

**PQC WORKSHOP 2017**  
**COHERENT EFFECTS IN PHYSICS AND CHEMISTRY**  
**April 28<sup>th</sup> (Fri), 12-5pm in MRGN 121/MANN**

***Quantum Coherent Transport  
in Atoms & Electrons***

[Yong P. Chen](#), Quantum Matter and Devices Laboratory

Dept. of Physics and School of Electrical & Computer Engineering,

Birck Nanotechnology Center & Purdue Quantum Center,

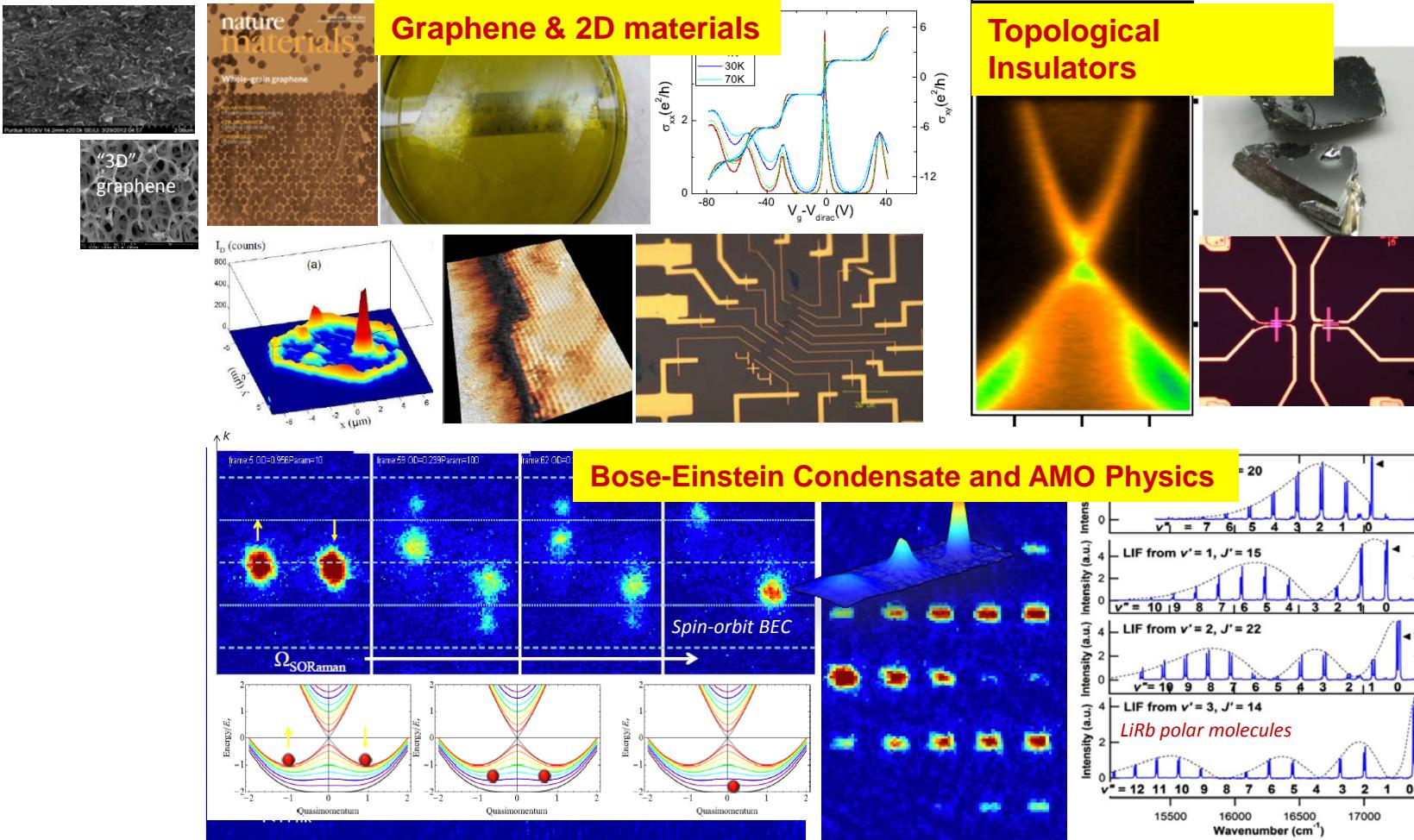
Purdue University, West Lafayette INDIANA 47907 USA

<http://www.physics.purdue.edu/quantum>

<http://engineering.purdue.edu/PQC/>

# Research in Quantum Matter and Device (QMD) Laboratory

<http://www.physics.purdue.edu/quantum> /QMD>=|physics>+|engineering>



General remark

**"Path"** can be in

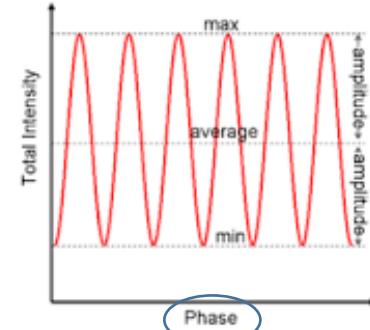
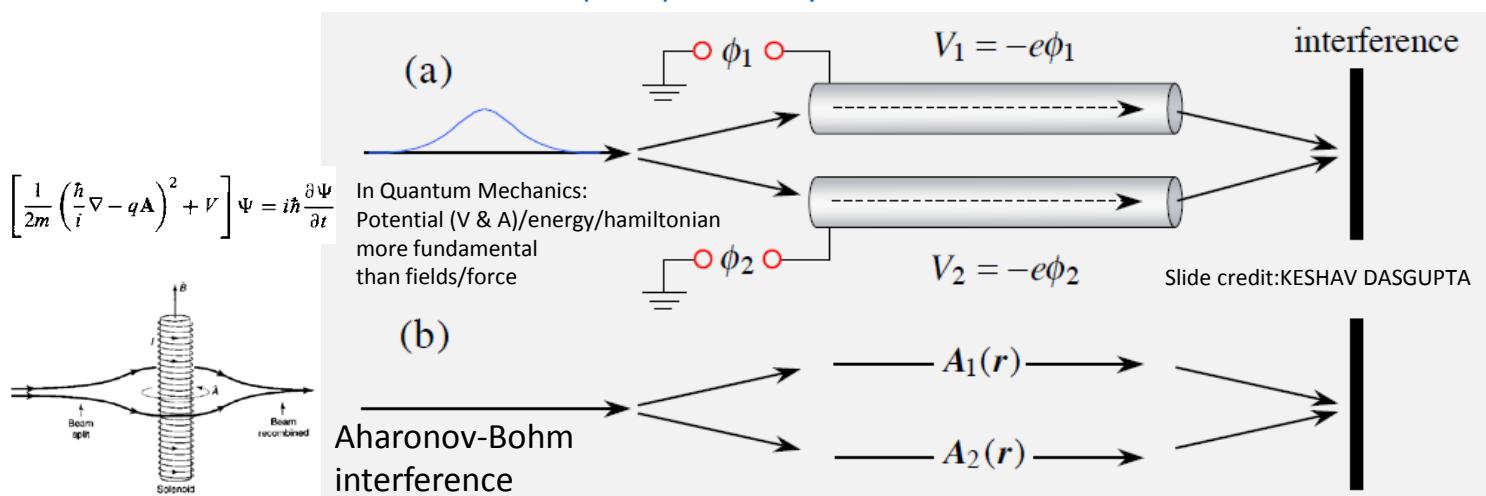
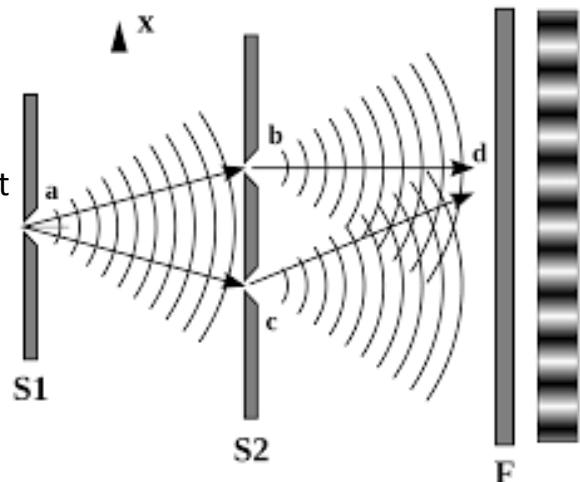
- Real ( $r$ ) – space
- Momentum ( $k$ ) – space
- More abstract/complicated “configuration”/Hilbert space....  
[sometimes related to “topology”]

**"Wave"** can be

- Light (classical E&M or quantum)
- electrons
- atoms
- other “matter wave” or more complex quantum systems

## Two-path interference

Young' double-slit interference  
(optics/light  
**wave**)



## Outline

- **Quantum Coherent Transport of Atoms (cold atom BEC): A Spin-resolved Atom Interferometer (Stueckelberg Interference)**

PHYSICAL REVIEW A **95**, 043623 (2017)

Stueckelberg interferometry using periodically driven spin-orbit-coupled Bose-Einstein condensates

Abraham J. Olson,<sup>1</sup> David B. Blasing,<sup>1</sup> Chunlei Qu,<sup>2,3</sup> Chuan-Hsun Li,<sup>4</sup> Robert J. Niffenegger,<sup>1</sup> Chuanwei Zhang,<sup>2</sup> and Yong P. Chen<sup>1,4,5,\*</sup>



<sup>1</sup>Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47907, USA

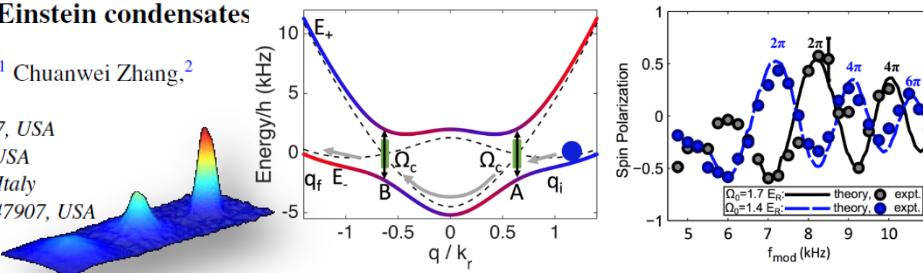
<sup>2</sup>Department of Physics, The University of Texas at Dallas, Richardson, Texas 75080, USA

<sup>3</sup>INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, Povo 38123, Italy

<sup>4</sup>School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47907, USA

<sup>5</sup>Purdue Quantum Center, Purdue University, West Lafayette, Indiana 47907, USA

\$ ARO-DURIP, NSF-GRF, Purdue OVPR



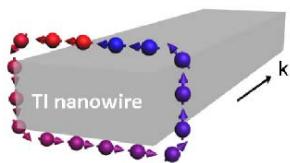
- **Quantum Coherent Transport of Electrons (spin-helical Dirac fermions on topological insulators): a “half-integer” Aharonov-Bohm Effect (electronic interferometer)**

nature  
nanotechnology

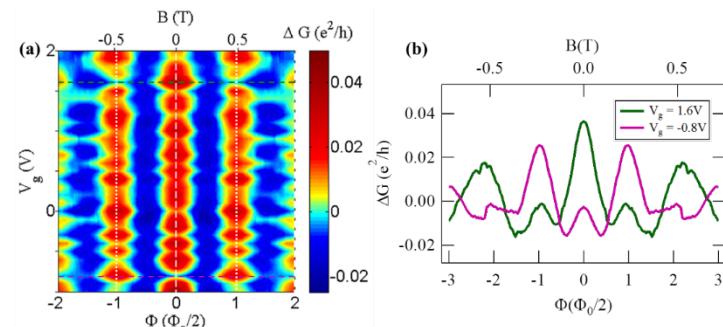
PUBLISHED ONLINE: 18 JANUARY 2016 | DOI: 10.1038/NNANO.2015.293

ARTICLES

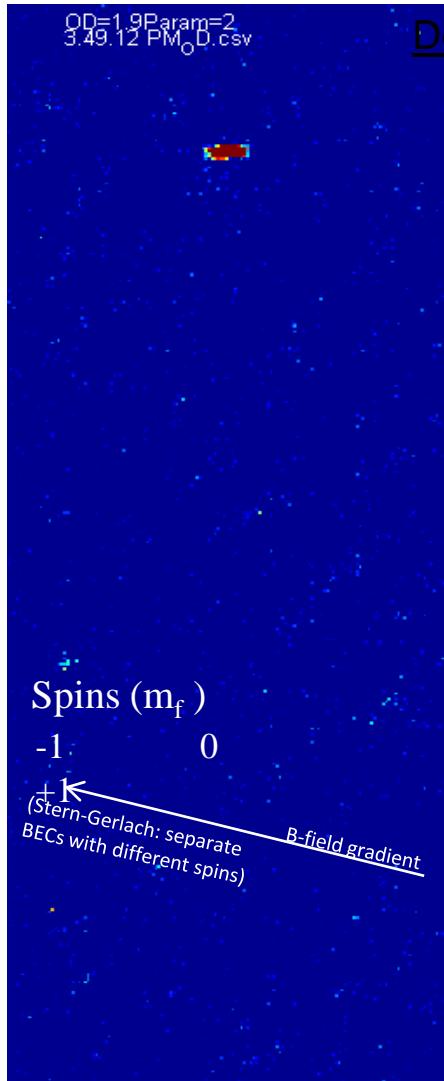
Magnetic field-induced helical mode and topological transitions in a topological insulator nanoribbon



Luis A. Jauregui<sup>1,2</sup>, Michael T. Pettes<sup>3,†</sup>, Leonid P. Rokhinson<sup>1,2,4</sup>, Li Shi<sup>3,5</sup> and Yong P. Chen<sup>1,2,4,\*</sup>

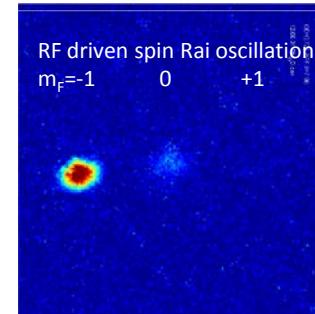
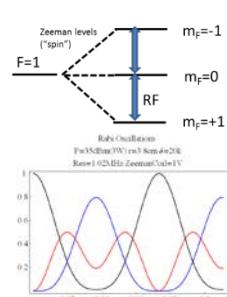
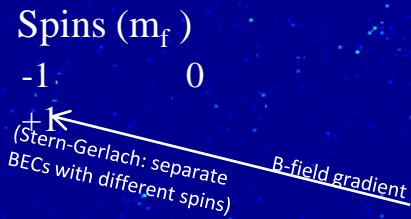


cold atoms/BEC --- “seeing” quantum mechanics & dynamics!  
 (“slowed down” and “blown up” so much that you can shoot photos & videos!)

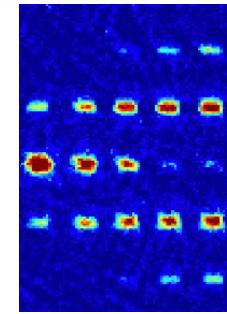


## Demo with our Bose-Einstein Condensation (BEC)

Purdue QMD’s “all-optical” Rb87 BEC apparatus  
 With synthetic gauge fields and spin-orbit coupling



coherent oscillation of BEC  
 between 3 spin states



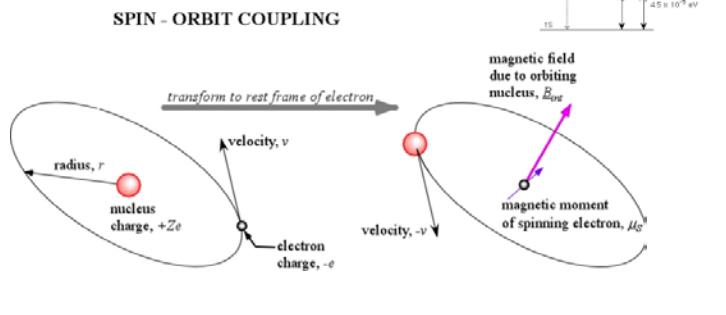
BEC (matter wave) diffraction from  
 laser standing wave (optical grating)

# How to create an atomic beam splitter -- a trick: “Spin Orbit Coupling” (SOC)

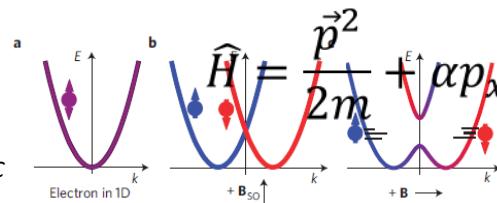
Review of electronic SOC:

a relativistic effect --- moving E-field acts as B-field:  $\vec{B}_{SO} \sim \left( \frac{\hbar}{mc^2} \right) \vec{v} \times \vec{E}$

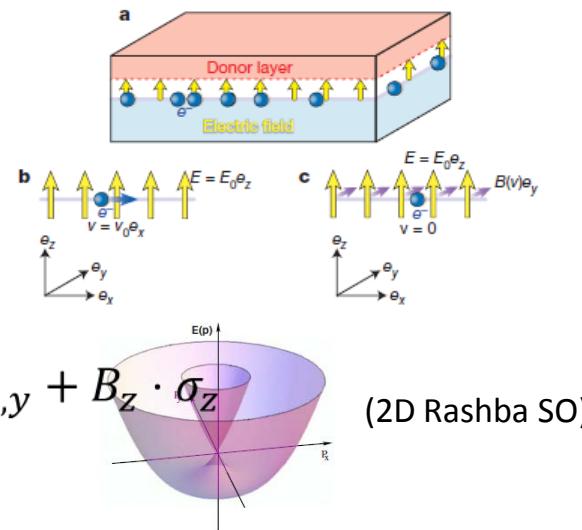
## Atomic Fine Structure



Can simulate these  
(solid state & beyond) SOC  
with neutral atoms



## Inversion Symmetry Breaking



(2D Rashba SO)

# REVIEW

doi:10.1038/nature11841

REVIEWS OF MODERN PHYSICS, VOLUME 83, OCTOBER–DECEMBER 2011

## Colloquium: Artificial gauge potentials for neutral atoms

Jean Dalibard\* and Fabrice Gerbier†

Laboratoire Kastler Brossel, CNRS, UPMC, Ecole normale supérieure,  
24 rue Lhomond, 75005, Paris, France

Gediminas Juzeliūnas‡

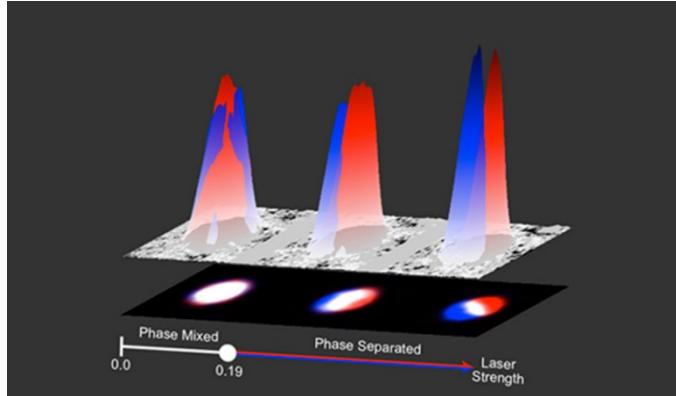
Institute of Theoretical Physics and Astronomy, Vilnius University,  
A. Goštauto 12, Vilnius 01108, Lithuania

Patrik Öhberg§

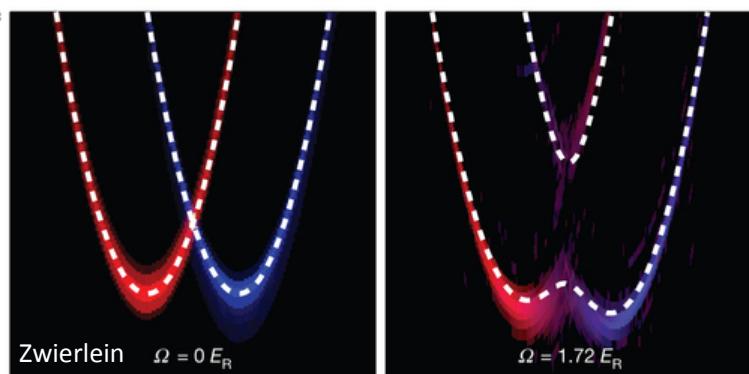
SUPA, Department of Physics, Heriot-Watt University,  
Edinburgh, EH14 4AS, United Kingdom

(published 30 November 2011)

Spin-orbit coupling links a particle's velocity to its quantum-mechanical spin, and is essential in numerous condensed matter phenomena, including topological insulators and Majorana fermions. In solid-state materials, spin-orbit coupling originates from the movement of electrons in a crystal's intrinsic electric field, which is uniquely prescribed in any given material. In contrast, for ultracold atomic systems, the engineered 'material parameters' are tunable: a variety of synthetic spin-orbit couplings can be engineered on demand using laser fields. Here we outline the current experimental and theoretical status of spin-orbit coupling in ultracold atomic systems, discussing unique features that enable physics impossible in any other known setting.



(spin-orbit BEC – I.Spielman/NIST)  
[also USTC, SXU, WSU, Purdue, ..]

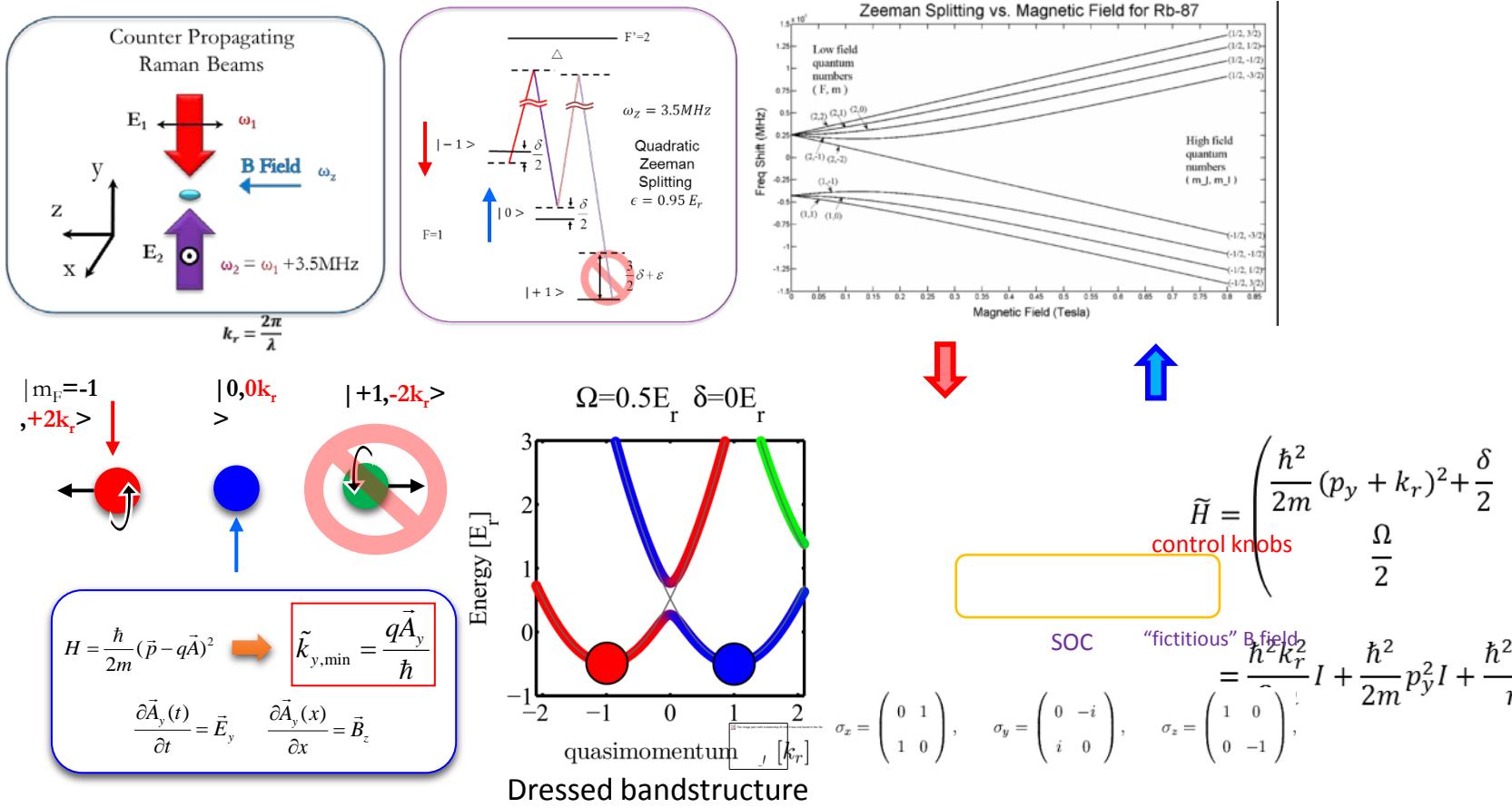


(spin-orbit fermi gas)  
[MIT, SXU, NIST, ..]

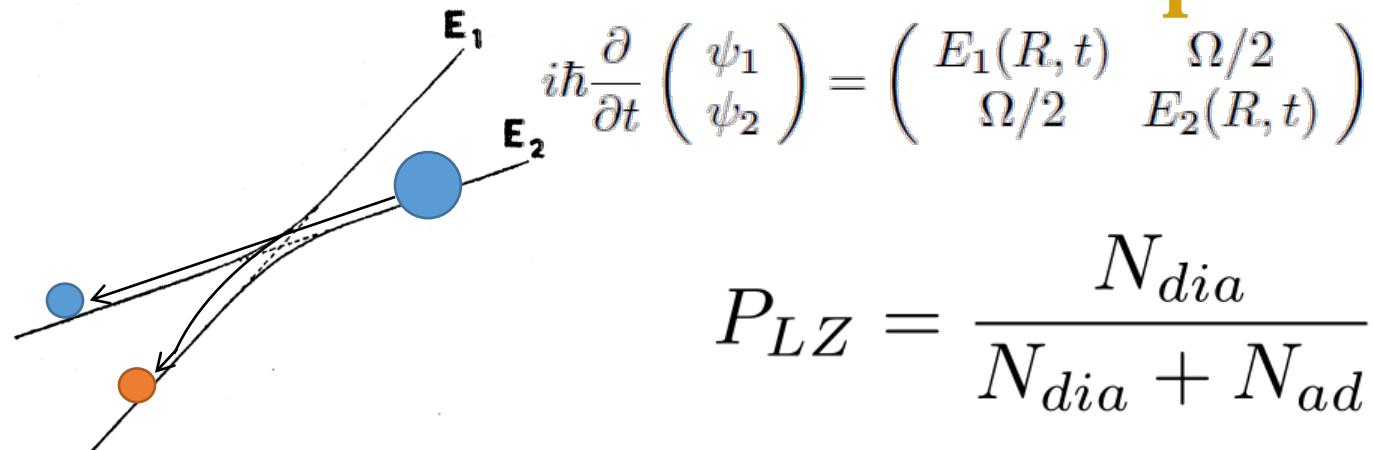
See also reviews by: H. Zhai'11; Y. Li-G.Martone-S.Stringari'14, J.Zhang'14 etc.

Note: there are several other ways to generate SOC and gauge fields (e.g., modulation..)

# Synthetic Spin Orbit Coupling (SOC) by Raman



# Landau-Zener Transitions: a quantum crossroad “beam splitter”



$$P_{LZ} = \frac{N_{dia}}{N_{dia} + N_{ad}}$$

$\mathbf{R}_o \quad \mathbf{R} \rightarrow$  —Crossing of energy levels in idealised problem.  $P_{LZ} = \exp \left[ -2\pi \frac{(\Omega/2)^2}{\hbar v \beta} \right]$

Full lines are adiabatic eigenwerte.

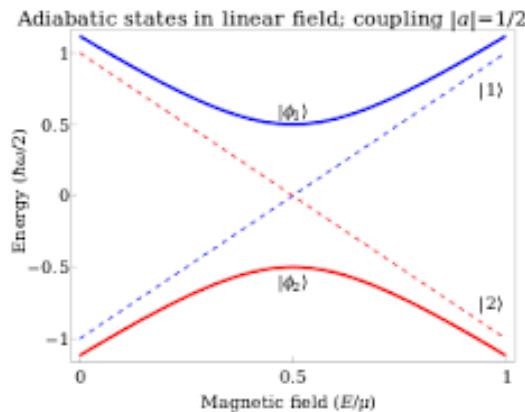
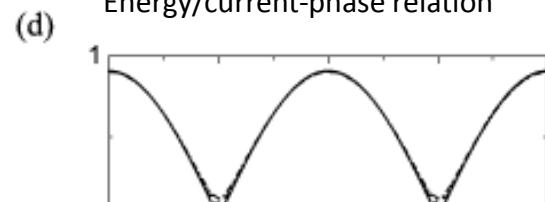
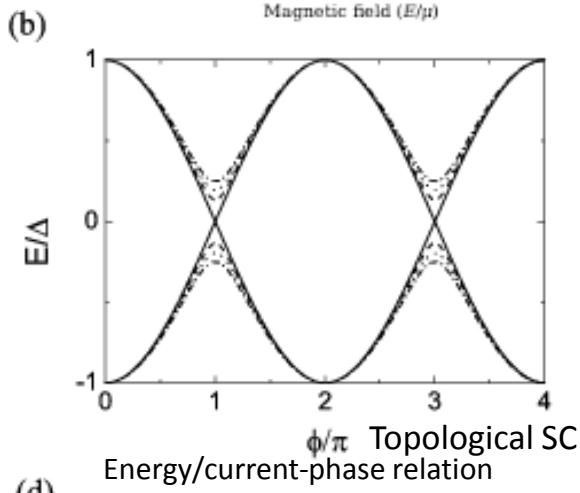
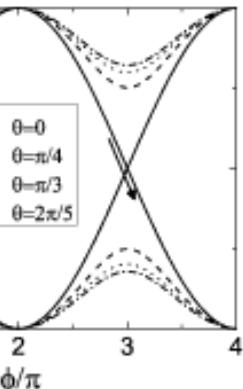
$$v = \frac{dR}{dt}$$

$$\beta = [\partial E_1(x)/\partial x - \partial E_2(x)/\partial x]_{x=x_c}$$

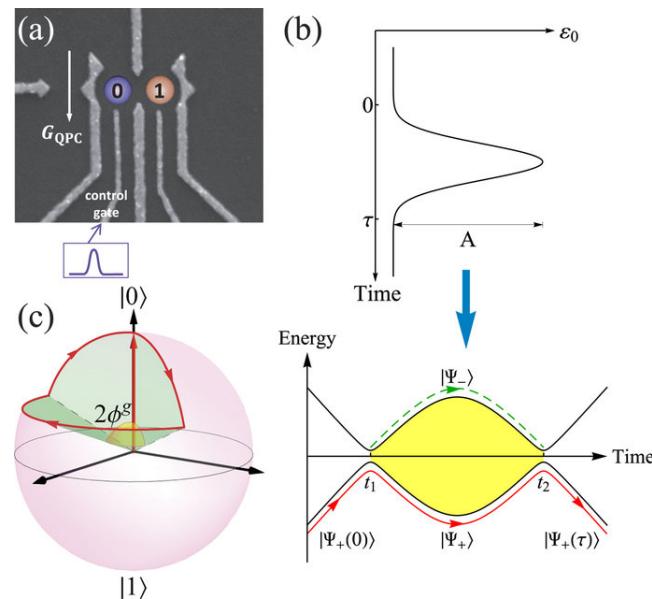
$\Omega/\omega$  — coupling between diabatic states

$$\beta = [\partial E_1(R)/\partial R - \partial E_2(R)/\partial R]_{R_c}$$

# Importance of Landau-Zener



Quantum dynamics,  
Quantum state control/transfer/measurement...



Guo et al'15

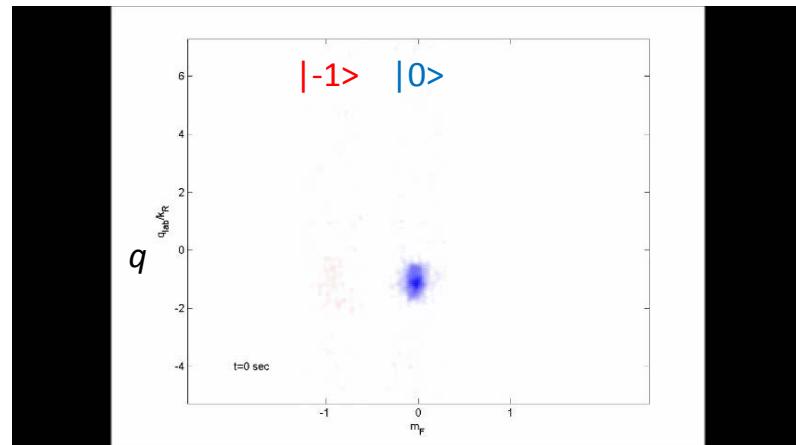
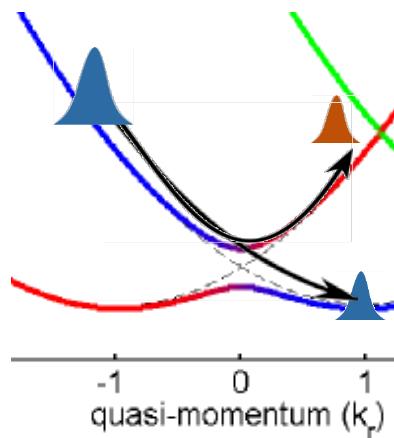
# LZ transition for a SO coupled BEC

$$P_{LZ} = \frac{N_{dia}}{N_{dia} + N_{ad}} = \exp \left[ -2\pi \frac{(\Omega/2)^2}{\hbar v \beta} \right]$$

$$v \equiv \frac{dq}{dt}$$

$\beta \equiv$  difference in slopes of  $E_{dia}(q)$

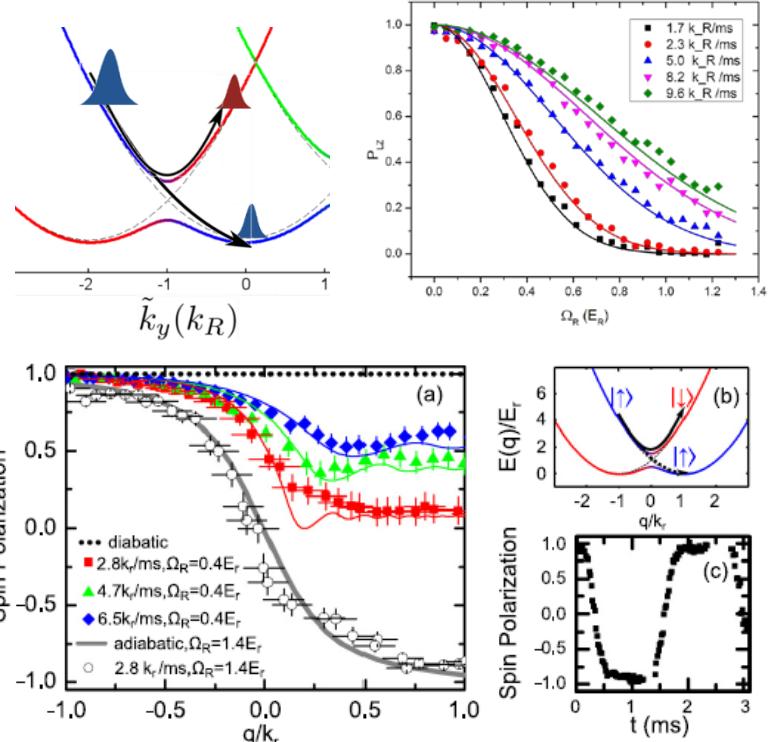
$\Omega/2 \equiv$  Raman coupling



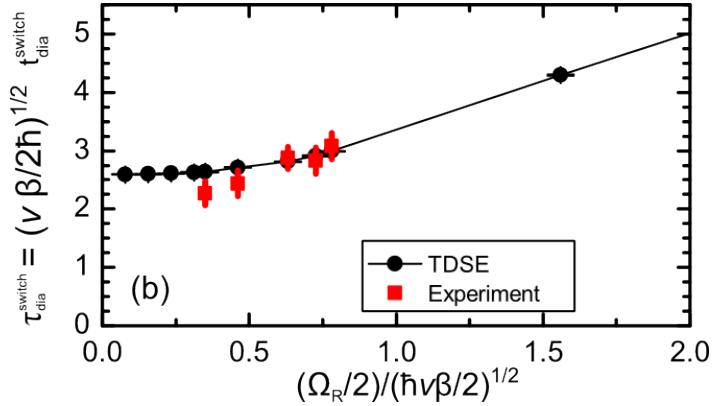
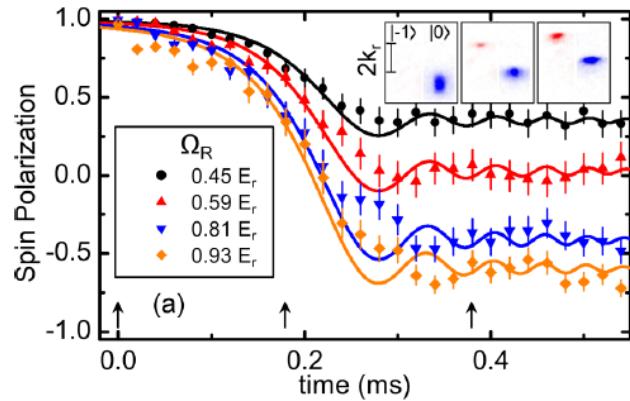
# Tunable Landau-Zener Transition (between SOC dressed bands)

- Landau-Zener model  
Varied all three parameters
- acceleration in SO gauge fields by gravity or trapping potential
- Spin dependent “atomtronic” transistor
- Non-adiabatic breakdown of spin-momentum locking in SOC BEC
- (spin-dependent) atomic **beam splitter** (*in momentum space*)

$$P_{LZ} = \exp \left[ -2\pi \frac{(\Omega/2)^2}{\hbar v \beta} \right]$$



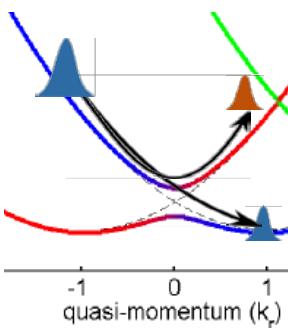
# Time-dependent LZ measurements



$$dq/dt = 5.0 k_r / \text{ms}$$

$$\beta = 4E_R/k_r$$

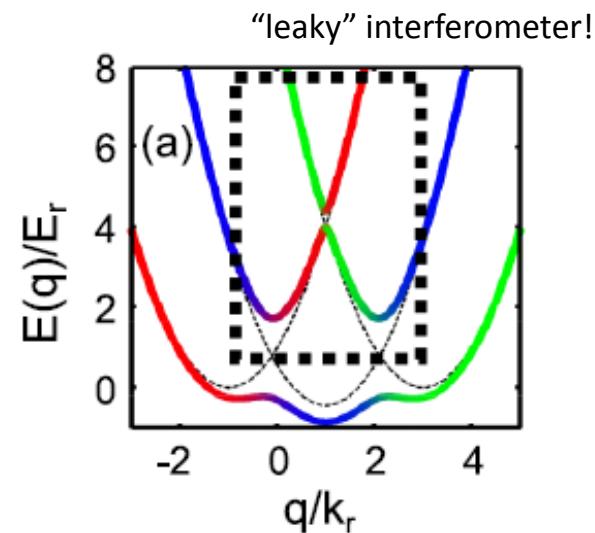
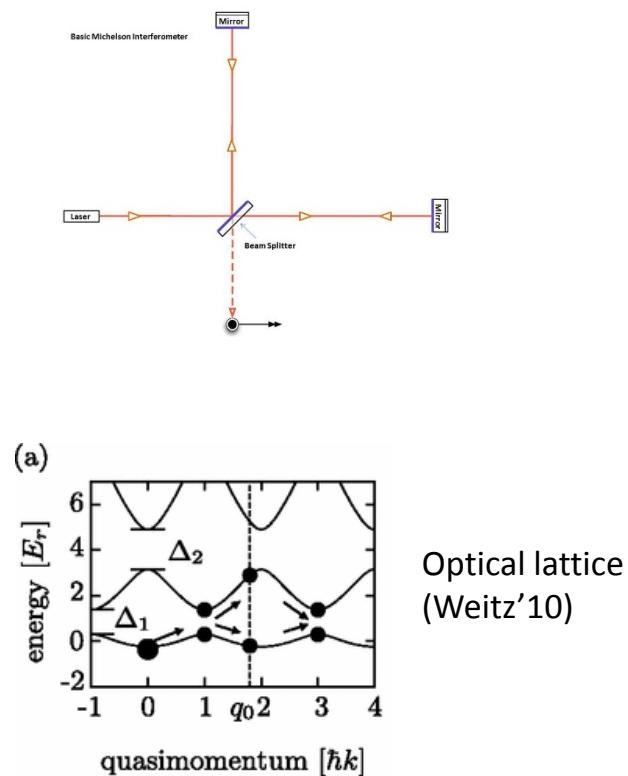
Adiabatic limit  $\sim 2.5$  agrees with  
N.V. Vitanov, Phys. Rev. A (1999)



t-dependent Schrodinger equation in k space (S.Wang & C. Greene)

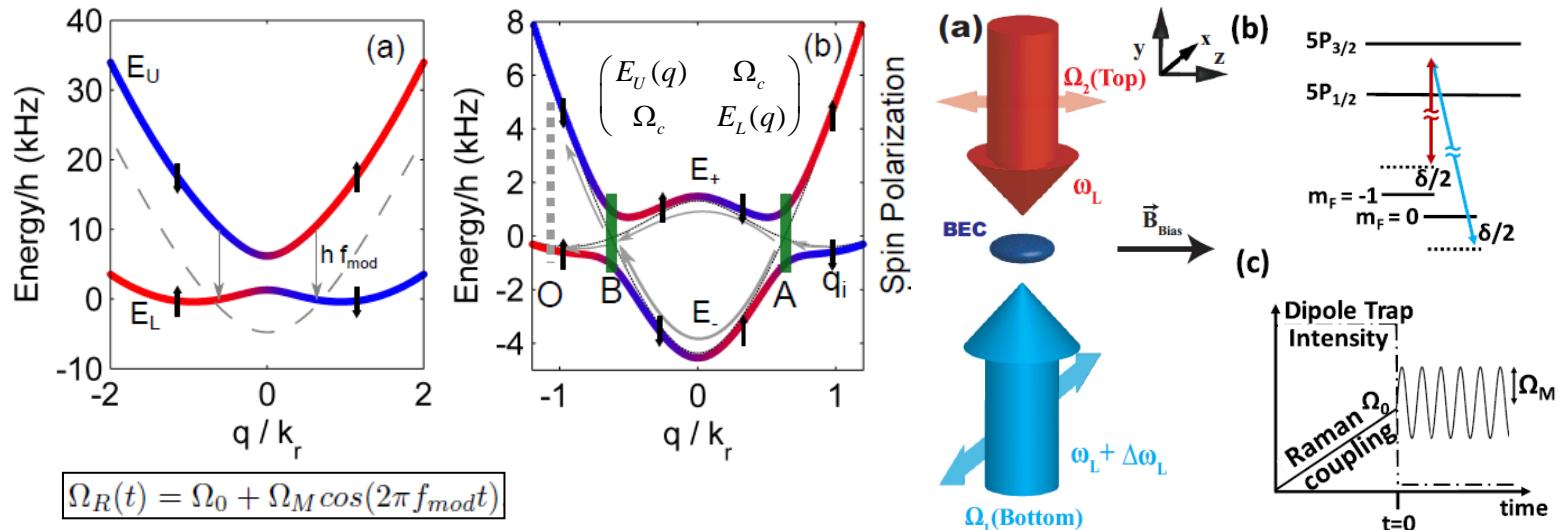
$$i\hbar\partial_t\Psi(q,t) = \hat{H}\Psi(q,t) = \left[ -\frac{1}{2}m\omega^2\frac{\partial^2}{\partial q^2} + \begin{pmatrix} \frac{\hbar^2}{2m}(q+k_R)^2 - \delta/2 & \Omega_R/2 \\ \Omega_R/2 & \frac{\hbar^2}{2m}(q-k_R)^2 + \delta/2 \end{pmatrix} \right] \Psi(q,t),$$

# Landau-Zener beamsplitter twice: Stuckelberg Interference



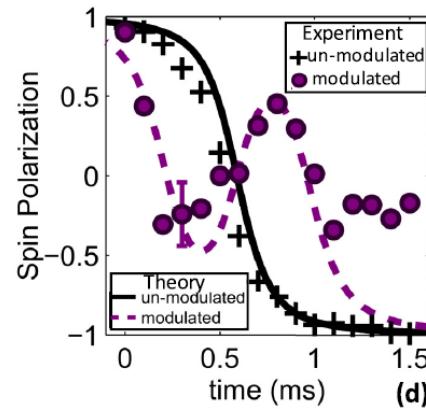
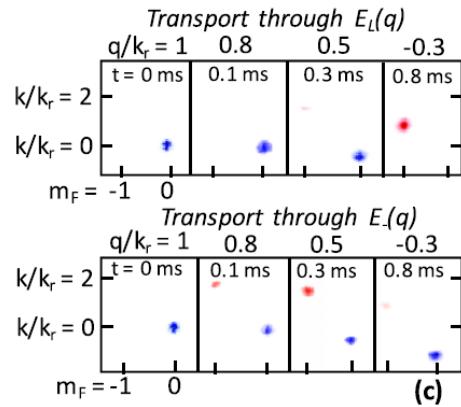
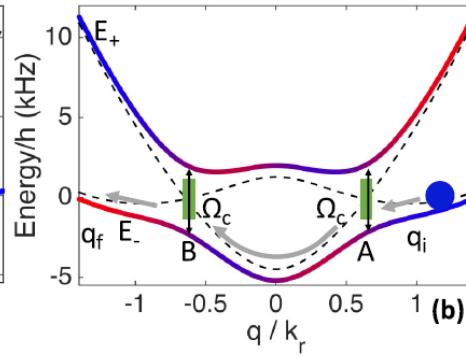
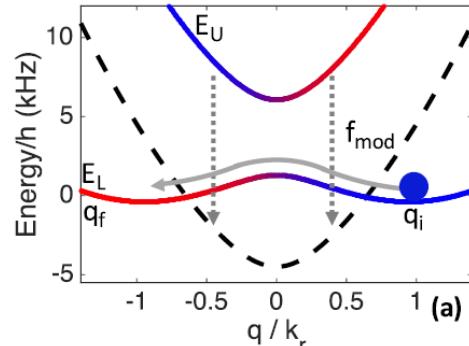
# Stueckelberg interference and engineering an atom-interferometer with light-induced synthetic gauge fields

Abraham J. Olson\* and , et al.

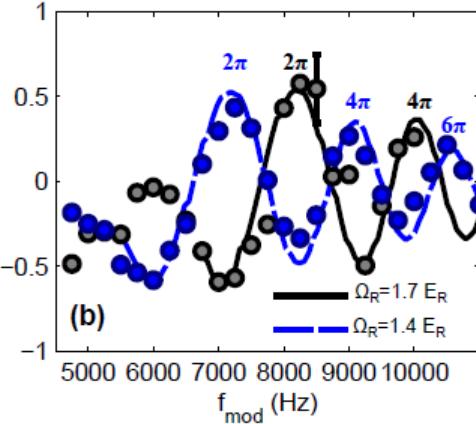
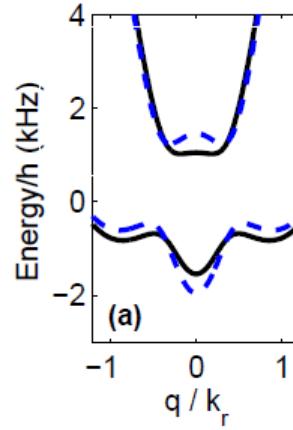
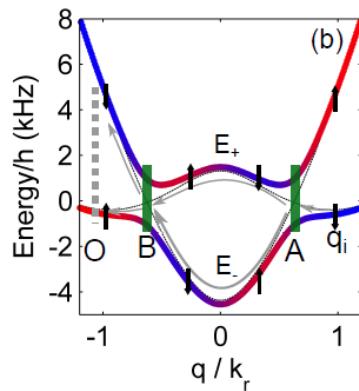


- Modulate Raman beam intensity → induce inter-dressed band coupling  
→ “dressing” dressed band → create “2<sup>nd</sup> generation” synthetic SOC/dressed bands
- Creates 2 atomic beam splitters in  $k$  space
- Stuckelberg Atom interferometer  
(oscillation of spin output)

# A new-spin-momentum texture!



# Observation of Stueckelberg interference in dressed bands: oscillating spin polarization

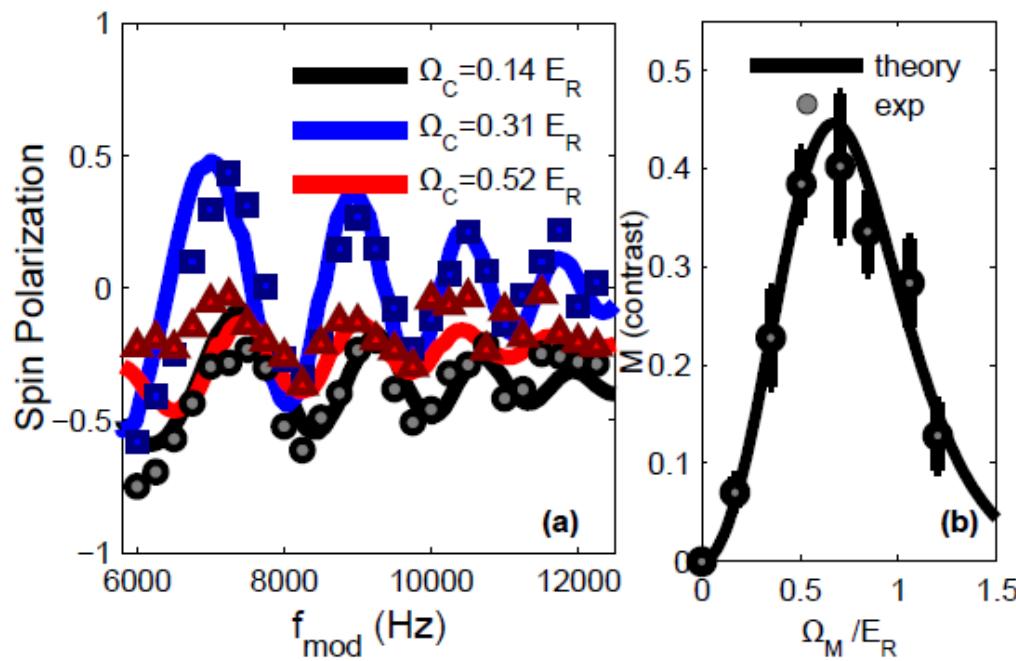


$$\Phi(\alpha) = \int_{q_A}^{q_B} [E_+(q) - E_-(q)] dq / (\hbar\alpha).$$

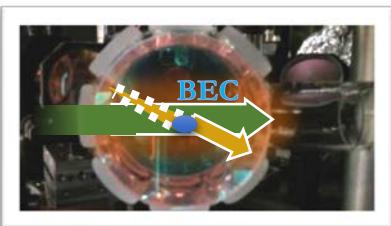
$$\mathcal{S}(\alpha) = 4[P_{LZ}(\alpha) - P_{LZ}(\alpha)^2] \cos[\Phi(\alpha)] - [1 - 2P_{LZ}(\alpha)]^2.$$

Compare to previous LZS interference in optical lattice (no spin): Arimondo'09; Weitz'10

# 'Fringe' contrast vs modulation

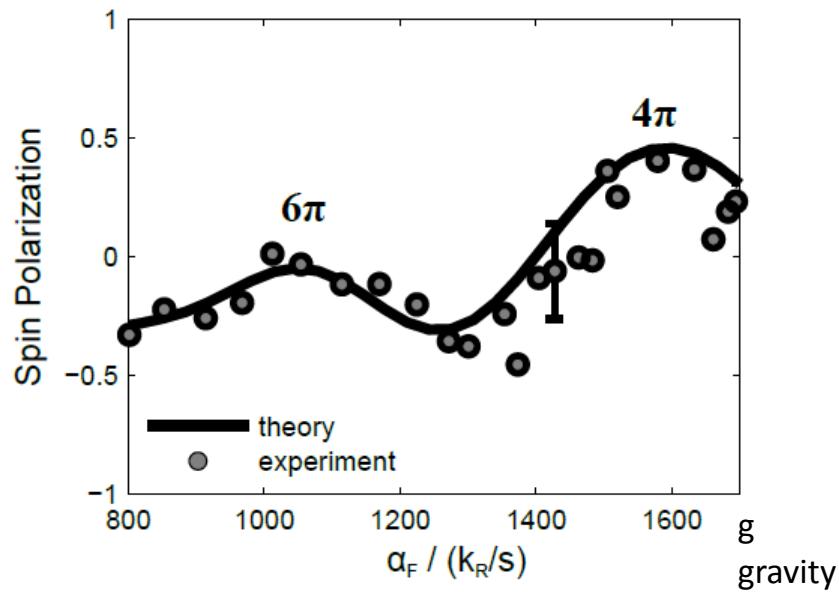


# Measuring acceleration (proof of principle)



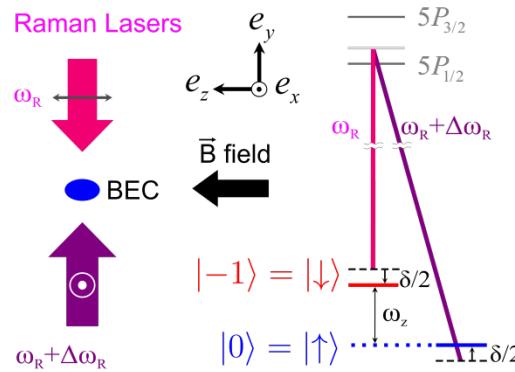
1550nm cross-beam optical trap  
(optical tweezer)

A.J. Olson *et al.*, Phys. Rev. A 87,  
053613 (2013)

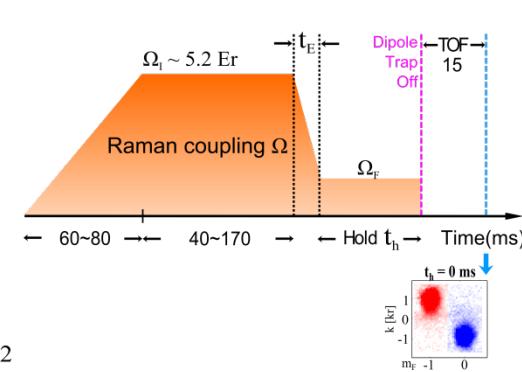


# New experiment: quantum quench and coherent atomic spin current

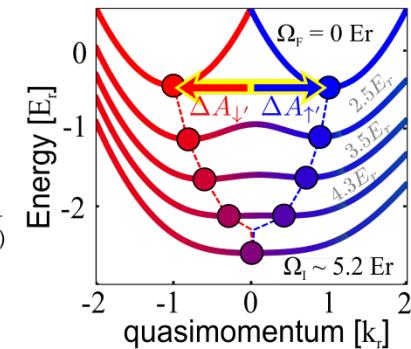
a Geometry and Raman coupling



b Experimental timing



c Band diagrams

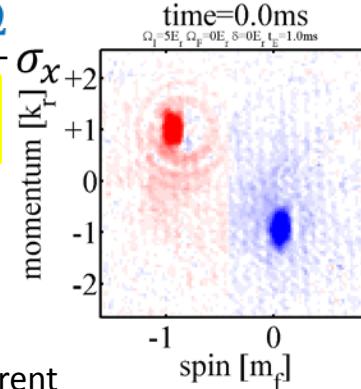


$$H = \frac{\hbar^2}{2m} p_y^2 I + \frac{\hbar^2 k_r}{m_A \sigma} p_y \sigma_z + \frac{\Omega}{2} \sigma_x$$

*"quantum quench"*

$$E_{\sigma} = \frac{m_A \sigma}{\delta t} \approx \frac{\Delta A_{\sigma}}{t_E}$$

"Collide 2 spinor BECs in trap"



SDM previously studied in non SOC quantum gases,  
eg. fermi gas: Sommer [Zwierlein] et al'11 by magnetic gradient  
bosons: Koller et al'12; Maddaloni et al'00  
theory (fermi gas): Stringari'99, etc.

What is the effect of SOC?

Spin Dipole Mode  
(SDM) --- AC spin current

## Outline

- Quantum Coherent Transport of Atoms (cold atom BEC): A Spin-resolved Atom Interferometer (Stueckelberg Interference)

PHYSICAL REVIEW A 95, 043623 (2017)

Stueckelberg interferometry using periodically driven spin-orbit-coupled Bose-Einstein condensates

Abraham J. Olson,<sup>1</sup> David B. Blasing,<sup>1</sup> Chunlei Qu,<sup>2,3</sup> Chuan-Hsun Li,<sup>4</sup> Robert J. Niffenegger,<sup>1</sup> Chuanwei Zhang,<sup>2</sup> and Yong P. Chen<sup>1,4,5,\*</sup>

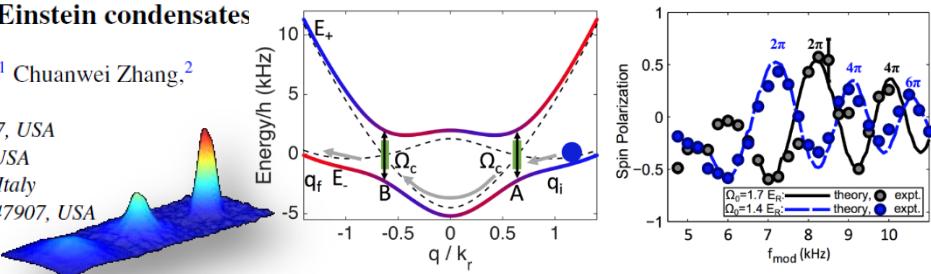
<sup>1</sup>Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47907, USA

<sup>2</sup>Department of Physics, The University of Texas at Dallas, Richardson, Texas 75080, USA

<sup>3</sup>INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, Povo 38123, Italy

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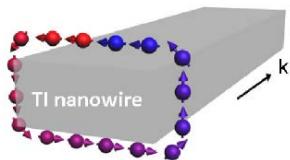


- Quantum Coherent Transport of Electrons (spin-helical Dirac fermions on topological insulators):  
a “half-integer” Aharonov-Bohm Effect (electronic interferometer)

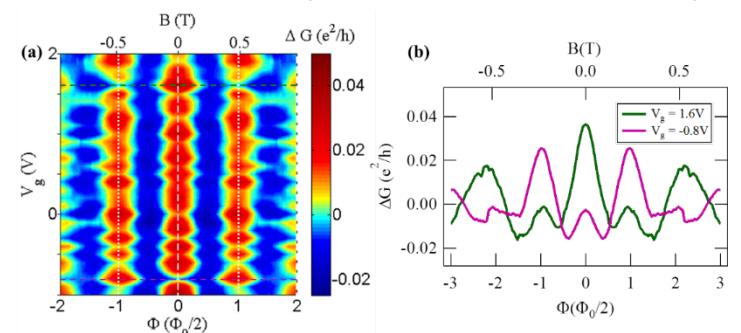
\$ DARPA, Intel, Purdue-PCTM



Magnetic field-induced helical mode and topological transitions in a topological insulator nanoribbon



Luis A. Jauregui<sup>1,2</sup>, Michael T. Pettes<sup>3,†</sup>, Leonid P. Rokhinson<sup>1,2,4</sup>, Li Shi<sup>3,5</sup> and Yong P. Chen<sup>1,2,4\*</sup>

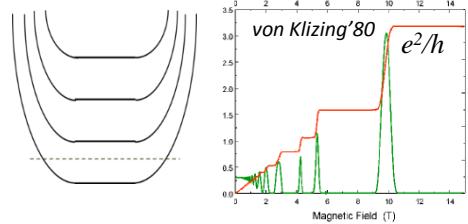
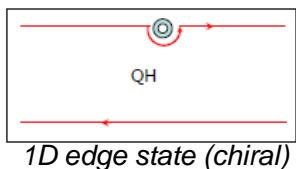


# From Quantum Hall Effect (QHE) to Topological Insulator (TI)



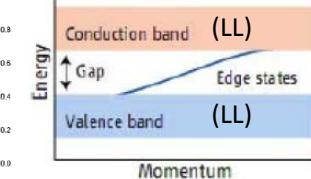
## 2D Quantum Hall

[time-reversal breaking:  
external  $B \neq 0$ ]



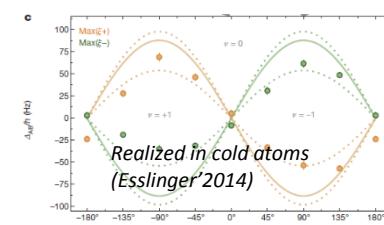
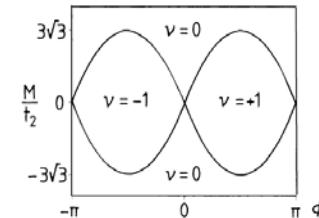
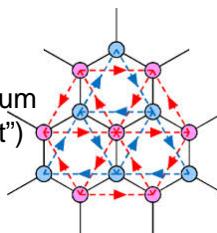
$$K = \frac{i}{2\pi} \times \int_{BZ} d^2k \left( \left( \frac{\partial \Phi_0}{\partial k_x} \right| \frac{\partial \Phi_0}{\partial k_y} - \left( \frac{\partial \Phi_0}{\partial k_y} \right| \frac{\partial \Phi_0}{\partial k_x} \right)$$

Thouless



## Haldane Model '88

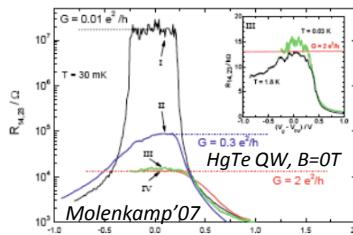
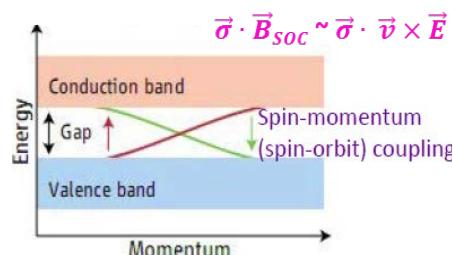
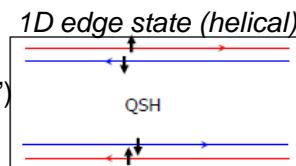
(2D "Chern insulator"/"quantum anomalous Hall (QAH) effect")  
[time-reversal breaking;  
 $B=0$  (no LL)]



## 2D Quantum Spin Hall

("2D topological insulator")  
[time-reversal invariant:  
 $B=0$ ] (2D TI)

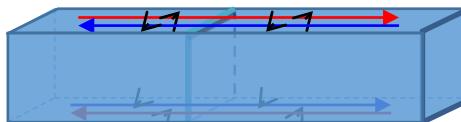
Kane-Mele'05; S.C.Zhang et al'06



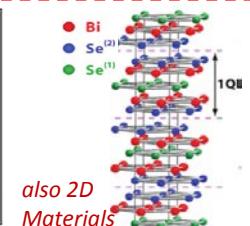
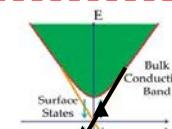
## 3D (strong) Topological Insulator

[time-reversal invariant:  $B=0$ ]

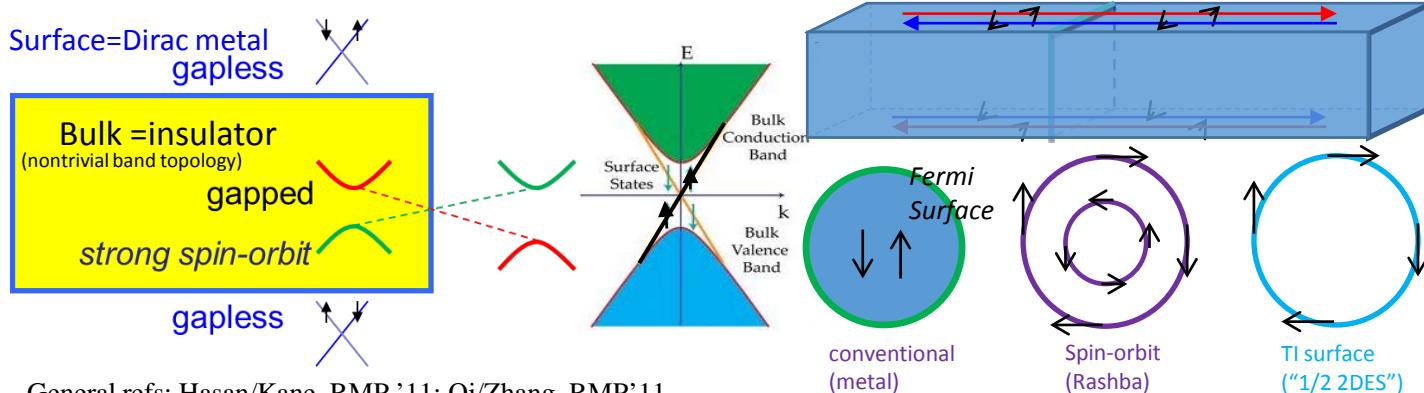
Fu-Kane'07; Moore-Balents'07; Roy'09; Qi-Zhang'08...



## 2D surface state ("spin-helical Dirac fermion")



## (3D) Topological Insulator & Topological Surface State



General refs: Hasan/Kane, RMP '11; Qi/Zhang, RMP'11

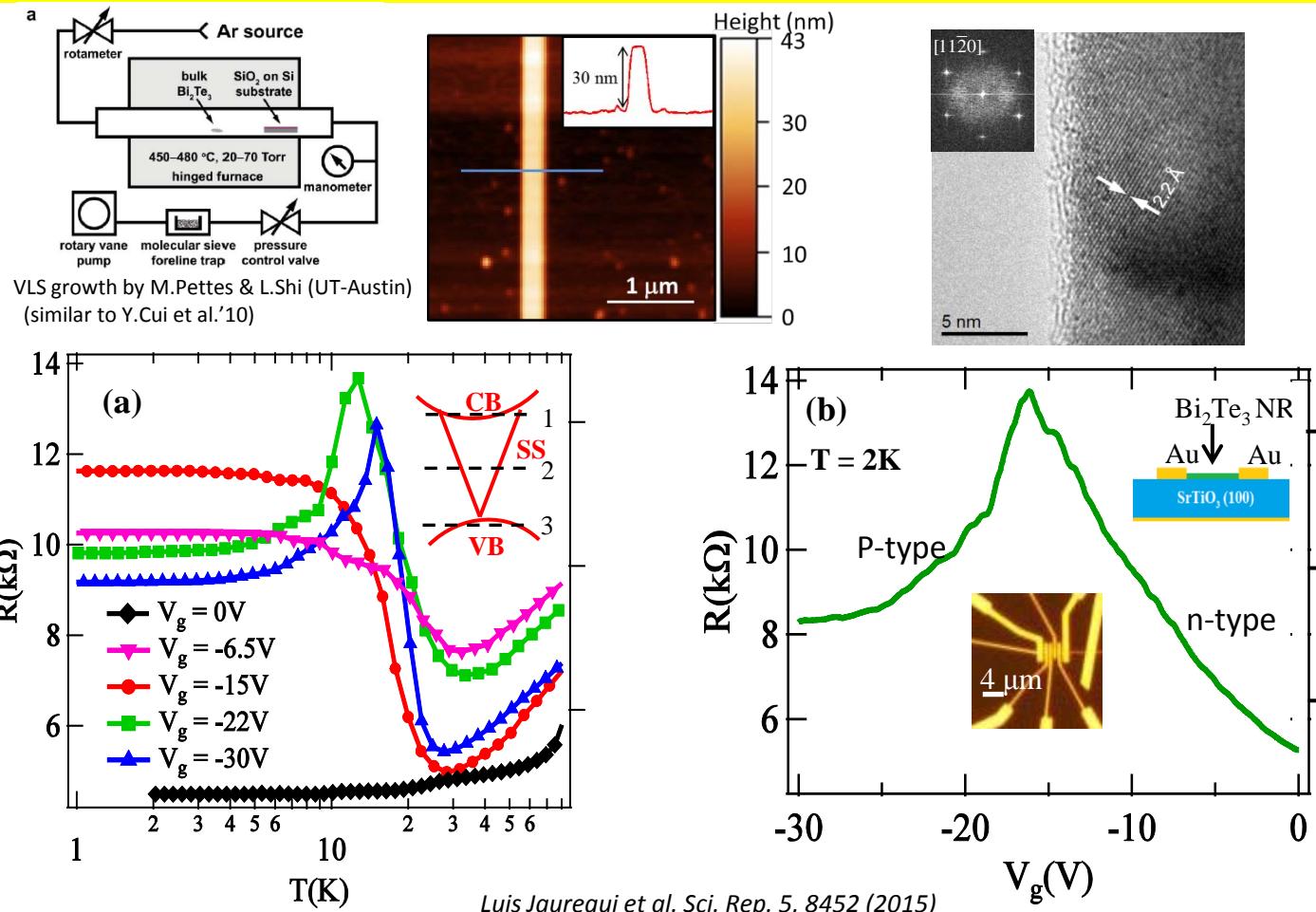
Key Properties (inter-related)	Benefits/Potential Device Applications
<b>Topological Protection</b> (reduce backscattering) [assuming time-reversal symmetry]	High mobility/conductivity (FET) [in absence of magnetic impurities]
<b>Dirac fermions</b> (linear E-k dispersion) $H(\mathbf{q}) = \hbar v_F \mathbf{q} \cdot \vec{\sigma} \times \hat{n}$ <i>real spin</i> [odd # Dirac cones]	"graphene-like" physics & devices (eg. Klein tunneling; electron "optics" etc.)
<b>Spin-momentum locking</b> (in-plane polarized) $\vec{S} \sim \vec{k} \times \hat{n} \sim \hat{n} \times \vec{j}$ [Berry phase real & k space]	Spin-polarized surface current (spintronics: all-electric spin injection etc.)
<b>"Axion" electrodynamics</b> $\Delta\mathcal{L} = \theta(e^2/2\pi\hbar)\mathbf{E} \cdot \mathbf{B}$ (topological magneto-electric effect)	E-field controlled magnetism (magnetoelectric & spintronic devices etc.)

Chen, Proc. SPIE 8373, 83730B (2012)

Material challenge: reduce bulk conduction

Physics challenge: what are hallmarks of "topological transport"? ["1/2"]

**Experiment#2 --- Access/reveal surface state Dirac fermions in TI Nanoribbons**  
**Bi<sub>2</sub>Te<sub>3</sub> nanoribbon: Gate-tuned bulk metal-insulator transition & ambipolar field effect**

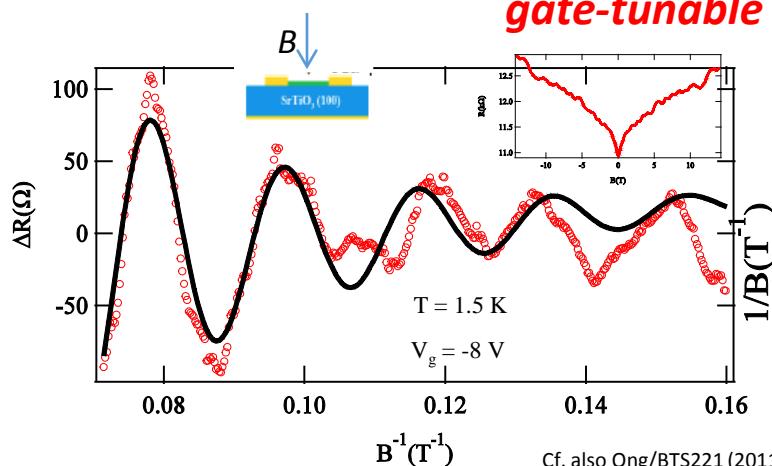


Luis Jauregui et al. Sci. Rep. 5, 8452 (2015)

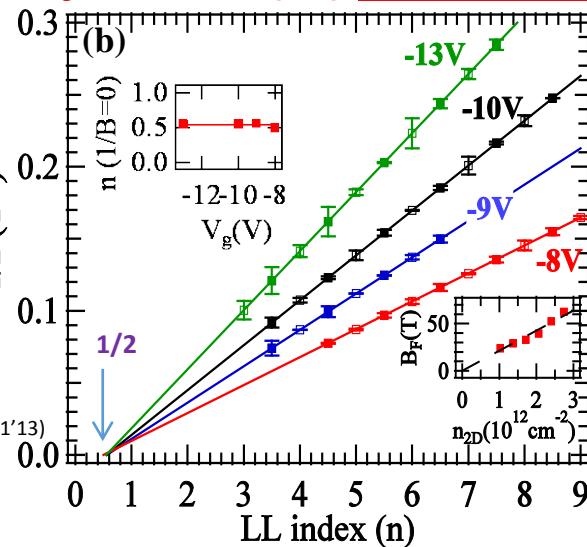
## Shubnikov-de Haas (SdH) oscillations :

Luis Jauregui et al. Sci. Rep. 5, 8452 (2015)

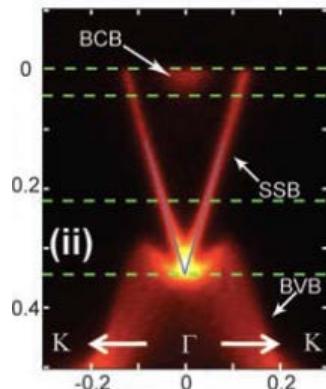
*gate-tunable surface state (SS) Dirac fermions*



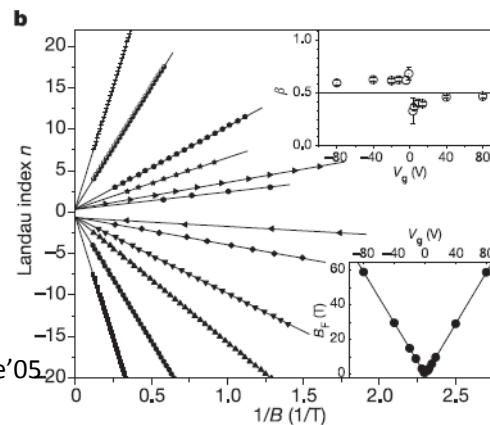
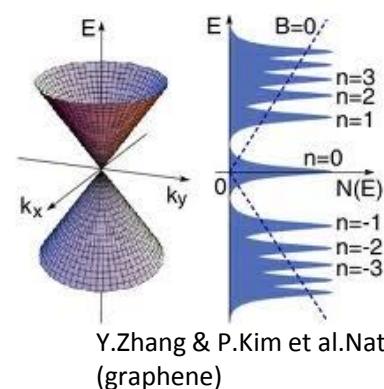
Cf. also Ong/BTS221 (2011'13)



Note: only n-type SS  
Dirac fermion accessible in Bi<sub>2</sub>Te<sub>3</sub>

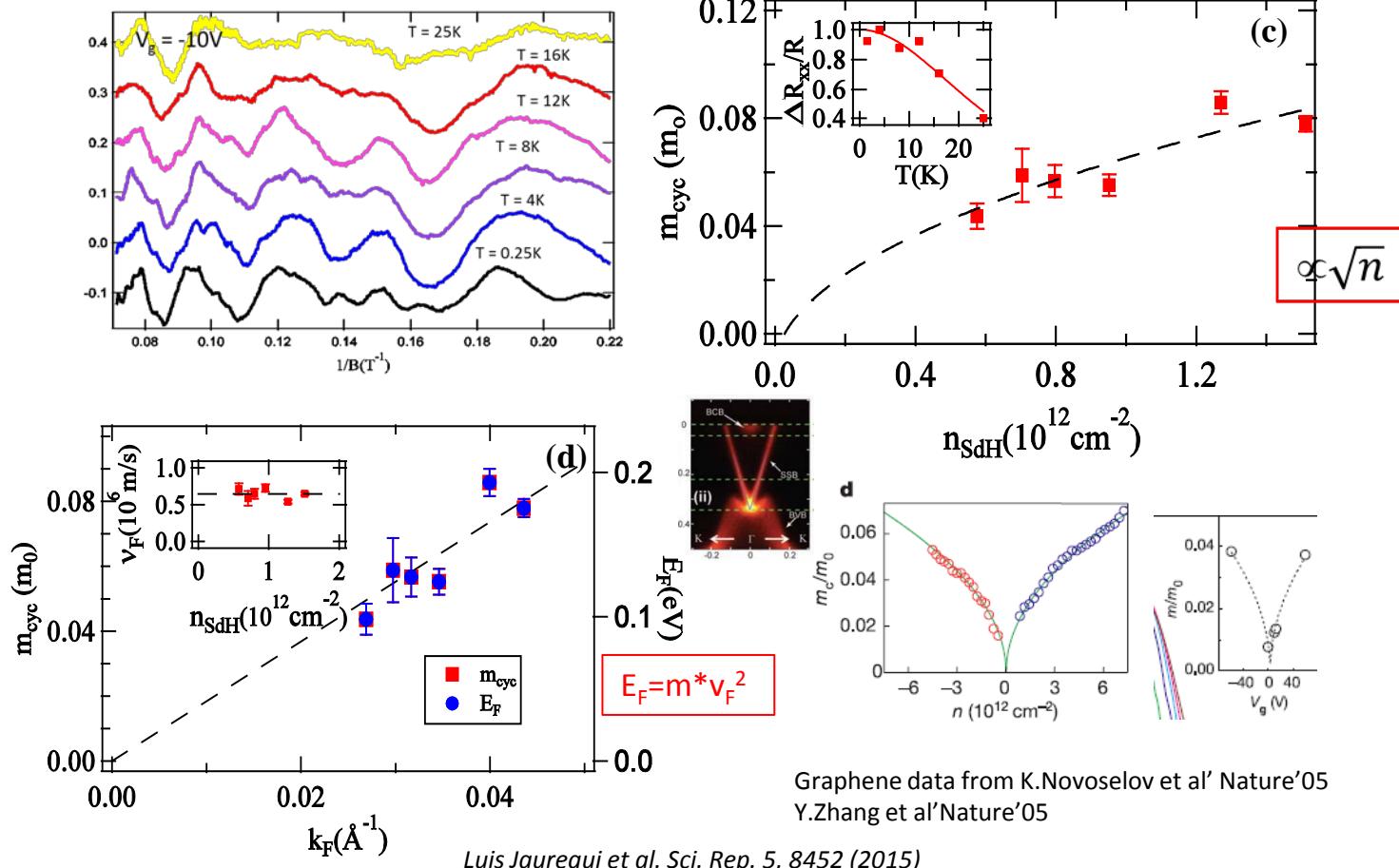


(YL.Chen/ZX Shen et al. Science'09)

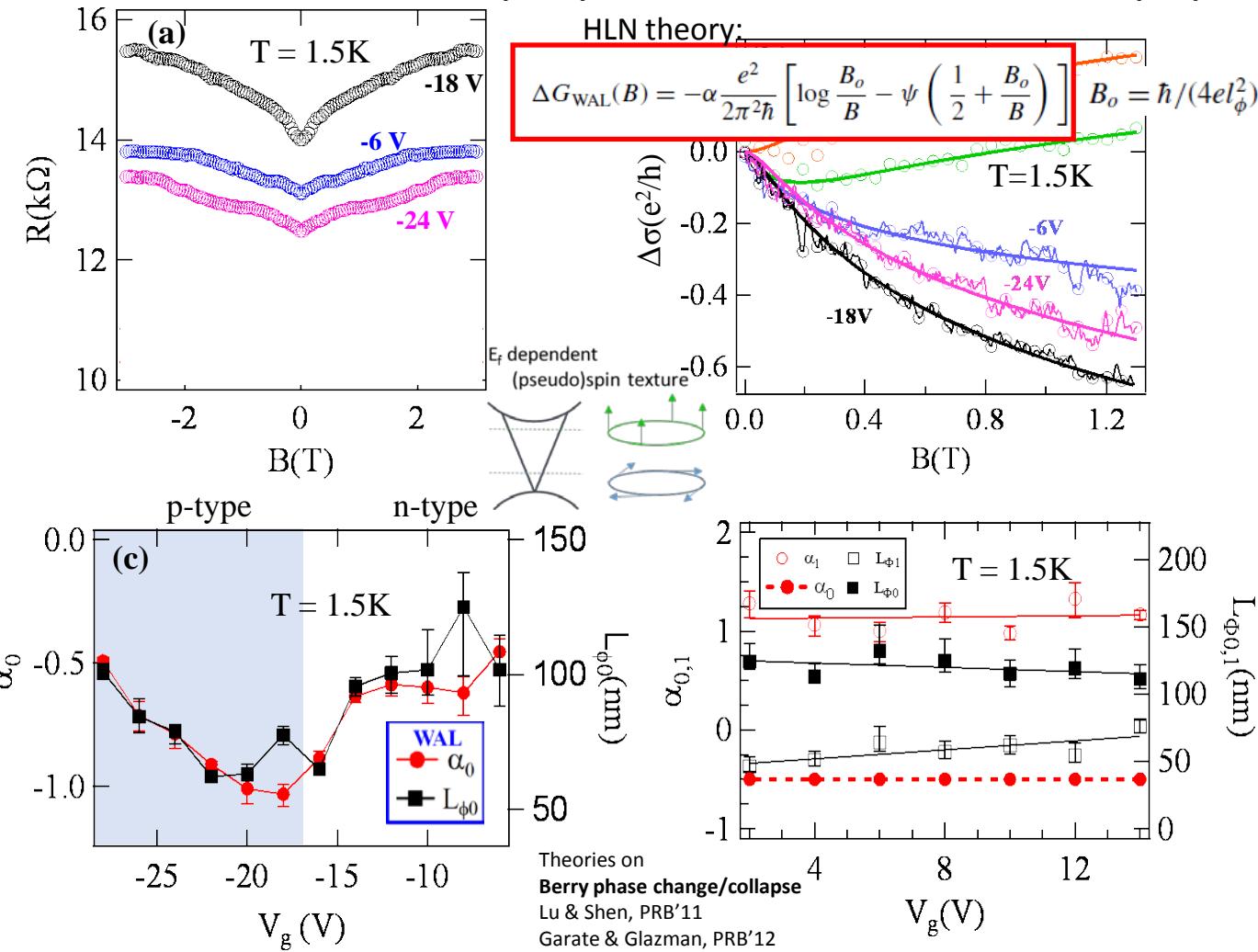


Gate tunable effective (cyclotron) mass ( $\propto \sqrt{n} \propto k_F$ )  
 → transport signature for linear E-k dispersion of SS Dirac fermi

L. Jauregui et al. Sci. Rep. '15

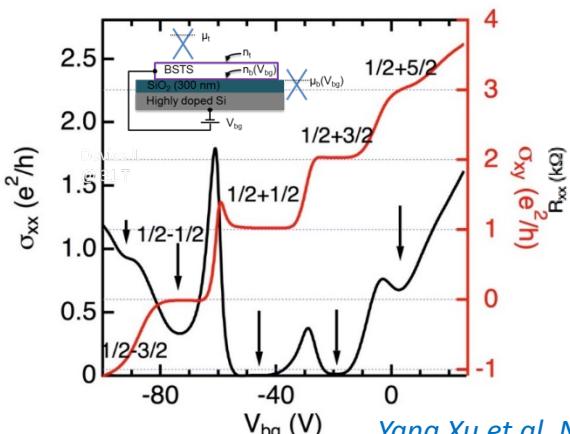


## Gate-tuned weak antilocalization (WAL) and transition to weak localization (WL)

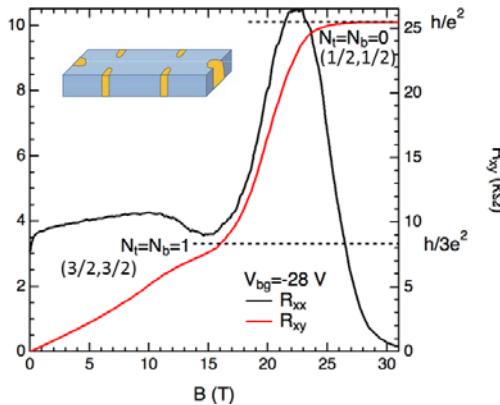


A Fingerprint:

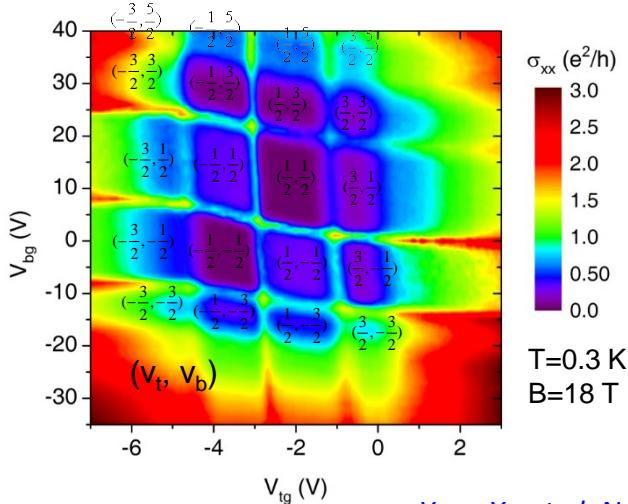
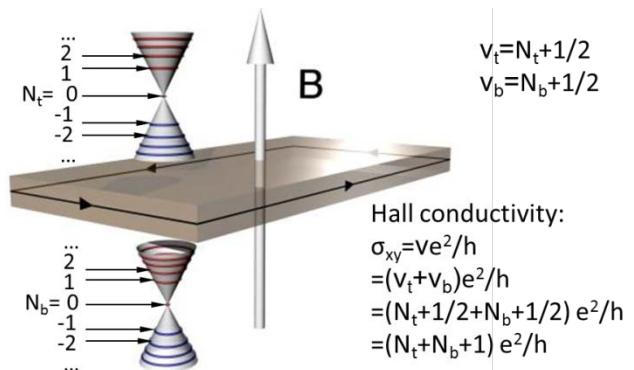
## “Half-integer” QHE --- of two-component (surface) Dirac fermions



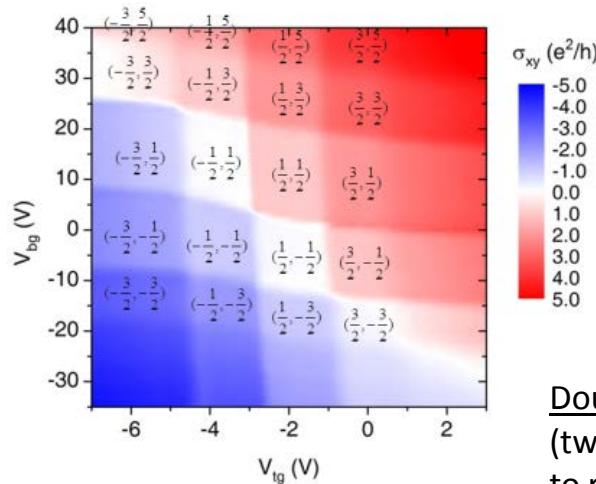
Yang Xu et al. Nature Physics 10, 956 (2014);



Each surface:  $\sigma_{xy} = (1/2)e^2/h$   
--- “1/4-graphene”



Yang Xu et al. Nature Comm. 7, 11434 (2016)

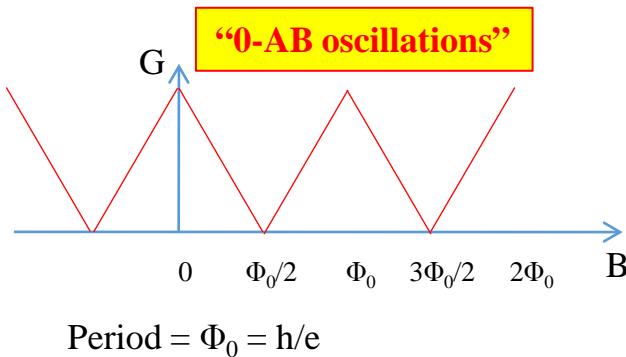
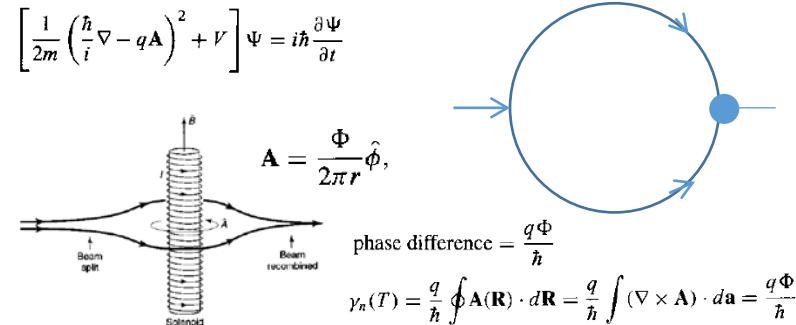


Dirac fermions:  $\sigma_{xy} = g(N + 1/2)e^2/h$   
 $g=4$  for graphene  
 $g=1$  for a TI surface

Also MBE:  
Y. Tokura'15 ( $\text{Bi}_x\text{Sb}_{2-x}\text{Te}_3$ );  
S. Oh'15 ( $\text{Bi}_2\text{Se}_3$ )  
(also HgTe:  
Molenkamp'11'15)

Double Gated TI QHE  
(two surfaces independently gated  
to resolve its contribution)

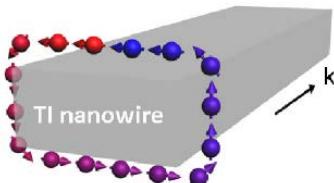
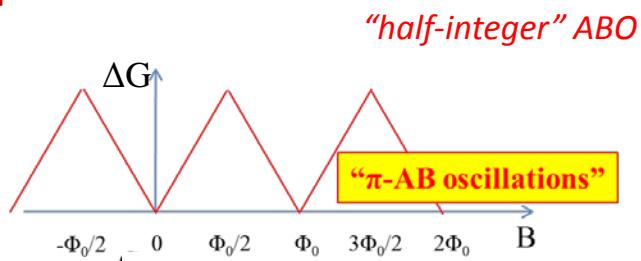
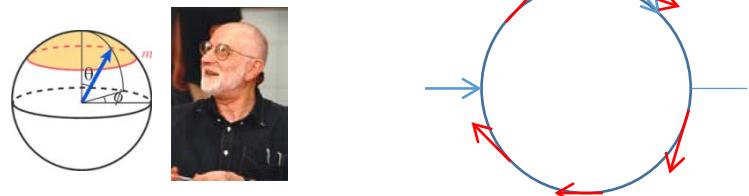
## Aharonov-Bohm quantum interference → oscillations (ABO)



$$\Psi = e^{ig\Psi'},$$

$$g(\mathbf{r}) \equiv \frac{q}{\hbar} \int_{\mathcal{O}}^{\mathbf{r}} \mathbf{A}(\mathbf{r}') \cdot d\mathbf{r}',$$

Spin helical electron → additional/Berry phase  $\pi$



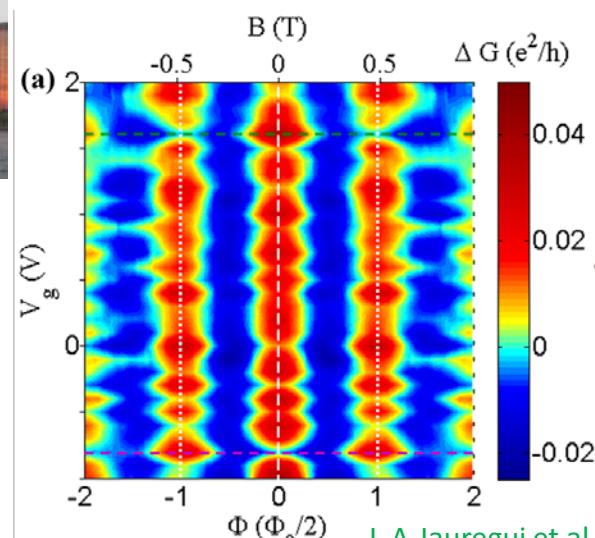
J. H. Bardarson, et al. PRL 105, 156803 (2010)  
Y. Zhang, et al. PRL 105, 206601 (2010)  
A. Cook, et al 2012 & 2011



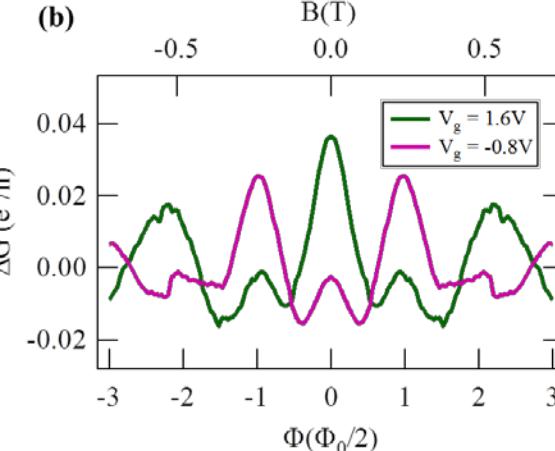
Luis (→Harvard)



in collab:  
M. Pettes/L.Shi (UT Austin)  
L. Rokhinson (Purdue)

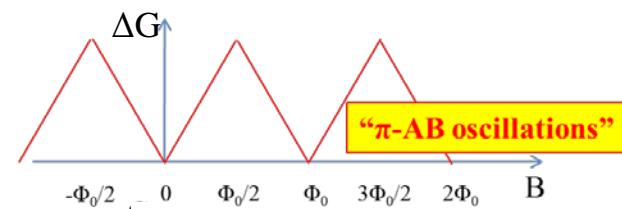
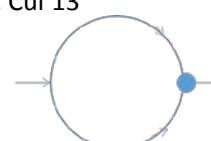
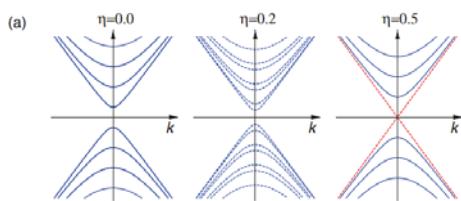


## Alternating $\pi/0$ ABO!



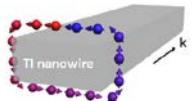
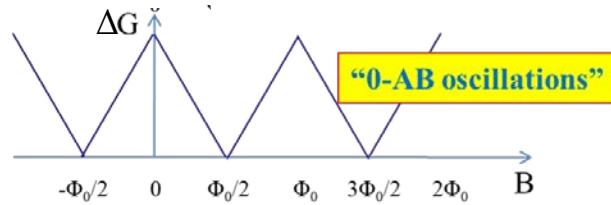
L.A.Jauregui et al. Nature Nano 11, 345 (2016)

***B*-driven topological transition in TSS sub-bands!**



$$\vec{S} \sim \vec{k} \times \hat{n}$$

Berry-phase  
(real space  
rotation)--  
not in CNT!



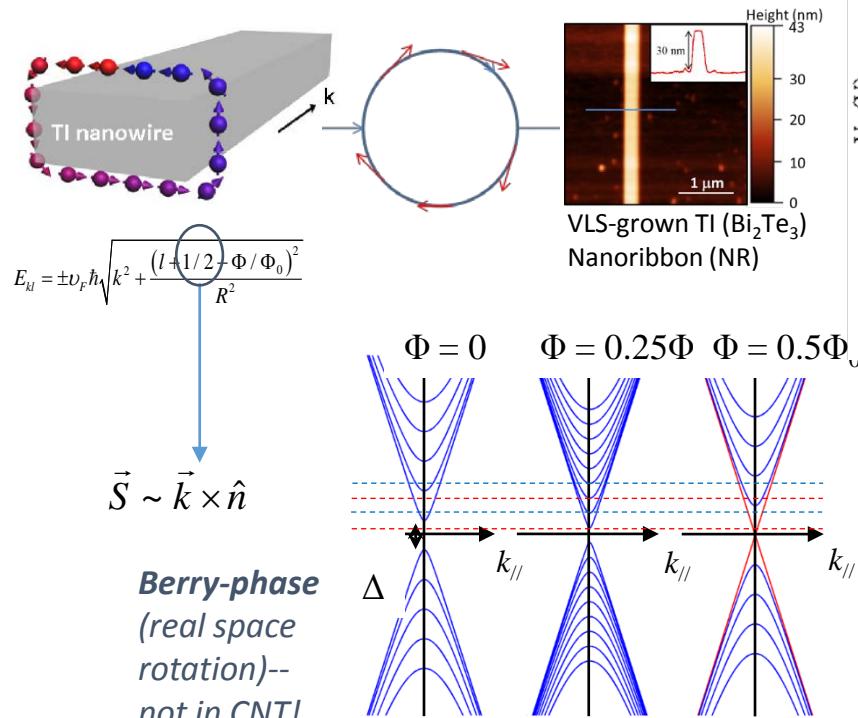
J. H. Bardarson, et al. PRL 105, 156803 (2010); Y. Zhang et al'2010; Cook/Franz '2011

8

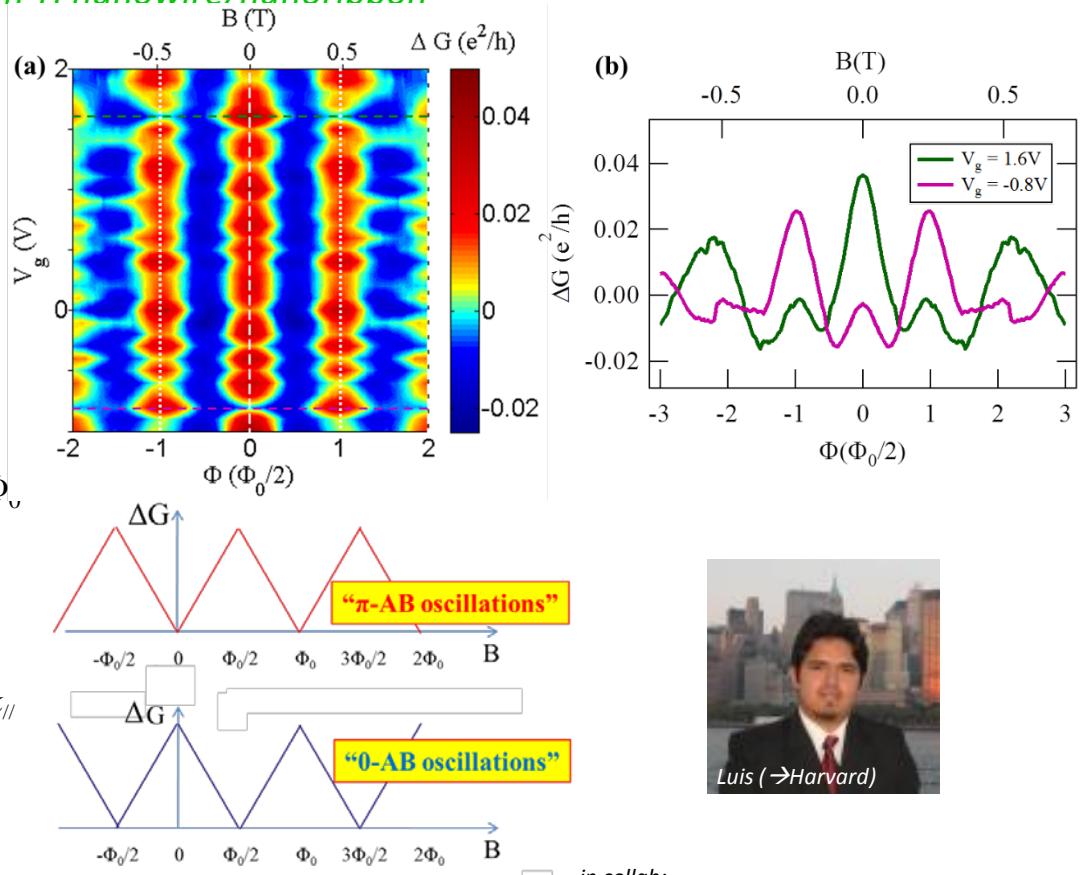
## Another unique transport signature of TI spin-helical surface Dirac fermions:

**"half-integer"(pi) AB oscillations & alternating 0-ABO/pi-ABO periodic in  $k_F$** --- due to quantized TSS subbands and  $B$  field driven topological transitions in TI nanowire/nanoribbon

Luis Jauregui, et al. *Nature Nanotechnology* (2016)

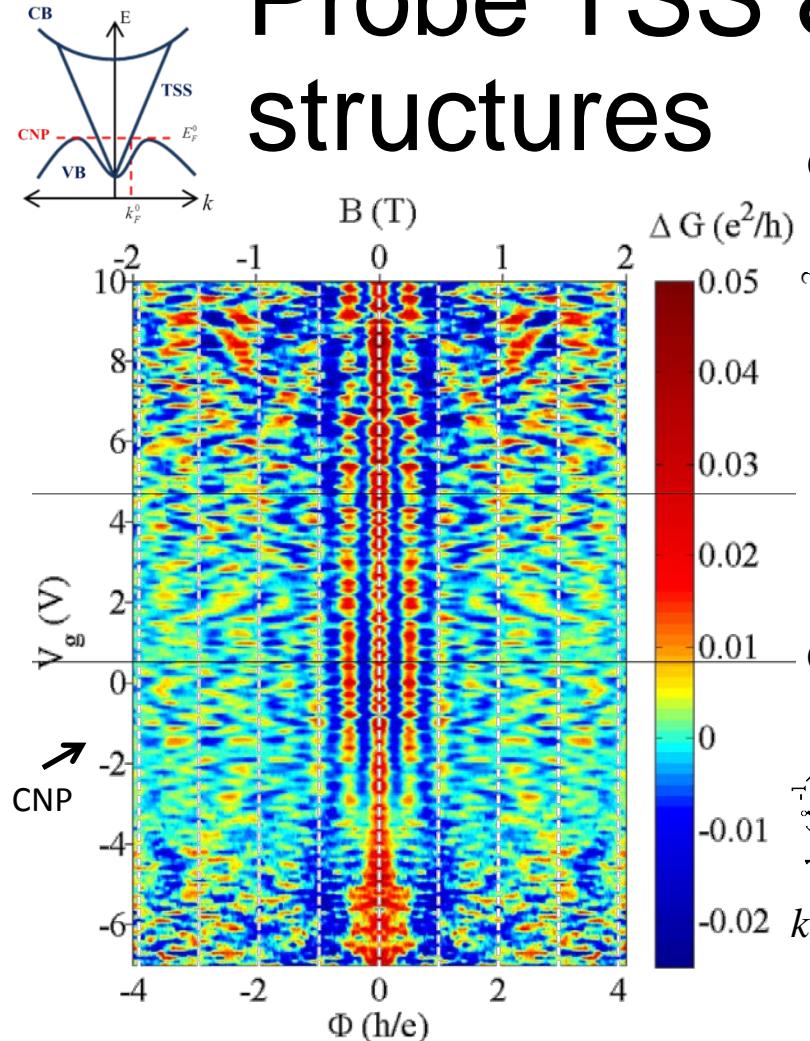


See also  $\text{Bi}_2\text{Se}_3$ : N. Mason'15 (exfoliated); Y. Cui'13 (VLS)



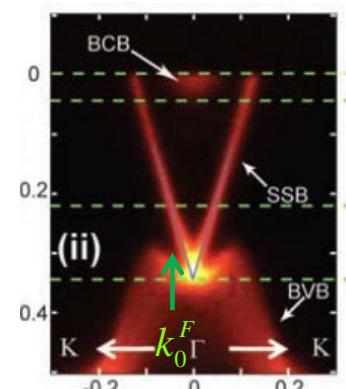
in collab:  
M. Pettes/L.Shi (UT Austin); L. Rokhinson  
(Purdue)

# Probe TSS & band structures

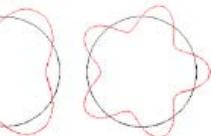


L.A.Jauregui et al. Nature Nano 11, 345 (2016)

(b)



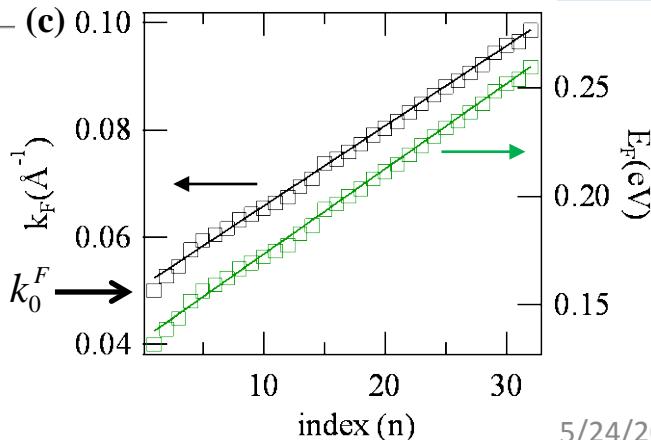
(Y.L.Chen/Z.X.Shen et al. Science'09)



circumferentially  
quantized  
Surface state  
electrons

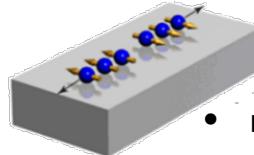
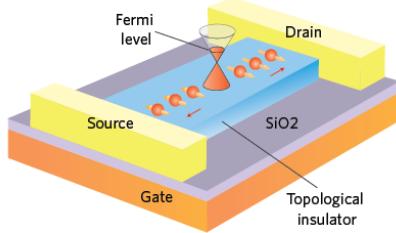
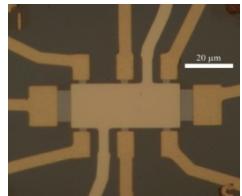
*Gate-dependent,  
 $k_F$ -periodic oscillation!*

(c)



# TI electronic transport: Rich physics and Potential Device Applications

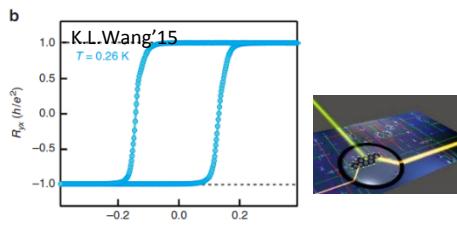
Want: Topological Surface State (TSS) Transport (spin –helical Dirac fermion)



- nanoelectronics
- spintronics
- Thermoelectrics ...

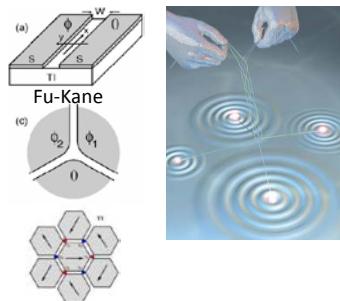
## TI surface state (Dirac fermion, spin polarized)

**TI + magnetism**  
Quantum Anomalous Hall  
[QK.Xue et al'13]  
Topological magnetoelectric  
“axion” eletrodynamics  
Topological Phase Transition

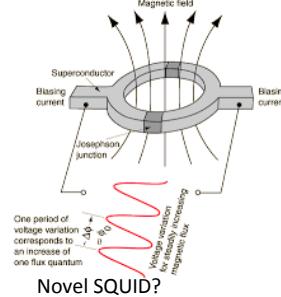


Ultralow power dissipation  
(dissipationless) interconnect/FFIT

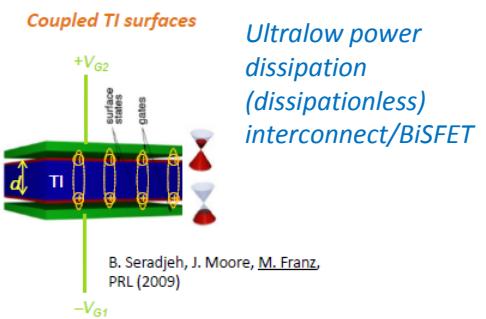
**TI + superconductor**  
(Majorana fermions)  
Non-Abelian statistics (1D/2D)



Topological quantum computing

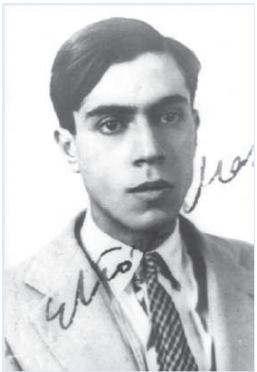


**TI + TI**  
(Excitonic condensate)  
[electronic superfluid]



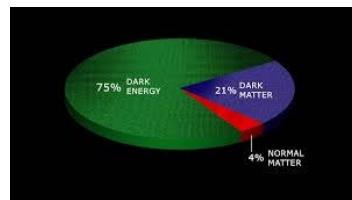
Ultralow power  
dissipation  
(dissipationless)  
interconnect/BiSFET

B. Seradjeh, J. Moore, M. Franz,  
PRL (2009)



## Majorana (fermion)

- Neutrino?
- Supersymmetric partner  
e.g. of photon: *photino*
- ...
- WIMPs (dark matter) ?...



## Axion (boson)

Peccei and Quinn QCD

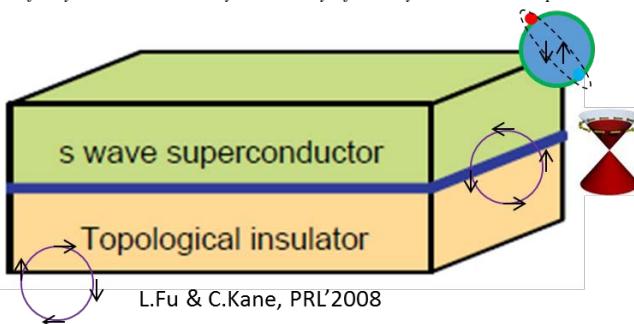


A light mass dark matter candidate

PRL 100, 096407 (2008)  
Superconducting Proximity Effect and Majorana Fermions  
at the Surface of a Topological Insulator

Liang Fu and C. L. Kane

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA



L.Fu & C.Kane, PRL'2008

QUANTA

Imagining science  
MAGAZINE

QUANTIZED: PHYSICS

Time's (Almost) Reversible Arrow

By Frank Wilczek  
January 7, 2016

## Dynamical axion field in topological magnetic insulators

NATURE PHYSICS | VOL 6 | APRIL 2010

Rundong Li<sup>1</sup>, Jing Wang<sup>1,2</sup>, Xiao-Liang Qi<sup>1</sup> and Shou-Cheng Zhang<sup>1\*</sup>

Axions are weakly interacting particles of low mass, and were postulated more than 30 years ago in the framework of the Standard Model of particle physics. Their existence could explain the missing dark matter of the Universe. However, despite

---

## Topological magnetoelectric effect

- Modified Maxwell equations by axion field

---

Modified Maxwell equations

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= \rho - \kappa \vec{\nabla} \theta \cdot \vec{B} \\ \vec{\nabla} \times \vec{E} &= -\partial \vec{B} / \partial t \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{B} &= \partial \vec{E} / \partial t + \vec{j} + \kappa (\partial \theta / \partial t \vec{B} + \vec{\nabla} \theta \times \vec{E})\end{aligned}$$

constitutive relations

$$\begin{aligned}\vec{D} &= \epsilon \vec{E} + \kappa \theta \vec{B} \\ \vec{H} &= \frac{1}{\mu} \vec{B} - \kappa \theta \vec{E}\end{aligned}$$

Slide credit:  
J. Ihm

$\theta = 0$  for NI and  $\pi$  for TI

F. Wilczek *PRL* **58**, 1799 (1987)

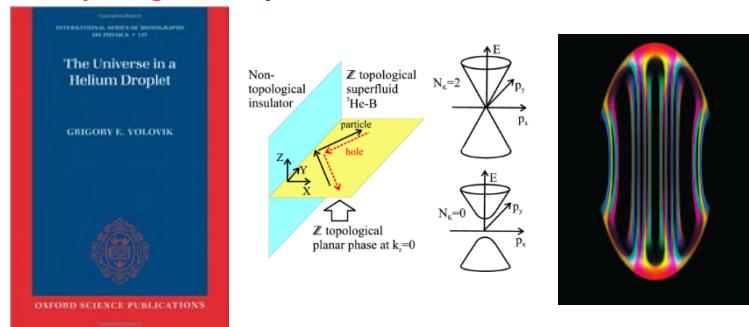
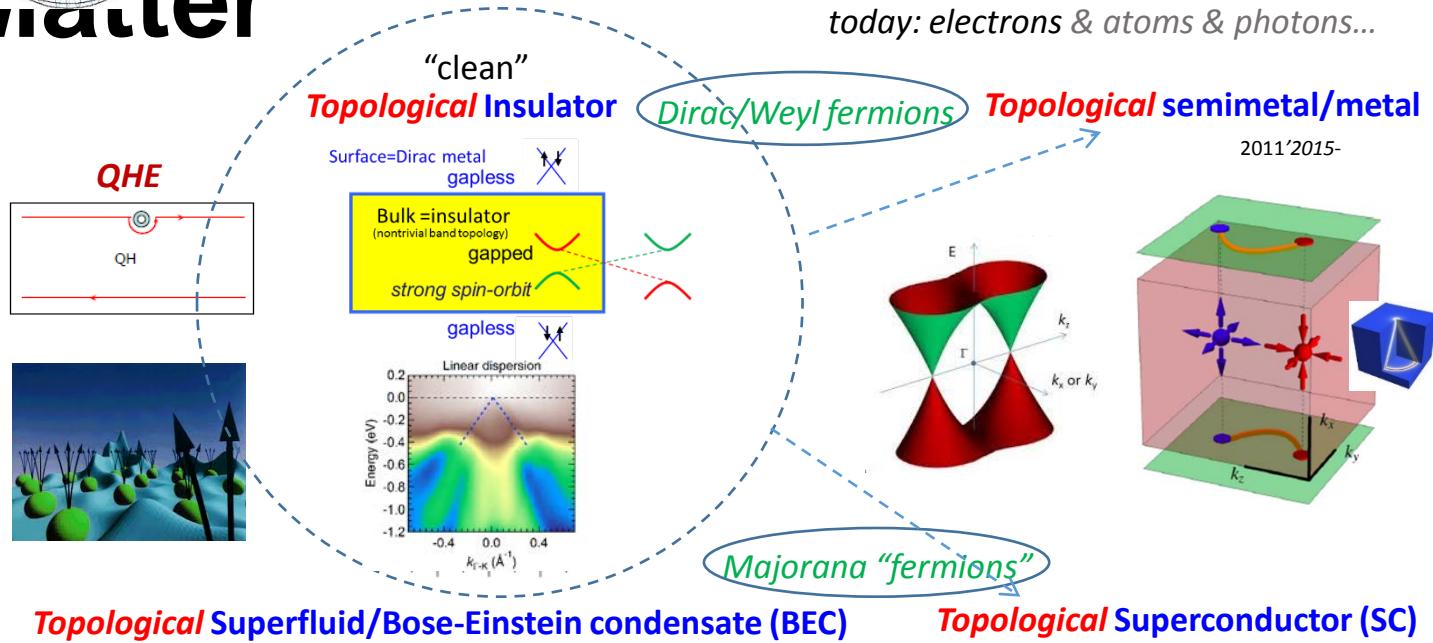
X.-L.Qi *et al.*, *PRB* **78**, 195424 (2008):  
S.-C.Zhang group

$-\kappa \vec{\nabla} \theta \cdot \vec{B}$  : topological charge ( $\rho_t$ )

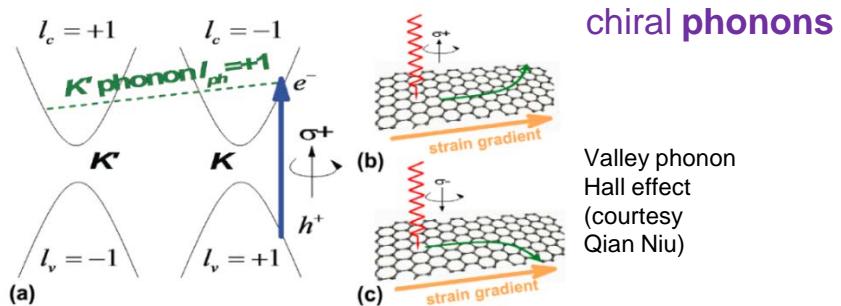
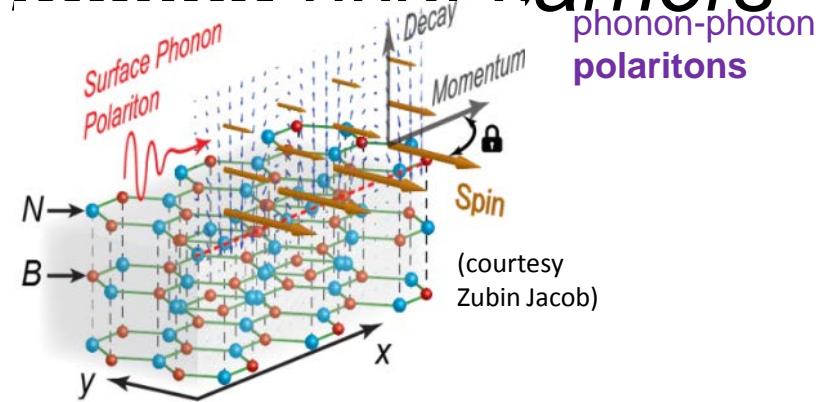
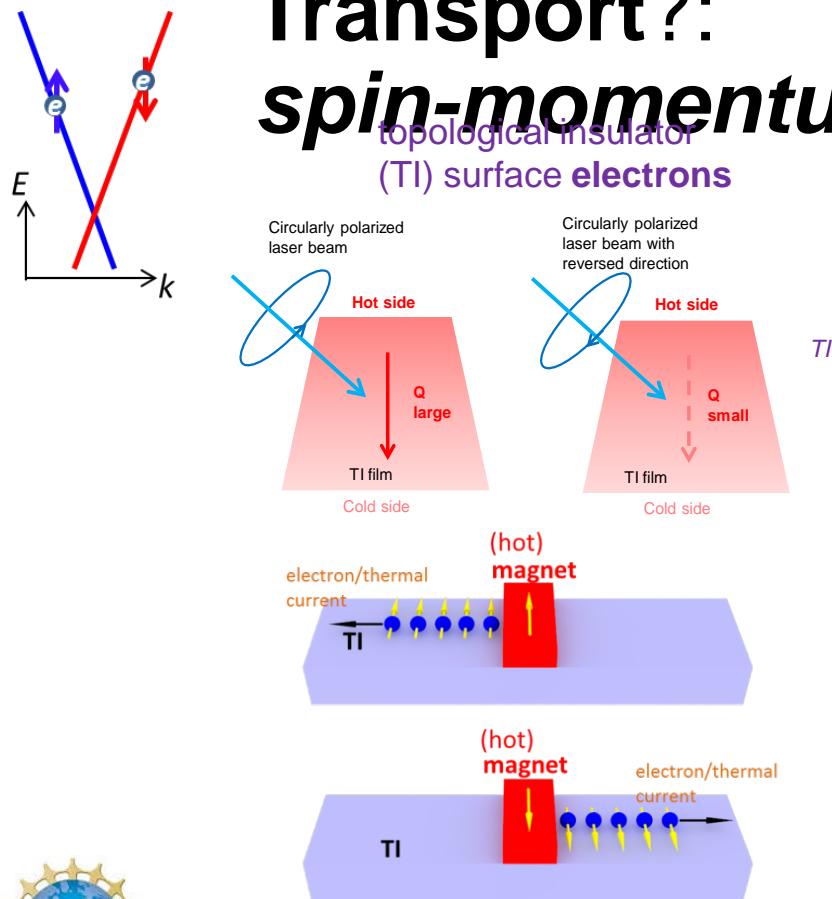
$\kappa (\partial \theta / \partial t \vec{B} + \vec{\nabla} \theta \times \vec{E})$  : topological current ( $\vec{j}_t$ )

Topological magnetoelectric effect can be described phenomenologically in terms of axion electrodynamics.

# Topological Quantum Matter



# Can we have Topological Thermal Transport?: *spin-momentum-locked heat carriers*



NSF EFRI “NewLaw” (2016): Yong P. Chen, Xianfan Xu, Zubin Jacob (Purdue) & Qian Niu (UT Austin)

## Outline

### • Quantum Coherent Transport of Atoms (cold atom BEC): A Spin-resolved Atom Interferometer (Stueckelberg Interference)

PHYSICAL REVIEW A **95**, 043623 (2017)

Stueckelberg interferometry using periodically driven spin-orbit-coupled Bose-Einstein condensates

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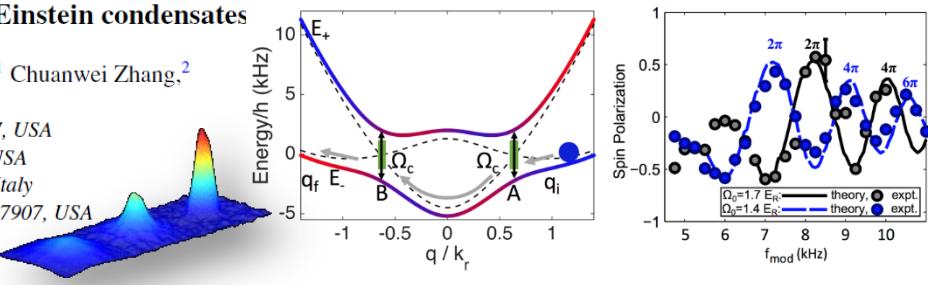
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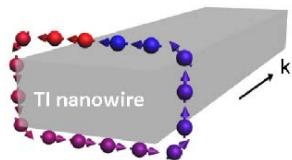
### • Quantum Coherent Transport of Electrons (spin-helical Dirac fermions on topological insulators): a “half-integer” Aharonov-Bohm Effect (electronic interferometer)

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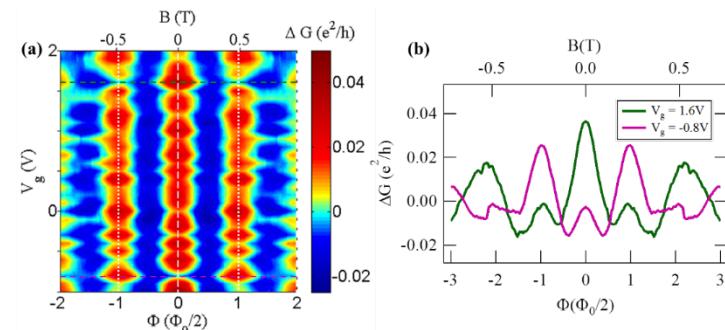
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ARTICLES

Magnetic field-induced helical mode and topological transitions in a topological insulator nanoribbon



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General remark

**"Path"** can be in

- Real ( $r$ ) – space
- Momentum ( $k$ ) – space
- More abstract/complicated “configuration”/Hilbert space....  
[sometimes related to “topology”]

**"Wave"** can be

- Light (classical E&M or quantum)
- electrons
- atoms
- other “matter wave” or more complex quantum systems

## Two-path interference

Young' double-slit interference  
(optics/light  
**wave**)

