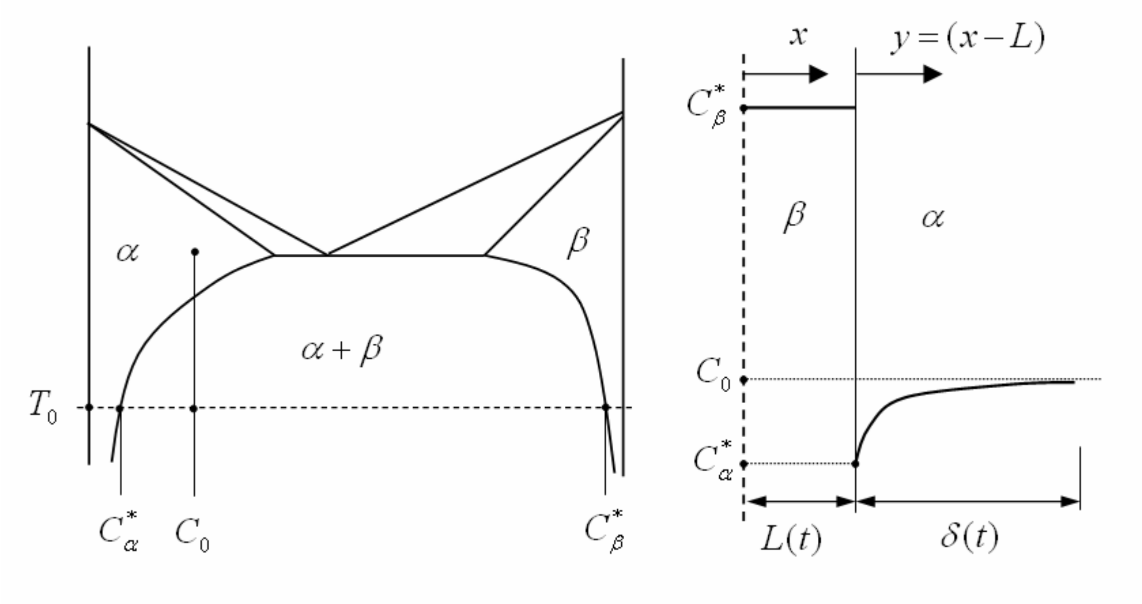
Consider the growth of a second phase during the second step of precipitation hardening (if you don't remember the process, look it up in Callister*, 4th edition, Sections 11.7-11.8). A binary phase diagram and the growing precipitate particle are shown in the figures below.



At some temperature (T_o) and overall composition (C_o), the phase diagram shows $\alpha + \beta$ in equilibrium. Due to the previous step in the hardening process, only the α phase is present and it has a composition of C_o. When raised to T_o, the β phase precipitate particles nucleate and grow. The temperature of the heat treatment is determined by the relationship between the growth rate of the β phase and the time allotted for the process. This relationship is controlled by mass diffusion of solute from the α phase to form the β solid.

Assume that the particles grow in the form of one-dimensional plates and that the distance between nucleation sites is larger that 2(L + δ), so that they do not interfere with one another. Also assume that the diffusion coefficient is not a function of composition.

(a) The concentration field in the δ solid can be found by solving for mass diffusion in the region x > L. The mass conservation equation (Fick's Second Law) and the boundary conditions:

$$D\frac{\partial^2 C}{\partial y^2} = \frac{\partial C}{\partial t} \qquad C(y=0) = C_\alpha^* \qquad C(y \to \infty) = C_o \qquad \left.\frac{\partial C}{\partial y}\right|_{y\to\infty} = 0$$

Normalize this system using:

$$\Gamma = \frac{C - C_\alpha^*}{C_o - C_\alpha^*} = C_1\eta^2 + C_2\eta + C_3 \qquad \eta = \frac{y}{\delta(t)} \qquad y = x - L(t)$$

Write out the normalized governing equation and the boundary conditions.

(b) Find the coefficients (C_1, C_2, C_3) in the concentration profile (϶).
(c) Integrate the governing equation found in part (a) from y = 0 to y = δ (η = 0 to η = 1). Find an expression for δ(t).
(d) Using the interface condition for a moving boundary between two phases,

$$-D\frac{\partial C_\beta}{\partial x}\bigg|_{x=L} + -D\frac{\partial C_\alpha}{\partial x}\bigg|_{y=0} = \left(C_\beta^* - C_\alpha^*\right)\frac{dL}{dt},$$

show that

$$L(t) = \frac{\delta(t)}{3}\left(\frac{C_o - C_\alpha^*}{C_\beta^* - C_\alpha^*}\right).$$

*W.D.Callister, Materials science and engineering: an introduction, 4-th edition, John Wiley & Sons, 1998