Analytical Solution of Microbes Interacting with Surfaces

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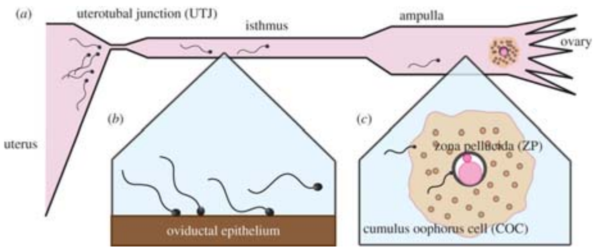
Introduction & Motivation

- There is a rising interest in studying the behavior of microbes and their interactions with surrounding fluid. This research could have important biological application, such as understand the behavior of sperm near the oviductal epithelium.



(Kenta I, Eamonn G, 2016)

- To explore the velocity fields, pressures and forces around the microbes, the solution of Stokes equations, which is called a Stokeslet, are introduced:

$$\mu\Delta \boldsymbol{u} = \nabla p - \boldsymbol{F} \qquad (Eq.1)$$
$$\nabla \cdot \boldsymbol{u} = 0 \qquad (Eq.2)$$

- The Stokeslet represents the velocity field due to a concentrated external force acting on fluid at a single point. This singularity could cause the expression of velocity not integrable. The Regularized Stokeslet Method and Method of Images are used to address this challenge.

Method

- To de-singularize the force exerted on the fluid, a new form of it is given:

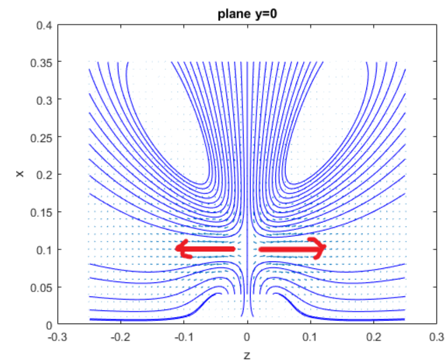
$$\boldsymbol{F}(x) = \boldsymbol{f}_0\varphi_\epsilon(x - x_0) \qquad (Eq.3)$$

Where φ_ϵ is a radially symmetric smooth function which controls the spread of the force.

- To generate the velocity field around the microorganism near an infinite planar wall, the boundary condition must be satisfied: the velocity at the wall must be zero. The image points of discretized points on the model are symmetrically generated On the other side of the wall to meet the boundary condition.

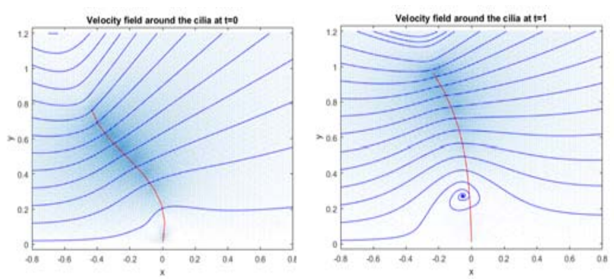
Numerical Results

1) The velocity field around a single microorganism near the wall (x = 0)



- **The microorganism is simplified as a force dipole with unit magnitude**

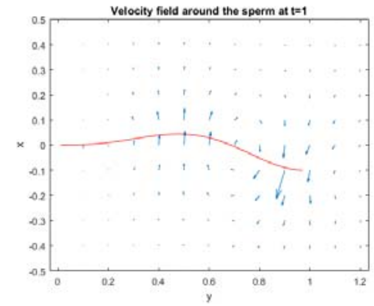
2) The velocity field around periodically-beating cilia near the wall (x = 0)



- **The position of each discretized point on the cilia is described by:**

$$\zeta(s,t) = \frac{1}{2}a_0(s) + \sum_{n=1}^{N_0} a_n(s)\cos(n\sigma t) + b_n(s)\sin(n\sigma t) \qquad (Eq.4)$$

3) The velocity field around a single sperm near the wall (y = 0)



- **The curvature function of the sperm configuration is given by:**

$$y(x,t) = b_0 x \sin(kx - \omega t) \qquad (Eq.5)$$

Conclusion

- The introduction of the Method of Regularized Stokeslet and its Image Method succeeds in diminishing the singularity of Stokeslet, which helps us to numerically construct the velocity field in fluid domain.

Further Research

- In future, the time-evolution of the motion of microorganism near the wall will be simulated rather than calculating the velocity instantly.

Reference

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