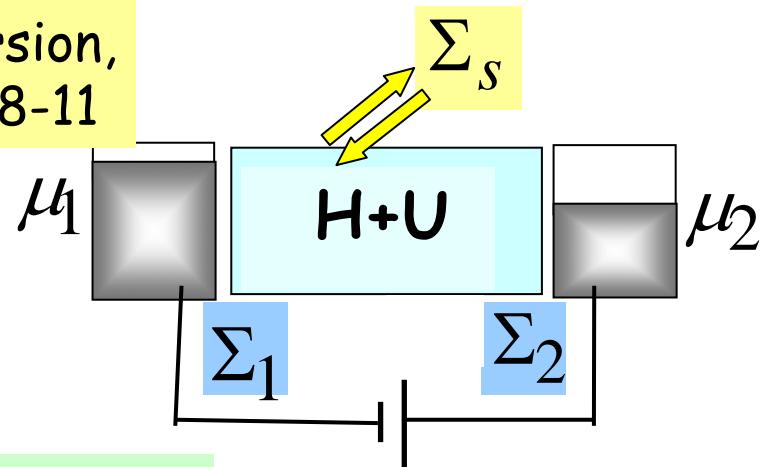
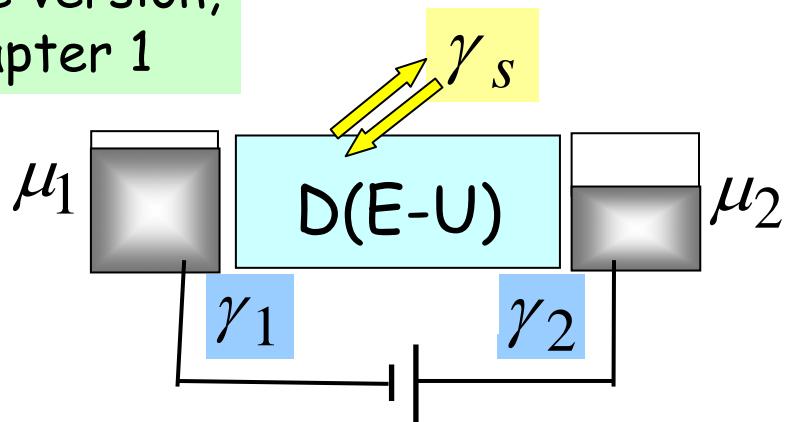


Matrix version,
Chapters 8-11

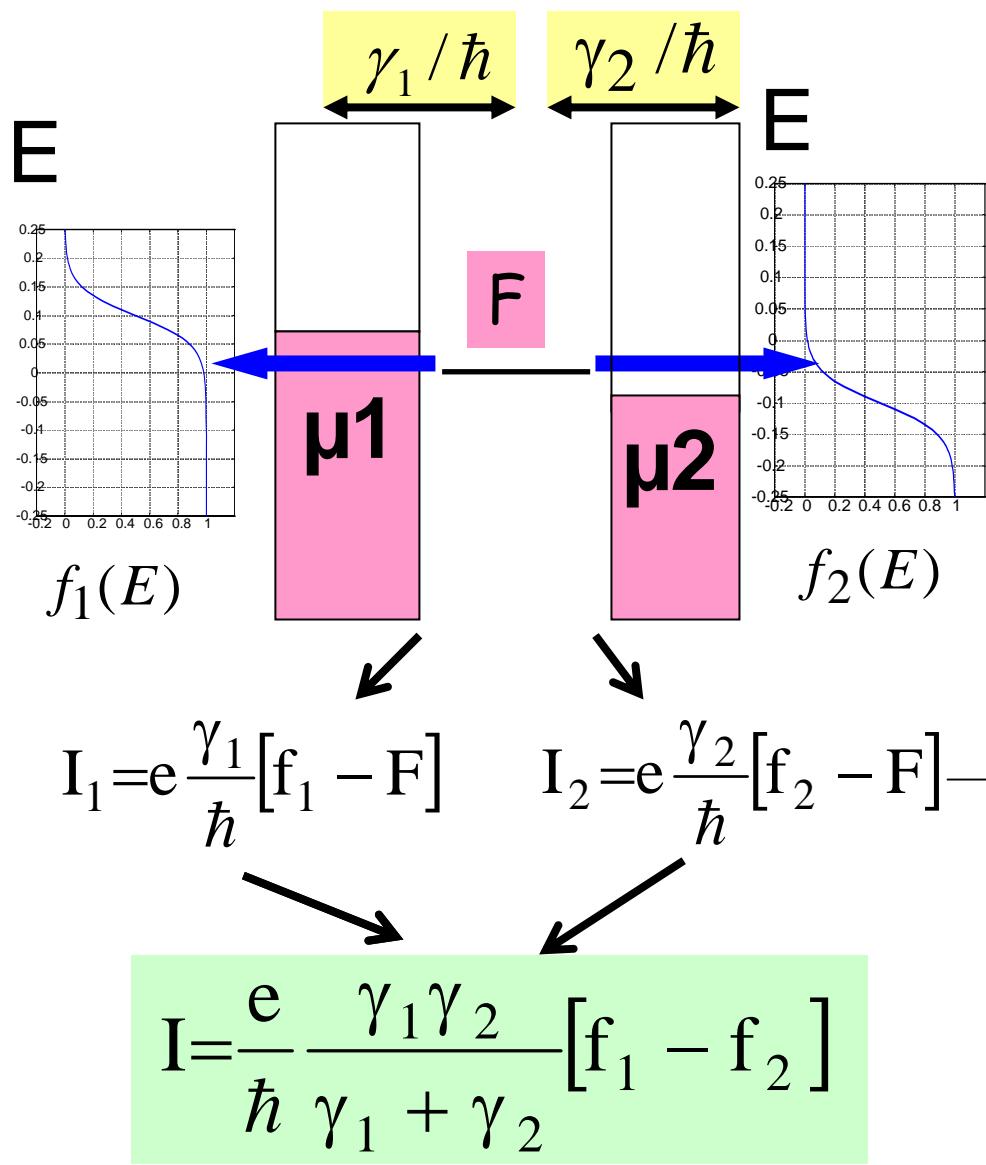


Simple version,
Chapter 1



For detailed write-up of Lecture I
See arXiv:condmat/0704.1623

Datta, Quantum Transport:
Atom to Transistor,
Cambridge (2005)



$$H = [\varepsilon]$$

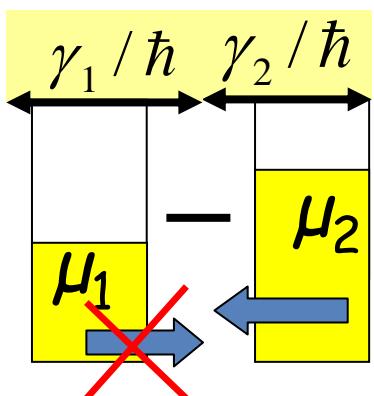
$$E\psi = \varepsilon\psi \longrightarrow [E - \varepsilon]\psi = 0$$

$$\left[E - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2} \right] \psi = 0$$

$$\left[i\hbar \frac{\partial}{\partial t} - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2} \right] \psi = 0$$

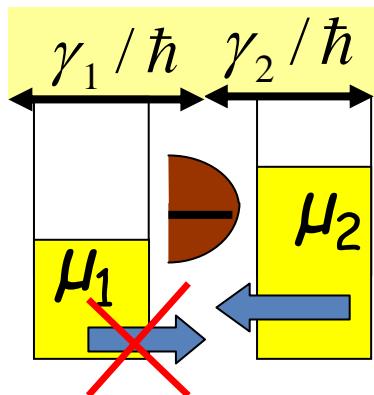
$$\psi = \exp\left(-\frac{i\varepsilon t}{\hbar}\right) \exp\left(-\frac{\gamma_1 t}{2\hbar}\right) \exp\left(-\frac{\gamma_2 t}{2\hbar}\right)$$

$$n = \psi\psi^* = \exp\left(-\frac{\gamma_1 t}{\hbar}\right) \exp\left(-\frac{\gamma_2 t}{\hbar}\right)$$



$$\left[E - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2} \right] \psi = s_2$$

Source



$$\left[E - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2} \right] \psi = s_2$$

Source

$$\psi = \frac{s_2}{E - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2}}$$

$$n = \psi \psi^*$$

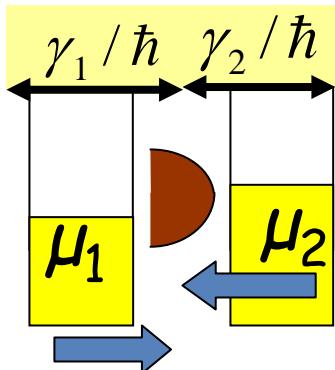
$$n(E) = \frac{s_2 s_2^*}{(E - \varepsilon)^2 + \left(\frac{\gamma_1}{2} + \frac{\gamma_2}{2} \right)^2}$$

$$N = \int \frac{dE}{2\pi} n(E) = \frac{\gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$\text{if } s_2 s_2^* = \gamma_2 f_2$$

$$n = \psi\psi^*$$

1 Level



$$s_1 s_1^* = \gamma_1 f_1$$

$$s_2 s_2^* = \gamma_2 f_2$$

$$\left[E - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2} \right] \psi = s_2 + \textcolor{red}{s_1}$$

Source

$$\psi = \frac{s_2 + \textcolor{red}{s_1}}{E - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2}}$$

WRONG !

$$n(E) = \frac{s_2 s_2^* + \textcolor{red}{s_1 s_1^*} + s_2 s_1^* + s_1 s_2^*}{(E - \varepsilon)^2 + \left(\frac{\gamma_1}{2} + \frac{\gamma_2}{2} \right)^2}$$

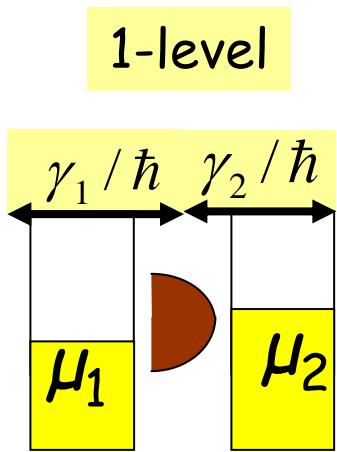
$$n(E) = \frac{s_2 s_2^* + s_1 s_1^*}{(E - \varepsilon)^2 + \left(\frac{\gamma_1}{2} + \frac{\gamma_2}{2} \right)^2}$$

$$N = \int \frac{dE}{2\pi} n(E) = \frac{\gamma_2 f_2 + \gamma_1 f_1}{\gamma_1 + \gamma_2}$$

ψ cannot be added

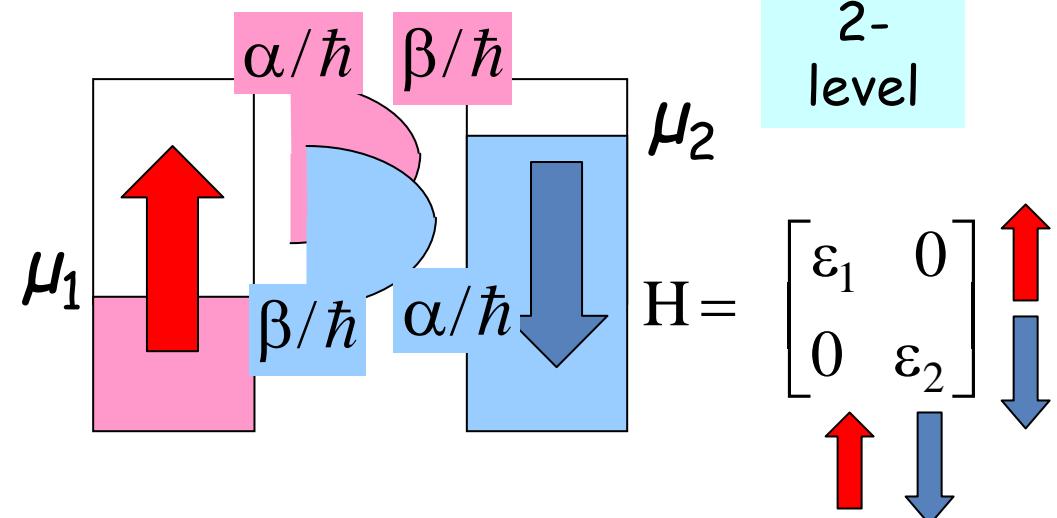
$n = \psi\psi^*$ can be

2 levels: Wavefunction approach



$$\left[E - \varepsilon + \frac{i\gamma_1}{2} + \frac{i\gamma_2}{2} \right] \psi = s_2$$

$$s_2 s_2^* = \gamma_2 f_2$$



$$\begin{bmatrix} E - \varepsilon_1 + \frac{i\alpha}{2} + \frac{i\beta}{2} & 0 \\ 0 & E - \varepsilon_2 + \frac{i\beta}{2} + \frac{i\alpha}{2} \end{bmatrix} \{\psi\} = \{s_2\}$$

2 levels: Wavefunction approach

$$H = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$$

"Self-energy" matrices

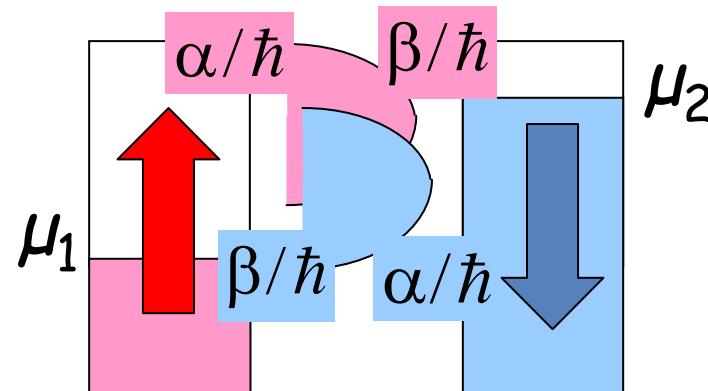
$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$

Broadening
matrices

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+]$$

$$\Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$



2-
level

$$\begin{bmatrix} E - \varepsilon_1 + \frac{i\alpha}{2} + \frac{i\beta}{2} & 0 \\ 0 & E - \varepsilon_2 + \frac{i\beta}{2} + \frac{i\alpha}{2} \end{bmatrix} \{\psi\} = \{S_2\}$$

$$[EI - H - \Sigma_1 - \Sigma_2] \{\psi\} = \{S_2\}$$

$$\{S_2\} \langle S_2 \}^+ = [\Gamma_2] f_2$$

$$H = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$$

"Self-energy" matrices

$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

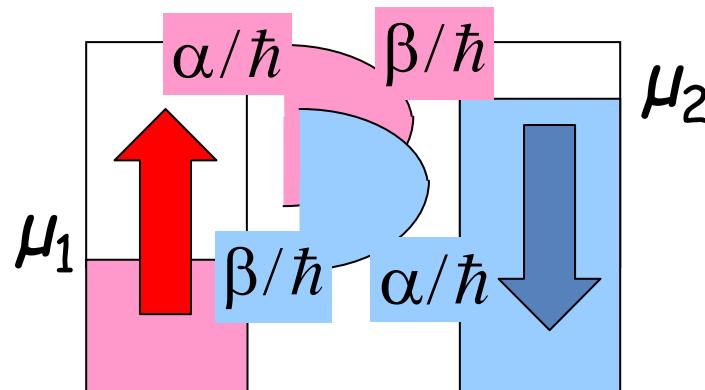
$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$

Broadening matrices

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^\dagger] \quad \Gamma_2 = i[\Sigma_2 - \Sigma_2^\dagger]$$

$$\langle S_2 \rangle \langle S_2 \rangle^+ = [\Gamma_2] f_2$$

$$[G] = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$



2-level

$$[EI - H - \Sigma_1 - \Sigma_2] \{\psi\} = \{S_2\}$$

$$\{\psi\} = [G] \{S_2\}$$

$$G^n = \langle \psi | \langle \psi \rangle^+ = [G] \overbrace{\{S_2\} \{S_2^+\}}^{[\Gamma_2] f_2} [G]^+$$

$$G^n = G \Gamma_2 G^+ f_2$$

Superpose contributions to correlation function from different sources

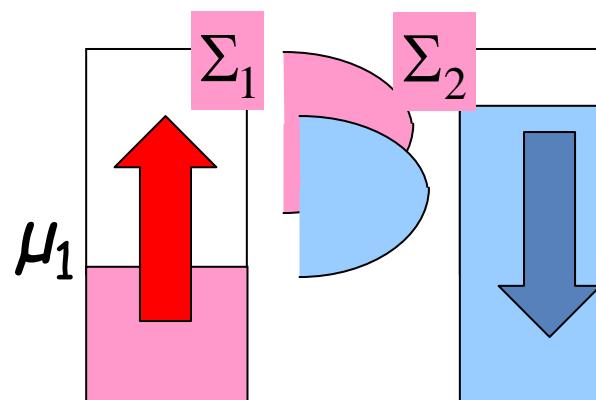
Correlation function

$$G^n (\equiv -iG^<) = \{\psi\} \{\psi\}^+$$

$$\begin{Bmatrix} \Psi_R \\ \Psi_B \end{Bmatrix} \begin{Bmatrix} \Psi_R^* & \Psi_B^* \end{Bmatrix}$$

$$= \begin{bmatrix} \Psi_R \Psi_R^* & \Psi_R \Psi_B^* \\ \Psi_B \Psi_R^* & \Psi_B \Psi_B^* \end{bmatrix}$$

Spectral function

 μ_2

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+]$$

$$\Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$

$$(1) [G] = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$

"Electron density"

$$(2) G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1$$

"Density of states"

$$(3) A = G\Gamma_2 G^+ + G\Gamma_1 G^+$$

$$A = i[G - G^+]$$

$$H = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$$

"Self-energy" matrices

$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

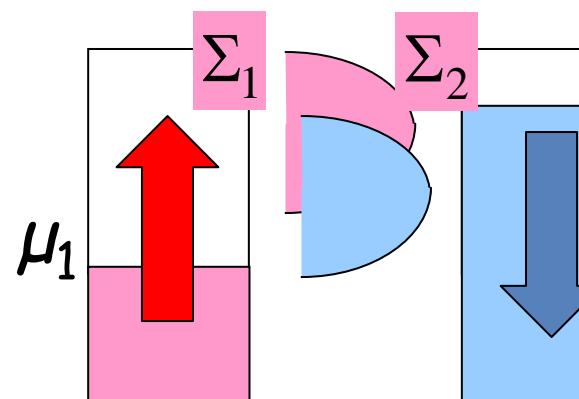
$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$

Broadening matrices

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+] = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

$$\Gamma_2 = i[\Sigma_2 - \Sigma_2^+] = \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$

If all matrices are diagonal ...



μ_2

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+]$$

$$\Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$

$$(1) [G] = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$

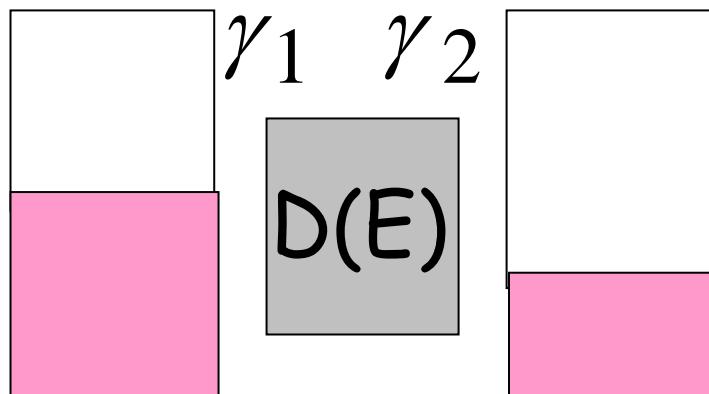
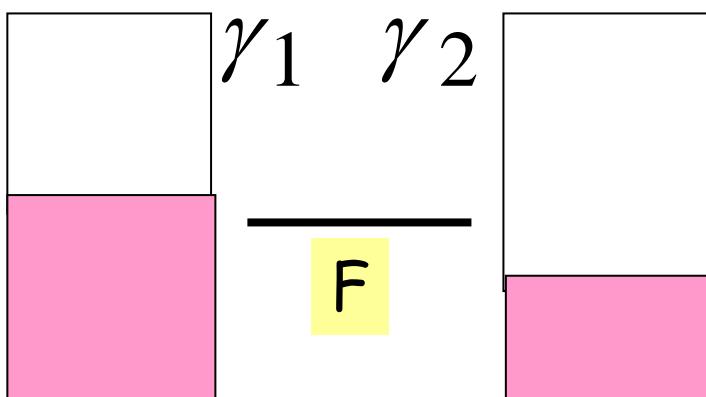
"Electron density"

$$(2) G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1$$

"Density of states"

$$(3) A = G\Gamma_2 G^+ + G\Gamma_1 G^+$$

$$A = i[G - G^+]$$



$$F = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$



$$I = \frac{e}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

$$n(E) = D(E) \left[\frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} \right]$$

$$I = dE \frac{e}{\hbar} D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1 - f_2]$$

"Self-energy" matrices

$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

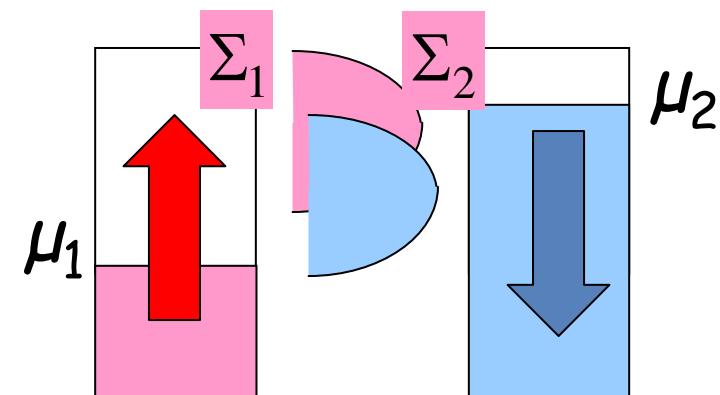
$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$

$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

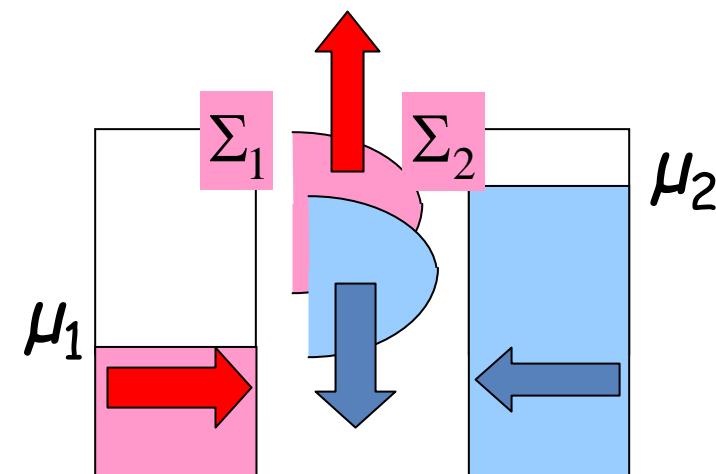
$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} a & -b \\ -b & a \end{bmatrix}$$

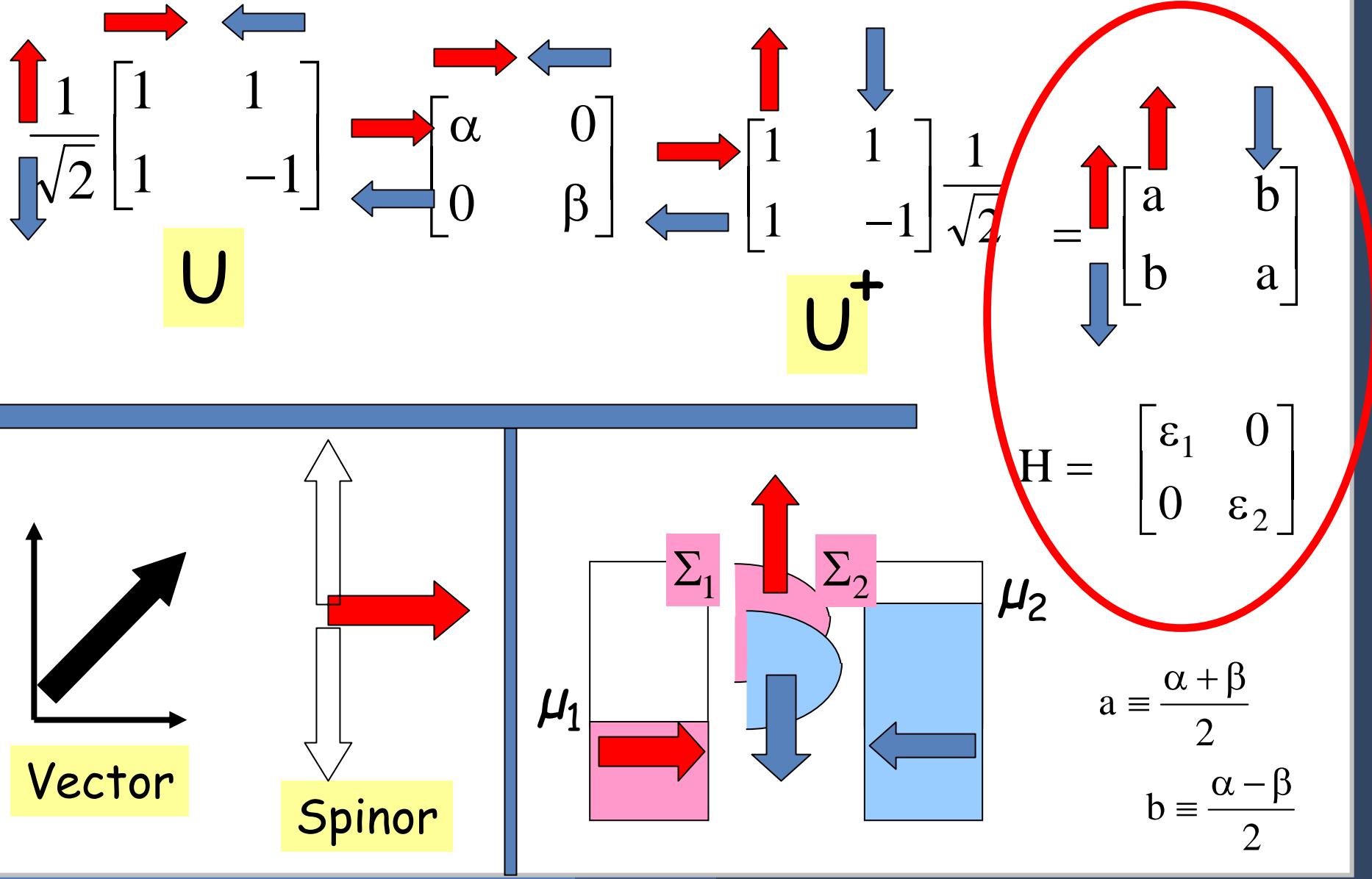
$$a \equiv \frac{\alpha + \beta}{2} \quad b \equiv \frac{\alpha - \beta}{2}$$

But all matrices need not be diagonal ...



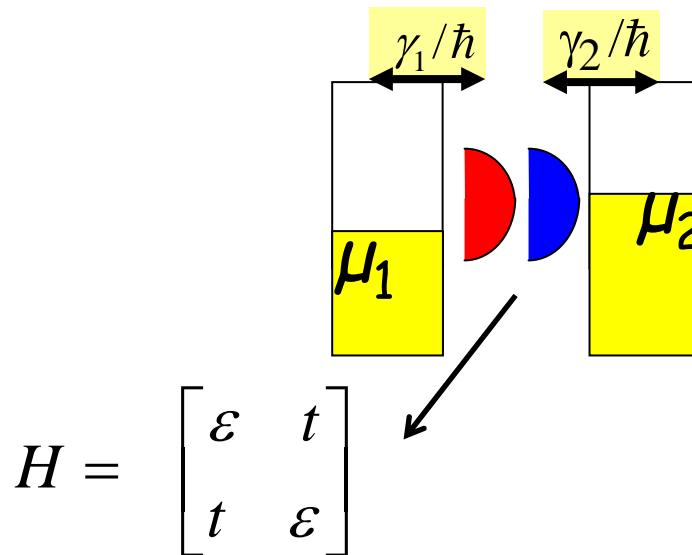
$$H = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$$





$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \gamma_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} 0 & 0 \\ 0 & \gamma_2 \end{bmatrix}$$



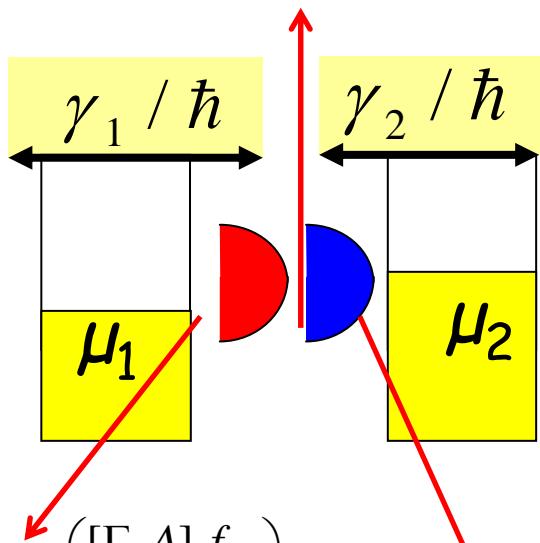
$$\Sigma_1 = -\frac{i}{4} \begin{bmatrix} \gamma_1 & \gamma_1 \\ \gamma_1 & \gamma_1 \end{bmatrix}$$

$$\Sigma_2 = -\frac{i}{4} \begin{bmatrix} \gamma_2 & -\gamma_2 \\ -\gamma_2 & \gamma_2 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon + t & 0 \\ 0 & \varepsilon - t \end{bmatrix} \rightarrow \begin{cases} +1 \\ +1 \end{cases}$$

$$\begin{bmatrix} \varepsilon + t & 0 \\ 0 & \varepsilon - t \end{bmatrix} \rightarrow \begin{cases} +1 \\ -1 \end{cases}$$

$$\frac{I_{12}}{q/\hbar} = \text{Trace} \left([G_{12}^n H_{21}] - [H_{12} G_{21}^n] \right)$$



$$\frac{I_1}{q/\hbar} = \text{Trace} \begin{pmatrix} [\Gamma_1^A] f_1 \\ -[\Gamma_1 G^n] \end{pmatrix}$$

cf.

$$\gamma_1 D f_1 - \gamma_1 n$$

$$\frac{I_2}{q/\hbar} = \text{Trace} \begin{pmatrix} [\Gamma_2^A] f_2 \\ -[\Gamma_2 G^n] \end{pmatrix}$$

$$\gamma_2 D f_2 - \gamma_2 n$$

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+] \quad \Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$

$$[G] = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$

"Electron density"

$$G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1$$

"Density of states"

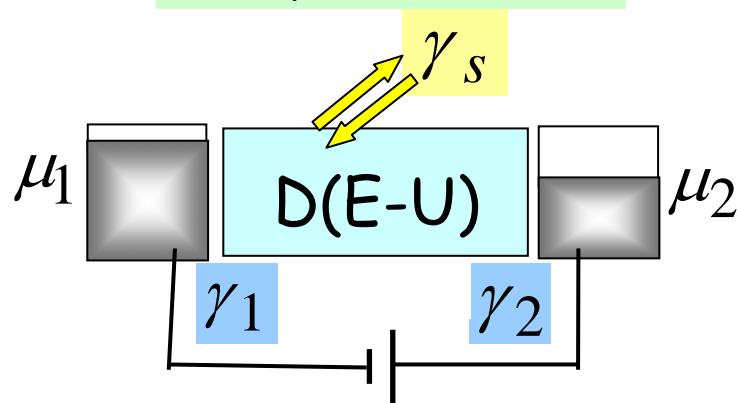
$$A = i[G - G^+]$$

Coherent transport

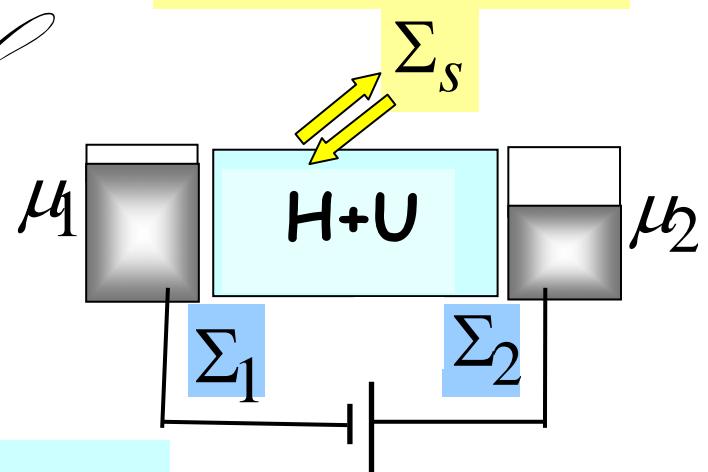
$$I = \frac{e}{h} \int dE \quad T(E) \quad [f_2 - f_1]$$

$$T(E) = \text{Trace} [\Gamma_1 G \Gamma_2 G^+]$$

Simple version



Matrix version



$$\varepsilon, U \Leftrightarrow [H], [U]$$

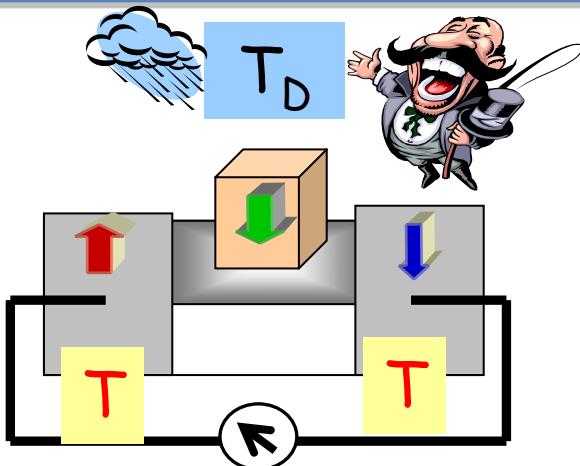
$$\gamma \Leftrightarrow [\Gamma], [\Sigma]$$

$$D(E) \Leftrightarrow [A(E)]$$

$$n(E) \Leftrightarrow [G^n(E)] \equiv -i G^<(E)$$

$$p(E) \Leftrightarrow [G^p(E)] \equiv +i G^>(E)$$

$N \times N$ matrices
 N : number of basis functions



$$I_{1R} = e \frac{\alpha}{\hbar} D(E) [f_1 - f_R]$$

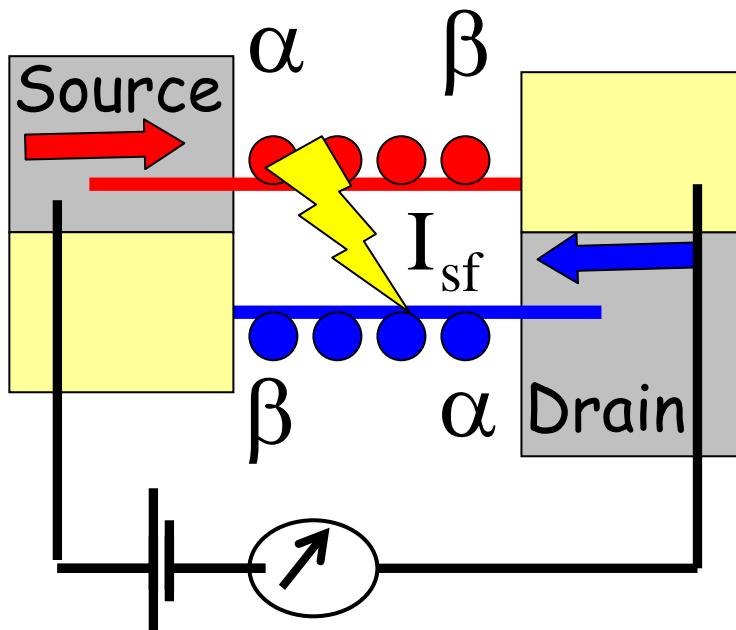
$$I_{1B} = e \frac{\beta}{\hbar} D(E) [f_1 - f_B]$$

$$I_{2R} = e \frac{\beta}{\hbar} D(E) [f_2 - f_R]$$

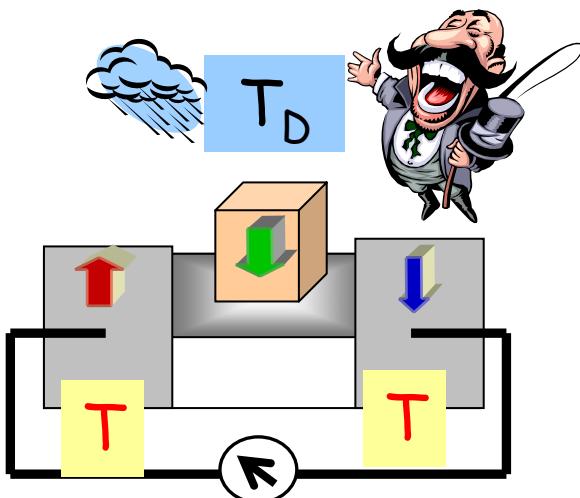
$$I_{2B} = e \frac{\alpha}{\hbar} D(E) [f_2 - f_B]$$

$$I_{2R} = I_{1R} - I_{sf}$$

$$I_{2B} = I_{1B} + I_{sf}$$

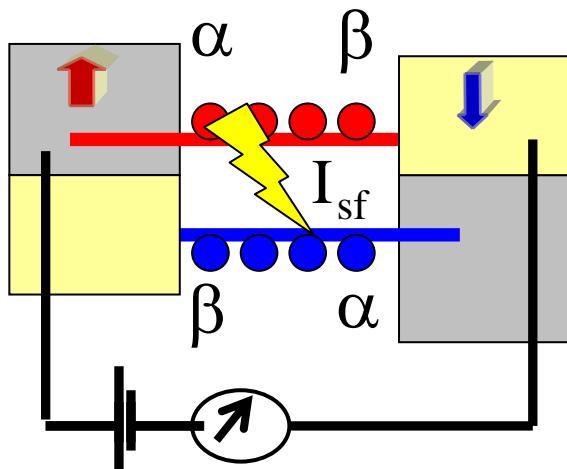


Electronic Maxwell's demon



$$I_{sf} = e \frac{\gamma_s}{\hbar} D_R(E) D_B(E - \varepsilon)$$

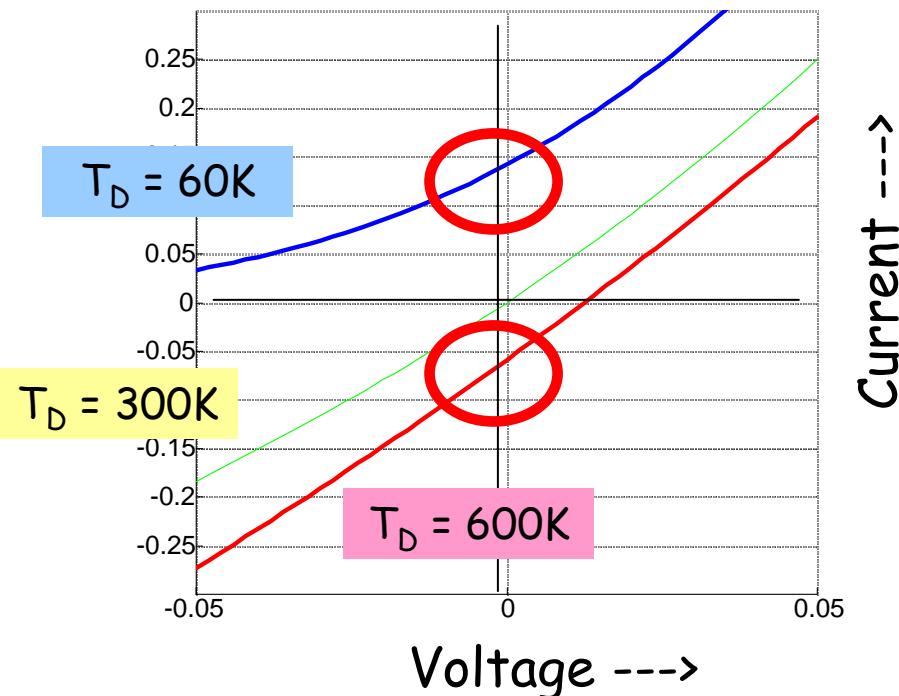
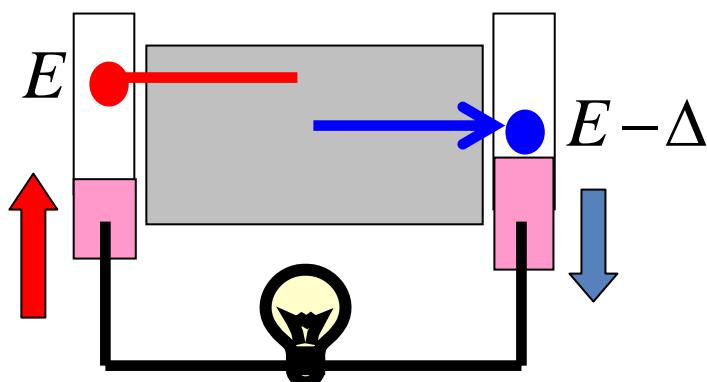
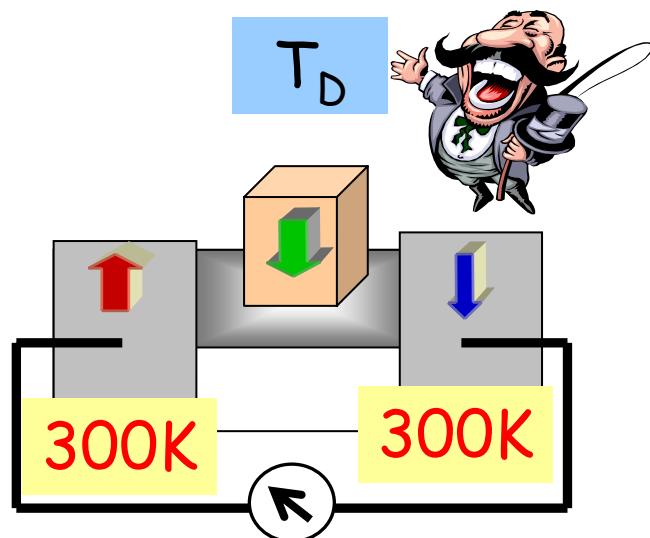
$$\left(\begin{array}{ccc} F_{out}(\varepsilon) & f_R(E) & (1 - f_B(E - \varepsilon)) \end{array} \right)$$



$$F_{in}(\varepsilon) = F_{out}(\varepsilon) \exp\left(-\frac{\varepsilon}{k_B T_D}\right)$$

If demon is in equilibrium
with temperature T_D

The nanoscale heat engine



Q_1 : heat from contacts
 Q_2 : heat to demon
 $Q_1 - Q_2$: useful work

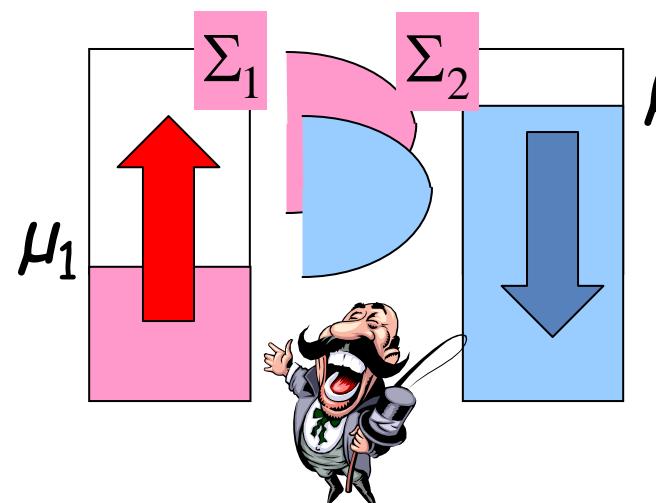
Incoherent transport: Matrix method

$$H = \begin{bmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}$$

"Self-energy" matrices

$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$

 μ_2

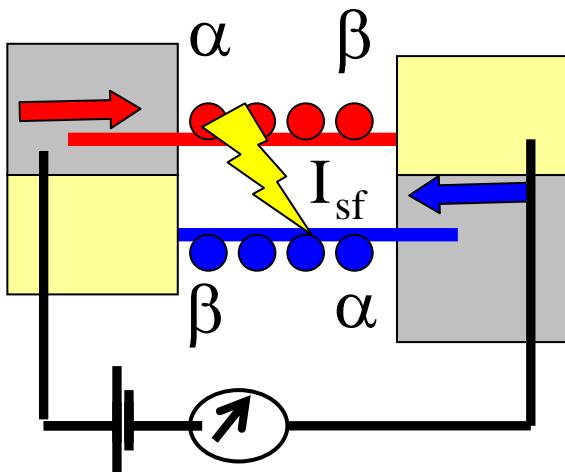
$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+]$$

$$\Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$

$$[G] = \begin{bmatrix} EI - H - \Sigma_1 - \Sigma_2 \\ -\Sigma_s \end{bmatrix}^{-1}$$

$$G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1 + G\Sigma_s^{\text{in}} G^+$$

Incoherent transport: Matrix method

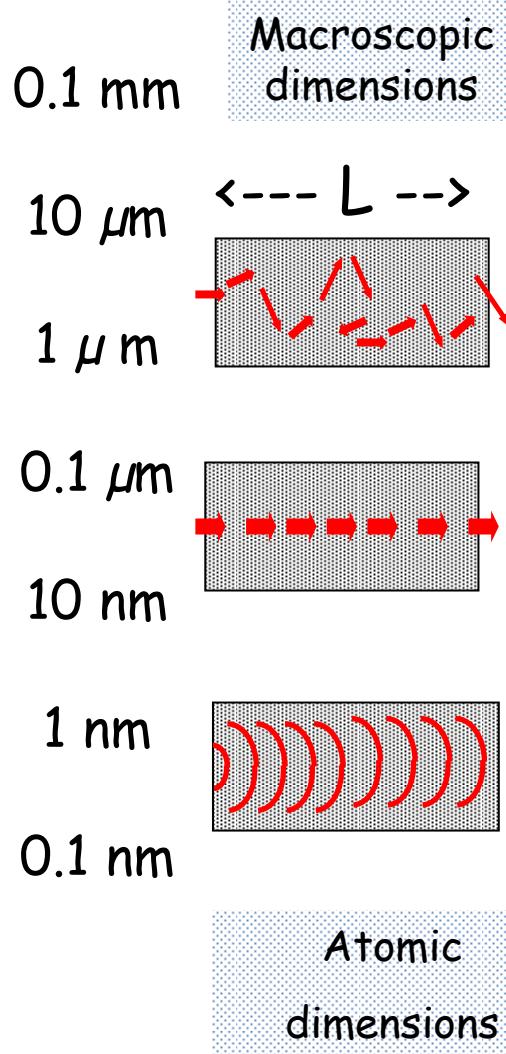


$$I_{sf} = e \frac{\gamma_s}{\hbar} \left(n_R(E) \begin{bmatrix} F_{out}(\varepsilon) & p_B(E-\varepsilon) \\ +F_{in}(\varepsilon) & n_B(E-\varepsilon) \end{bmatrix} - D_R(E) \begin{bmatrix} F_{in}(\varepsilon) & n_B(E-\varepsilon) \end{bmatrix} \right)$$

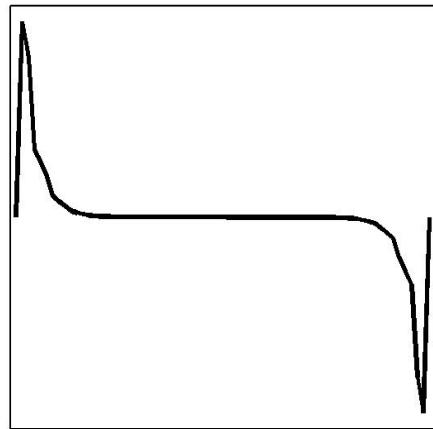
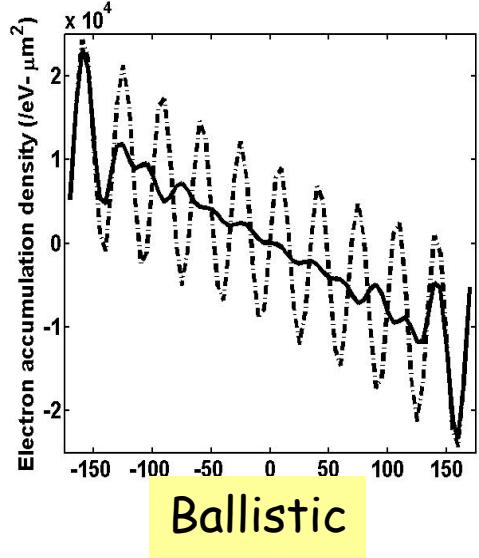
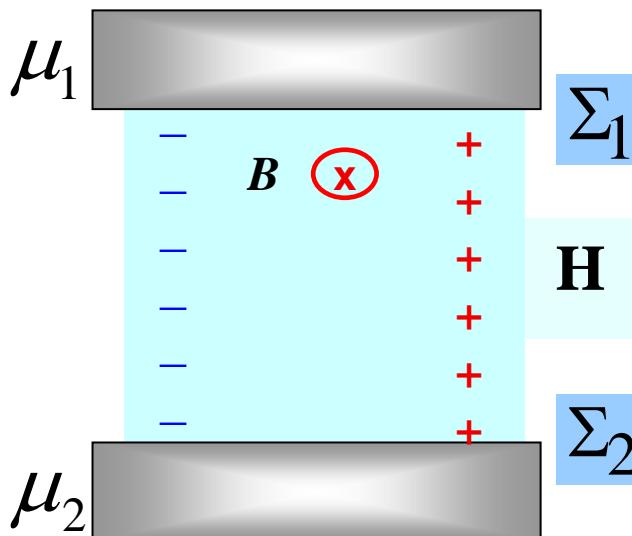
$$\begin{aligned} D_{in}(i, j; k, l; \varepsilon) &= \\ D_{out}(l, k; j, i; \varepsilon) & \\ \exp\left(-\frac{\varepsilon}{k_B T_D}\right) & \end{aligned}$$

$$\Gamma_s = e \frac{\gamma_s}{\hbar} \begin{bmatrix} D_{out}(\varepsilon) & G^p(E-\varepsilon) \\ +D_{in}(\varepsilon) & G^n(E-\varepsilon) \end{bmatrix}$$

$$\Sigma_s^{in} = e \frac{\gamma_s}{\hbar} [D_{in}(\varepsilon) \quad G^n(E-\varepsilon)]$$

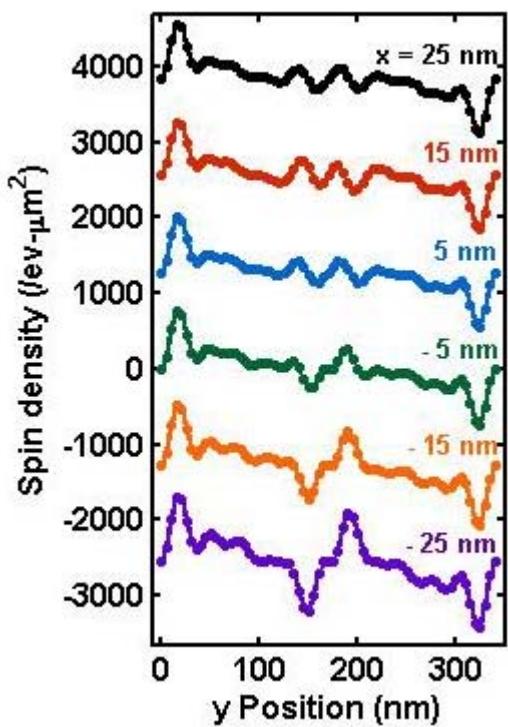


From Roksana
Golizadeh-
Mojarad

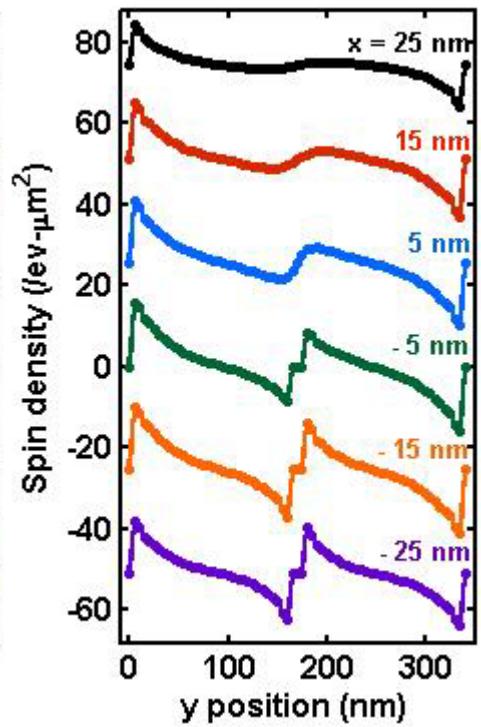


Theory: Golizadeh-Mojarad and Datta, Spin-Hall Effect:
From the Ballistic to the Diffusive Regime, Condmat 0703280

Ballistic

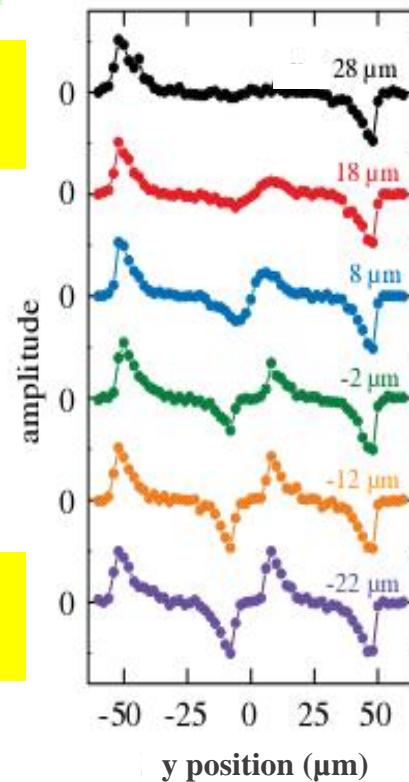


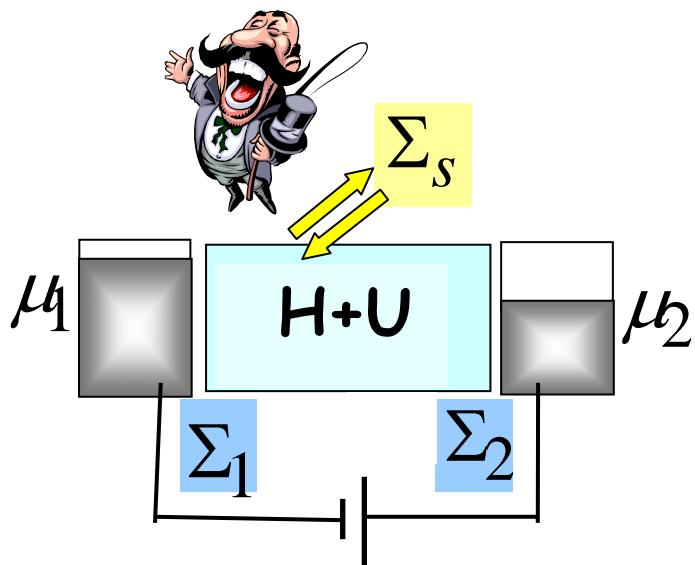
Diffusive


 Σ_1
 H
 Σ_2

GaAs

Experiment:
Sih and Awschalom et.al.
PRL 97, (2006)





U: Self-consistent Field
describing
electron-electron
interactions

$$[G] = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$

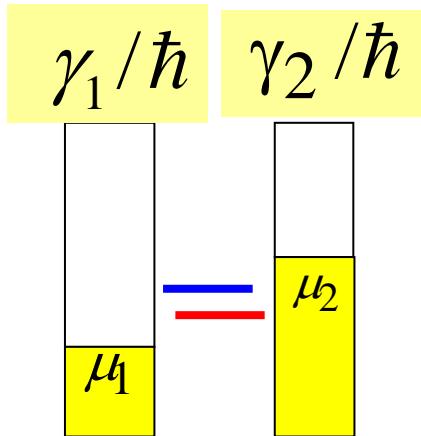
$$[G] = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1}$$

$$G^n = G\Gamma_2 G^+ f_2 + G\Gamma_1 G^+ f_1 + G\Sigma_s^{in} G^+$$

$$\Gamma_s = e \frac{\gamma_s}{\hbar} \begin{bmatrix} F_{out}(\varepsilon) & G^p(E-\varepsilon) \\ & +F_{in}(\varepsilon) & G^n(E-\varepsilon) \end{bmatrix}$$

$$\Sigma_s^{in} = e \frac{\gamma_s}{\hbar} \begin{bmatrix} F_{in}(\varepsilon) & G^n(E-\varepsilon) \end{bmatrix}$$

Coulomb blockade and strong correlation



f_R and f_B

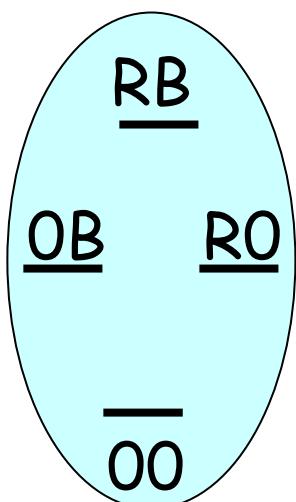
"UNCORRELATED"

$$P_{00} = (1 - f_R) * (1 - f_B)$$

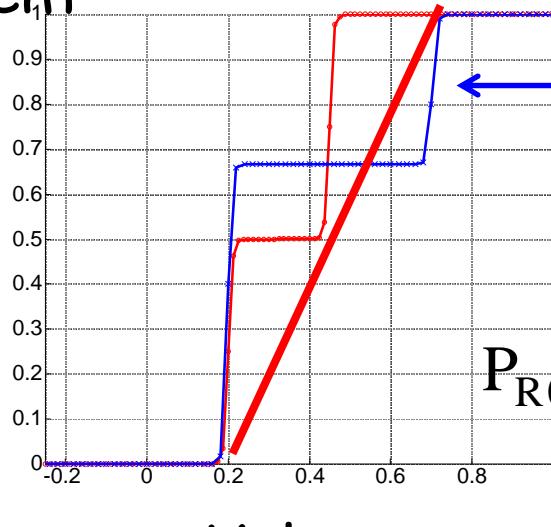
$$P_{R0} = f_R * (1 - f_B)$$

$$P_{0B} = (1 - f_R) * f_B$$

$$P_{RB} = f_R * f_B$$



Current



STRONGLY
CORRELATED

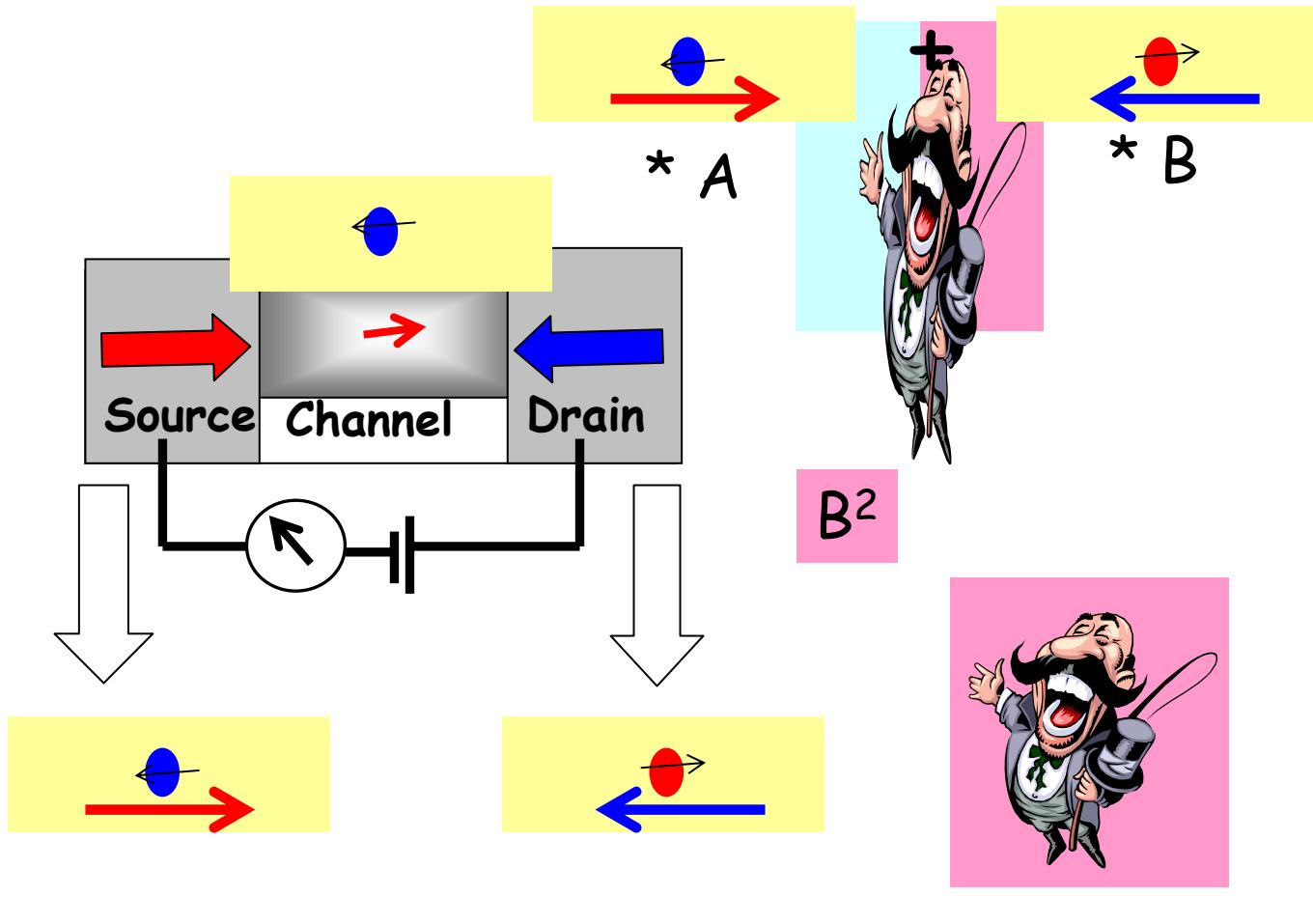
$$P_{RB} = 0$$

$$P_{R0} = P_{0B} = \gamma_2 / (\gamma_1 + 2\gamma_2)$$

$$P_{00} = \gamma_1 / (\gamma_1 + 2\gamma_2)$$

Correlated / Entangled "demon"

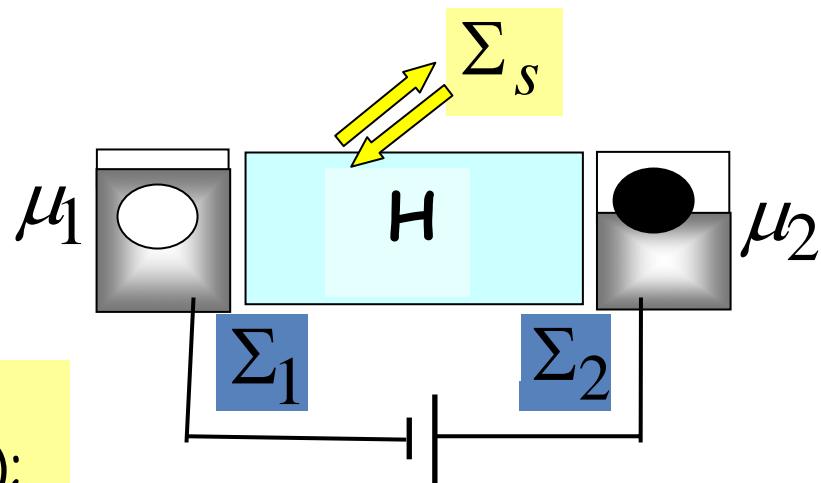
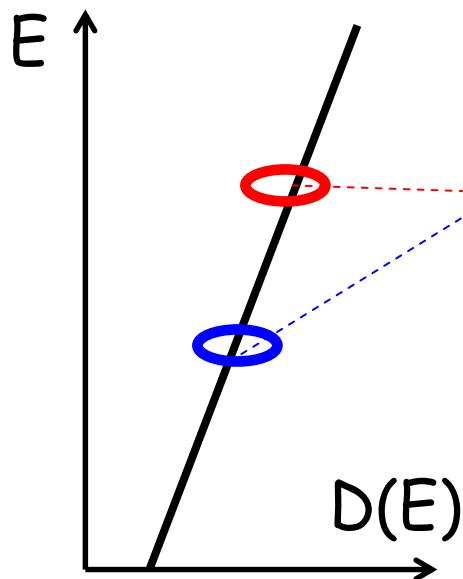
Entangled !



"Reservoir"

"System"

Down > Up



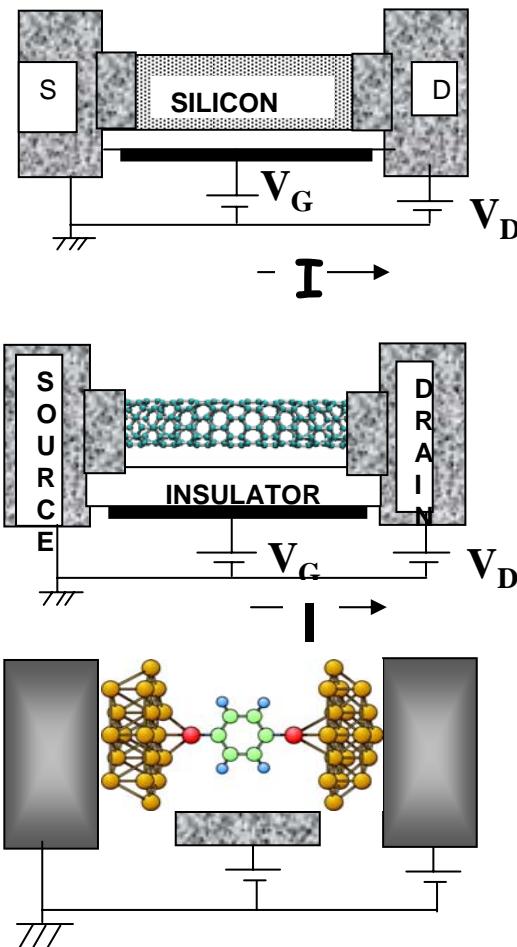
Model based on

- (1) Datta, Phys. Rev. B 40, 5830 (1989);
J. Phys. Cond. Matt. 2, 8023 (1990).
- (2) Meir and Wingreen,
Phys. Rev. Lett. 68, 2512 (1992).

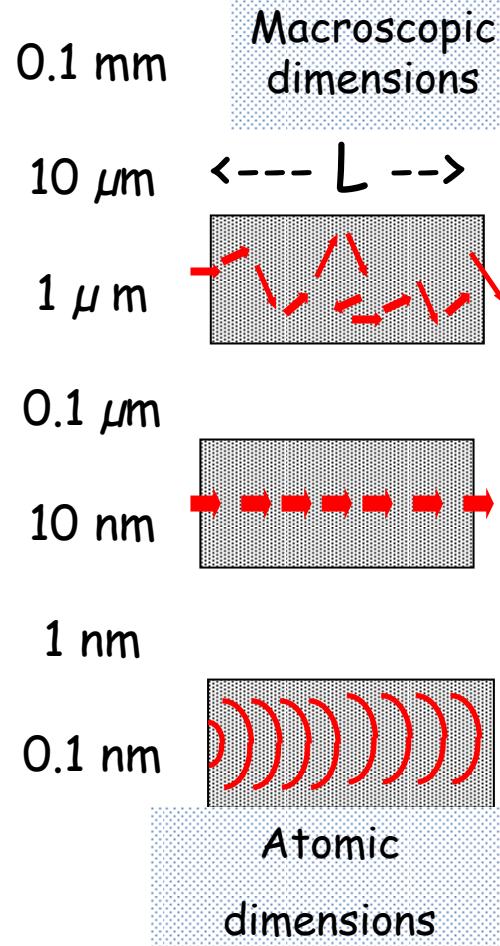
Non-Equilibrium Green Function (NEGF) + Landauer

Quantum Transport far from Equilibrium

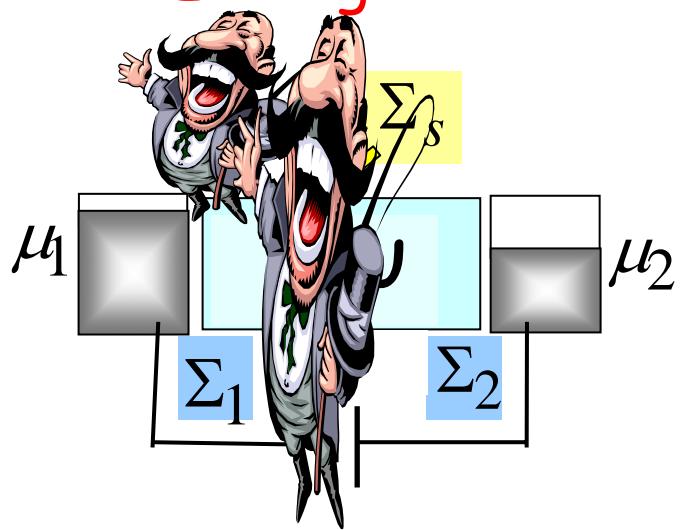
Materials



Transport Regimes



Entangled!



Non-Equilibrium
Green Function
(NEGF) + Landauer

Quantum Transport:
Atom to Transistor,
Cambridge (2005)