1. Spreadsheet finite difference model of 1-D unsteady conduction

   The design for some optical device calls for a poly(methyl methacrylate) (PMMA, a.k.a. plexiglass) plate 5 mm thick at 30°C to be immersed in a hot liquid at 80°C. Because the index of refraction will change with temperature, the designer wants to know how the temperature varies across the plate as a function of time.

   Let x represent the distance into the PMMA plate from one side.

   Data:

   - PMMA thermal conductivity: $0.21 \frac{\mathrm{W}}{\mathrm{m \cdot K}}$
   - PMMA density: $1190 \frac{\mathrm{kg}}{\mathrm{m^3}}$
   - PMMA specific heat: $1470 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$
   - Fluid heat transfer coefficient: $9000 \frac{\mathrm{W}}{\mathrm{m^2 \cdot K}}$

   Data source: `http://www.asahi-kasei.co.jp/plastic/e/technical/pmma/bussei_doukou.htm`

   (a) Calculate the Biot number for this situation. What kind of boundary condition approximation can you make at the two sides of the plate?

   (b) For the 1-D explicit form of the finite difference method, give the temperature of an interior node in terms of the mesh Fourier number and the temperatures of the adjacent nodes at the previous timestep.

   (c) For a mesh with six evenly-spaced nodes in the x-direction (so $n = 5$ intervals), what is the maximum allowed time step size Δt in the explicit scheme? How does this change for eleven nodes ($n = 10$)?

   (d) Starting with the spreadsheet provided, insert the difference equations into the temperature cells based on your answer to part 1b to calculate the temperature profiles at times from zero to the steady-state timescale using the finite difference method with $n = 5$.

   (e) Repeat the finite difference calculation in part 1d for $n = 10$ (on the second sheet).

   For fun: increase the mesh Fourier number in the spreadsheet and watch what happens to the temperatures.