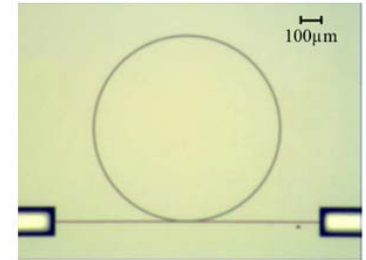


Frequency Bin Quantum Photonics

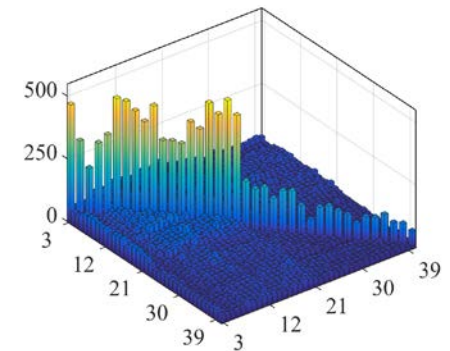
Andrew M. Weiner
Purdue University



Prestige Lecture Series, NSF Science of Information STC, Purdue University, April 8, 2019

Outline

- Introductory
- Frequency bin entanglement
- Manipulating frequency encoded quantum states



- An experimental perspective; looking forward to making new connections with quantum information people

Credits

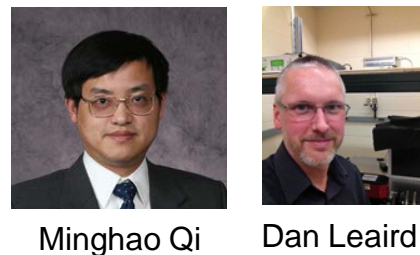
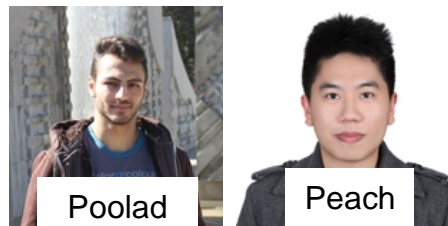
Graduated students

Jose Jaramillo
Ogaga Odele



Current students

Mohammed Al alshaykh
Poolad Imany
Navin Lingaraju
Hsuan-Ho (Peach) Lu
Alex Moore
Nathan O'Malley
Oscar Sandoval
Suparna Seshadri



Faculty and staff

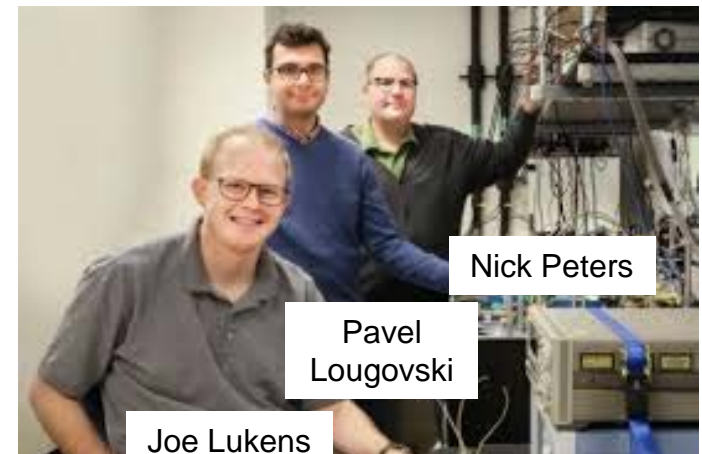
Prof. Sabre Kais
Dr. Dan Leaird
Prof. Minghao Qi
Dr. Yi Xuan

Oak Ridge National Lab

Pavel Lougovski
Joseph Lukens
Nick Peters
Brian Williams

Army Research Lab

Misha Brodsky



Sponsors



Bits and Qubits

Binary or two-level systems



Classical:

Bits:

0,1; heads or tails

- *Either in 0 or 1, never in both simultaneously*

Quantum mechanical:

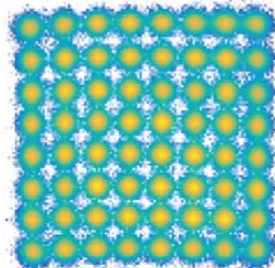
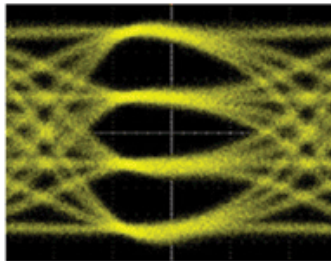
Qubits:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle) ; |\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |T\rangle)$$

- *Generally in a superposition of 0 and 1*
- *Intrinsic randomness in measurement*
- *Interference phenomena; phase matters*

Higher-level encoding; higher dimensionality

Multi-level communications

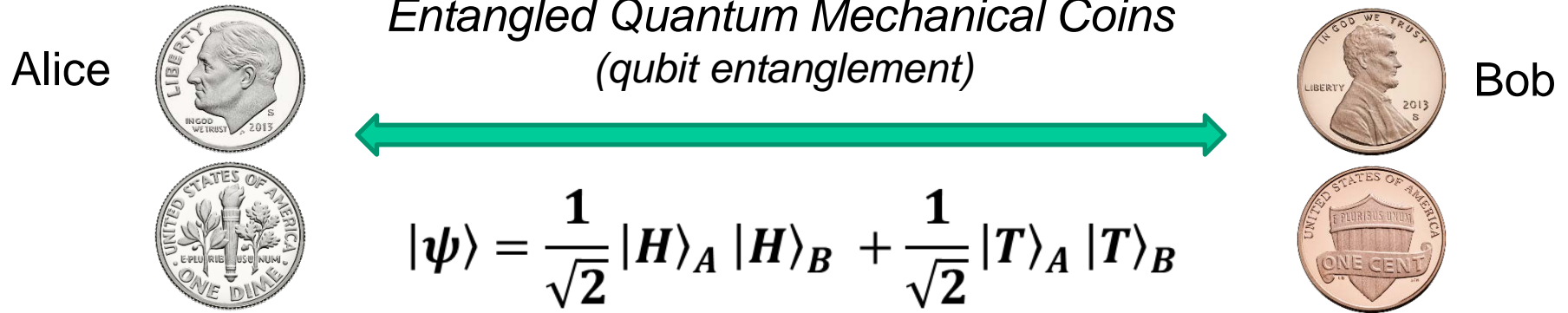


Qu^dits: d-level quantum states

$$|\psi\rangle \sim c_0|0\rangle + c_1|1\rangle + \dots + c_{d-1}|d-1\rangle$$

- *Potential for multiple qubits per particle*

Quantum Entanglement



- **Classically:** Alice has heads or tails; Bob has heads or tails; these are independent.
- **Quantum mechanically:** Alice's and Bob's outcomes can be highly correlated.
 - Correlations are nonlocal; measuring Alice's coin affects Bob's measurement immediately and at a distance.
 - Phase matters!
 - An important resource in quantum information



Qudit entanglement is also possible

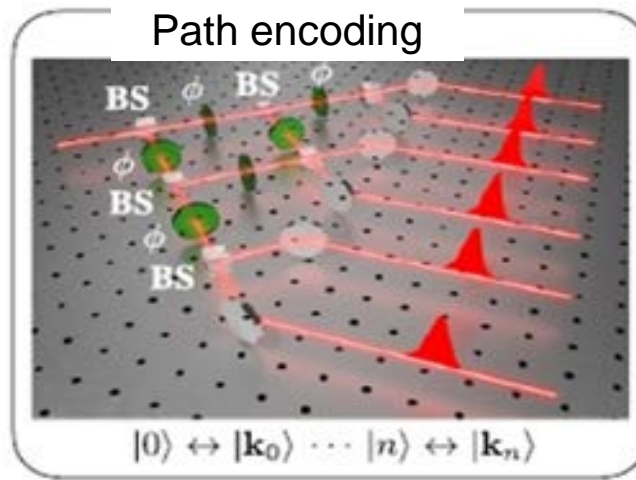
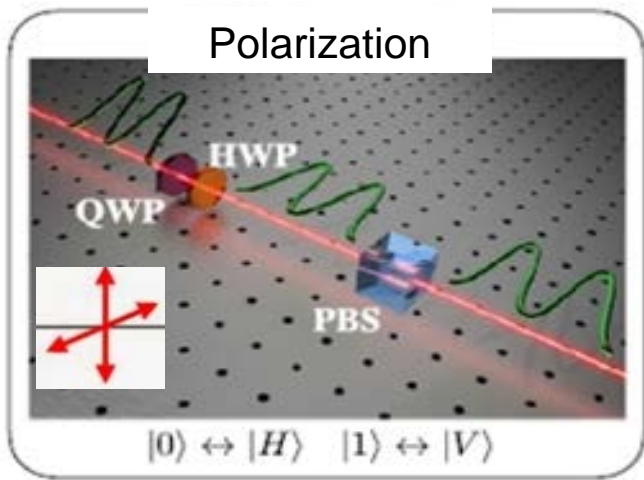


A large green double-headed arrow pointing left and right, indicating a connection between the two dice.

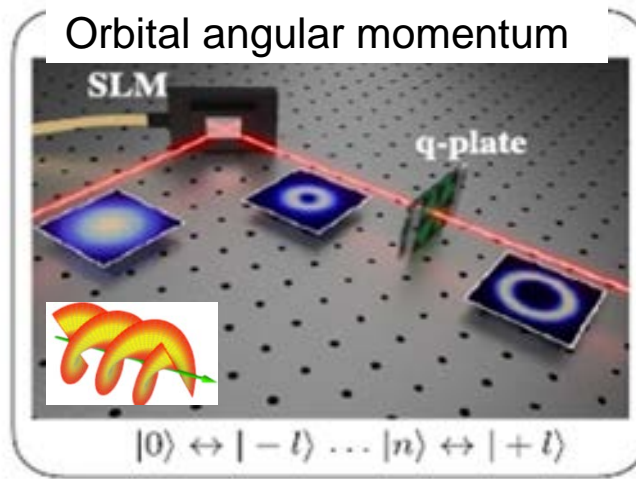
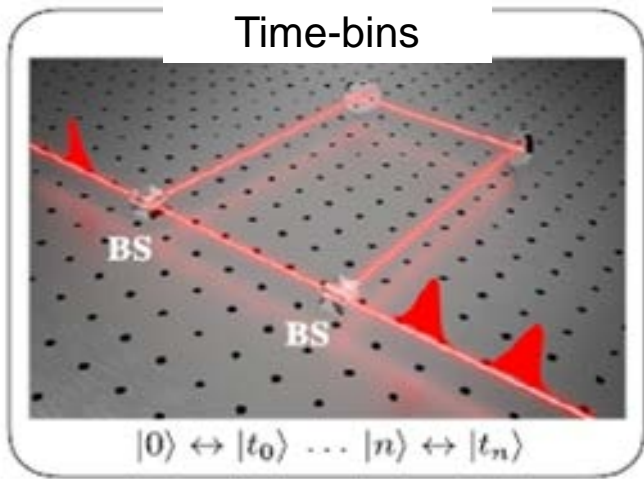


Encoding Quantum Information in Photons

- Offers many degrees of freedom, most supporting high dimensionality 😊
- Excellent for communications 😊
- Weak interactions – little decoherence 😊 but very hard to make two photon gates 😞
- Lack of deterministic photon sources 😞

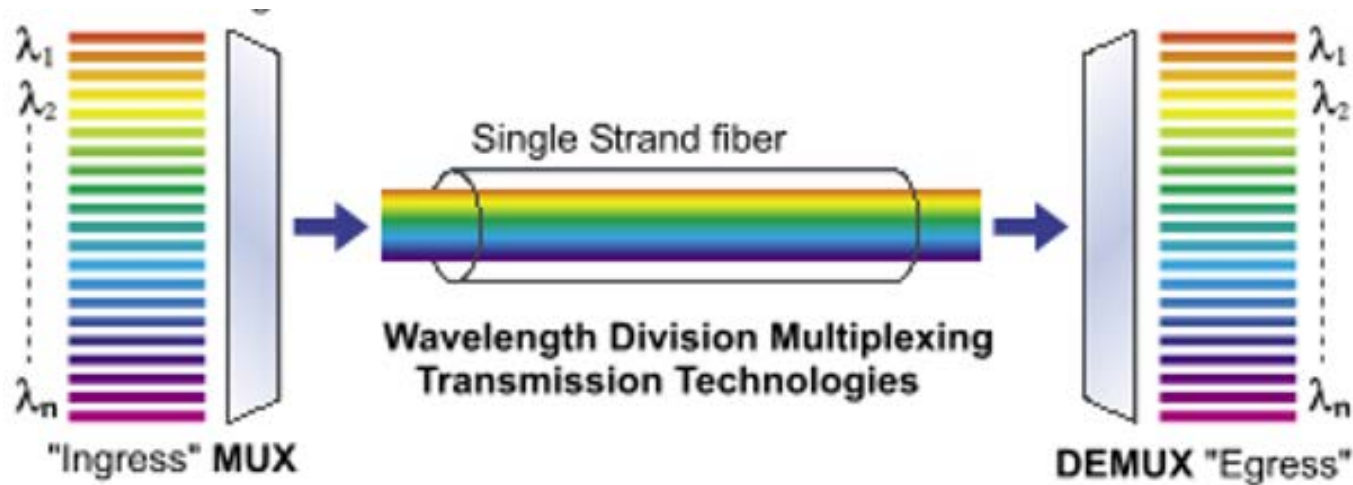


Optical frequency??



F. Flamini, *Reports on Progress in Physics* 82.1 (2018): 016001.

What About Encoding & Entanglement in Optical Frequency?



- Frequencies are robust and compatible with transmission over fibers, but only recently becoming popular for quantum*
- Potential for high dimensionality – processing with *qudits* – more information per photon
- Ability to perform routing based on optical frequency
- Manipulation in parallel in frequency domain
- Chip-scale microresonator sources
 - Naturally generate high dimensional photon entanglement in a single-spatial mode
 - Compatible with photonic integration
- Prospects for hyper-entanglement with frequency and time

*Recent review: Kues, Reimer, Lukens, Munro, Weiner, Moss, and Morandotti, "Quantum optical microcombs," Nature Photonics **13**, 170 (2019)

Some Classical Background: Femtosecond Pulse Shaping

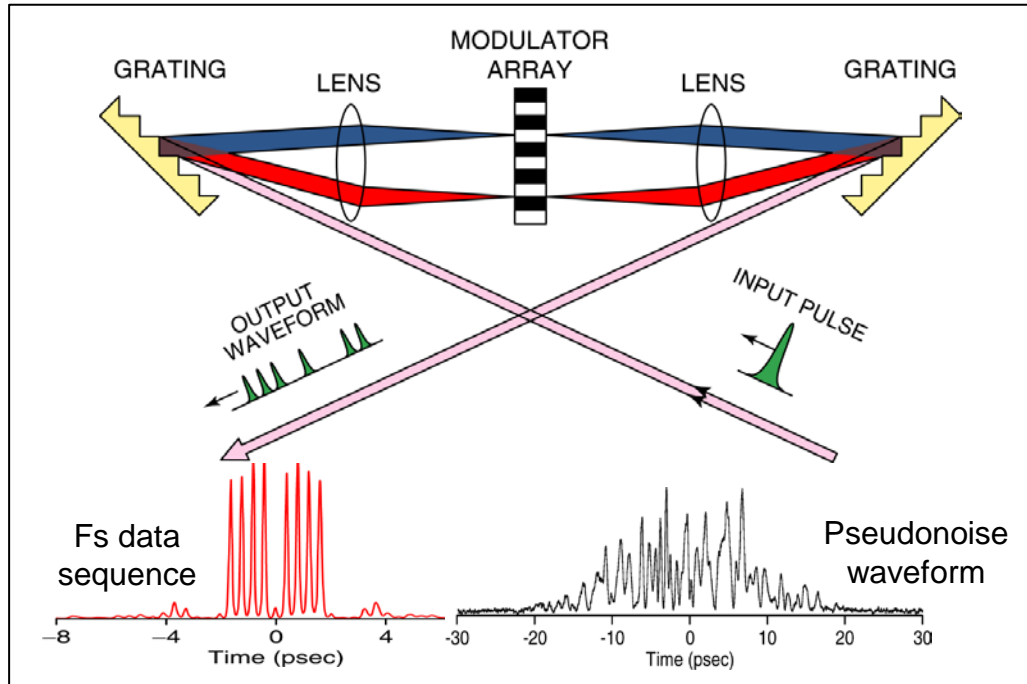
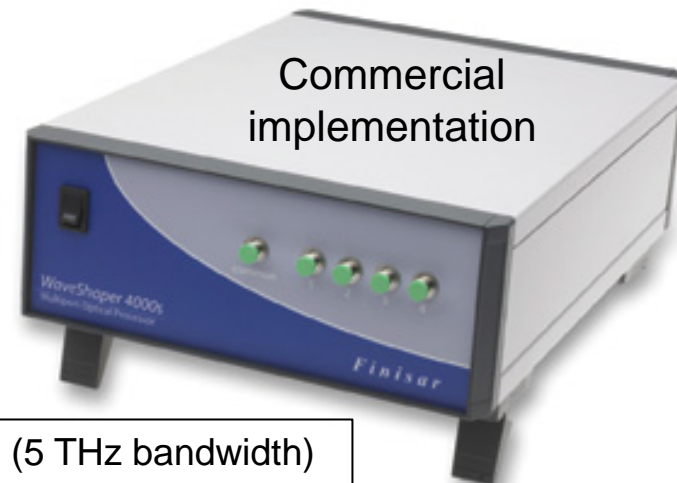
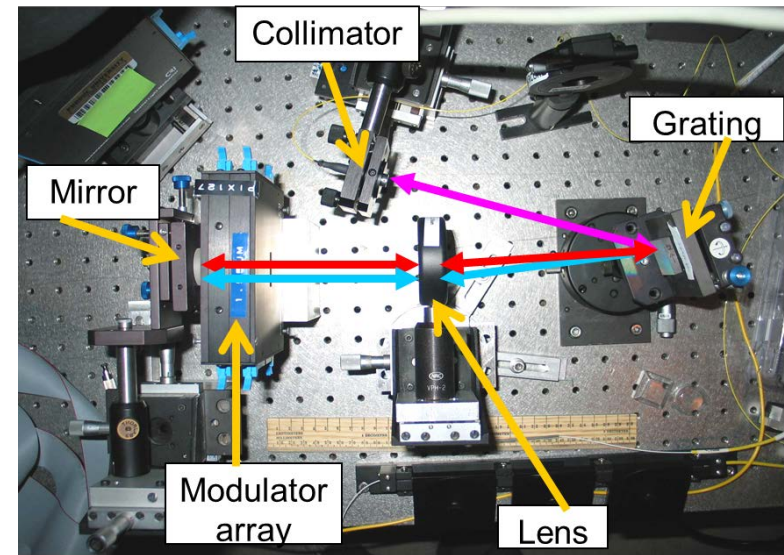


Table top setup

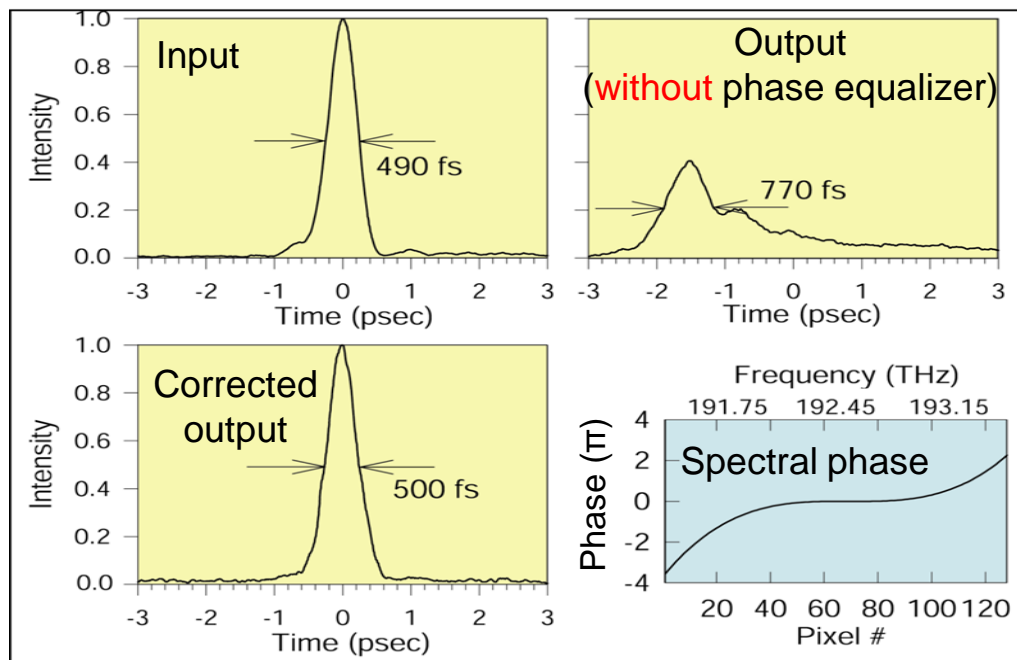
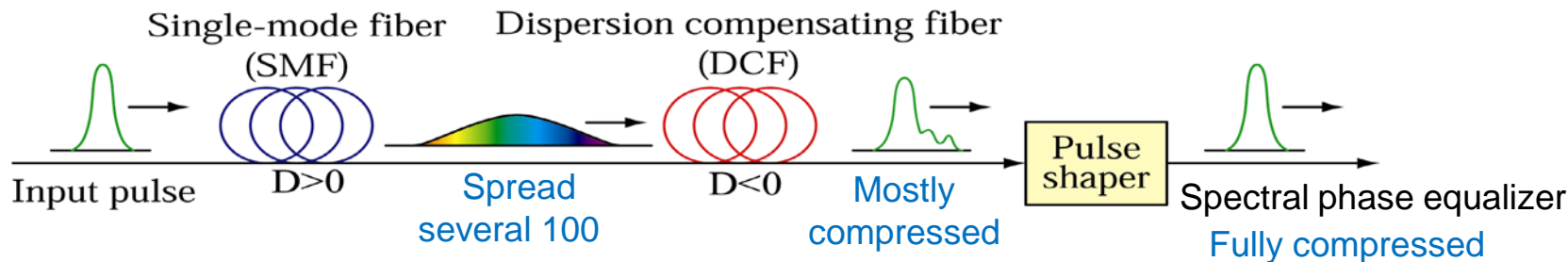


- **Fourier synthesis** via parallel spatial/spectral modulation
- **Full programmability** for user-defined waveform generation
- **Diverse applications:** fiber communications & ultrabroadband radio-frequency photonics to coherent quantum control

Here simply a programmable, arbitrary amplitude & phase filter

- 1530-1570 nm (5 THz bandwidth)
- 10 GHz resolution
- 500 resolvable control elements

Programmable Fiber Dispersion Compensation Using a Pulse Shaper: Subpicosecond Pulses



Higher-order dispersion compensation using a **pulse shaper** as a **programmable spectral phase equalizer**

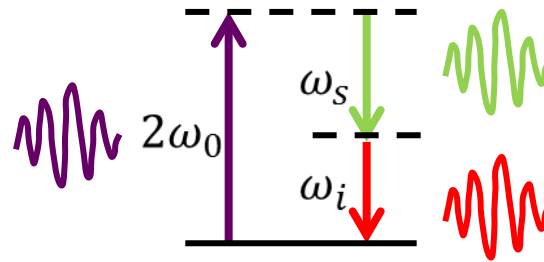
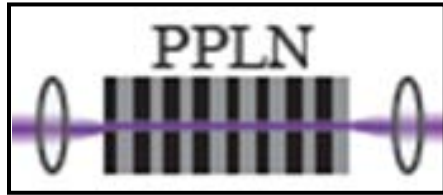
$$\tau(\omega) = \frac{-\partial\psi(\omega)}{\partial\omega}$$

Frequency dependent delay \longleftrightarrow Slope of frequency-dependent phase

Frequency Bin Entanglement

Time-Frequency Entangled Photons (Biphotons)

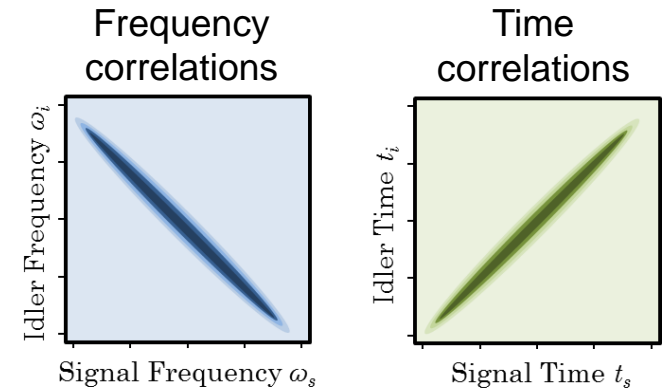
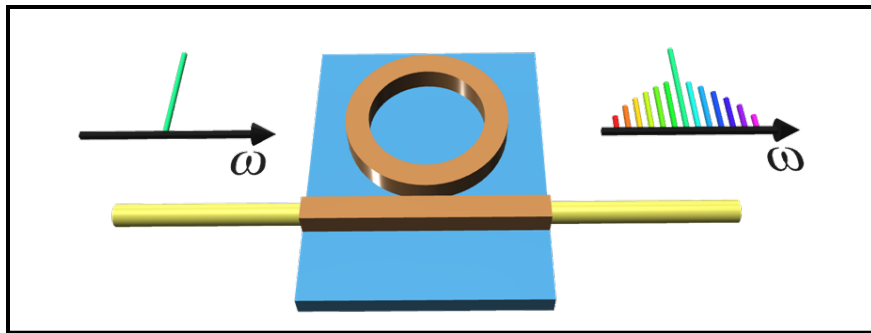
Spontaneous parametric down-conversion (SPDC)



$$|\Psi\rangle = \int d\Omega \phi(\Omega) |\omega_0 + \Omega\rangle_s |\omega_0 - \Omega\rangle_i$$

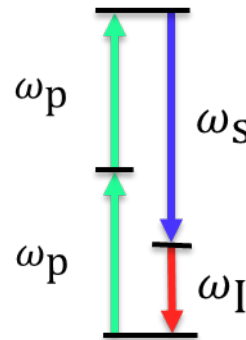
- Broadband, continuous spectrum (>5 THz) – *but can be filtered to form a discrete spectrum*

Spontaneous four-wave mixing (SFWM), in a microresonator



$$|\Psi\rangle = \sum_{k=1}^N \alpha_k |k, k\rangle_{SI}$$

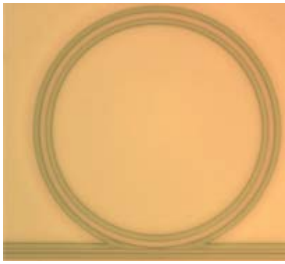
$$|k, k\rangle_{SI} = \int d\Omega \Phi(\Omega - k\Delta\omega) |\omega_p + \Omega, \omega_p - \Omega\rangle_{SI}$$



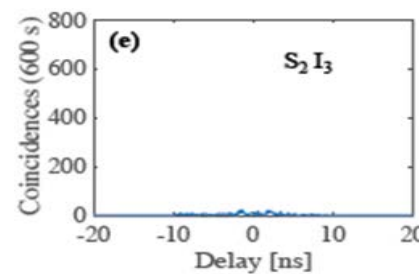
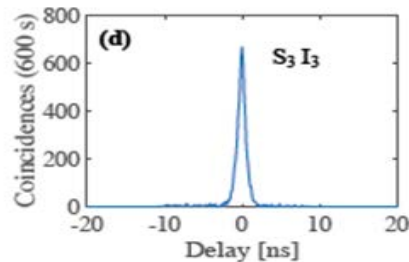
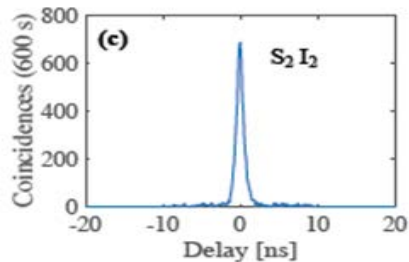
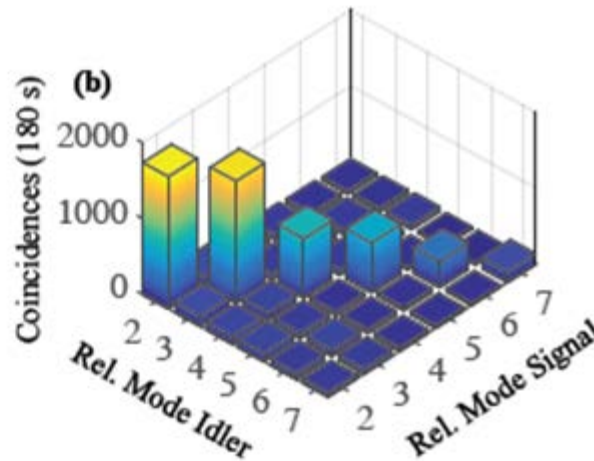
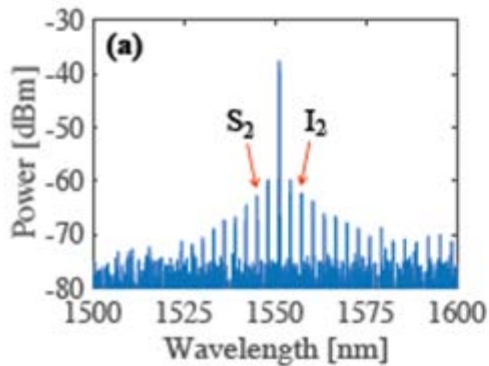
- Biphoton frequency comb
- May be broadband, but made of narrow (discrete) frequency modes
- Frequency bin entanglement: analogies to classical WDM?

Biphoton Frequency Combs (via microring resonators)

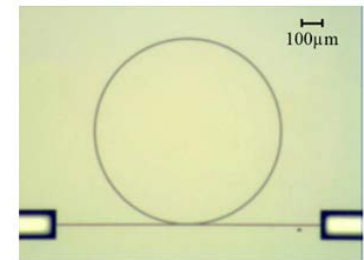
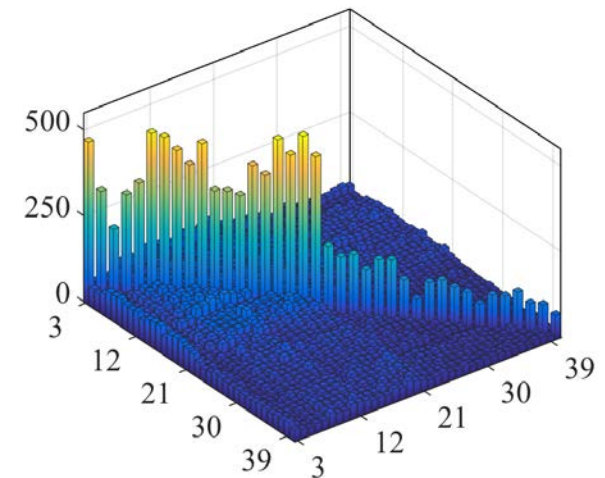
Potential for very high dimensional entanglement



Small microring (~ 380 GHz FSR)



Larger microring (~ 50 GHz FSR)
Strong correlations out to 40th line pair!



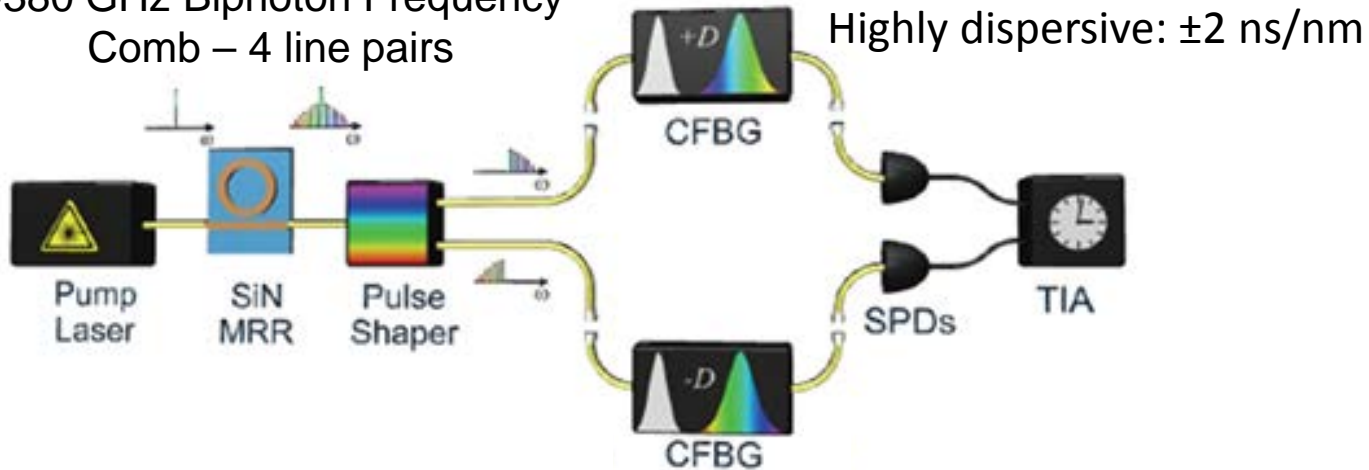
Imany, et al, Opt. Exp. **26**,
1825 (2018)

Jaramillo, et al, Optica **4**, 655 (2017)

Nonlocal Dispersion Compensation

Frequency-to-time mapping of biphoton combs

~380 GHz Biphoton Frequency
Comb – 4 line pairs

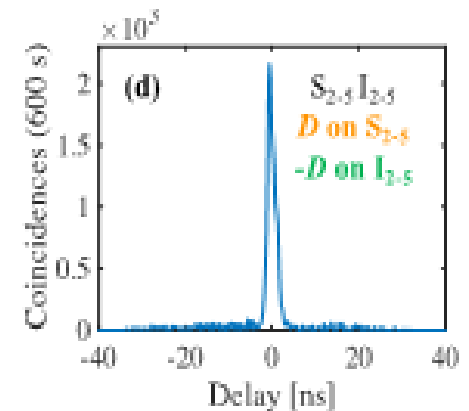
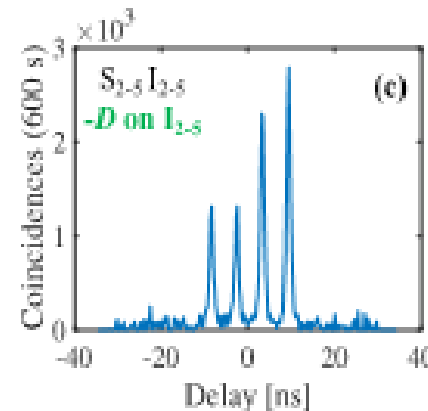
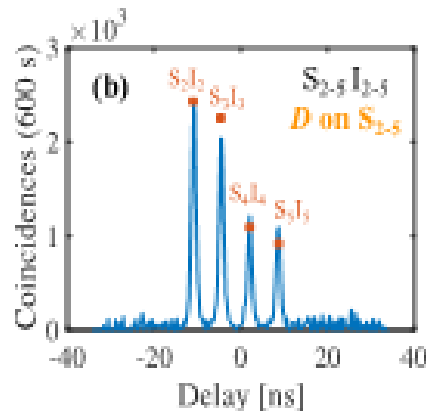
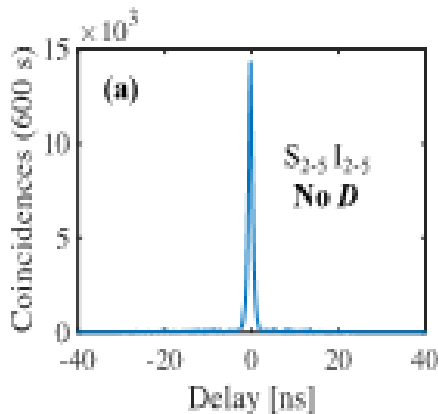


Undispersed

Signal dispersed

Idler dispersed

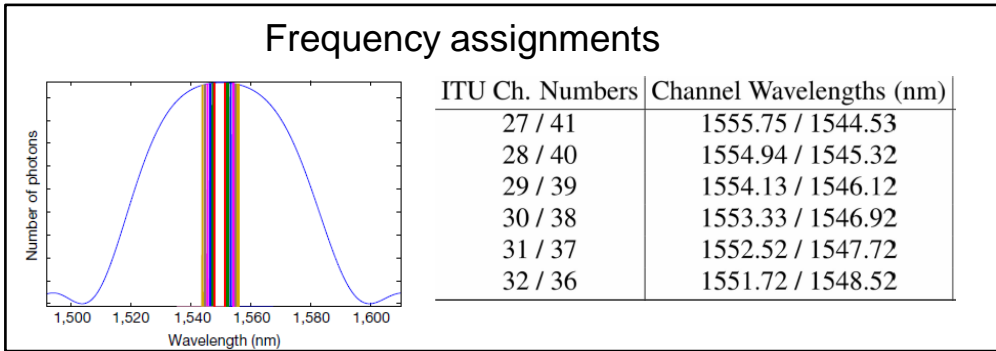
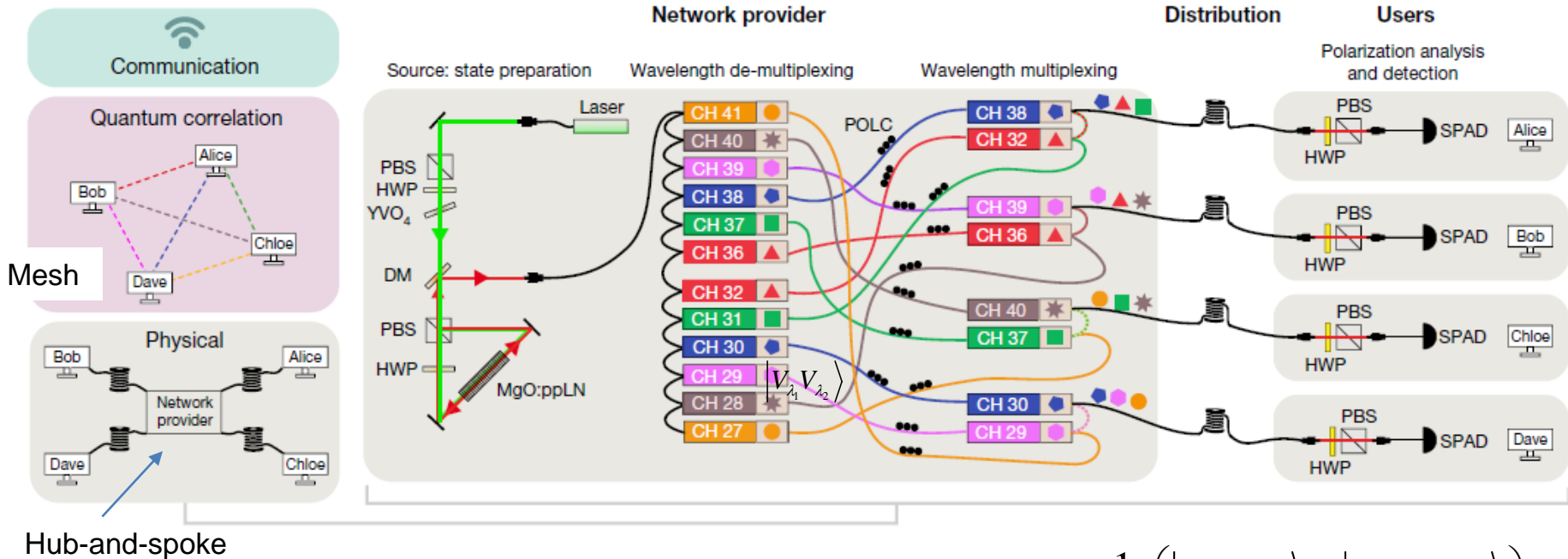
Both dispersed:
compensation



An entanglement-based wavelength-multiplexed quantum communication network

Sören Wengerowsky^{1,2*}, Siddarth Koduru Joshi^{1,2,4}, Fabian Steinlechner^{1,2,5,6}, Hannes Hübel³ & Rupert Ursin^{1,2*}

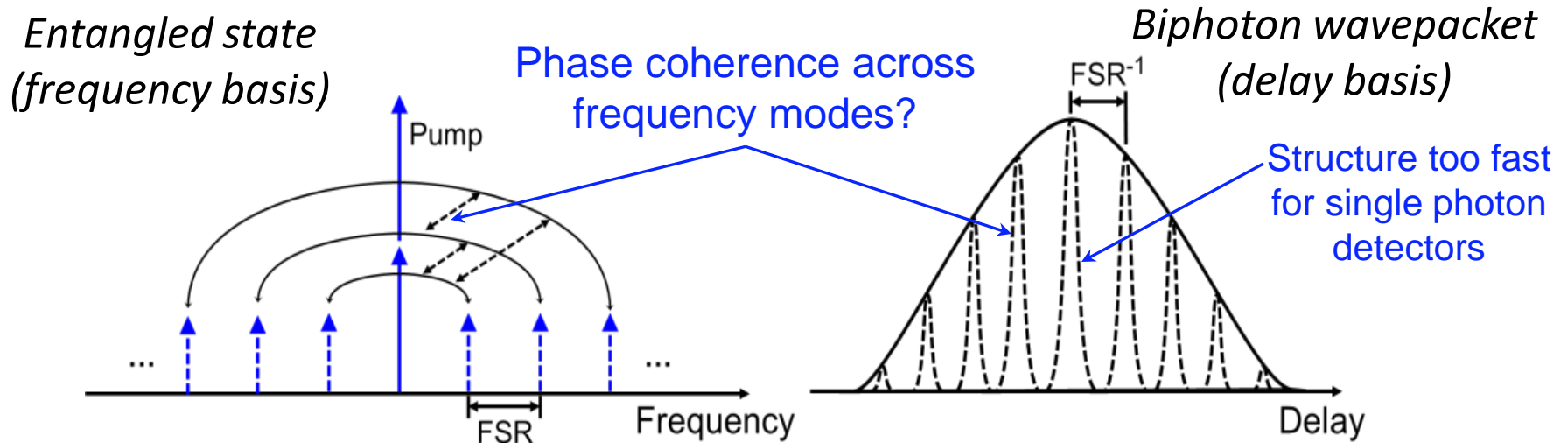
13 DECEMBER 2018 | VOL 564 | NATURE | 225



$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} \left(|V_{\lambda_1} V_{\lambda_2}\rangle + |H_{\lambda_1} H_{\lambda_2}\rangle \right)$$

- Carves optical frequency channels from a broadband down-conversion source for pair-wise distribution of polarization entanglement between 4 users
- Exploits frequency correlations, but NOT frequency bin entanglement or coherence

Biphoton Frequency Comb Entangled State



Complex amplitude (both magnitude and phase)

How to prove the phase coherence for frequency-bin entangled photons?

Sum over frequency bins

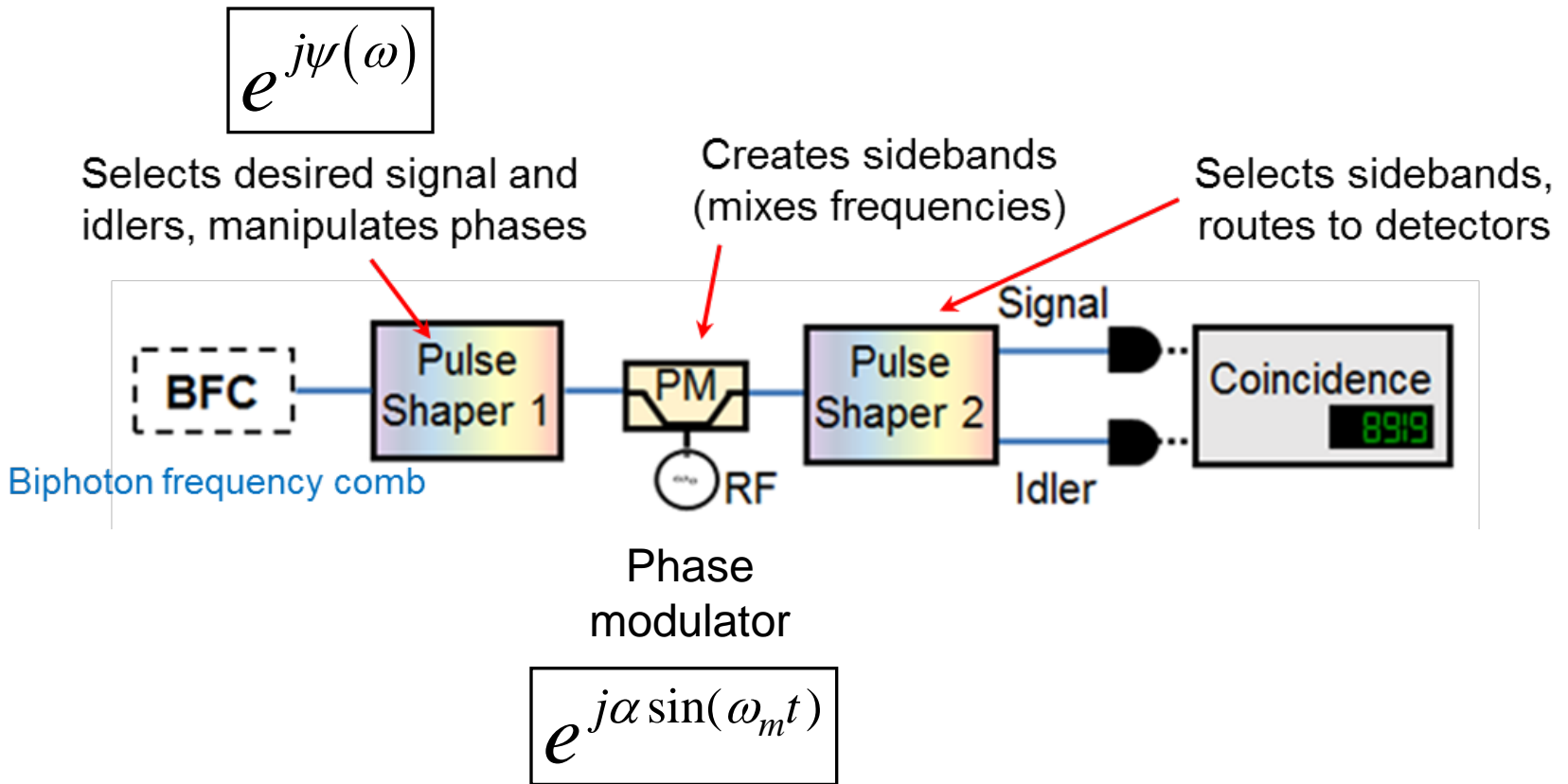
$$|\Psi\rangle = \sum_{k=1}^N \alpha_k |k, k\rangle_{SI}$$

$$|k, k\rangle_{SI} = \int d\Omega \Phi(\Omega - k\Delta\omega) |\omega_p + \Omega, \omega_p - \Omega\rangle_{SI}$$

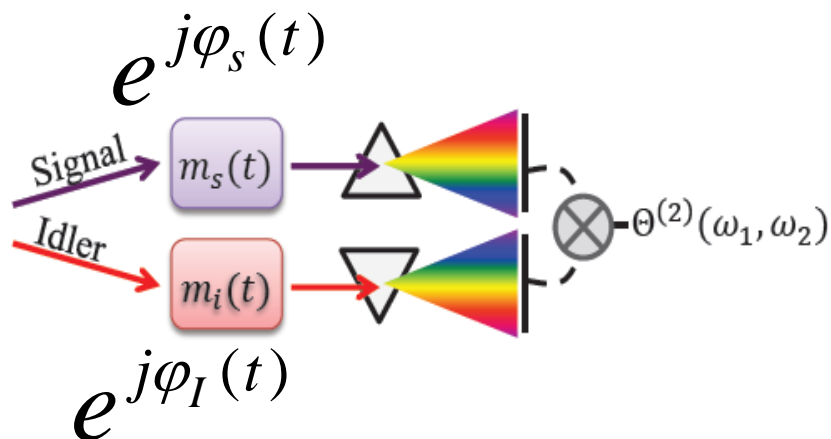
Lineshape function

How to Prove Frequency Bin Entanglement?

Use a phase modulator to project a single frequency into multiple sidebands

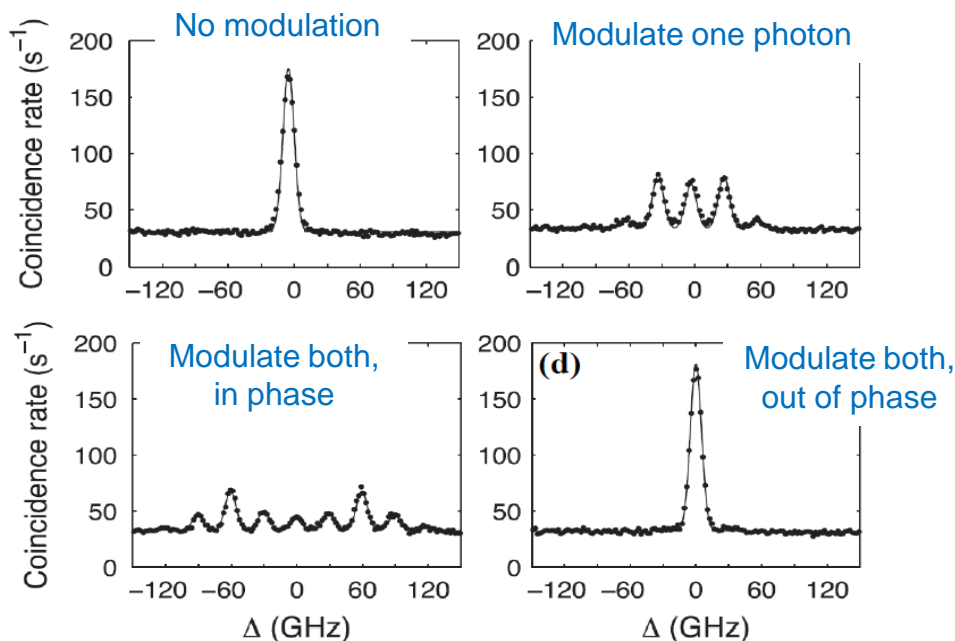


Phase Modulation Applied to Frequency Entanglement



Manipulating frequency correlations

Nonlocal modulation compensation – a frequency dual of nonlocal dispersion compensation



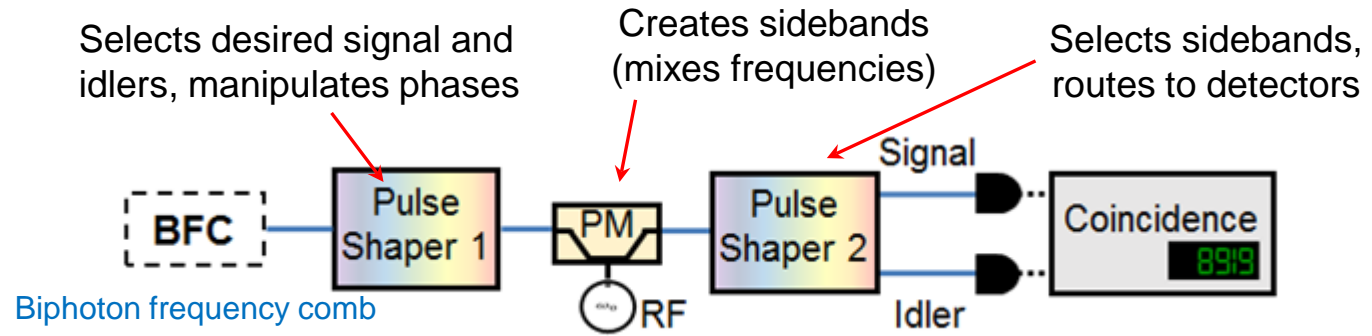
How to Prove Frequency Bin Entanglement?

Use a phase modulator to project a single frequency into multiple sidebands

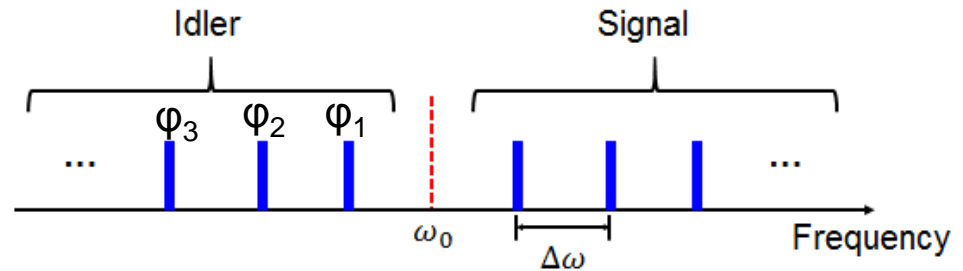
How to prove stable (coherent) phase?

$$|\Psi\rangle = \sum_{k=1}^N \alpha_k |k, k\rangle_{SI}$$

Complex amplitude (both magnitude and phase)

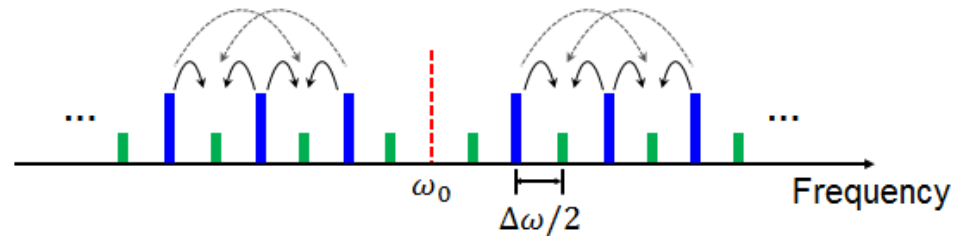


Start with BFC:
& sweep spectral phase

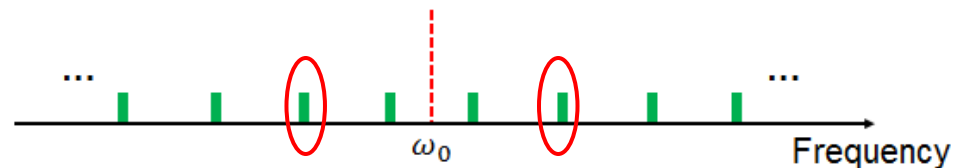


After phase modulation:

$$(\omega_m = \frac{\Delta\omega}{2})$$



Pick overlapped sidebands:

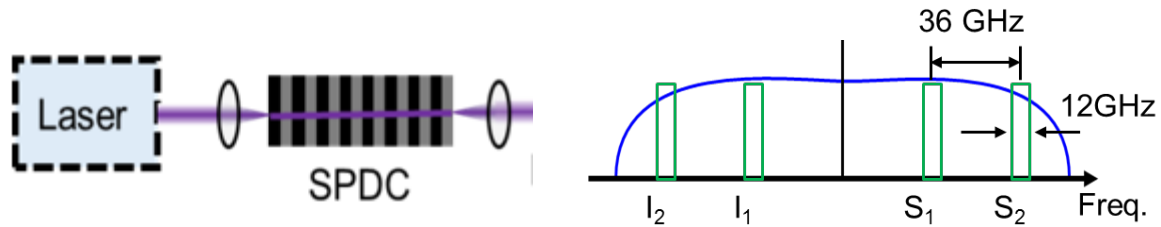


Interference will be observable even with slow single-photon detectors

Imany, et al, Opt. Exp. **26**, 1825 (2018), Phys. Rev. A **97**, 103813 (2018)

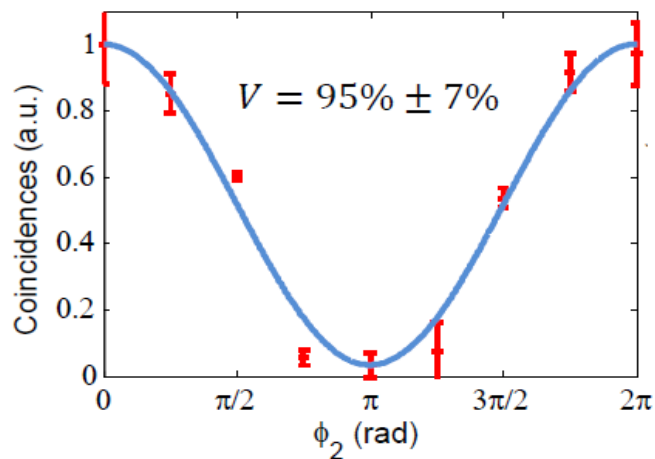
See also: Kues et al, Nature **546** (2017) (INRS)

Two Dimensional Frequency Bin Entanglement

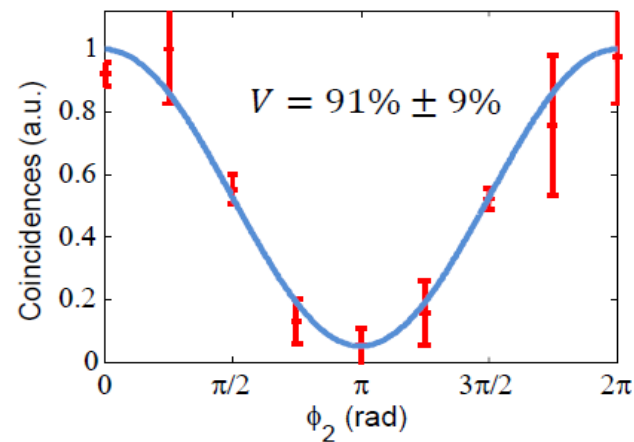


Classical Limit
 Visibility $< \frac{1}{\sqrt{2}}$

S_{12}, I_{12} : $|\psi\rangle = |SI\rangle_1 + e^{i\phi_2} |SI\rangle_2$

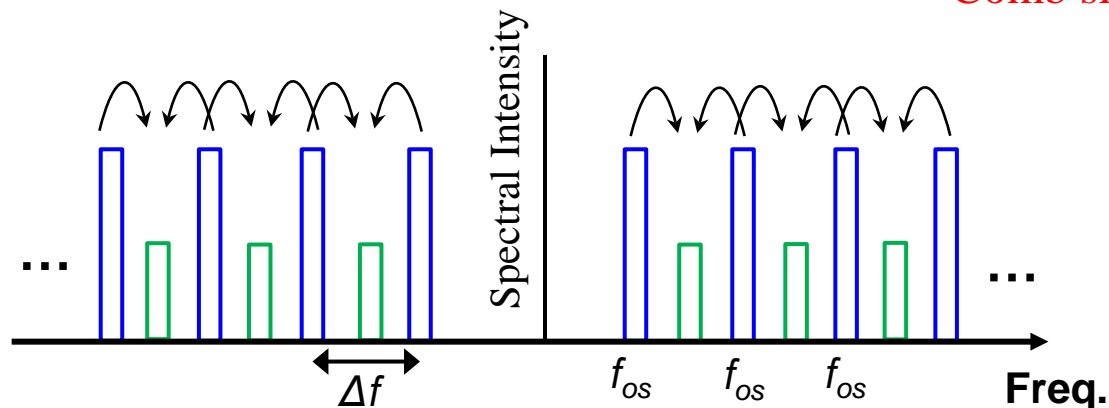


S_{23}, I_{23} : $|\psi\rangle = e^{i\phi_2} |SI\rangle_2 + |SI\rangle_3$



Dispersion Measurement with Biphoton Frequency Comb

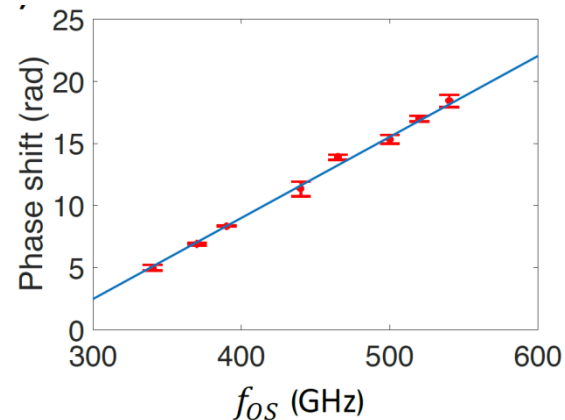
Comb sliced from broadband SPDC spectrum



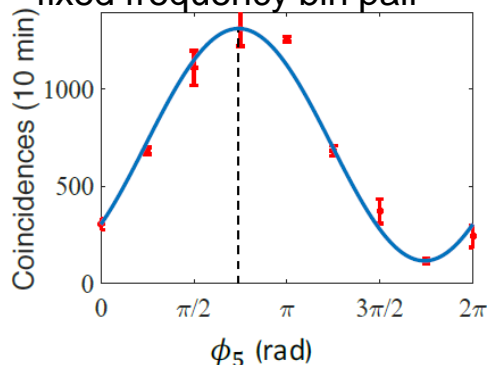
Imany, Odele, et al,
Phys. Rev. A 97, 103813 (2018)

Phase shift due to fiber dispersion

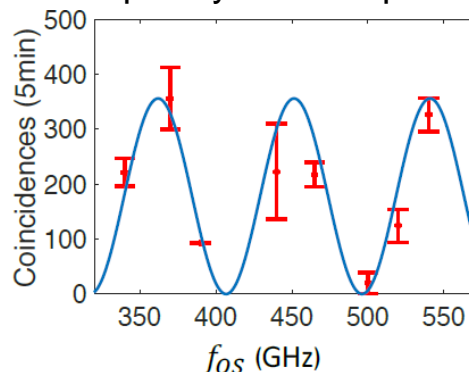
$$\phi_{shift} = -(2\pi)^2 \beta_2 L \Delta f (2f_{os} + \Delta f)$$



Coincidences vs. phase for fixed frequency bin pair



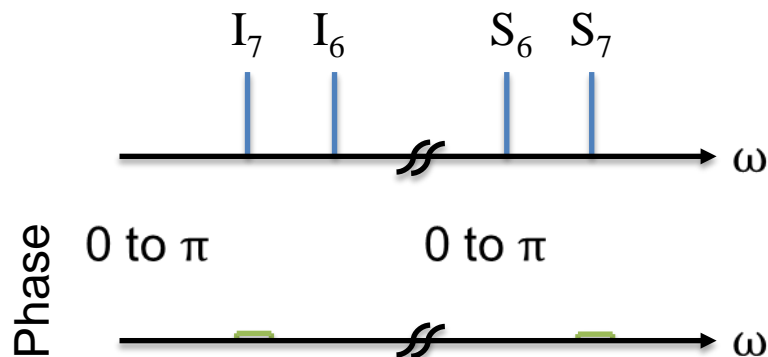
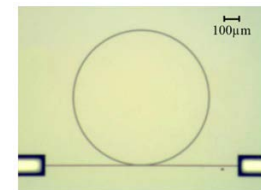
Coincidences vs. frequency for fixed phase



Slope gives $\beta_2 = -2.03 \times 10^{-2} \text{ ps}^2/\text{m}$
Expected value of $\beta_2 = -2.06 \times 10^{-2} \text{ ps}^2/\text{m}$

Testing Frequency Bin Entanglement

50 GHz FSR microring resonator



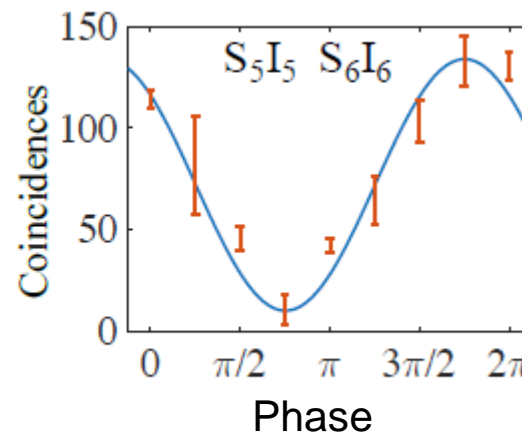
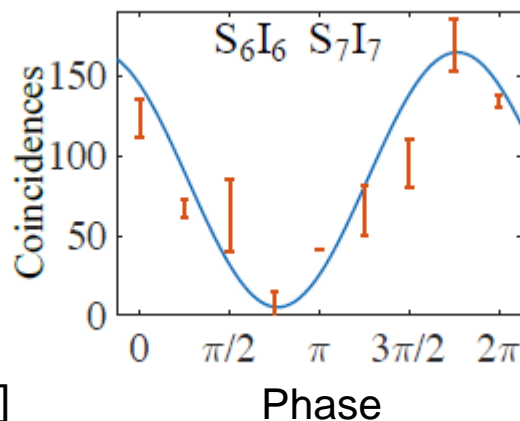
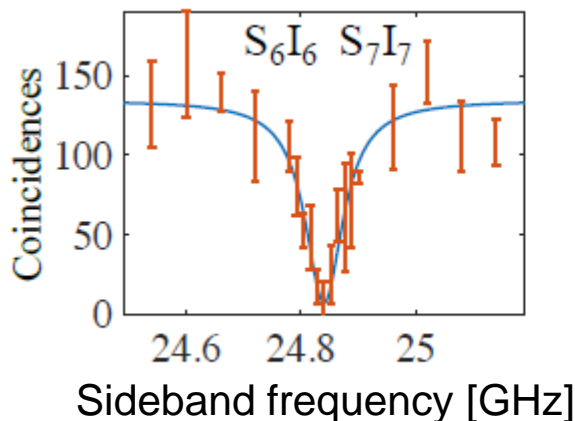
Classical Limit
 Visibility $< \frac{1}{\sqrt{2}}$

R. T. Thew, *et al.*
Phys. Rev. Lett. **93** (2004)

Tuning RF frequency
 under destructive
 2-photon interference

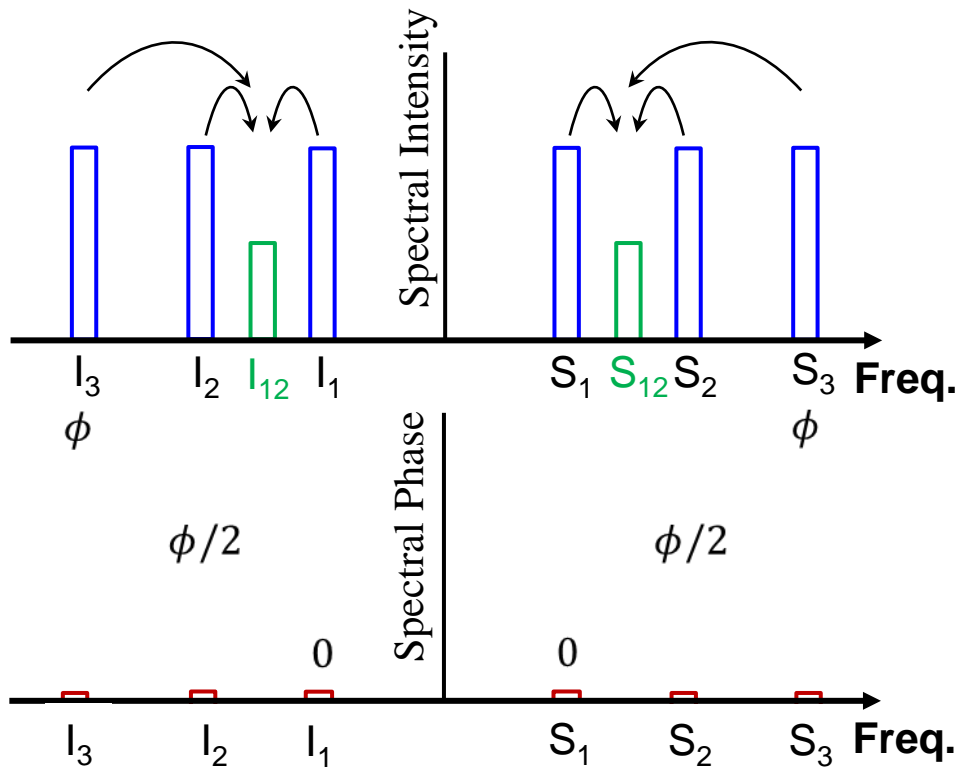
Visibility: $93\% \pm 13\%$

Visibility: $86\% \pm 11\%$



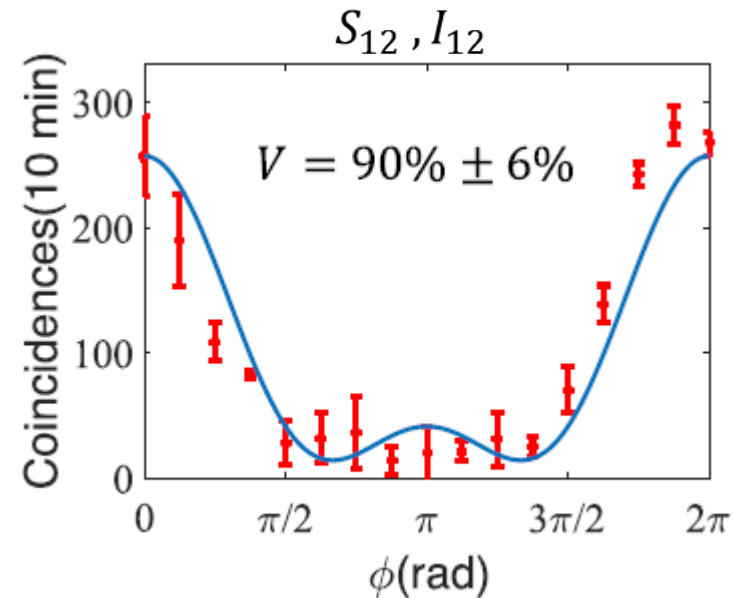
Three-Dimensional Frequency Bin Entanglement [SPDC]

Selecting a symmetric set of phase modulation sidebands



$$|\psi\rangle = |SI\rangle_1 + e^{i\phi} |SI\rangle_2 + e^{i2\phi} |SI\rangle_3$$

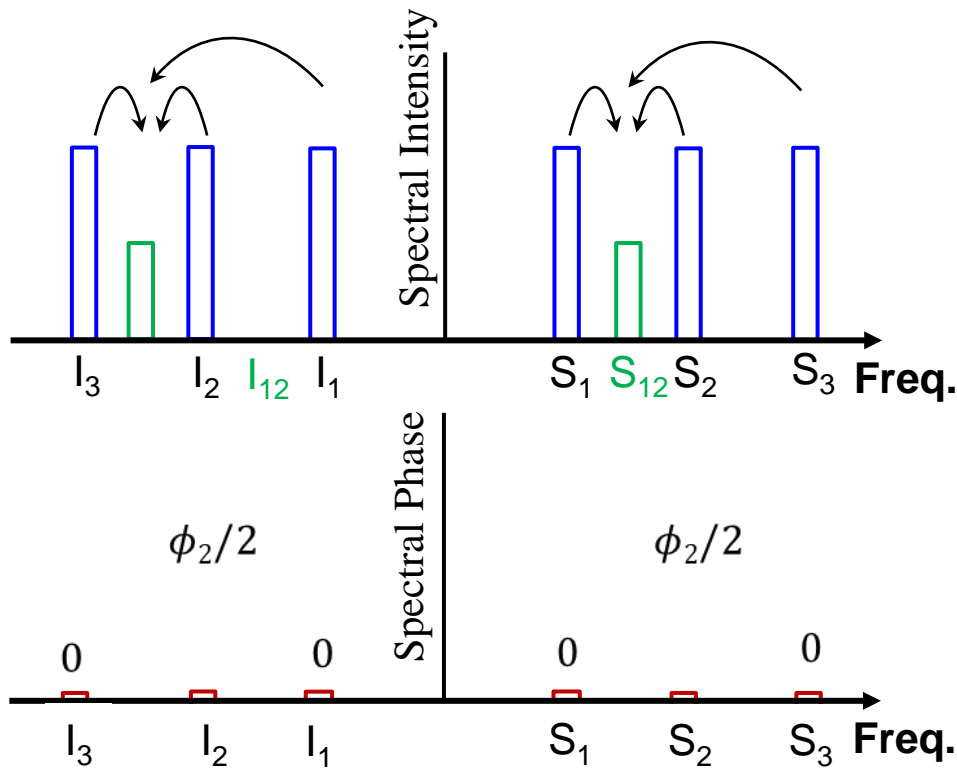
- Equalized signal & idler lines
- Equal PM contributions in each sideband
 - Linear phase



This interference pattern gives us evidence of phase coherence (entanglement) between all three comb line pairs simultaneously

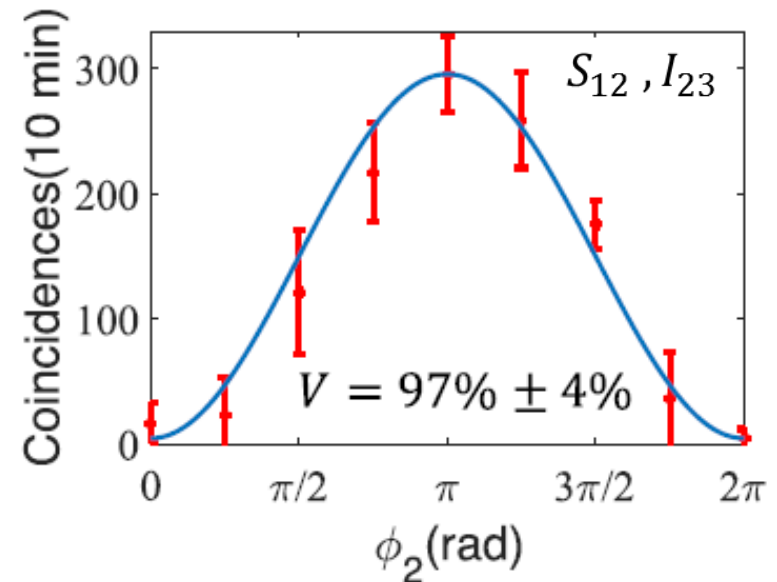
Three-Dimensional Frequency Bin Entanglement [SPDC]

Another example, with asymmetric set of phase modulation sidebands



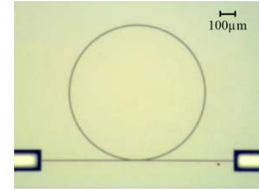
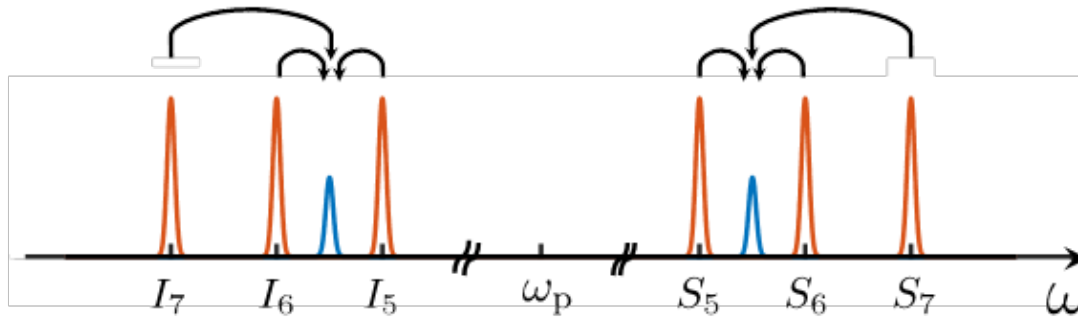
$$|\psi\rangle = \frac{1}{2} |SI\rangle_1 - \exp(i\phi_2) |SI\rangle_2 + \frac{1}{2} |SI\rangle_3$$

- Equalized signal & idler lines
- 3rd sideband = $\frac{1}{2}$ 1st sideband
 - Phase on $S_2 I_2$ only



Illustrates flexible control of two-photon quantum interference in three frequency dimensions

Three-Dimensional Frequency Bin Entanglement [Microring]



8 measurements with different specific spectral phase settings

CGLMP inequality (3D Bell inequality)

$$I_3 = 3 [P^{11}(0,0) + P^{21}(0,1) + P^{22}(0,0) + P^{12}(0,0)] - 3 [P^{11}(0,1) + P^{21}(0,0) + P^{22}(0,1) + P^{12}(1,0)] \leq 2$$

Classical upper bound

Term	Coincidences	Term	Coincidences

$$P^{a,b}(x,y) = \frac{\text{Coinc.}}{\text{Max coinc.}}$$

Max coinc. = 160 ± 18

$I_3 = 2.63 \pm 0.2$ (sufficient to establish qutrit entanglement)

R. Thew, et al, *Phys. Rev. Lett.* **93**, 010503 (2004);

C. Bernhard, et al, *J. Phys. A: Math. Theor.* **47**, 424013 (2014).

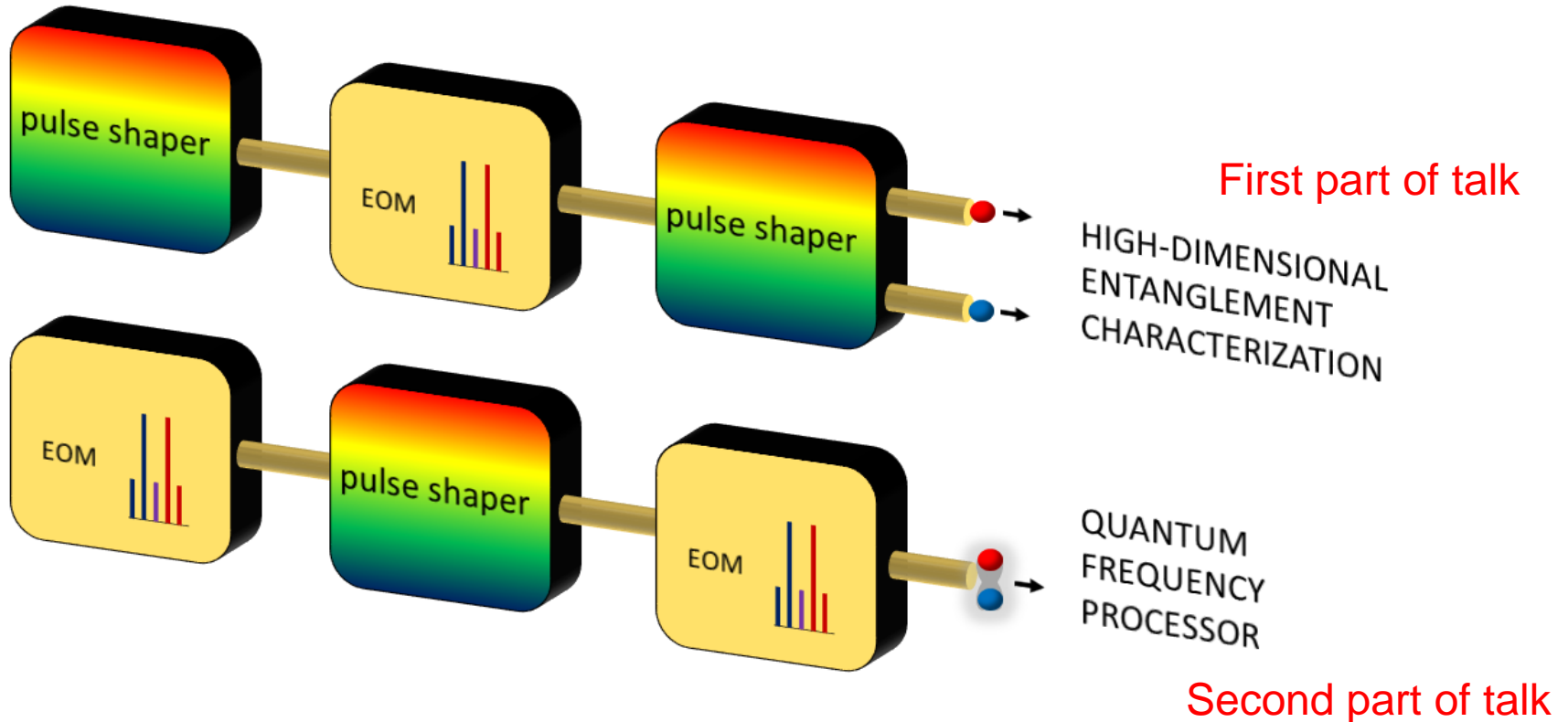
Imany, et al, *Opt. Exp.* **26**, 1825 (2018)

Manipulating Frequency Encoded Photons

(Gates)

Quantum state manipulation for frequency encoded photons

An Interesting Duality: Same components, different order for entanglement characterization & state manipulation



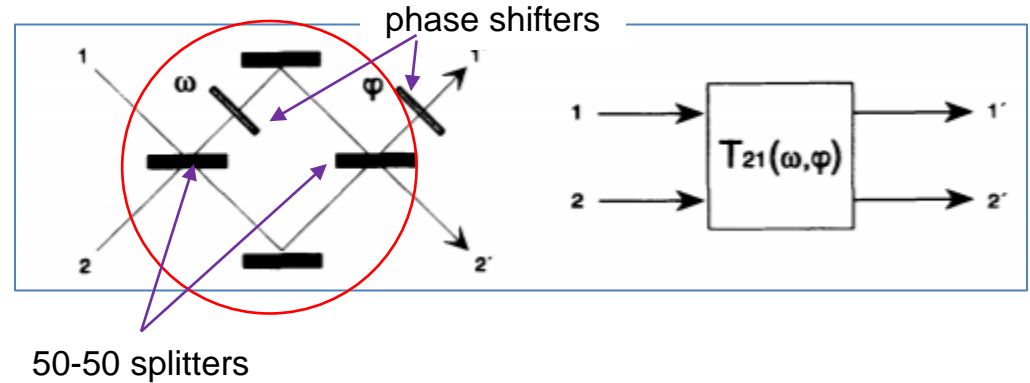
Decomposition of $N \times N$ Unitary Operators into 2×2 Variable Beamsplitters

Basis for integrated photonic quantum chips based on path encoding

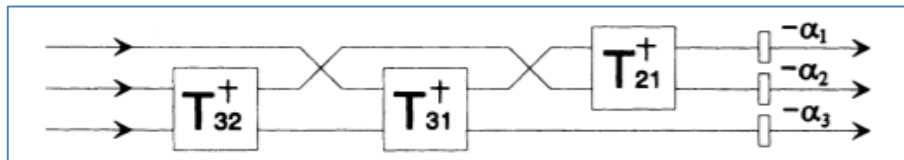
2×2 unitary: variable beam splitter + phase shifter

$$\begin{pmatrix} k'_1 \\ k'_2 \end{pmatrix} = \begin{pmatrix} e^{i\phi} \sin \omega & e^{i\phi} \cos \omega \\ \cos \omega & -\sin \omega \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

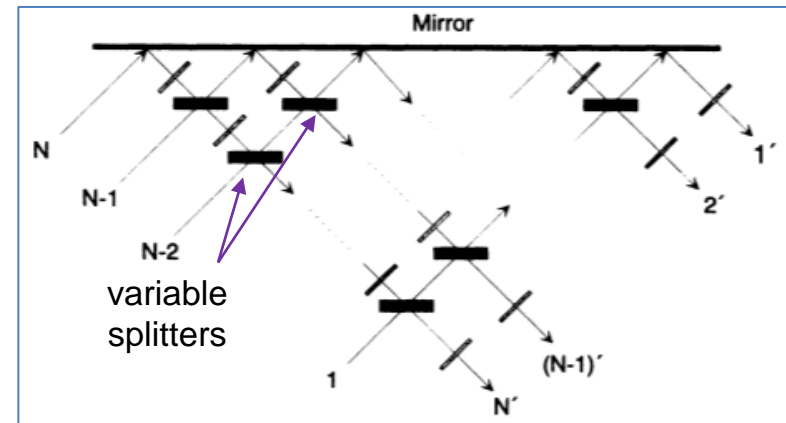
Implementation using Mach-Zehnder interferometer



3×3 unitary: constructed from $3 \times 2 \times 2$'s



$N \times N$ construction

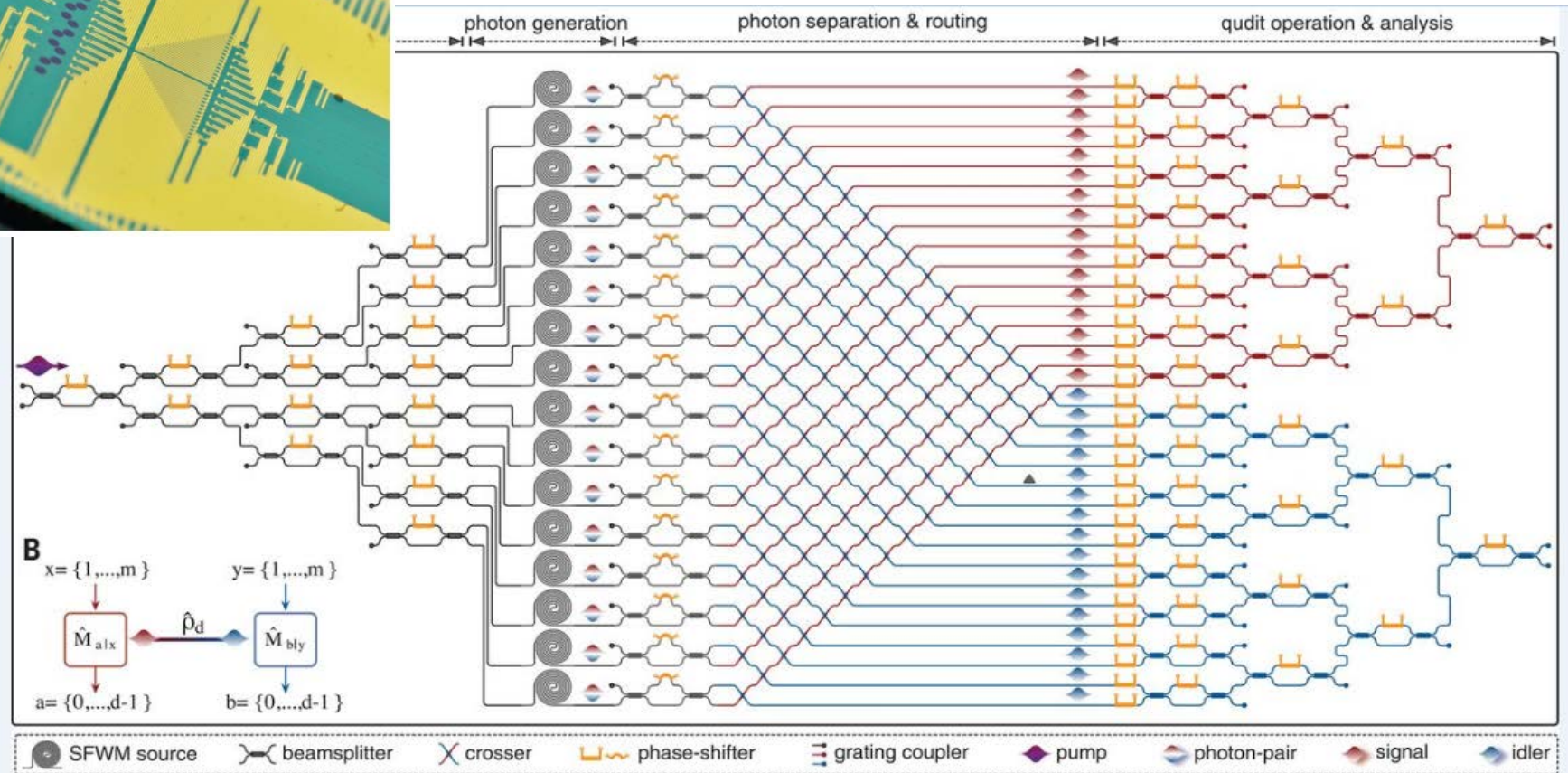
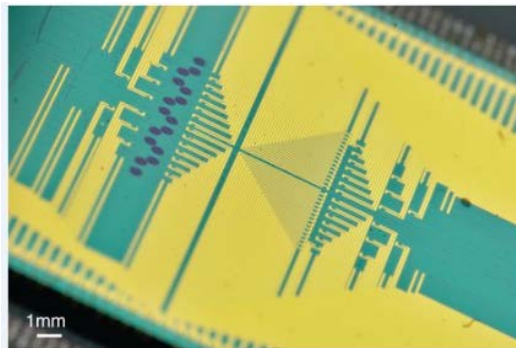


- Algorithmic decomposition of $N \times N$ matrix into at most $N(N-1)/2$ 2×2 's

Multidimensional quantum entanglement with large-scale integrated optics

J. Wang, S. Paesani et al, Science **360**, 285 (2018)

- State-of-the art in multi-dimensional QIP: programmable two-party entanglement up to dimensionality of 15×15
- Realized in a silicon photonic chip with >500 photonic components
- Path encoding: compatible with on-chip implementations, but not fiber optic transmission



Gate Construction for Frequency-Encoded Qubits (Qudits)

Based on alternating phase control in time and frequency

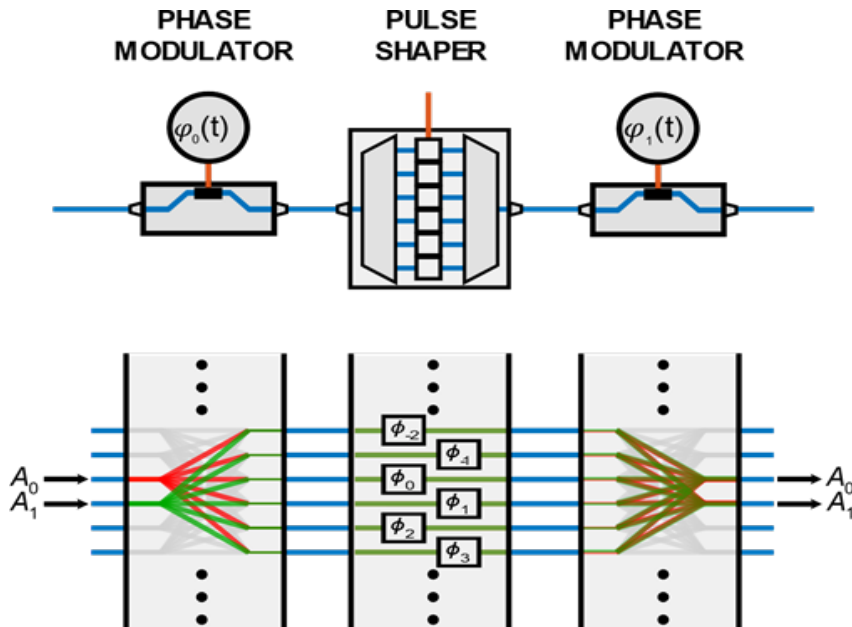
$$V = F \tilde{D}_R F^\dagger D_R \cdots F \tilde{D}_2 F^\dagger D_2 F \tilde{D}_1 F^\dagger D_1$$

F, F^\dagger : Fourier transforms

Diagonal phase in frequency (pointing to $\tilde{D}_R, \tilde{D}_2, \tilde{D}_1$)
 Diagonal phase in time (pointing to D_R, D_2, D_1)

Two phase modulator, one pulse shaper example

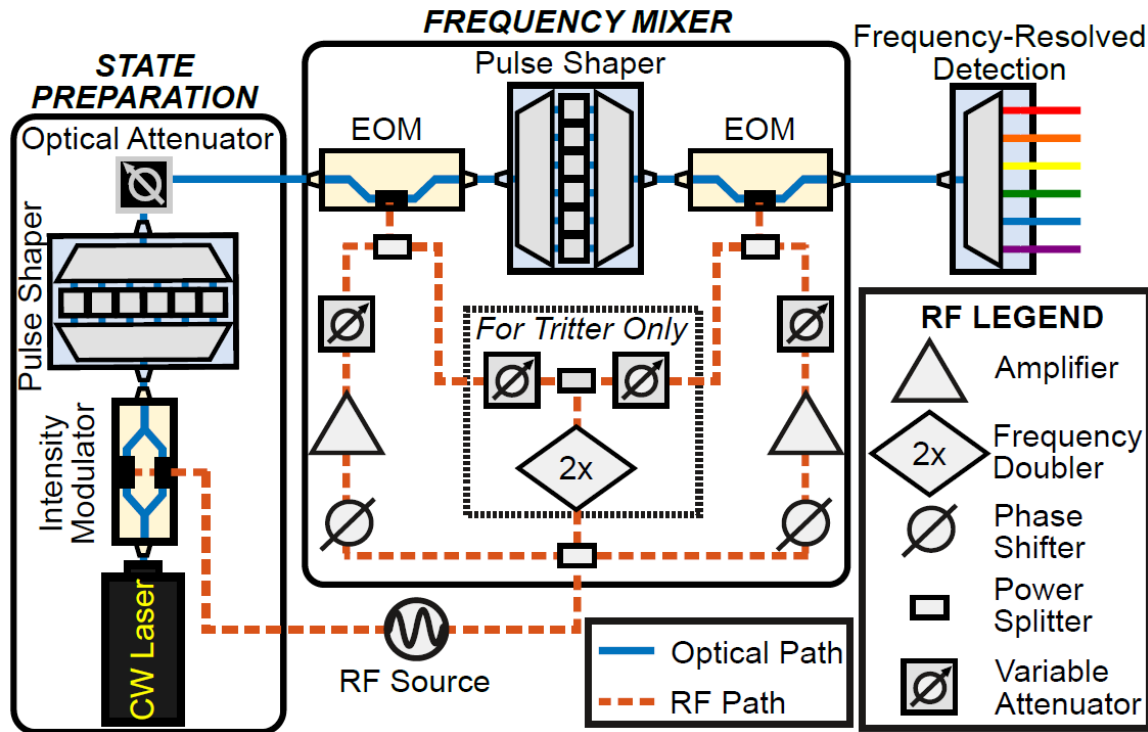
Lukens and Lougovski, *Optica* **4**, 8 (2017)



- Phase modulators and pulse shapers for diagonal phase in time and frequency, respectively
- Phase functions designed via optimization approach; algorithmic decomposition unknown
- May enable greater functionality for given number of elements with more complex control signals
- Practical considerations constrain number of RF harmonics and physical elements

High-Fidelity Gates for Frequency Encoded Photons

Phase modulator – pulse shaper – phase modulator frequency mixer architecture



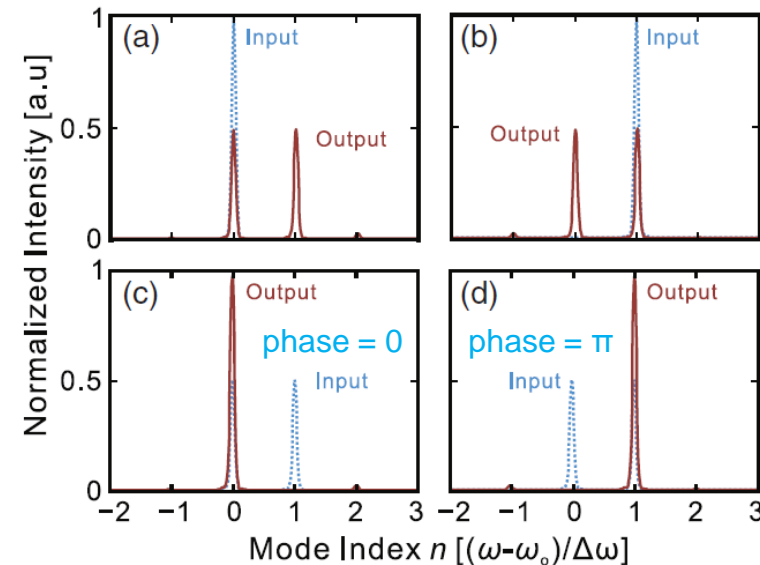
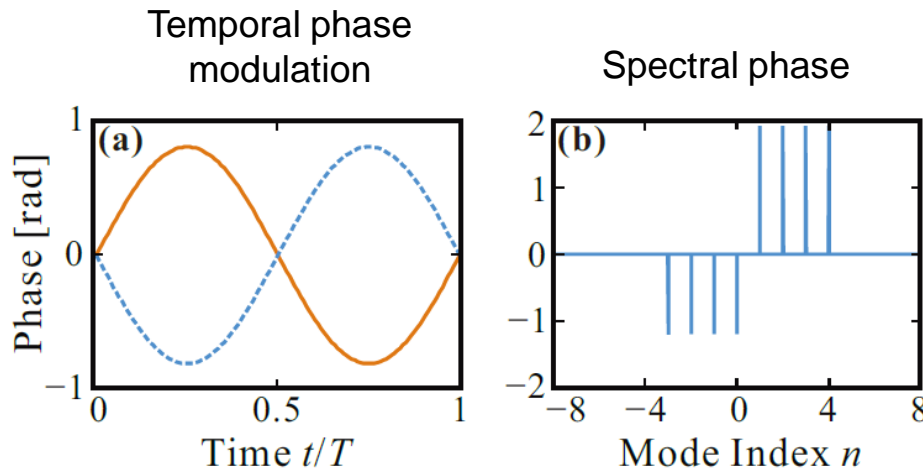
- Temporal and spectral phase waveforms designed for high fidelity and success probability
 - Circumvents “scattering” of frequencies out of the computational space
- Extra RF harmonics or additional cascaded modulators and shapers also possible

High-Fidelity Gates for Quantum Information Processing

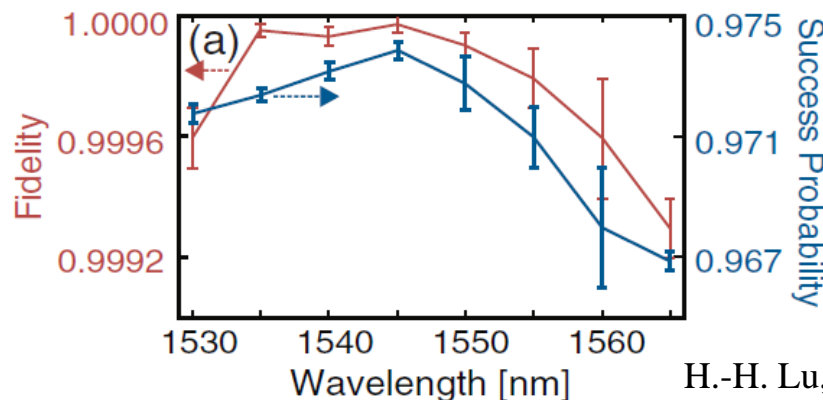
“2 2” coupler (frequency beam splitter) – Hadamard gate

$$U_{2 \times 2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Experiments with classical light;
similar results with weak
coherent states



Capability for parallel operation



Design: $F = 0.9999$, $P = 0.9760$

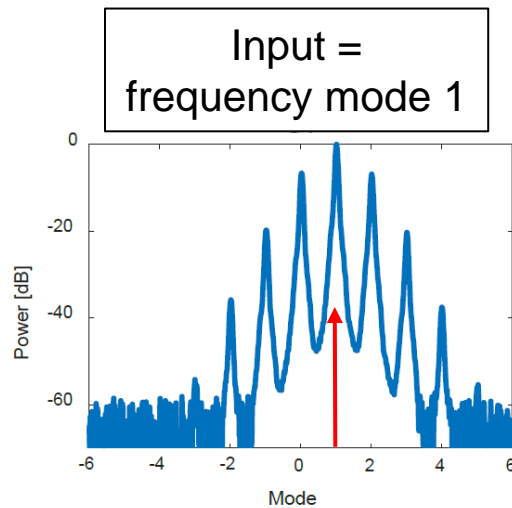
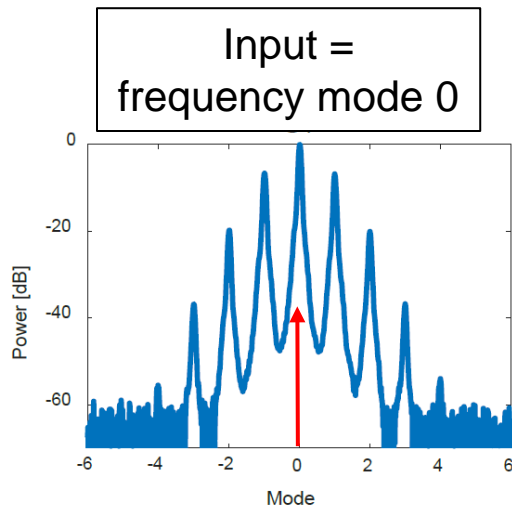
Experiment: $F = 0.9999$, $P = 0.9739$

12.5 dB insertion loss

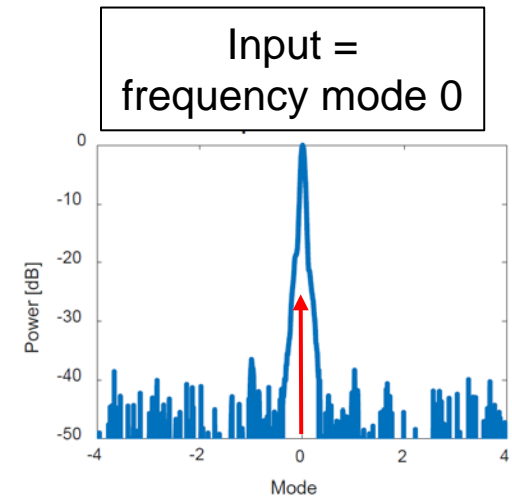
“2 2” coupler (frequency beam splitter)

Further detail on spectral occupancy

Spectra after first phase modulator



Spectrum after modulator – shaper – modulator (shaper phase set to 0)



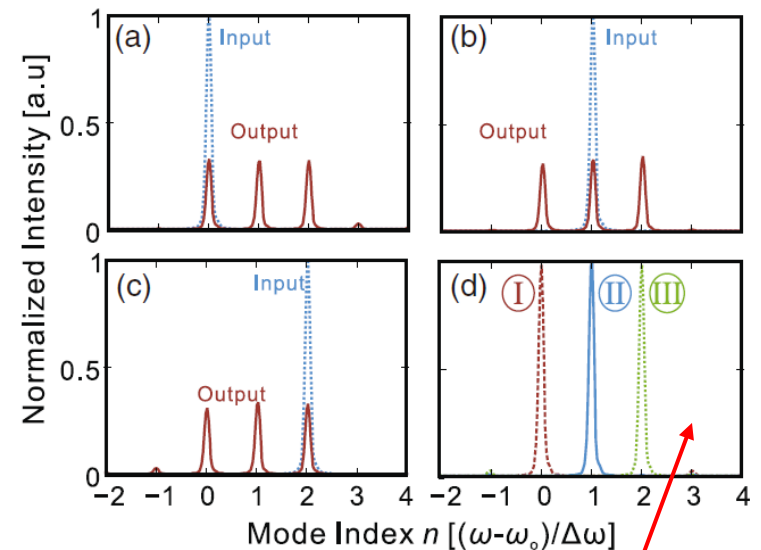
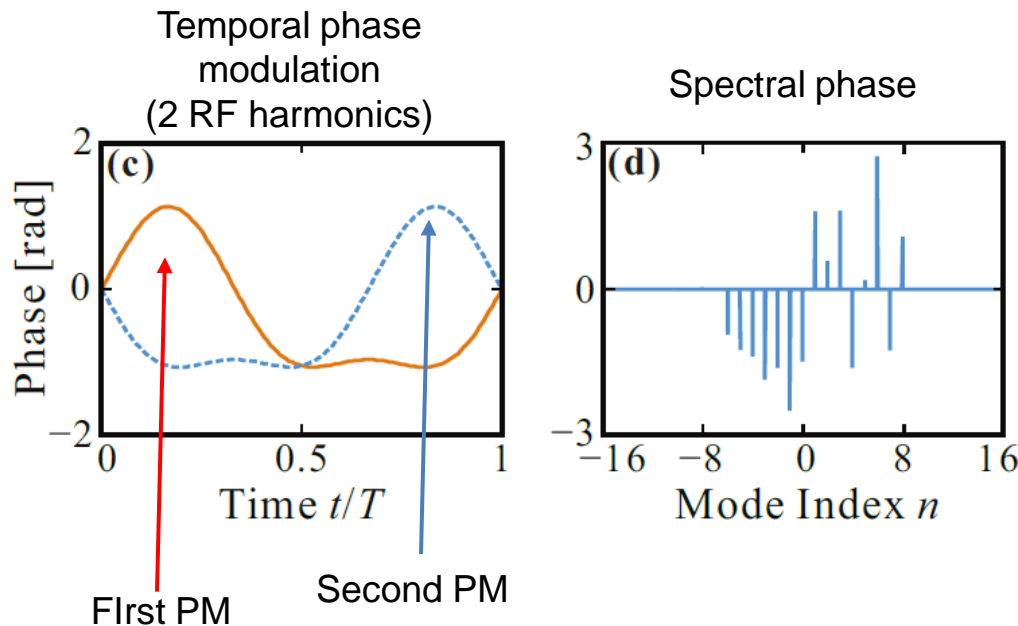
- Beam splitter action turned off depending on pulse shaper phase
- Parallel Hadamard operations possible with 4-frequency-bin guard bands

High-Fidelity Gates for Quantum Information Processing

“3” 3” coupler (frequency tritter) – discrete Fourier transform

$$U_{3 \times 3} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{2\pi i/3} & e^{4\pi i/3} \\ 1 & e^{4\pi i/3} & e^{2\pi i/3} \end{pmatrix}$$

Experiments with classical light;
similar results with weak
coherent states



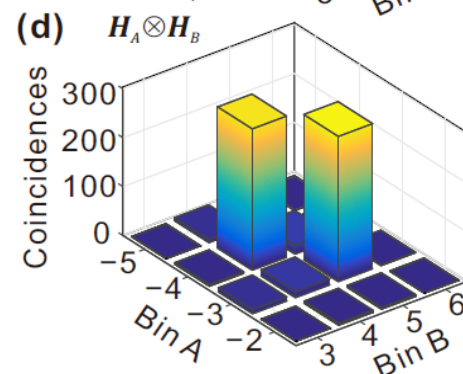
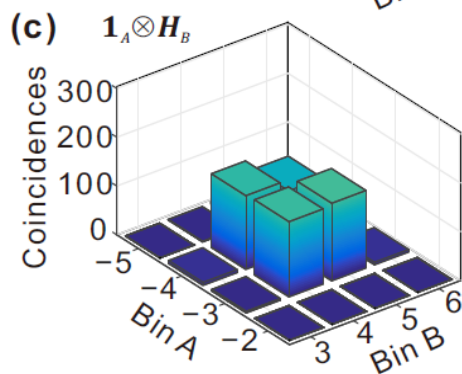
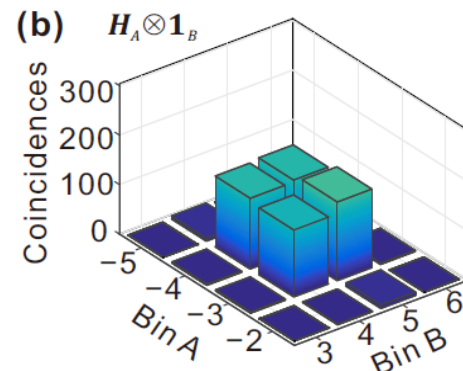
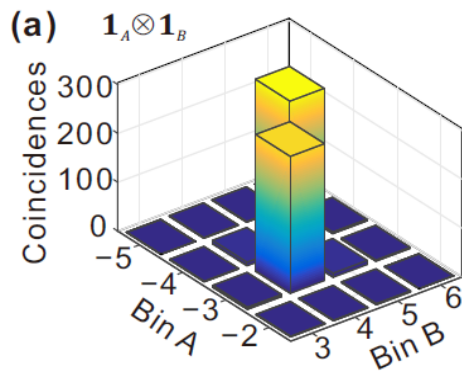
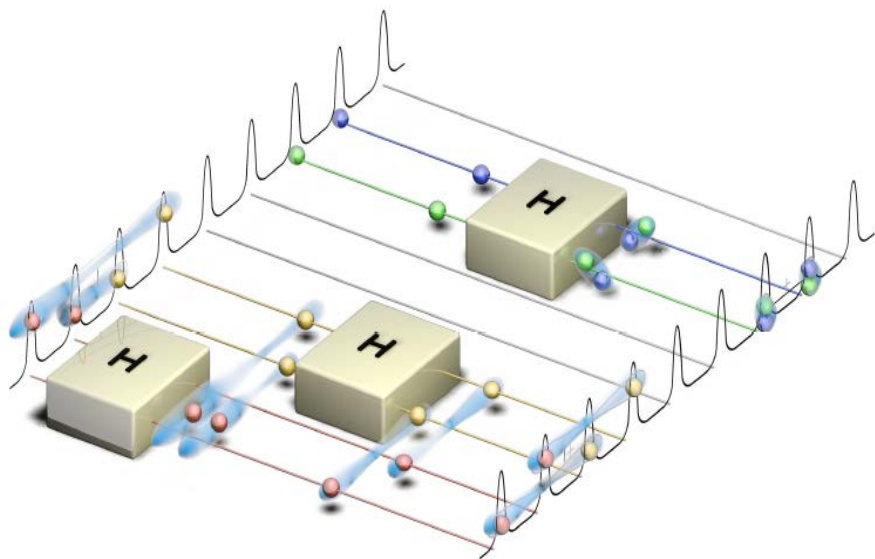
Design: $F = 0.9999$, $P = 0.9733$
Experiment: $F = 0.9989$, $P = 0.9730$

Outputs for superposition state input $\sim |\psi\rangle = |\omega_0\rangle + e^{-i\varphi}|\omega_1\rangle + e^{-i2\varphi}|\omega_2\rangle$,
for $\varphi =$ (I) 0, (II) $2\pi/3$, (III) $4\pi/3$

Coherent Processing of Entangled Frequency-Bin Qubits

Quantum frequency processor operating on 2 photons, each with 2 frequency bins, independently and in parallel

H: Hadamard transform
(2-dimensional discrete Fourier transform)



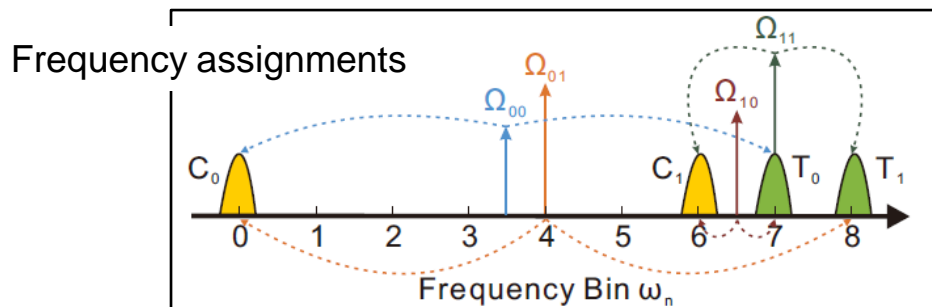
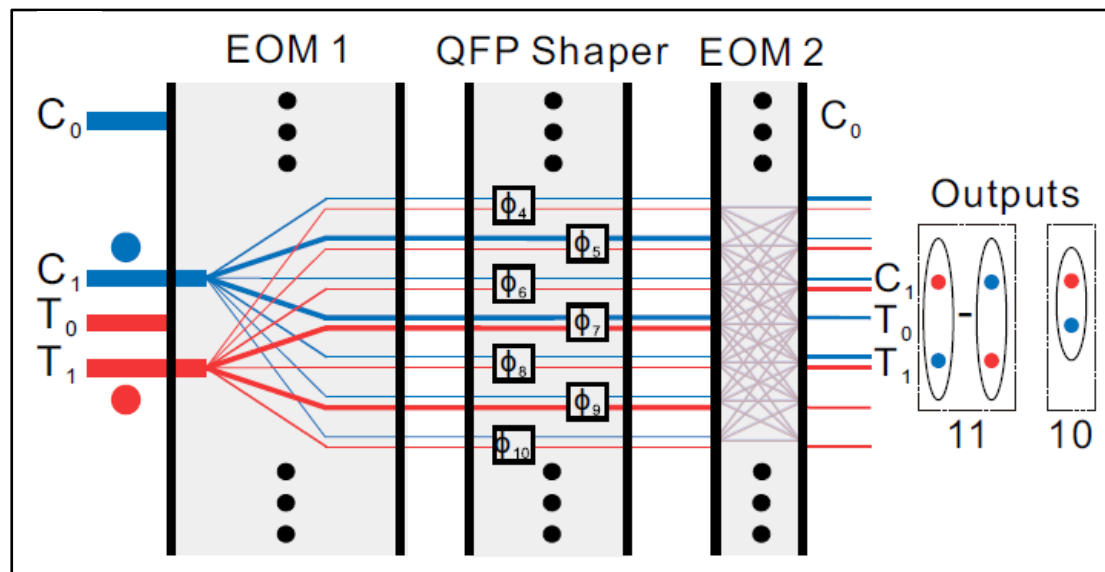
- Novel transformation: swapping the correlations between frequency bins
- Illustrates the power of parallel, independently configured frequency bin operations

Lu, Lukens, Peters, Williams, Weiner and Lougovski, Optica **5**, 1455 (2018)

Frequency-Encoded, Probabilistic Two Photon Gate

Linear optical computing paradigm: realize controlled gates via quantum interference and post-selection

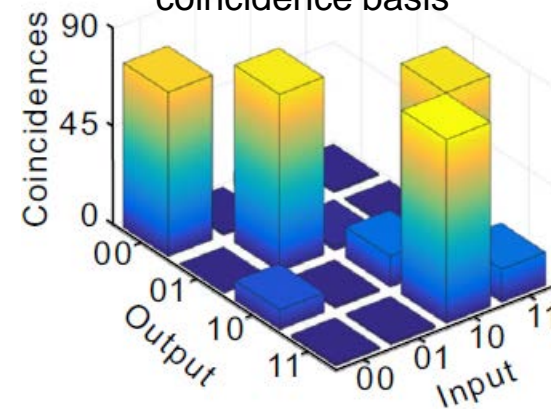
Quantum interference illustration (C_1T_1 input). 0.045% success probability achieved with simple EOM-PS-EOM configuration



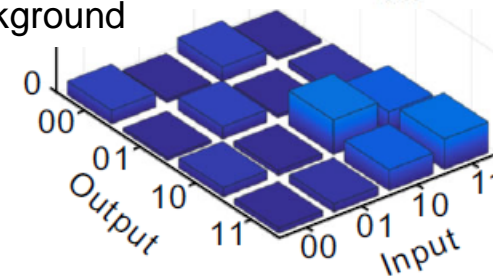
CNOT

$$U_{\text{ideal}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Experimental results: coincidence basis



Background

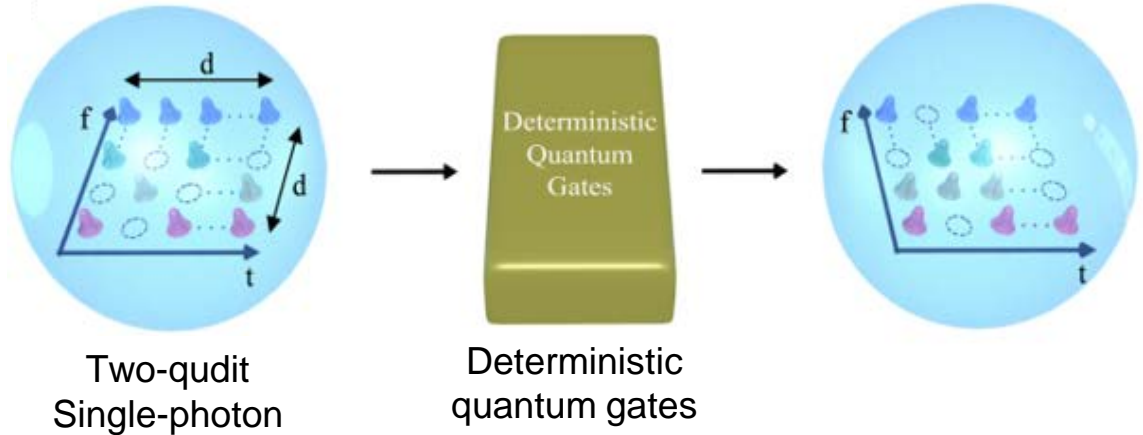


Time-Frequency Hyper-entanglement

Deterministic optical quantum logic
with multiple high-dimensional degrees of freedom in a single photon

High-dimensional optical quantum logic in large operational spaces [PURDUE]

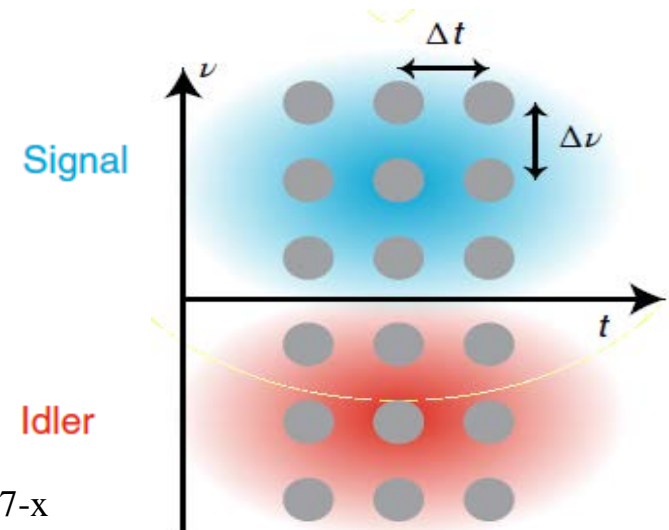
- Enables construction of *deterministic* photonic gates, scalable in dimension
- Experiments in 3 3 and 16 16 dimensions with a single photon
- Readily extended to two photons



Imany, Jaramillo-Villegas, Lukens, Alshaykh, Odele, Moore, Leaird, Qi, and Weiner, arXiv:1805.04410

High-dimensional one-way quantum processing implemented on d -level cluster states [INRS]

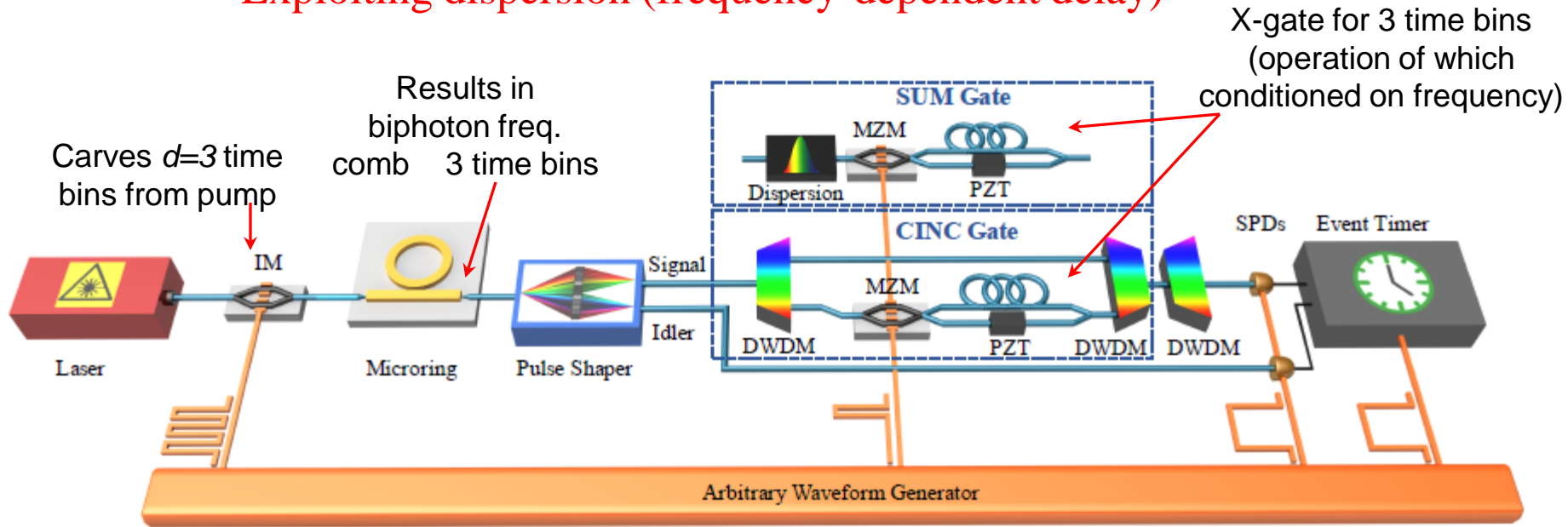
- Realization and characterization of four-partite, 3 dimensional cluster states via time-frequency encoding of signal-idler pair



Reimer, et al, Nature Physics (2018), <https://doi.org/10.1038/s41567-018-0347-x>

Deterministic 2-Qudit Time-Frequency Gates: Experiments

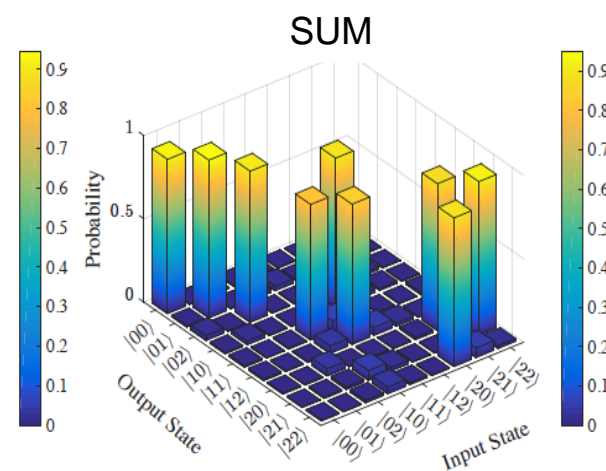
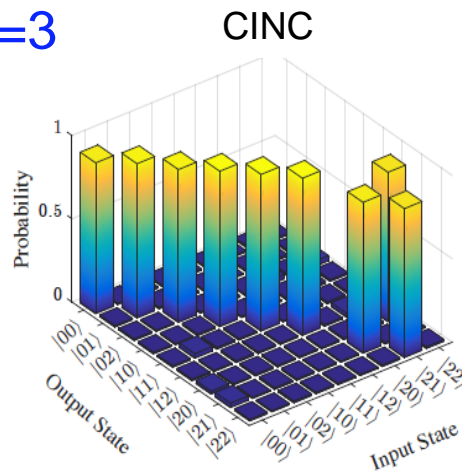
Exploiting dispersion (frequency-dependent delay)



Dimensionality: $d=3$

Frequency as control,
time as target

Controlled X-gate

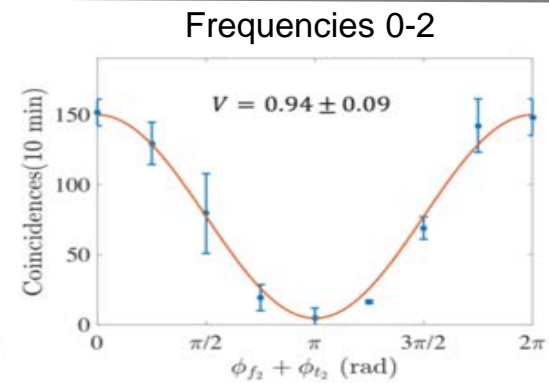
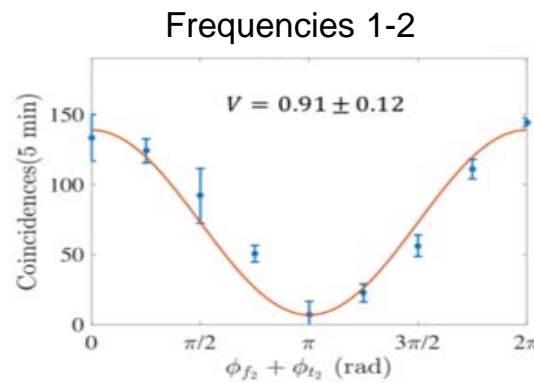
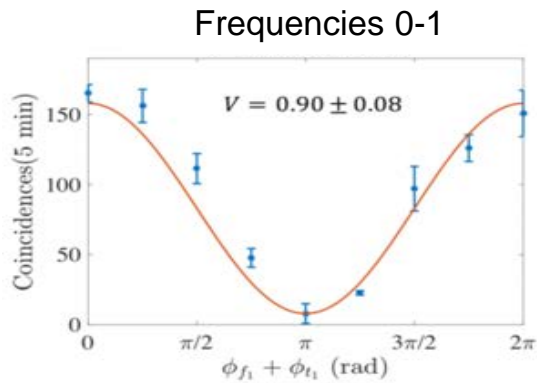
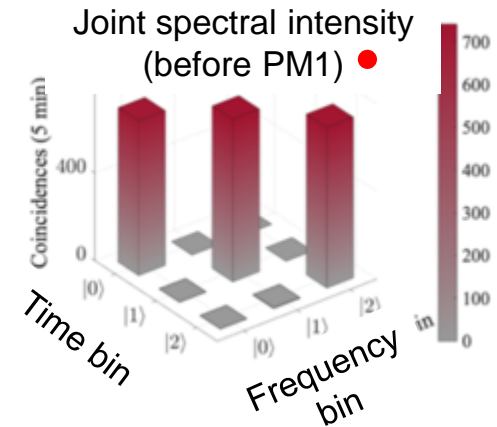
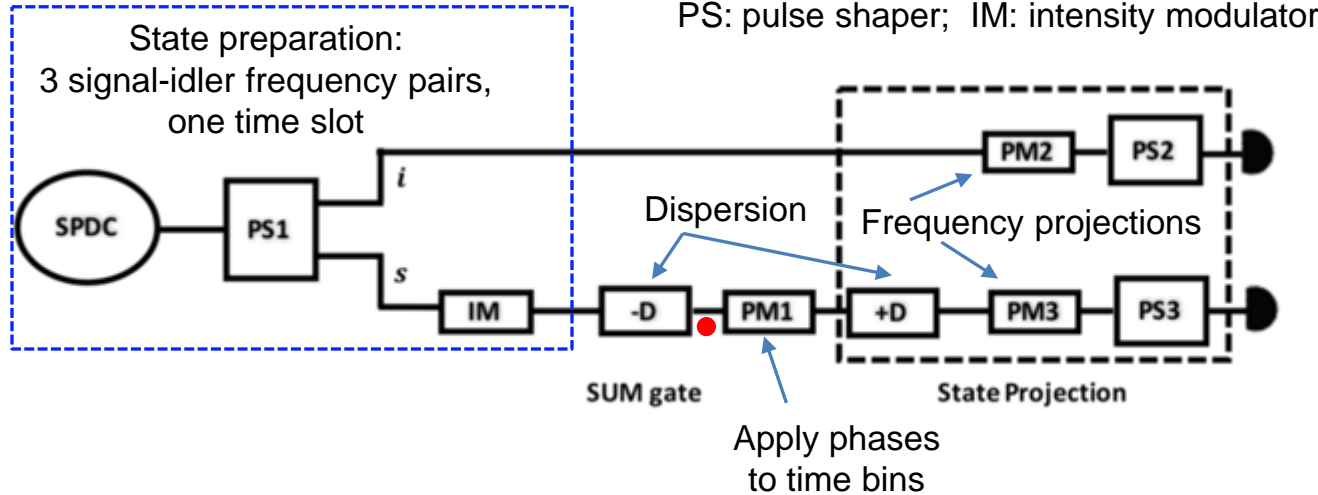


Control + target,
modulo d

Testing the Coherence of the Sum Gate

Quantum interference using superposition states

Filtered SPDC source
Dimensionality: 3 3
Frequency as control,
time as target



Entanglement of formation lower bounded at $EoF \geq 1.19 \pm 0.12$ ebits, confirming dimensionality >2

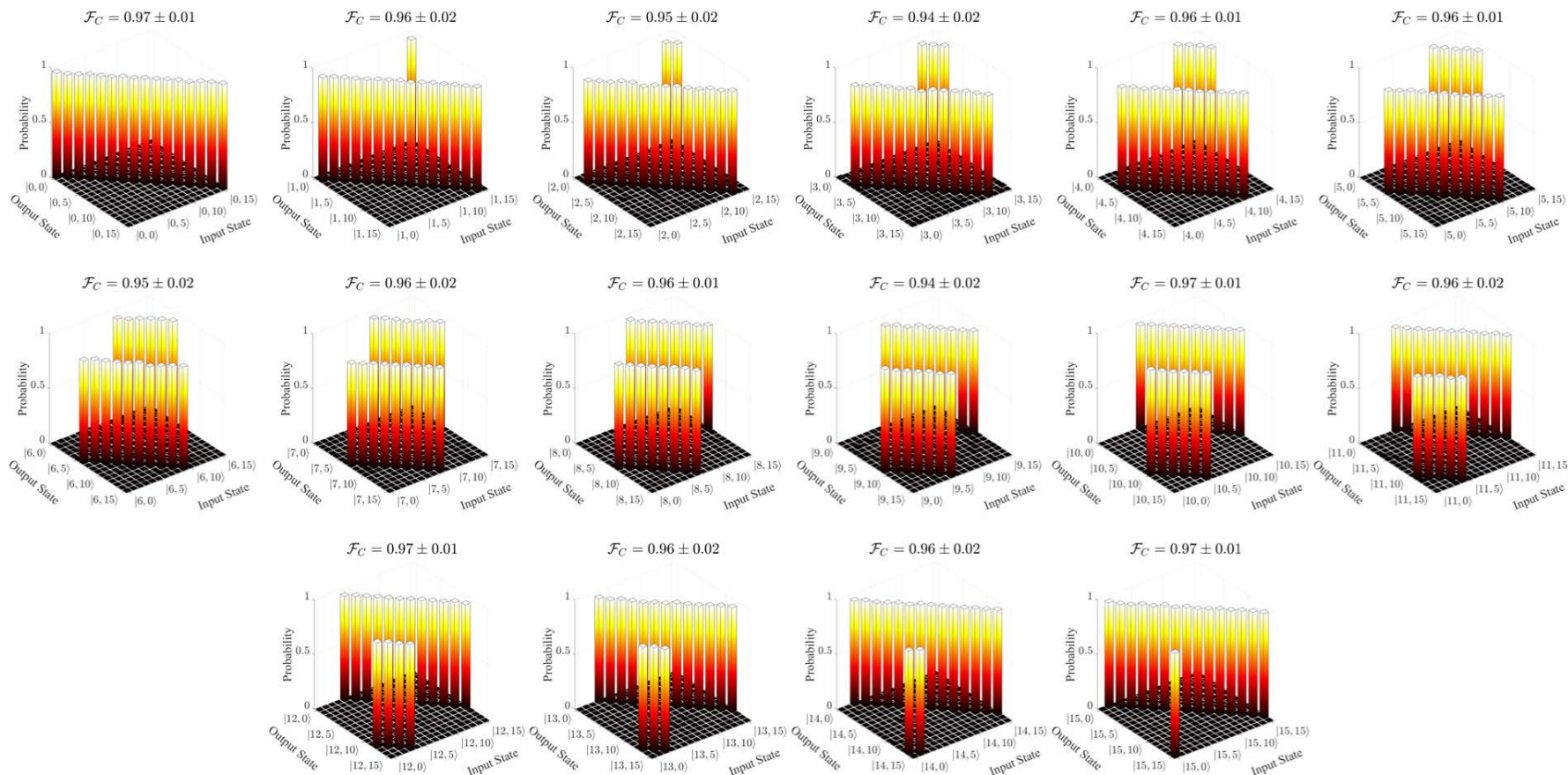
Scaling the Sum Gate to 16 16

16 frequency bins, 16 time bins
Equivalent to 8 qubits in a single photon

Filtered SPDC source
Frequency as control,
time as target

16 16 diagonal blocks
corresponding to individual
frequencies

Control + target,
modulo d



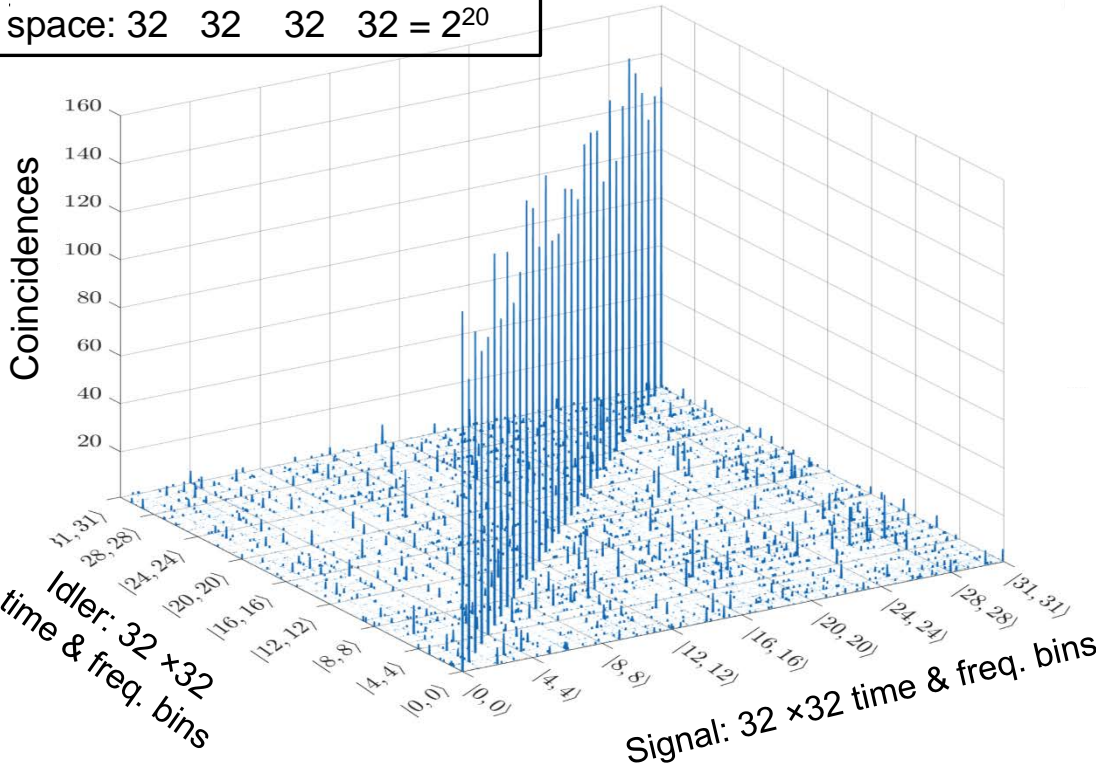
4-party, 32-dimensional Greenberger-Horne-Zeilinger (GHZ) states via SUM gate operating on entangled signal-idler pair

GHZ states consist of >2 parties, entangled with each other such that measurement of one party in the computational basis determines the state of all the other parties.

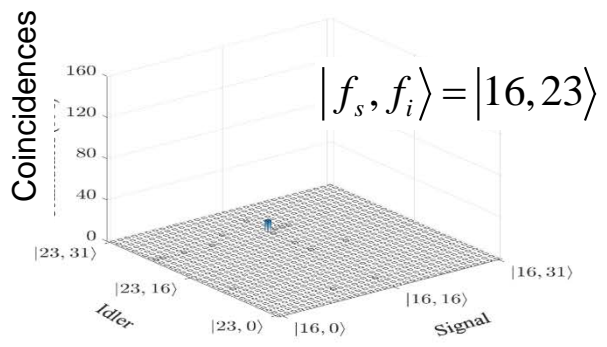
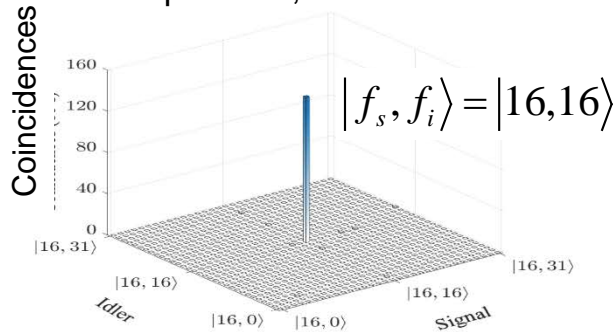
- Proposed applications include: quantum secret sharing, open-destination quantum teleportation, generation of connected networks of cluster states for photonic quantum computation

$$|\psi\rangle = 1/\sqrt{32} \sum_{m=0}^{31} |m, m, m, m\rangle_{f_s t_s f_i t_i}$$

Hilbert space: 32 32 32 32 = 2²⁰

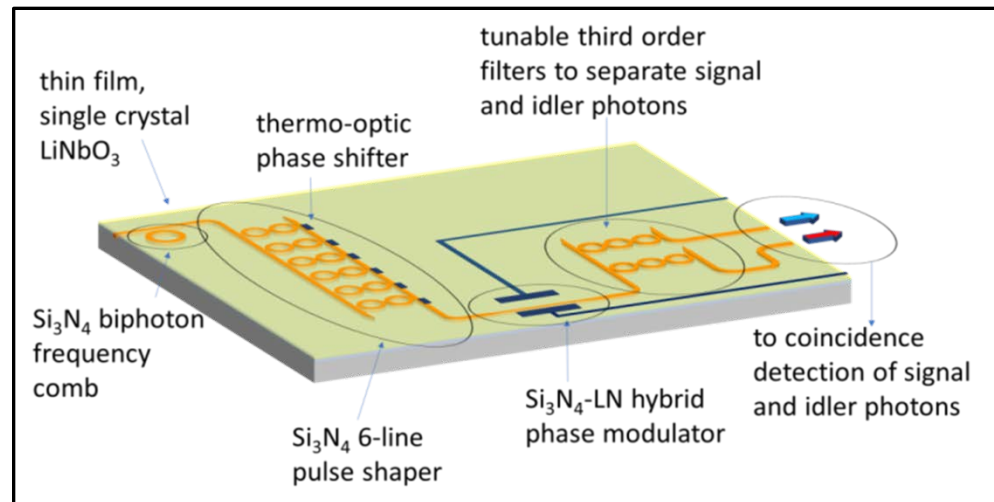


Submatrices:
individual signal and idler frequencies, 32 32 time bins



What Are We Interested in Going Forward?

- Characterize and manipulate entanglement in much higher dimension
- Probabilistic photon-photon two-qudit gates and qudit-teleportation using time-frequency hyperentangled photons
- Encoding qubits in qudits for error correction
- Qudits application for phase estimation and quantum simulation (with Prof. Sabre Kais)
- Photonic integration for on-chip quantum frequency technologies (with Prof. Minghao Qi)



Thanks to many students, collaborators, & sponsors!

- Proving the phase coherence / entanglement for frequency bin photons, including those from on-chip microring resonator sources
- Exploring quantum state manipulation for frequency-encoded photons

