1. Heat conduction and diffusion in alloy casting

   In the die-casting of a relatively thick roughly plate-shaped metal alloy part, the liquid metal alloy cools and reaches a roughy uniform distribution at the melting point at time $t = 0$, then solidifies with a plane front from the sides. The rate of solidification is limited by two types of heat transfer: conduction through the mold and to the environment can be represented by $q_x = h(T_s - T_{env})$ (T_s is the outer surface metal temperature), and conduction through the already-solidified metal of thickness Y.

   (a) Sketch the temperature profile (T vs. x, where x is the distance from one side of the mold) across the solid metal shell and liquid metal interior for short times (small Bi) and long times (large Bi).

   (b) Derive a simple expression for the growth rate dY/dt which is valid for short times (small Bi).

   (c) Derive a simple expression for the growth rate dY/dt which is valid for long times.

   (d) Sketch the relationship between solidified shell thickness Y and time t, showing the transition from convection- to conduction-limited growth.

   As the metal solidifies, the lower solubility of the alloying element (solute) results in its "rejection" into the liquid. At steady-state, the concentration of the solute in the solid is that of the liquid C_L, but in the liquid at the liquid-solid interface, the concentration is much higher, say $5C_L$.

   (e) Assuming that solidification front velocity U is constant, that diffusion is slow enough that the diffusion boundary layer is much smaller than the part thickness, and that the effect of fluid flow is negligible, derive the steady-state general equation for concentration as a function of distance into the liquid from the moving interface x' (that is, in the frame of reference of the moving interface).

   (f) Fit this general solution to the boundary conditions in the liquid at $x' = 0$ and $x' = \infty$ to give the particular concentration profile here.

   (g) Approximately how thick is the solute-enriched layer in the liquid (an expression, not a number)?