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ECE695: Reliability Physics of Nano-Transistors

Lecture 34A: Appendix

- Variability by Bootstrap Method

Muhammad Ashraful Alam

alam@purdue.edu

Appendix: Variability by Bootstrap method

Ref. courses.washington.edu/matlab2/Lesson_6.html

Uncertainty in parameters: Least Square

Is the error in W Gaussian distributed ?

$$W \equiv \beta \ln t + c \qquad \ln t \equiv \beta^{-1} W - \beta^{-1} c = a^* W + b^*$$

Inverse fitting is more appropriate … $x = a^* + b^* y$

$$a^* = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum y_i^2 - \left(\sum y_i\right)^2}$$

$$b^* = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$\sigma_\beta^2 = \left(\frac{\delta\beta}{\delta a}\right)^2 \sigma_a^2 + \left(\frac{\delta\beta}{\delta b}\right)^2 \sigma_b^2$$

$$\sigma_c^2 = \left(\frac{\delta c}{\delta a}\right)^2 \sigma_a^2 + \left(\frac{\delta c}{\delta b}\right)^2 \sigma_b^2$$

MLE estimator for Weibull: Did not discuss variability

Recall $$f(t;\alpha,\beta)=\frac{\beta}{\alpha^{\beta}}\cdot t^{\beta-1}\cdot e^{-\left(t/\alpha\right)^{\beta}}$$

$$\ln L=\sum_{i=1}^{n}\ln f(t_i,\alpha,\beta)$$

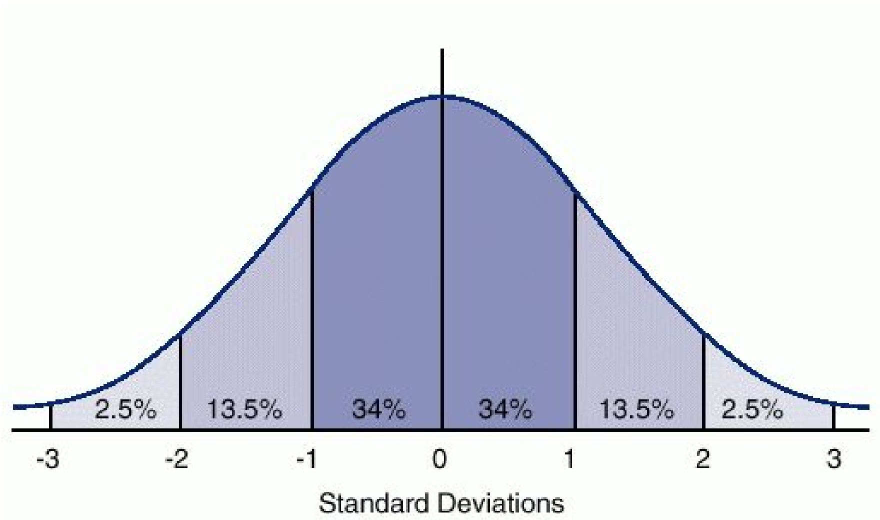
$$=n\ln\beta-n\ln\alpha+(\beta-1)\sum_{i=1}^{n}\ln t_i/\alpha-\sum_{i=1}^{n}\left(t_i/\alpha\right)^{\beta}$$

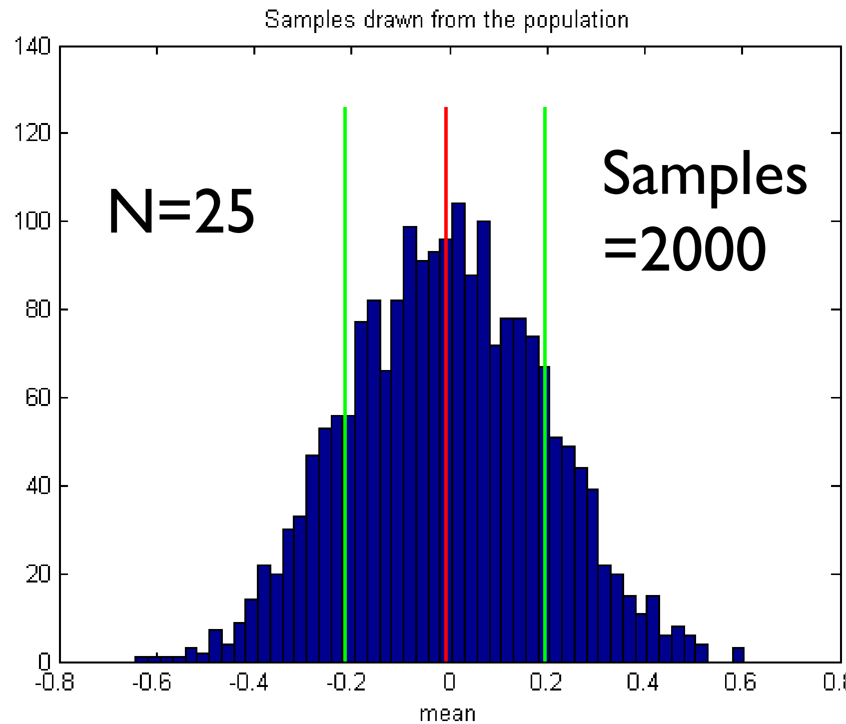
$$\frac{d\ln L}{d\alpha}=0\qquad\frac{d\ln L}{d\beta}=0$$

$$\left(\sum_{i=1}^{n}t_i^{\alpha}\ln(t_i)^{\beta}\Big/\sum_{i=1}^{n}t_i^{\beta}\right)-\frac{1}{n}\sum_{i=1}^{n}\ln(t_i)^{\beta}=1\qquad\alpha=\left[\frac{1}{n}\sum_{i=1}^{n}t_i^{\beta}\right]^{\frac{1}{\beta}}$$

Solve for unknowns α, β

(1) Bootstrap method: Introduction





68% between +0.2 to -0.2
95% between +0.4 to -0.4

$$s = \sqrt{\frac{\sum_{j=1, N=25} (t_j - \langle t \rangle)^2}{N-1}} \sim \sqrt{\frac{1}{24}} \sim 0.2$$

Ref. courses.washington.edu/matlab2/Lesson_6.html

Working with a single sample

0.2 -0.1 0.5 0.3 -0.6 All you have is a single sample ..

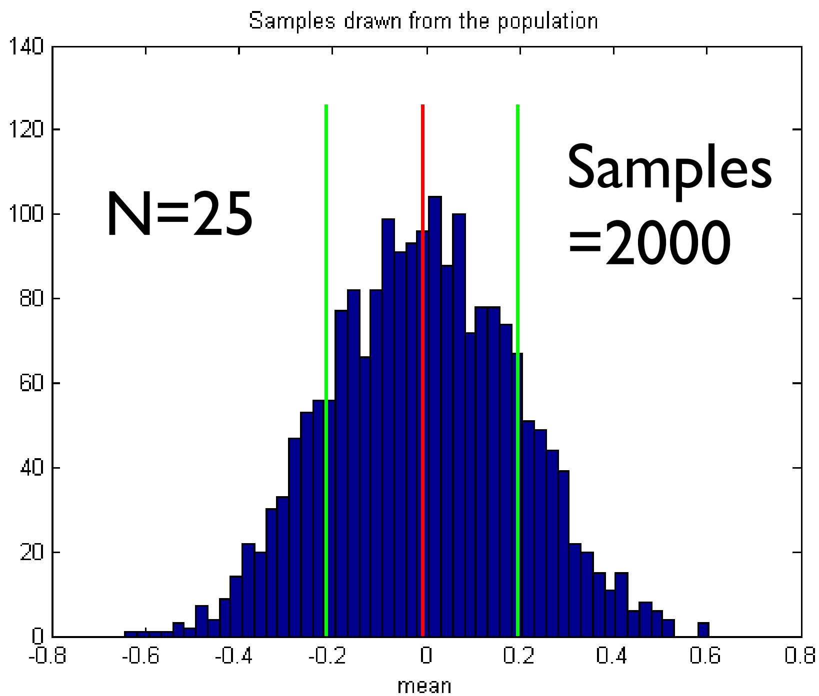
Generate synthetic samples from the original (with replacement)

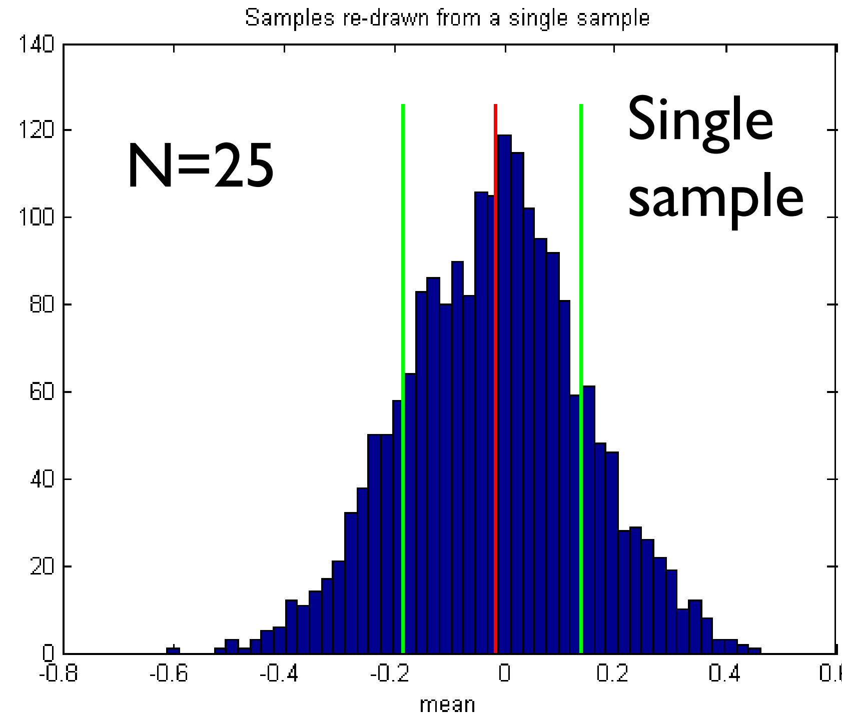
0.2 -0.1 -0.6 -0.1 0.5 Synthetic sample 1

0.3 0.2 -0.6 0.2 0.5 Synthetic sample 2

0.5 -0.1 0.5 0.2 0.3 Synthetic sample 3

Multiple sample vs. single sample





Bootstrap average is not zero!
And yet, the s~0.18, just from a single sample.
The success of the method relies on precision measurement

Parametric vs. non-parametric Bootstrap

0.2 -0.1 0.5 0.3 -0.6

Fit the distribution of your choice by Maximum likelihood estimators (MLE) (obtain parameters, i.e. η_0, β_0)

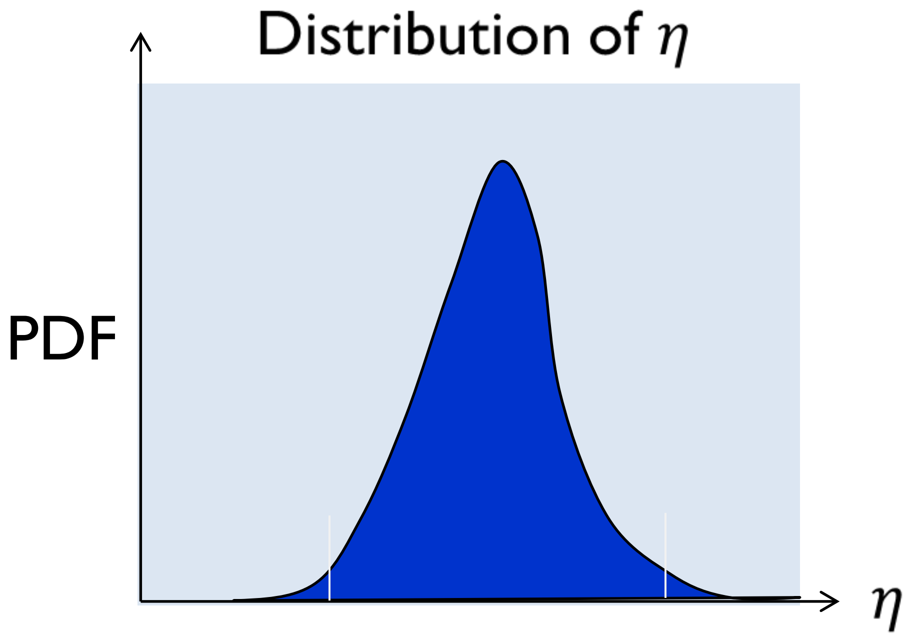
Generate synthetic samples based on the parametric distribution

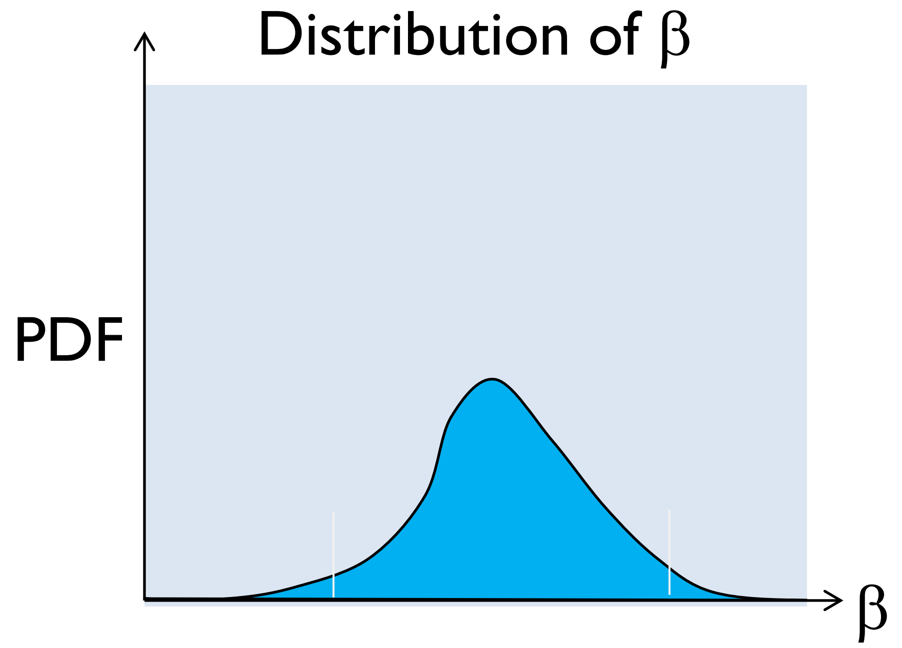
0.12 -0.17 -0.44 -0.71 0.52 Synthetic sample 1 (new η_1, β_1)

0.32 0.21 -0.69 0.23 0.58 Synthetic sample 2 (new η_2, β_2)

Plot distribution of statics η_i, β_i

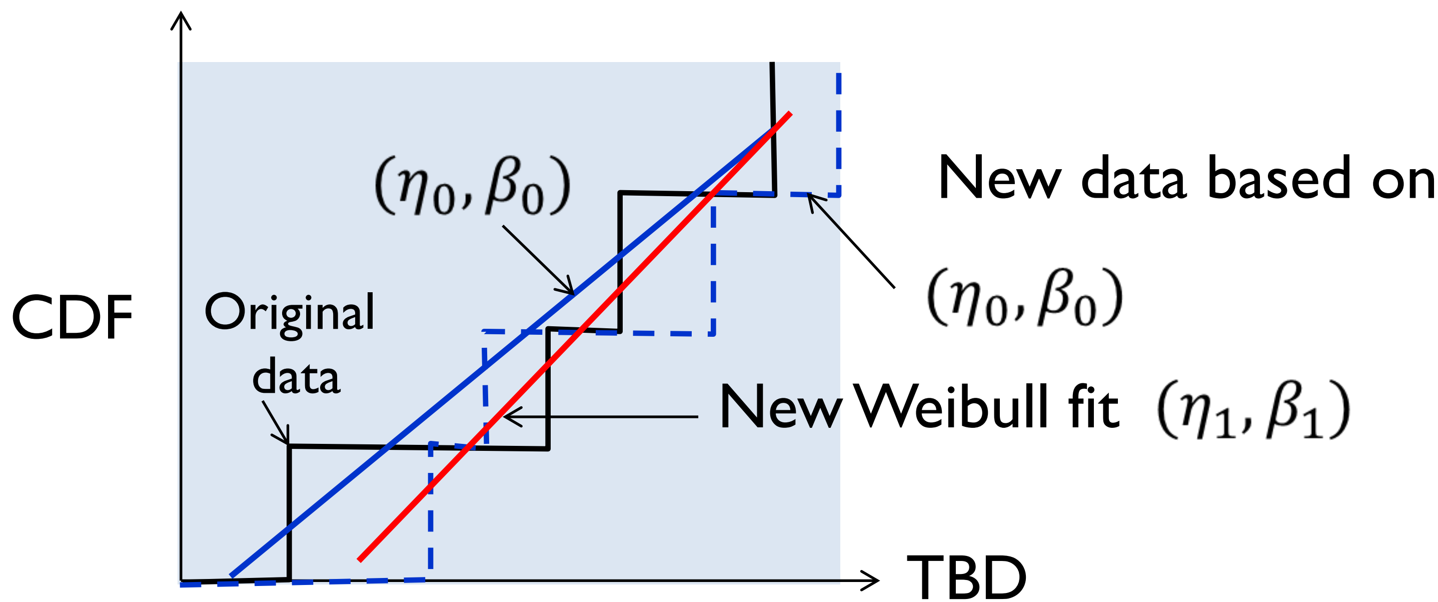
Distribution of α and β





Same technique for polling and tenure rate of faculty!

Why resampling from the same distribution generates new fit parameters



Samples taken from the same distribution (η_0, β_0) generates datapoints that are fitted with new (η_i, β_i)

References

1. "Detecting Novel Associations for large scale dataset", D. Reshef et al. , Science 334, p. 1418, 2011.
2. "Survival Analysis of Faculty Retention in Science and Engineering", D. Kaminski et al., Science, 335, 864, 2012.
3. Clauset, C. R. Shalizi, and M. E. J. Newman, "Power-Law Distributions in Empirical Data," SIAM Review, vol. 51, no. 4, p. 661, Nov. 2009.