lumped section model (with port 2 shorted to ground)



Solution (7) Distributed RC Analysis

$$R = r \cdot L, \quad C = c \cdot L$$

$$I(x) = -\frac{1}{r} \cdot \frac{\partial V}{\partial x}$$



$$c \cdot \delta x \cdot \frac{\partial V(x)}{\partial t} = \frac{V(x-\delta x) - V(x)}{r \cdot \delta x} + \frac{V(x+\delta x) - V(x)}{r \cdot \delta x} = \frac{V(x-\delta x) - 2V(x) + V(x+\delta x)}{r \cdot \delta x}$$

$$\frac{\partial^2 V}{\partial x^2} = rc \cdot \frac{\partial V}{\partial t} = \frac{\gamma^2}{L^2} \cdot V, \quad \gamma = \sqrt{j\omega RC}$$

$$V = \cosh\left(\gamma \frac{x}{L}\right) - \operatorname{ctnh}(\gamma) \sinh\left(\gamma \frac{x}{L}\right)$$

$$Y_{in} = I(0) = \frac{\gamma \operatorname{ctnh}(\gamma)}{R} \approx \frac{1}{R}\left(1 + \frac{\gamma^2}{3}\right)$$

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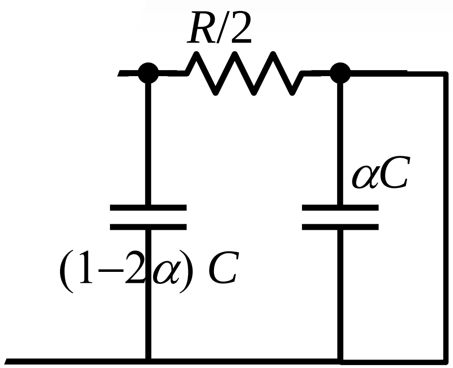
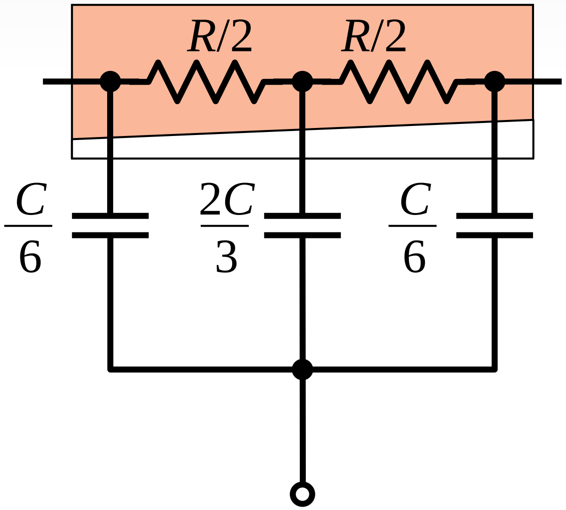
Solution (7)





$$y_1 = \frac{2}{R} + j\omega(1-2\alpha)C = \frac{2}{R}\left(1 + j\omega\frac{(1-2\alpha)}{2}RC\right)$$

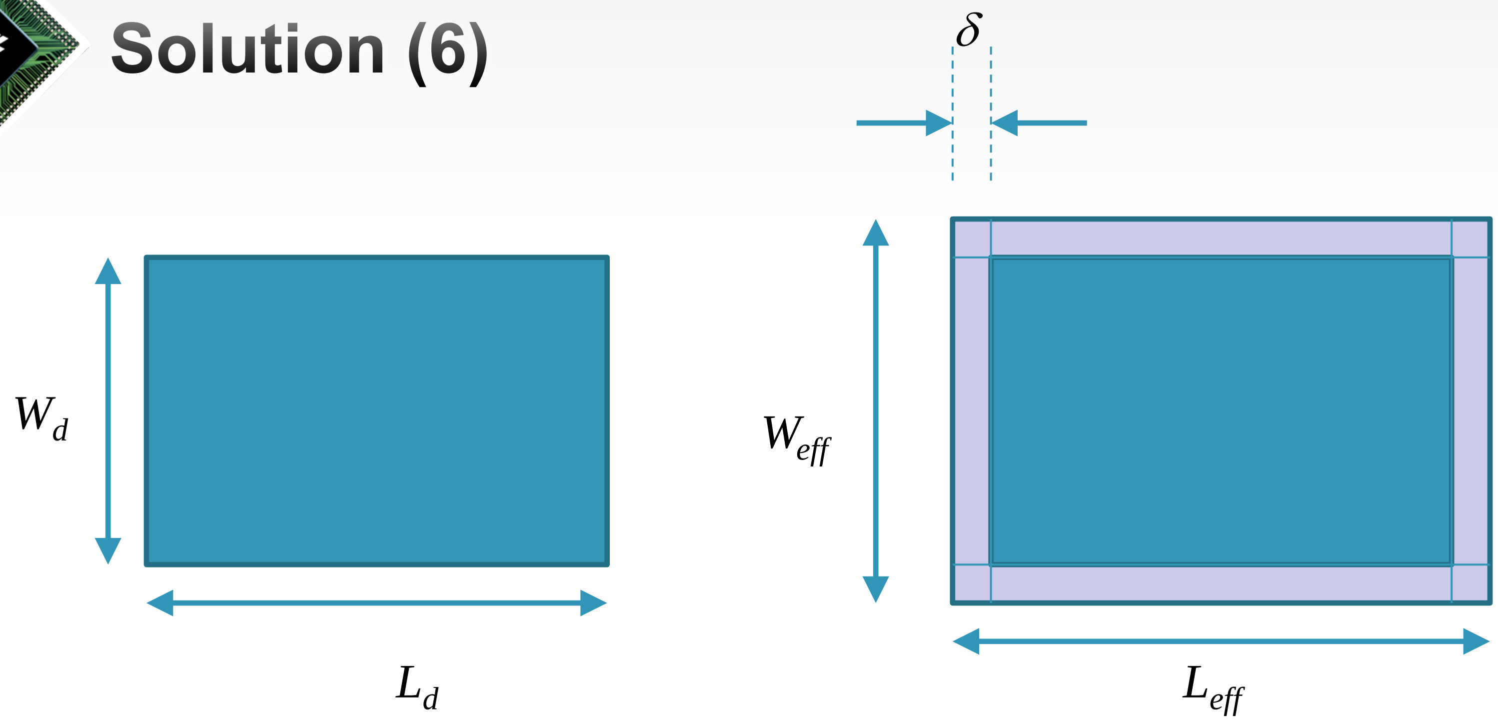
$$z_2 = \frac{R}{2} + \frac{R}{2}\frac{1}{1 + j\omega\frac{(1-2\alpha)}{2}RC} \approx R\left(1 - j\omega\frac{(1-2\alpha)}{4}RC\right)$$

$$Y_{in} \approx j\omega\alpha C + \frac{1}{R}\left(1 + j\omega\frac{(1-2\alpha)}{4}RC\right) = \frac{1}{R}\left(1 + j\omega\frac{(1+2\alpha)}{4}RC\right) \Rightarrow \frac{1+2\alpha}{4} = \frac{1}{3}, \quad \alpha = \frac{1}{6}$$

Homework (6)

- Show that for quantities that scale geometrically with fixed (corner), perimeter, and area components there is no need to consider effective, as compared to drawn dimensions; i.e. that the difference between these can be handled by appropriate modification of the coefficients.

Solution (6)



$$
\begin{aligned}
C &= 4C'_c + 2\left(L_{eff} + W_{eff}\right)\cdot C'_p + \left(L_{eff}\cdot W_{eff}\right)\cdot C'_a \\
&= 4C'_c + 2\left(L_d + W_d + 4\delta\right)\cdot C'_p + \left(L_d + 2\delta\right)\cdot\left(W_d + 2\delta\right)\cdot C'_a \\
&= 4\left(C'_c + 2\delta\cdot C'_p + \delta^2\cdot C'_a\right) + 2\left(L_d + W_d\right)\cdot\left(C'_p + \delta\cdot C'_a\right) + \left(L_d\cdot W_d\right)\cdot C'_a
\end{aligned}
$$

Difference between drawn and effective dimensions simply accounted for by modification of perimeter and corner capacitance component coefficients

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Homework (5)

- The solution for V_{sat} becomes numerically unstable when the amount of depletion pinching (i.e. df) becomes small, as happens in poly resistors. Look in the `r3` code and see how that situation is handled.
  - why is the error this introduces of no practical consequence?

$$V_{sat} \text{ is solution of } \frac{\partial}{\partial V}\left(V \frac{1 - d_f\sqrt{d_{pe} + V}}{1 + \dfrac{\frac{V}{L} - E_{co}}{E_{cr}}} \right) = 0 \quad \text{where} \quad d_{pe} = d_p - 2V_{c1}, \quad L_{DE} = L\left(E_{cr} - E_{co}\right)$$

$$\frac{d_f^2}{4L_{DE}^2} V_{sat}^4 + \frac{3d_f^2}{2L_{DE}} V_{sat}^3 + d_f^2\left(\frac{9}{4} + \frac{p_e}{L_{DE}}\right) V_{sat}^2 + \left(3d_f^2 p_e - 1\right) V_{sat} + p_e\left(d_f^2 p_e - 1\right) = 0$$

Solution (5)

$$\frac{d_f^2}{4L_{DE}^2}V_{sat}^4+\frac{3d_f^2}{2L_{DE}}V_{sat}^3+d_f^2\left(\frac{9}{4}+\frac{p_e}{L_{DE}}\right)V_{sat}^2+\left(3d_f^2p_e-1\right)V_{sat}+p_e\left(d_f^2p_e-1\right)=0$$

```
   if (iecrit>0.0) begin \
           a0        =  dfsq*dpe*dpe-dpe; \
           a1        = -1.0+3.0*dfsq*dpe; \
           a2        =  dfsq*(9.0/4.0+dpe/lde); \
           a3        =  1.5*dfsq/lde; \
           a4        =  4.0*lde*lde/dfsq; \
dfmin    =  sqrt(dp)/(dp+1.0e4); // min value of df for stable Vsat calculation
df       =  (sw_lin) ? 0.0 : dfinf+(dfw*len+dfl*wid+dfwl)/(len*wid);
if (df<dfmin) begin                // for highly linear (e.g. poly) resistors limit
    df       = `MAX(df,0.0);       // dfsq, which is only used for Vsat calculation
    dfsq     =  dfmin*dfmin;       // this underestimates Vsat, but that is not of
end else begin                     // consequence as it is very large anyway
    dfsq     =  df*df;
end
```

- “Fudged” (very small, but finite) `dfsq` is used
  - V_{sat} is **so** big it is well beyond any reasonable range of V
  - has no discernable affect
  - very small `df` means device never gets near pinch-off

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Homework (4)

- In `r3`, V_{sat} is calculated as the value of V at which

$$g_o = \frac{\partial}{\partial V}\frac{I_{depl}}{\mu'_{red}} = 0$$

  but using an asymptotic model for μ_{red} that is **less** than the actual μ_{red} model used to calculate I. This would seem to imply that when used with the actual μ_{red} model, which degrades the mobility **more**, g_o (in the absence of self-heating) at V_{sat} would be negative. However, this is not the case, it is guaranteed that $g_o(V_{sat})$ is positive. Prove this.
  - hint: write down an expression for g_o and see how this changes when the μ_{red} model, but not V_{sat}, is changed
  - do you think the μ_{red} form used in `r3` makes the velocity-field relationship monotonic or non-monotonic? (Note: it is non-trivial to *prove* which it is, you do not need to generate a proof)

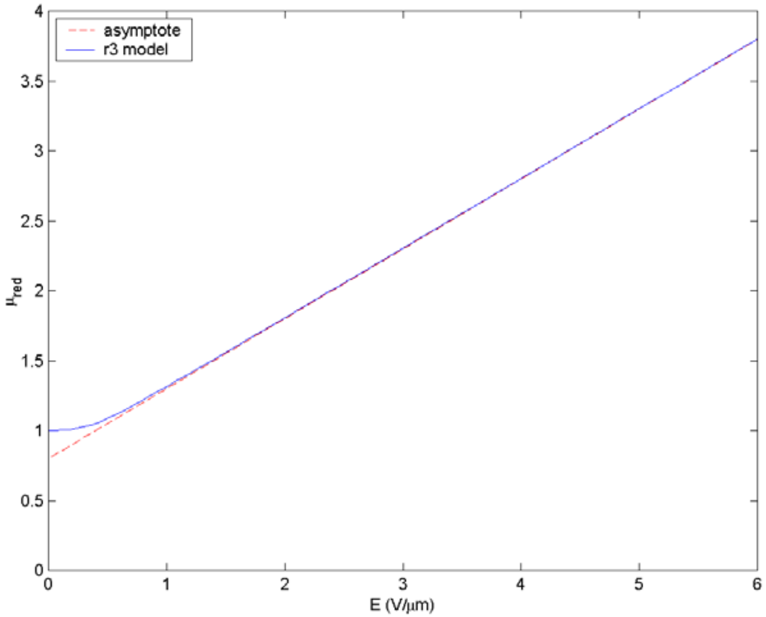
Solution (4)

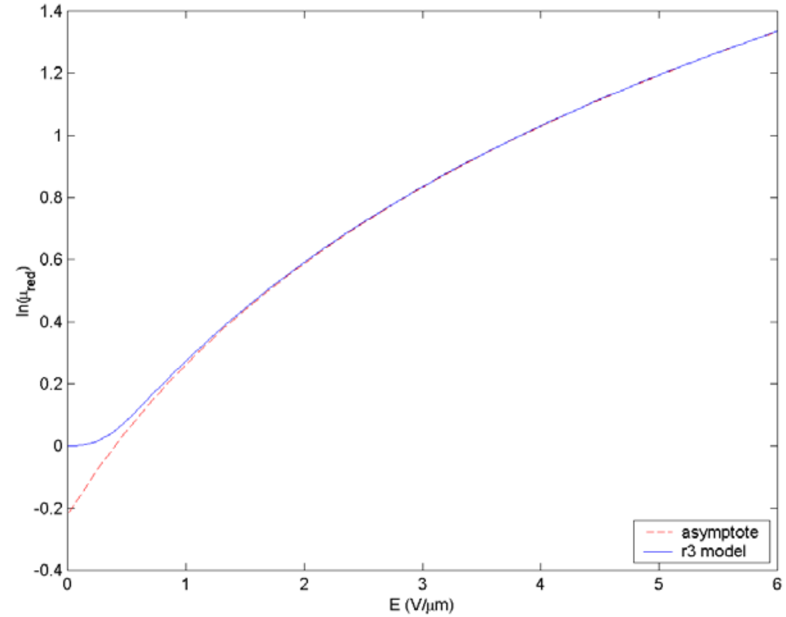
- Let I_{depl} be the current without velocity saturation, add a superscript prime for asymptotic mobility reduction model

$$V_{sat} = V \,|\, \frac{\partial}{\partial V}\frac{I_{depl}}{\mu'_{red}} = \frac{g_{depl} - I_{depl}\dfrac{d\ln(\mu'_{red})}{dV}}{\mu'_{red}} = 0, \quad g_{depl} = \frac{\partial I_{depl}}{\partial V}$$

- If $\dfrac{d\ln(\mu_{red})}{dV} \le \dfrac{d\ln(\mu'_{red})}{dV}$ then $g_o(V_{sat}) \ge 0$ as desired

Solution (4)





- Velocity-field relationship **is** monotonic provided $E_{co}<E_{cr}$

Homework (3)

- When sw_lin=1, is r3 **exactly** linear?
  - hint: look at how sw_lin affects DP, VS, and SH
  - what else could possibly induce a non-linearity
- Run the script simulateLinear.pl

```
if (iecrit>0.0) begin \
    fctrm    =  0.5*((Vbeff/leffE_um)-ecrneff)*iecrit; \
    fctrp    =  0.5*((Vbeff/leffE_um)+ecrneff)*iecrit; \
    sqrtm    =  sqrt(fctrm*fctrm+dufctr); \
    sqrtp    =  sqrt(fctrp*fctrp+dufctr); \
    rmu      =  sqrtm+sqrtp-uoff; \
end else begin \
    rmu      =  0.0; \
end \
dpfctr   =  1.0-df*sqrt(dpe+Vbeff); \
geff     =  gf*dpfctr/(1.0+rmu); \
Ib       =  sdFlip*geff*Vbeff; \
```

Solution (3)

- Left out one important line (top line below)
  - makes $V_{beff}(V_b)$ nonlinear
  - even though limiting is sqrt() rather than log(exp()), V_{sat} is still so big it introduces negligible nonlinearity

```
Vbeff    =  2.0*Vbi*Vsat/ \
   (sqrt((Vbi-Vsat)*(Vbi-Vsat)+atspo)+sqrt((Vbi+Vsat)*(Vbi+Vsat)+atspo)); \
if (iecrit>0.0) begin \
    fctrm    =  0.5*((Vbeff/leffE_um)-ecrneff)*iecrit; \
    fctrp    =  0.5*((Vbeff/leffE_um)+ecrneff)*iecrit; \
    sqrtm    =  sqrt(fctrm*fctrm+dufctr); \
    sqrtp    =  sqrt(fctrp*fctrp+dufctr); \
    rmu      =  sqrtm+sqrtp-uoff; \
end else begin \
    rmu      =  0.0; \
end \
dpfctr   =  1.0-df*sqrt(dpe+Vbeff); \
geff     =  gf*dpfctr/(1.0+rmu); \
Ib       =  sdFlip*geff*Vbeff; \
```

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Homework (2)

- Look over and run the “simulatePoly.pl” script, which
  - reads data from a L/W=42μm/4.2μm poly resistor
  - sets up parameters extracted from that resistor
    - not exactly the same parameters used in the TSM paper
  - runs r3 simulations
    - not exactly for the same biases
  - plots the measured data and simulation results
- Modify these parameters (one at a time), see what happens
  - rsh, dfinf, tc1, tc2, gtha
- Explain qualitatively why the curves change the way they do

Solution (2)

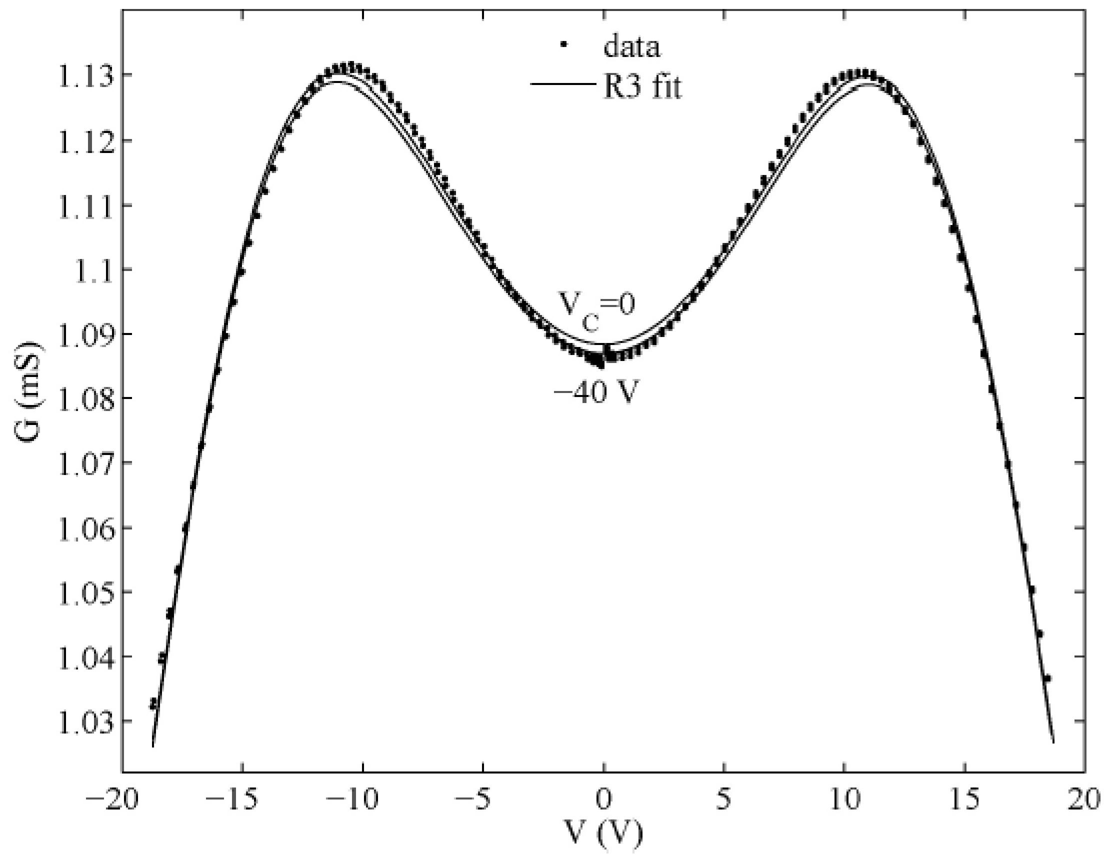

Fig. 13. R3 $G(V)$ for an 87 $\Omega/\square$ poly resistor, $L/W = 42\ \mu\text{m}/4.2\ \mu\text{m}$. Inset shows temperature rise.

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Homework (1)

- Go through the code of the core r3 macro
  - do **not** worry about the pinch-off and limiting code
  - ignore “vpo” and “swaccpo” code, they are complex
- Identify
  - where the flipping of terminals 1 and 2 is done
  - where DIBL is included as a shift in the effective control voltage
  - where the saturation voltage is calculated
  - where the limiting to V_{sat} is done
  - where the depletion factor and r_{μ} are calculated
  - where CLM is applied
- Where is self-heating handled?

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