Fiber Optic Communications
Lecture 2

- Fiber Modes
- System components
- Modulation
- Multiplexing
Maxwell’s Equations

**Divergence equations**

\[ \nabla \cdot D = \rho \]
\[ \nabla \cdot B = 0 \]

Flux lines start and end on charges or poles

**Curl equations**

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times H = \frac{\partial D}{\partial t} + J \]

Changes in fluxes give rise to fields
Currents give rise to \( H \)-fields

*Note: No constants such as \( \mu_0 \varepsilon_0, \mu \varepsilon, c, \chi \ldots \ldots \) appear when written this way*
*They are hidden in \( B \) and \( D \)*

**From Maxwell’s Equations: Existence of EM waves**
*(no need for charges, materials)*

![Diagram of electromagnetic waves](image)

**Curl equations:** Changing \( E \)-field results in changing \( H \)-field results in changing \( E \)-field...
Wave equation:

$$\nabla^2 U(r, t) = \frac{1}{v^2} \frac{\partial^2 U(r, t)}{\partial t^2}$$

for $E$ and $H$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

SOLUTION:

Waves propagating with a (phase) velocity $v$

$$U(r, t) = \text{Re}\left\{ U_0(r) \exp(i\omega t) \right\}$$
Wave Equation in Vacuum

\[ \nabla^2 U(r, t) = \frac{1}{v^2} \frac{\partial^2 U(r, t)}{\partial t^2} \]

(1809–1858). Equivalently, nowadays \( \mu_0 \) is assigned a value of \( 4\pi \times 10^{-7} \text{ m kg/C}^2 \) in SI units, and one can determine \( \varepsilon_0 \) directly from simple capacitor measurements. In any event,

\[ \varepsilon_0 \mu_0 = (8.85 \times 10^{-12} \text{ s}^2 \text{ C}^2/\text{m}^3 \text{ kg})(4\pi \times 10^{-7} \text{ m kg/C}^2) \]

or

\[ \varepsilon_0 \mu_0 \approx 11.12 \times 10^{-18} \text{ s}^2/\text{m}^2. \]

And now the moment of truth—in free space, the predicted speed of all electromagnetic waves would then be

\[ v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8 \text{ m/s}. \]

This theoretical value was in remarkable agreement with the previously measured speed of light (315,300 km/s) determined by Fizeau. The results of Fizeau's experiments, performed in 1849 with a rotating toothed wheel, were available to Maxwell and led him to comment:

This velocity [i.e., his theoretical prediction] is so nearly that of light, that it seems we have strong reason to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.

This brilliant analysis was one of the great intellectual triumphs of all time.
Wave Equation: Cylindrical Coordinates

Wave equation:

\[
\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n^2k_0^2 E_z = 0,
\]

for \( E \) and \( H \)

\[
E_z(\rho, \phi, z) = F(\rho) \Phi(\phi) Z(z)
\]

\[
\frac{d^2 Z}{dz^2} + \beta^2 Z = 0
\]

\[
\frac{d^2 \Phi}{d\phi^2} + m^2 \Phi = 0
\]

\[
\frac{d^2 F}{d\rho^2} + \frac{1}{\rho} \frac{dF}{d\rho} + \left( n^2k_0^2 - \beta^2 - \frac{m^2}{\rho^2} \right) F = 0
\]
Wave Equation: Cylindrical Coordinates

Solution:

\[ E_z = \begin{cases} AJ_m(p\rho)\exp(i\rho\phi)\exp(i\beta z) \quad & \rho \leq a, \\ CK_m(q\rho)\exp(i\rho\phi)\exp(i\beta z) \quad & \rho > a. \end{cases} \]

\[ H_z = \begin{cases} BJ_m(p\rho)\exp(i\rho\phi)\exp(i\beta z) \quad & \rho \leq a, \\ DK_m(q\rho)\exp(i\rho\phi)\exp(i\beta z) \quad & \rho > a. \end{cases} \]

\[ p^2 = n_1^2k_0^2 - \beta^2 \]
\[ q^2 = \beta^2 - n_2^2k_0^2 \]

\[ \bar{n} = n_2 + b(n_1 - n_2) \approx n_2(1 + b\Delta) \]

\[ b(V) \approx (1.1428 - 0.9960/V)^2 \]
Fiber Losses

Linear absorption:

\[ P_{out} = P_{in} \exp(-\alpha L) \]

Stimulated Brillouin Scattering:

\[ \frac{dI_p}{dz} = -g_B I_p I_s - \alpha_p I_p, \]
\[ -\frac{dI_s}{dz} = +g_B I_p I_s - \alpha_s I_s, \]

Stimulated Raman Scattering:

\[ \frac{dI_p}{dz} = -g_R I_p I_s - \alpha_p I_p, \]
\[ \frac{dI_s}{dz} = g_R I_p I_s - \alpha_s I_s, \]

Self-Phase Modulation

\[ \beta' = \beta + k_0 \bar{n}_2 P/A_{eff} = \beta + \gamma P, \]

Cross-Phase Modulation

\[ \frac{\partial A}{\partial z} + i\beta_2 \frac{\partial^2 A}{\partial t^2} = -\frac{\alpha}{2} A + i\gamma |A|^2 A \]
Group Velocity Dispersion

\[ v_g = (d\beta / d\omega)^{-1}. \]

\[ \tilde{n}_g = \tilde{n} + \omega (d\tilde{n} / d\omega) \]

\[ \Delta T = \frac{dT}{d\omega} \Delta \omega = \frac{d}{d\omega} \left( \frac{L}{v_g} \right) \Delta \omega = L \frac{d^2 \beta}{d\omega^2} \Delta \omega = L\beta_2 \Delta \omega \]

\[ \beta_2 = \frac{d^2 \beta}{d\omega^2} \]

\[ D = \frac{d}{d\lambda} \left( \frac{1}{v_g} \right) = -\frac{2\pi c}{\lambda^2} \beta_2 \]

\( D \) is called the dispersion parameter and is expressed in units of ps/(km-nm).
Material Dispersion

\[ n^2(\omega) = 1 + \sum_{j=1}^{M} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2} \]

\[ D_M \approx 122 \left( 1 - \frac{\lambda_{ZD}}{\lambda} \right) \]

Table 2.1 Characteristics of several commercial fibers

<table>
<thead>
<tr>
<th>Fiber Type and Trade Name</th>
<th>( A_{\text{eff}} ) (( \mu \text{m}^2 ))</th>
<th>( \lambda_{ZD} ) (nm)</th>
<th>( D ) (C band) [ps/(km-nm)]</th>
<th>Slope ( S ) [ps/(km-nm^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corning SMF-28</td>
<td>80</td>
<td>1302–1322</td>
<td>16 to 19</td>
<td>0.090</td>
</tr>
<tr>
<td>Lucent AllWave</td>
<td>80</td>
<td>1300–1322</td>
<td>17 to 20</td>
<td>0.088</td>
</tr>
<tr>
<td>Alcatel ColorLock</td>
<td>80</td>
<td>1300–1320</td>
<td>16 to 19</td>
<td>0.090</td>
</tr>
<tr>
<td>Corning Vascade</td>
<td>101</td>
<td>1300–1310</td>
<td>18 to 20</td>
<td>0.060</td>
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<tr>
<td>Lucent TrueWave-RS</td>
<td>50</td>
<td>1470–1490</td>
<td>2.6 to 6</td>
<td>0.050</td>
</tr>
<tr>
<td>Corning LEAF</td>
<td>72</td>
<td>1490–1500</td>
<td>2 to 6</td>
<td>0.060</td>
</tr>
<tr>
<td>Lucent TrueWave-XL</td>
<td>72</td>
<td>1570–1580</td>
<td>–1.4 to –4.6</td>
<td>0.112</td>
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<tr>
<td>Alcatel TeraLight</td>
<td>65</td>
<td>1440–1450</td>
<td>5.5 to 10</td>
<td>0.058</td>
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</tbody>
</table>
Limitations on Bit Rate

\[ B\Delta T < 1 \]

\[ BL|D|\Delta \lambda < 1 \]

\[ BL|D|\sigma_\lambda \leq \frac{1}{4} \]

\[ \sigma^2 = \sigma_0^2 + \frac{1}{2}(\beta_3 L\sigma_\omega^2)^2 \equiv \sigma_0^2 + \frac{1}{2}(SL\sigma_\lambda^2)^2 \]
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Optical Telecommunications: Basic System Components

Input

Optical Transmitter

Communication Channel

Optical Receiver

Output

Electrical input

Driver

Optical Source

Modulator

Channel Coupler
Optical Telecommunications: Basic System Components

- **Input**
- **Optical Transmitter**
- **Communication Channel**
- **Optical Receiver**
- **Output**

- **Channel Coupler**
- **Photodetector**
- **Electronics**
- **Demodulator**
Optical Telecommunications: Physical Components

- **Source**: Laser or LED
- **Signal**: information in the form of electrical signal – analog or digital
- **E/O**: Modulator – modulates the light from source according to the signal
- **Fiber**: Optical fiber – multimode or single mode fibers
- **OA**: Optical amplifier – boost the intensity of light
- **O/E**: Photodetector – converts light to electricity
- **Receiver**: Extracts information from the received light
Specific Communication System Components by Channel

<table>
<thead>
<tr>
<th>Wavelength $\lambda_o$ (nm)</th>
<th>O</th>
<th>E</th>
<th>S</th>
<th>C</th>
<th>L</th>
<th>U</th>
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<tbody>
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<td>800</td>
<td>LED</td>
<td>AlGaAs</td>
<td>InGaAsP</td>
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<tr>
<td>900</td>
<td>LASER</td>
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<td>1000</td>
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<td>1600</td>
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<tr>
<td>1700</td>
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</tbody>
</table>

**Source:**
- LED
- LASER

**Detector:**
- $p-i-n$
- APD

**Amplifier:**
- SOA
- OFA

**Fiber:**
- MMF: SI / GRIN
- SMF

- Silica glass
- DSF
- EDFA
- REFA
- RFA
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Optical Modulation

• Information is coded in the light wave itself by optical modulation
• Many modulation techniques
  – Field modulation
  – Intensity modulation
  – Pulse Code Modulation (PCM)
  – Frequency or Phase Shift Keying (PSK)
Field Modulation

Modulation similar to microwave modulation: amplitude, frequency and phase modulation

Difficult to implement with light: requires
- extremely stable laser source
- extreme coherence
- polarization controlled transmission
- heterodyne receiver
Intensity Modulation

- Simple implementation
- LED or laser sources can be used
- WDM is implemented using this modulation scheme
Pulse Code Modulation (PCM)

Amplitude of the signal is proportional to number of pulses within each sample-period.
ON-OFF Keying (OOK)

Frequency shift keying (FSK) and Phase Shift keying (PSK) are variants of OOK.

(a) Optical intensity

(b) Optical field
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Multiplexing

- Providing (dynamic) rerouting of channels
- Electronic multiplexing – signals from different channels are added before optical modulation
- Optical multiplexing – signals from different channels are coded into light before multiplexing
- Different schemes
  - Frequency Division Multiplexing (FDM)
  - Time Division Multiplexing (TDM)
  - Code Division Multiplexing (CDM)
Multiplexing schemes

- FDM – each channel is assigned to a different frequency
- TDM – each channel is transmitted in a different time interval
- CDM – each channel is encoded differently
Code Division Multiplexing

- Each channel is encoded differently using different keys.
- Decoding requires a key which selects only a particular channel.
Hierarchical multiplexing

- Multiplexing many channels together is often performed in a hierarchy.
Wavelength division multiplexing (WDM)

- Different channels are transmitted at different optical frequencies
- Multiplexer and demultiplexers are frequency selective routers
Wavelength Division Multiplexing

Also known as frequency division multiplexing (FDM).

Each channel is assigned to a different frequency band.

Demultiplexing is performed by spectral filtering.

Minimum cross-talk between channels.
MUX and DEMUX for WDM

(a) Prism

(b) Diffraction gratings

(c) Thin-film filters

(d) Fiber Bragg grating (FBG)

(e) Microring resonators
WDM Types + Specifications

• Coarse WDM (CWDM) – channels are spaced wide apart (typical: 20 nm apart)
• Dense WDM (DWDM) – channels are closely spaced, more channels can be transmitted
• Typical channel spacing: 25 to 100 GHz (0.2 to 0.4 nm)
• DWDM requires extremely stable light source
OADM (Optical Add-Drop Mux)

- Uses a multiplexer-demultiplexer pair
- Mux adds the signals, demux drops one channel at each stage.
- One particular channel can be accessed at each stage.

Microring resonator based

Fiber Bragg-grating based
MZI demultiplexer

- Mach-Zehnder interferometer: light split into two paths interferes
- Output is high if the interference is constructive, low if destructive
- Constructive interference at one wavelength may be arranged to produce destructive at the other wavelength

\[
\text{phase difference } \phi = \frac{2\pi d}{\lambda} \\
\Delta d = q_1 \lambda_1 / 2 \text{ and } \Delta d = q_2 \lambda_2 / 2.
\]

\[
P_1 = \cos^2\left(\frac{\pi \Delta d}{\lambda}\right), \quad P_2 = \sin^2\left(\frac{\pi \Delta d}{\lambda}\right).
\]