Entanglement, discord and other quantum correlations

A. R. P. Rau, Physics & Astronomy, Louisiana State Univ

Entanglement, use in quantum information, decay and restoration;

Bell Inequalities; Entropic Inequalities;

Quantum Discord, entropic and geometric;

Calculations for 7-parameter X-states of two qubits;

X- and extended X-states, qubit-qudit or N qubits;

Geometrical generalization of Bloch Sphere to two-qubit states and their evolution : a decomposition of SU(N) analogous to that of SU(2), provides a minimal unitary operator useful in calculating discord

Dmitry Uskov, Sai Vinjanampathy, Mazhar Ali & Gernot Alber

J. Phys. A 42, 412002 (‘09); 45, 095303 (‘12); PRA 78, 022331 (‘08); 82, 032336 (‘10);
Quantum Info Pro 17, 216 (1-18) (‘18); arXiv:1905.06914

Kirkman's Schoolgirls and Quantum Spin Pairs

A 175 year long thread through mathematics and physics

Combinatorics, Design theory, Coding theory, Latin Squares, SuDoKu, Finite Geometries, Experimental Statistics, Lie and Clifford algebras of multiple quantum spins, …

A. R. P. Rau, J. Biosci. **34**, 353 (2009)

Kirkman's schoolgirls, Pair of Qubits, Rainbow of four primary colors: Common Patterns and Designs

Different aspects of the number 15

3 X 5

$2^4 - 1$

Pascal: 1 + 4 + 6 + 4 + 1

quadruple binary string 1111

Dirac gamma matrices
four-V, T, A, P(seudo)
H atom's O(4, 2):
$\vec{r}, \vec{p}, \vec{\ell}, \vec{A}, r, p^2, \vec{r} \cdot \vec{p}$
ARPR & G Alber, J Phys B 50, 242001 (2017)

7 days of the week, a 7-dimensional subgroup of SU(4)

John Paul Marceaux, LSU Honors Thesis 2018
designs in color and sound

Kirkman's Schoolgirl problem:

"15 young ladies in a school walk out three abreast for seven days in succession. It is required to arrange them daily, so that no two will walk twice abreast". W.S.B. Woolhouse, Lady's and Gentleman's Diary, Prize question 1733 (1844)

Solved by Rev. Thomas Kirkman, a Lancashire clergyman Lady's and Gentleman's Diary **147**, 48 (1850) and Cambridge Dublin Math. J. 5, 255 (1850)

Designs, Balanced Incomplete Block Designs (BIB), Steiner Triple Systems: J. Steiner, J.Reine Angew.Math. **45**,181 (1853); F. Yates, Ann. Eugenics **7**, 121 (1936), R.A. Fisher, The Design of Experiments (1935).

Two sets, allot one's to the second's with specified conditions

First set of v symbols (varieties)	$v = 15$ ladies
To be put into b blocks	Kirkman's solution with $b = 35$
Each block to have k symbols	$k = 3$ ladies
Each symbol to occur in r different blocks	$r = 7$ days
Every pair ($t = 2$) to occur together in λ blocks	$\lambda = 1$ no repeats

(v, b, r, k, λ) design

Block design with $k = 3$: Triple Systems

Designs with $\lambda = 1$: Steiner Systems $S(t, k, v)$

If $k = 3$: Steiner triple systems $S(2, 3, v)$

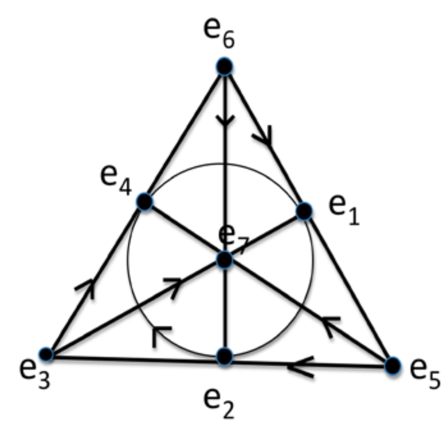
Steiner triple systems exist iff $v = 1$ or $3 \bmod 6$

M. Reiss, J. Reine Angew. Math. **56**, 326 (1859).

For any BIB: vr = bk, λ(v-1) = r(k-1)
if triple system k =3 : r = λ(v-1)/2, b= λv(v-1)/6

BIB denoted t – (v, k, λ) so that Steiner triple: 2 – (v, 3, 1)
Kirkman: 2 – (15, 3, 1)
An even smaller Steiner triple is 2 – (7, 3, 1)
BIB is symmetrical if v = b, r = k as above: v = b = 7, r =k = 3
Number and classification of Steiner triple systems becomes complicated with increasing v.

Points and lines in geometry in place of symbols and blocks. The 2 – (7, 3, 1) design is 7 points and 7 lines such that each point lies on 3 lines and each line passes through 3 points: Fano Plane Projective Geometry: Duality between points and lines



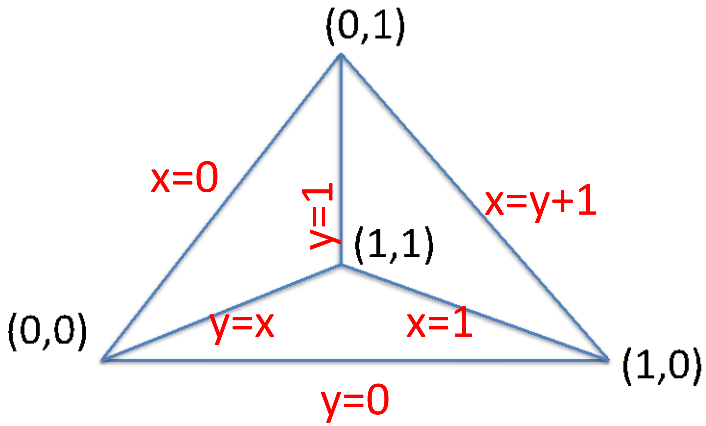
Finite Geometries

Euclidean geometries EG(n, s) with s^n points

EG(2, 2): 4 points and 6 lines:

Lines either meet in a point or not, then "parallel"

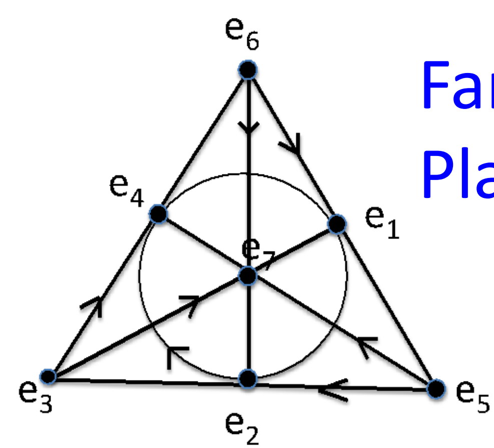
Points (x, y)



Projective geometries PG(n, s)

Add "finite" or points at infinity to make $(s^{n+1} -1)/(s -1)$ points

PG(2, 2): 7 points with 3 new points at infinity and a new line at infinity: duality



Fano Plane

Every pair of lines contains a common point: Projective Plane

Connection between Designs and Finite Projective Geometries

Symmetric BIB or Steiner triple system S(2, 3, 7) with v = b = 7, r = k = 3, λ = 1, denoted 2 – (7, 3, 1), is PG(2, 2) or Fano Plane

Kirkman design v = 15, b = 35, r = 7, k = 3, λ = 1, denoted 2 – (15, 3, 1), is PG(3, 2) not a symmetric BIB

Another symmetric BIB with v = b = 13, r = k = 4, λ = 1 or 2 – (13, 4, 1) design is PG(2, 3) not a Steiner triple system, r and k now 4 and representing number of lines on a point.

Design Theory, Coding Theory, error correcting codes in classical and quantum cryptography, packing problem

R. A. Fisher Indian Science Congress, Bombay, 1938 R. C. Bose P. C. Mahalanobis, S. S. Bose S. N. Roy F. W. Levi

H. Gropp : The birth of a mathematical theory in British India, Coll. Math. Soc. Janos Bolyai **60**, 315 (1992); A. R. P. Rau: R A Fisher, Design Theory and the Indian Connection, J. Biosci. **34**, 353 (2009)

Quantum Spin   Qubit for Quantum Information

Three Pauli operators σ_x σ_y σ_z with Lie algebra $[\sigma_x , \sigma_y] = 2i\sigma_z$

and Clifford algebra $\sigma_x \sigma_y = i\sigma_z$ Cyclic (x, y, z)

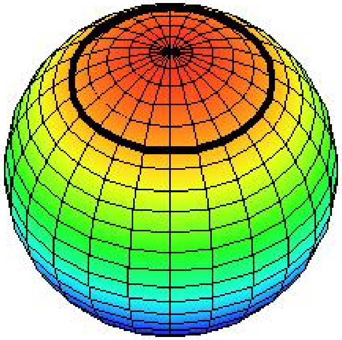
SU(2) group or su(2) algebra   3 components

Two-state System

$$\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

State	Wave Function	Density Matrix
$\|0\rangle$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$\|1\rangle$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\|\pm\rangle$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & \pm\frac{1}{2} \\ \pm\frac{1}{2} & \frac{1}{2} \end{pmatrix}$
Mixture	$\frac{1}{\sqrt{2}}(e^{i\theta_0}\|0\rangle + e^{i\theta_1}\|1\rangle)$	$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

Bloch Sphere



$SU(2) \cong S^2 \times U(1)$

Fiber Bundle

two spins qubit pair

I σ_i τ_i σ_i τ_j 15 operators SU(4)

O_X	σ_z	τ_z	$\sigma_z\tau_z$	σ_x	σ_y	$\sigma_x\tau_z$	$\sigma_y\tau_z$	τ_x	τ_y	$\sigma_z\tau_x$	$\sigma_z\tau_y$	$\sigma_x\tau_x$	$\sigma_y\tau_y$	$\sigma_x\tau_y$	$\sigma_y\tau_x$
σ_z	0	0	0	$i\sigma_y$	$-i\sigma_x$	$i\sigma_y\tau_z$	$-i\sigma_x\tau_z$	0	0	0	0	$i\sigma_y\tau_x$	$-i\sigma_x\tau_y$	$i\sigma_y\tau_y$	$-i\sigma_x\tau_x$
τ_z	0	0	0	0	0	0	0	$i\tau_y$	$-i\tau_x$	$i\sigma_z\tau_y$	$-i\sigma_z\tau_x$	$i\sigma_x\tau_y$	$-i\sigma_y\tau_x$	$-i\sigma_x\tau_x$	$i\sigma_y\tau_y$
$\sigma_z\tau_z$	0	0	0	$i\sigma_y\tau_z$	$-i\sigma_x\tau_z$	$i\sigma_y$	$-i\sigma_x$	$i\sigma_z\tau_y$	$-i\sigma_z\tau_x$	$i\tau_y$	$-i\tau_x$	0	0	0	0
σ_x	$-i\sigma_y$	0	$-i\sigma_y\tau_z$	0	$i\sigma_z$	0	$i\sigma_z\tau_z$	0	0	$-i\sigma_y\tau_x$	$-i\sigma_y\tau_y$	0	$i\sigma_z\tau_y$	0	$i\sigma_z\tau_x$
σ_y	$i\sigma_x$	0	$i\sigma_x\tau_z$	$-i\sigma_z$	0	0	$-i\sigma_z\tau_z$	0	0	0	$i\sigma_x\tau_y$	$-i\sigma_z\tau_x$	0	$-i\sigma_z\tau_y$	0
$\sigma_x\tau_z$	$-i\sigma_y\tau_z$	0	$-i\sigma_y$	0	$i\sigma_z\tau_z$	0	$i\sigma_z$	$i\sigma_x\tau_y$	$-i\sigma_x\tau_x$	0	0	$i\tau_y$	0	$-i\tau_x$	0
$\sigma_y\tau_z$	$i\sigma_x\tau_z$	0	$i\sigma_x$	$-i\sigma_z\tau_z$	0	$-i\sigma_z$	0	$i\sigma_y\tau_y$	$-i\sigma_y\tau_x$	0	0	0	$-i\tau_x$	0	$i\tau_y$
τ_x	0	$-i\tau_y$	$-i\sigma_z\tau_y$	0	0	$-i\sigma_x\tau_y$	$-i\sigma_y\tau_y$	0	$i\tau_z$	0	$i\sigma_z\tau_z$	0	$i\sigma_y\tau_z$	$i\sigma_x\tau_z$	0
τ_y	0	$i\tau_x$	$i\sigma_z\tau_x$	0	0	$i\sigma_x\tau_x$	$i\sigma_y\tau_x$	$-i\tau_z$	0	$-i\sigma_z\tau_z$	0	$-i\sigma_x\tau_z$	0	0	$-i\sigma_y\tau_z$
$\sigma_z\tau_x$	0	$-i\sigma_z\tau_y$	$-i\tau_y$	$i\sigma_y\tau_x$	$-i\sigma_x\tau_x$	0	0	0	$i\sigma_z\tau_z$	0	$i\tau_z$	$i\sigma_y$	0	0	$-i\sigma_x$
$\sigma_z\tau_y$	0	$i\sigma_z\tau_x$	$i\tau_x$	$i\sigma_y\tau_y$	$-i\sigma_x\tau_y$	0	0	$-i\sigma_z\tau_z$	0	$-i\tau_z$	0	0	$-i\sigma_x$	$i\sigma_y$	0
$\sigma_x\tau_x$	$-i\sigma_y\tau_x$	$-i\sigma_x\tau_y$	0	0	$i\sigma_z\tau_x$	$-i\tau_y$	0	0	$i\sigma_x\tau_z$	$-i\sigma_y$	0	0	0	$i\tau_z$	$i\sigma_z$
$\sigma_y\tau_y$	$i\sigma_x\tau_y$	$i\sigma_y\tau_x$	0	$-i\sigma_z\tau_y$	0	0	$i\tau_x$	$-i\sigma_y\tau_z$	0	0	$i\sigma_x$	0	0	$-i\sigma_z$	$-i\tau_z$
$\sigma_x\tau_y$	$-i\sigma_y\tau_y$	$i\sigma_x\tau_x$	0	0	$i\sigma_z\tau_y$	$i\tau_x$	0	$-i\sigma_x\tau_z$	0	0	$-i\sigma_y$	$-i\tau_z$	$i\sigma_z$	0	0
$\sigma_y\tau_x$	$i\sigma_x\tau_x$	$-i\sigma_y\tau_y$	0	$-i\sigma_z\tau_x$	0	0	$-i\tau_y$	0	$i\sigma_y\tau_z$	$i\sigma_x$	0	$-i\sigma_z$	$i\tau_z$	0	0

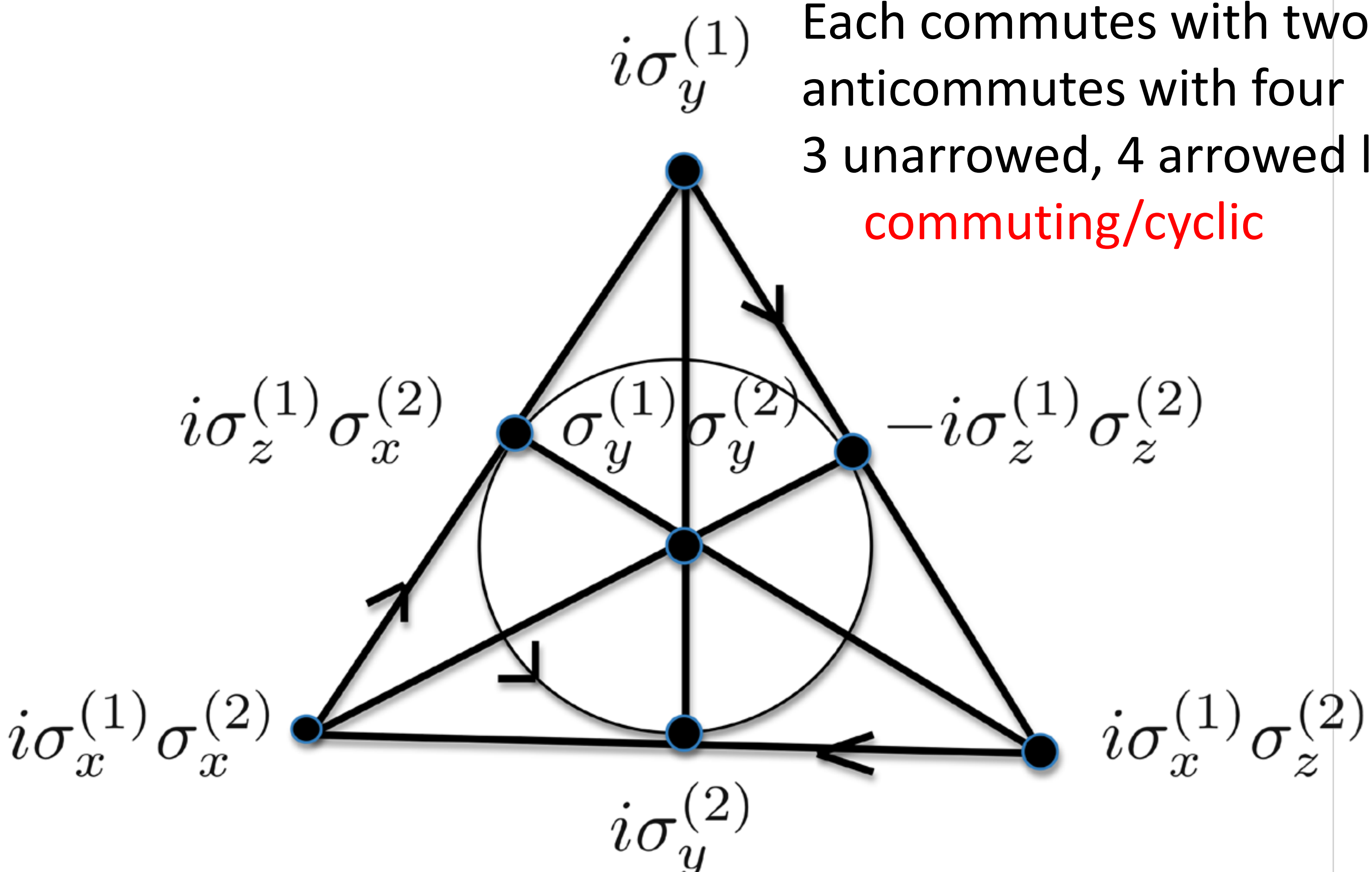
Table of Commutators

Each row has 7 zeroes, each O_X commutes with 6, anti- with 8

Subgroup of SU(4) : SU(2) X SU(2) X U(1)

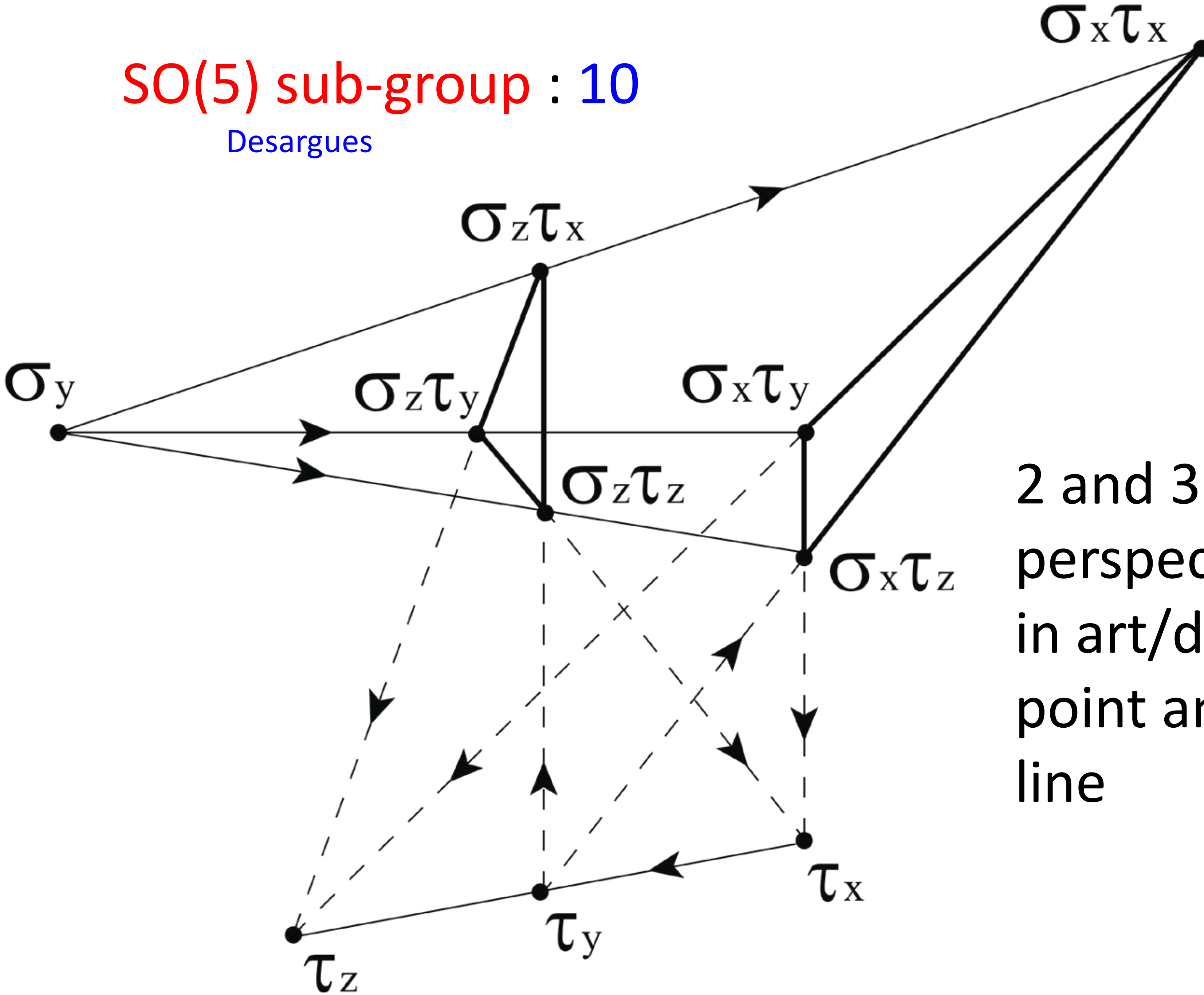
Seven generators; choose any one of 15, 6 others : 7 in all

Each commutes with two others,
anticommutes with four
3 unarrowed, 4 arrowed lines
commuting/cyclic



SO(5) sub-group : 10

Desargues



2 and 3 d, perspective in art/design, point and line

Connection of qubit states/operators to Kirkman

Full su(4) of two qubits: 15 operators, triplet multiplication rules
Each operator commutes with 6 others, anticommutes with 8

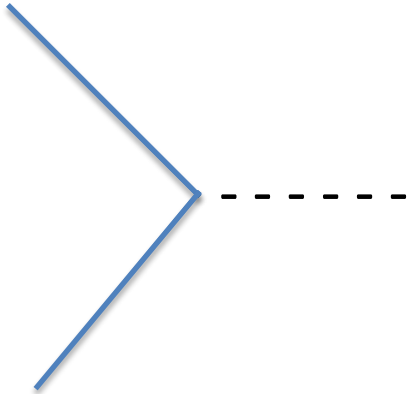
Kirkman's 15 schoolgirls meant 105 pairs. Ranks of 3 meant 35 such, for 5 rows each for 7 days: v = 15, b = 35, r = 7, k = 3. Note vr = bk.

15 operators also for triplet multiplication (Lie or Clifford) mean 105 and 35 lines. These divide, however, into two sets. A commuting set has 15 points on 15 lines, each point on 3 lines: v = 15 = b, r = 3 = k. An anti-commuting set has 15 points on 20 lines, each point on 4 lines: v = 15, b = 20, r = 4, k = 3. First set diagram has unarrowed lines, second set arrowed.

Note also that 3-qubit X states also involve 15 operators with similar analogical features. Thus qubit algebra ≅ Kirkman problem.

N qubits' general states: PG(2N-1,2) X- : PG(N,2)

Sai Vinjanampathy and ARPR Phys. Rev. A 82, 032336 (2010)



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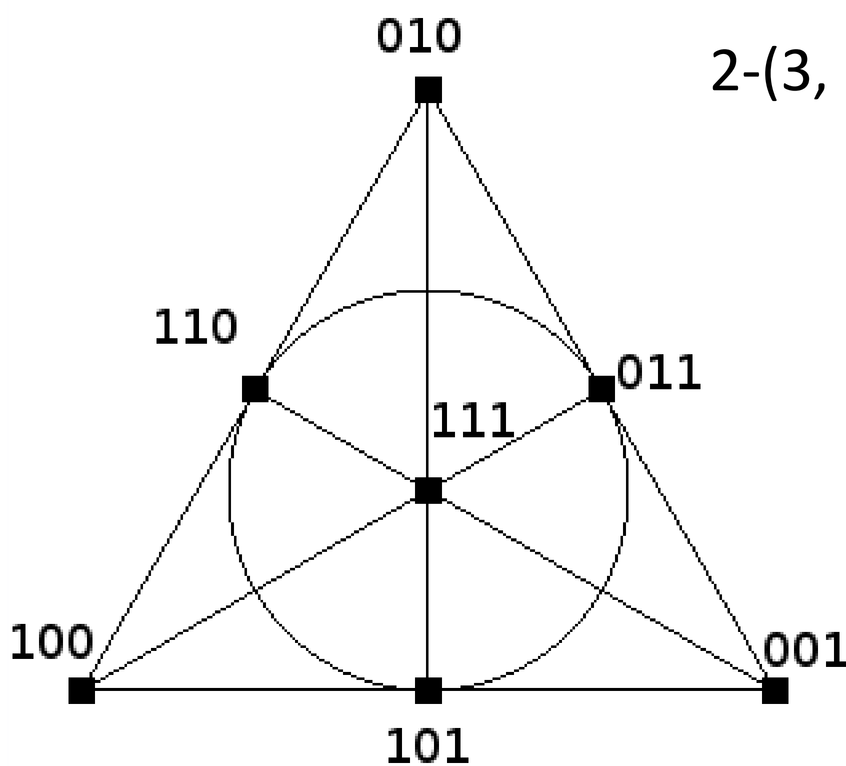
Basic Interaction,
Matrix element,
……

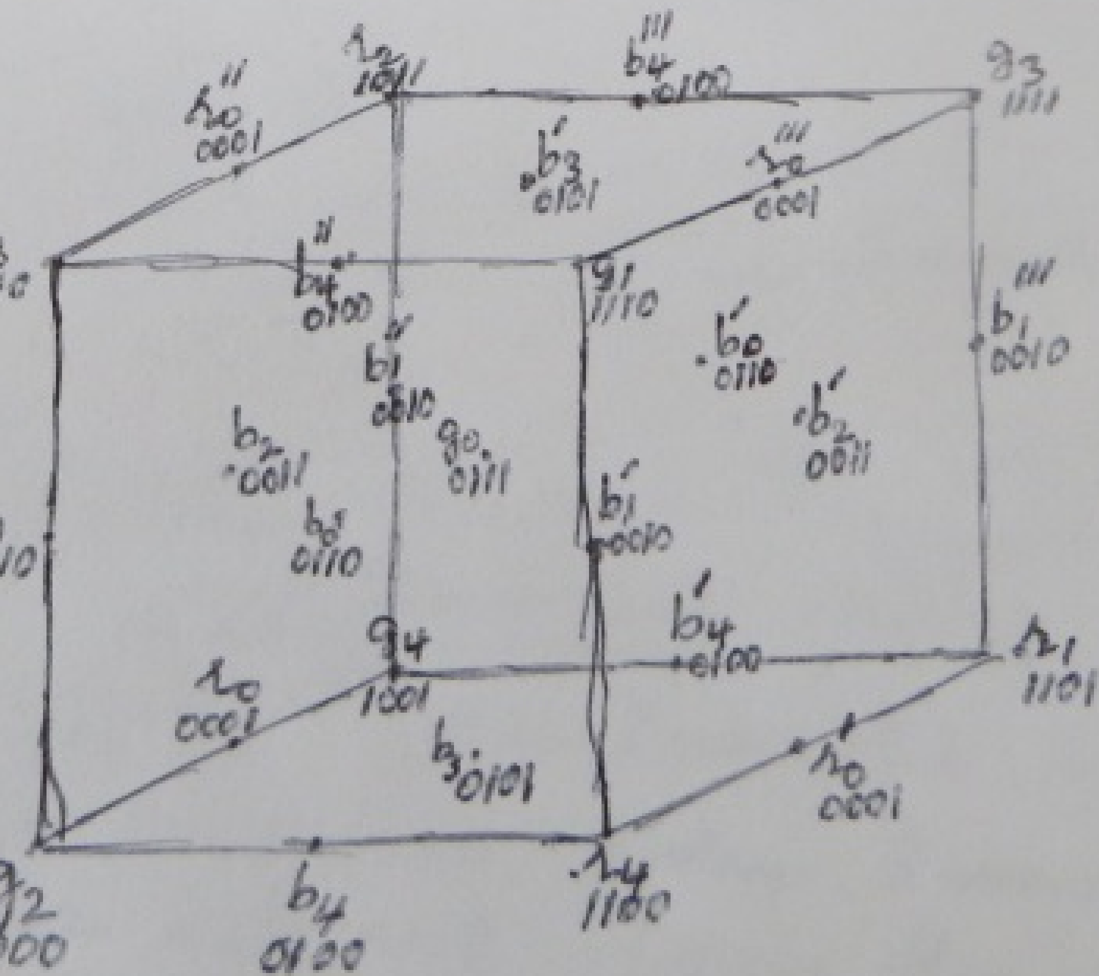
Fano Plane PG(2, 2), 2-(7, 3, 1) STS, 3-string binary

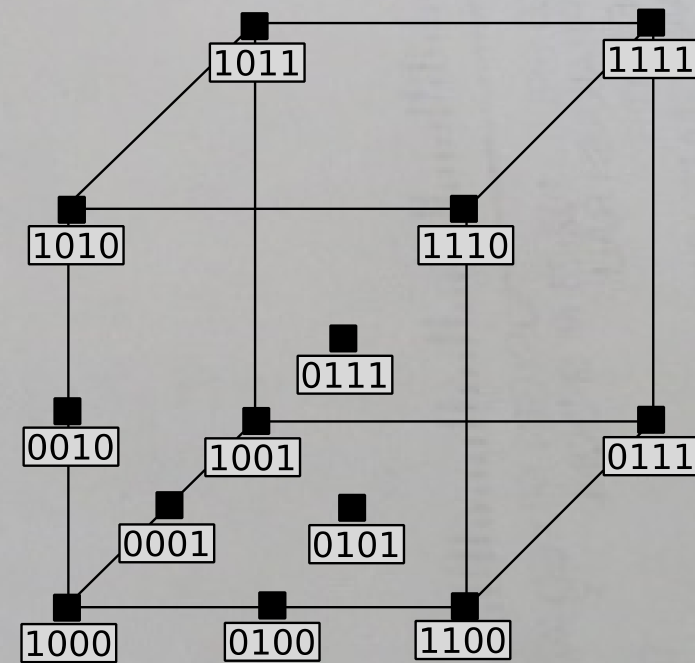


2-(3, 3, 1) : (01, 10, 11)

7 triplets/lines
7 points/girls

Cube: 8 vertices/schoolgirls 28 lines: 12 edges + 12 face diagonals + 4 body diagonals





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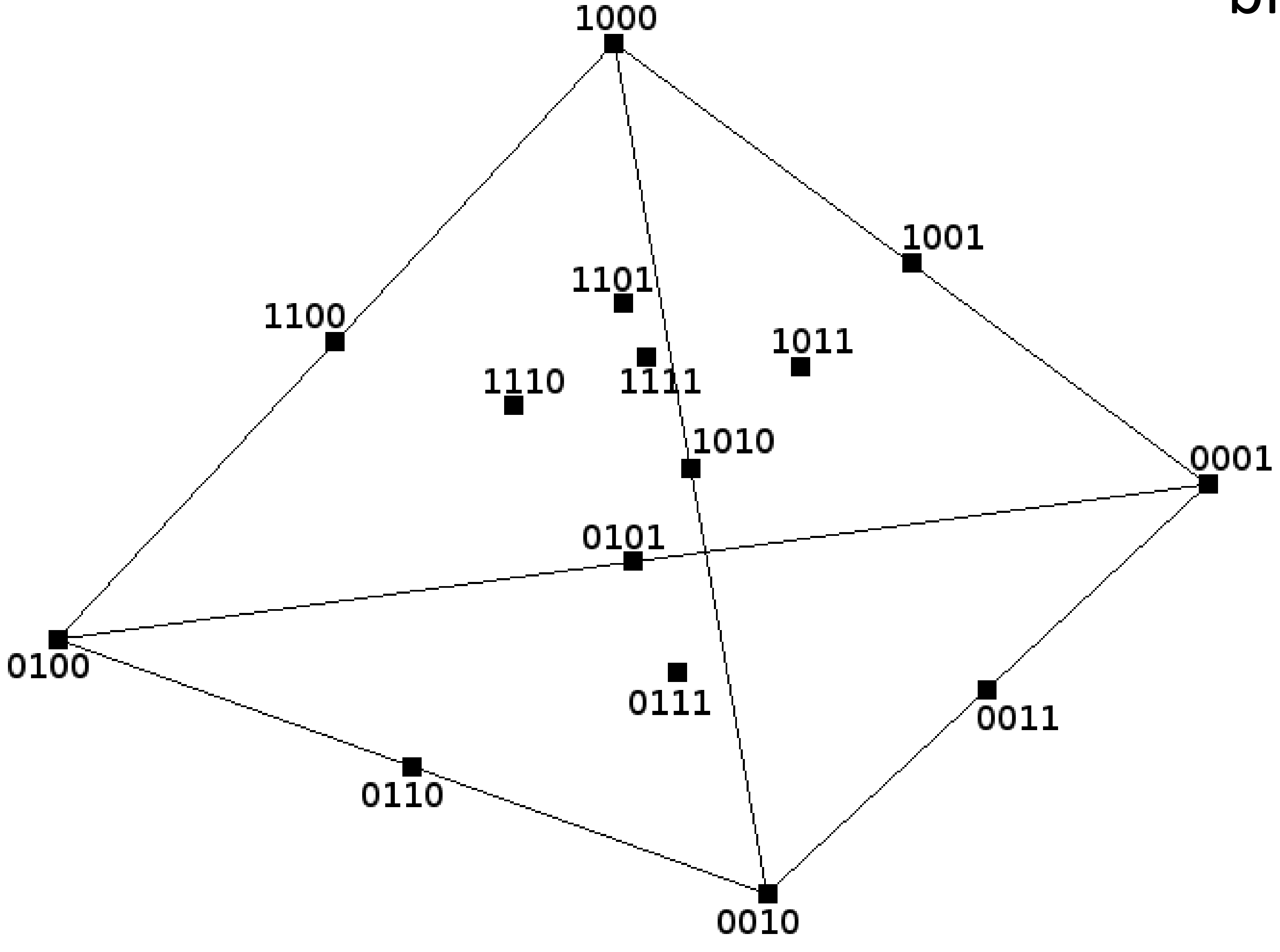
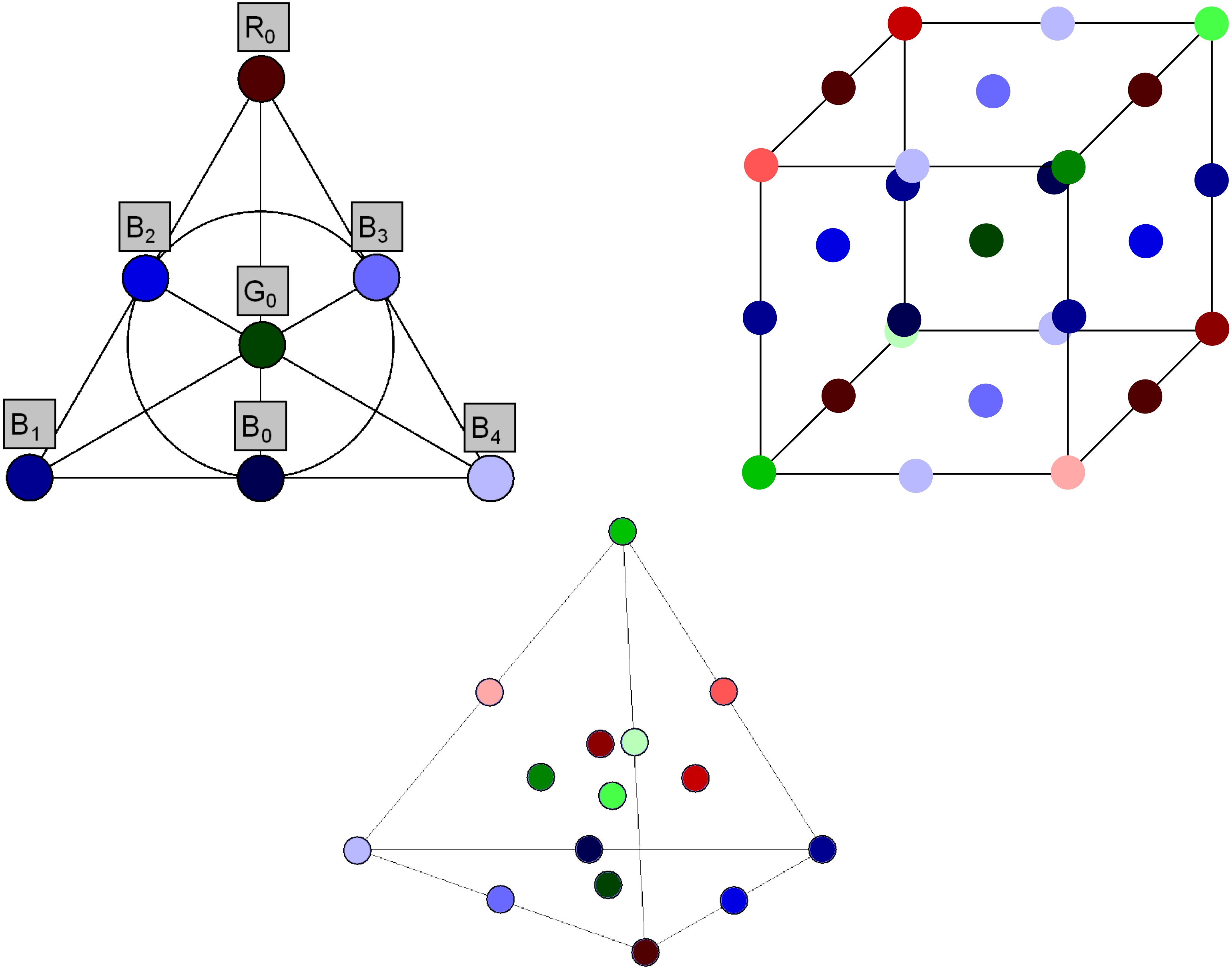
Tetrahedron: 15 points, 4 Fano Plane faces, 4-string binary



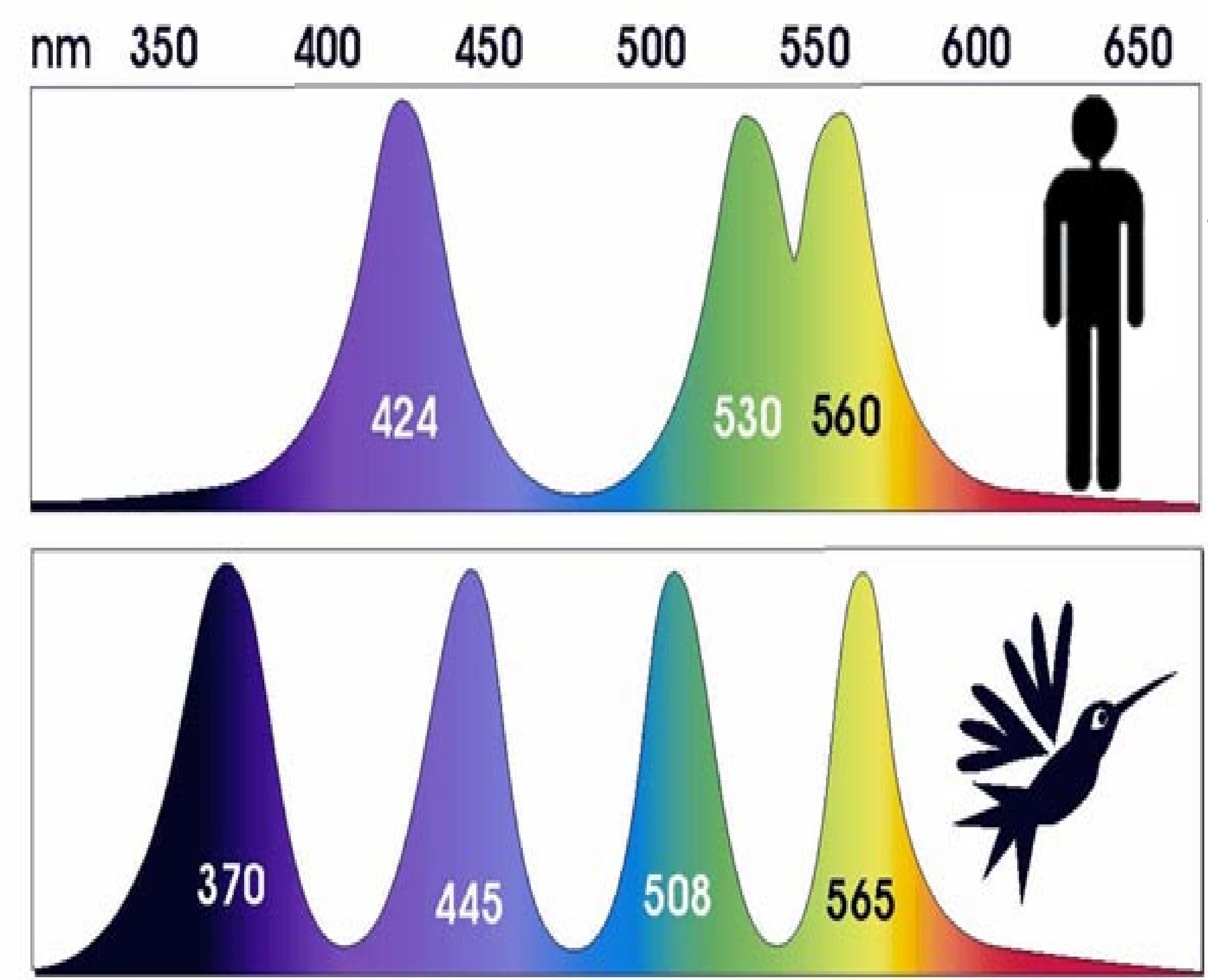

R_0
B_2
B_3
G_0
B_1
B_0
B_4

Constructing a Kirkman Arrangement

Choose any of the 15 O_i operators of qubit pair
place as centre + 6 it commutes with in Fano triangle,
also place as body centre and 8 others in the cube,
OR construct the equivalent tetrahedron;
place seven triplet lines from triangle in top row with same binary for days of the week, place other 28 lines to complete the table, the 4 "left out" of triangle as the central element

	SUN			MON			TU			WED			TH			FRI			SAT	
	111			001			010			011			100			101			110	
R_0	G_0	B_0	B_2	R_0	B_1	G_0	B_1	B_3	B_3	B_2	B_0	G_0	B_2	B_4	B_4	B_3	R_0	B_1	B_0	B_4
R_1	B_2'	G_1	R_2	B_0'	R_1	R_2	R_0''	R_3	R_4	R_0'	R_1	R_3	B_0	R_4	R1	G_0	R_3	R_4	G_0	R_2
R_3	B_1	G_2	G_1	G_0	G_4	G_1	B_0	G_2	G_3	G_0	G_2	G_2	R_0	G_4	G_4	B_0'	G_3	G_3	R_0'''	G_1
R_2	B_4'''	G_3	G_2	B_4	R_4	R_4	B_2'	G_3	G_1	B_4'''	R_3	G_1	B_3'	R_2	G_1	B_1'	R_4	G_2	B_3	R_1
R_4	B_3	G_4	G_3	B_3'	R_3	G_4	B_4'	R_1	G_4	B_1''	R_2	G_3	B_1'''	R_1	G_2	B_2	R_2	G_4	B_2	R_3

Color-flavor scheme: 3 colors, 5 flavors λ



(B, G, R):

7 colours rainbow

VIBGYOR

111

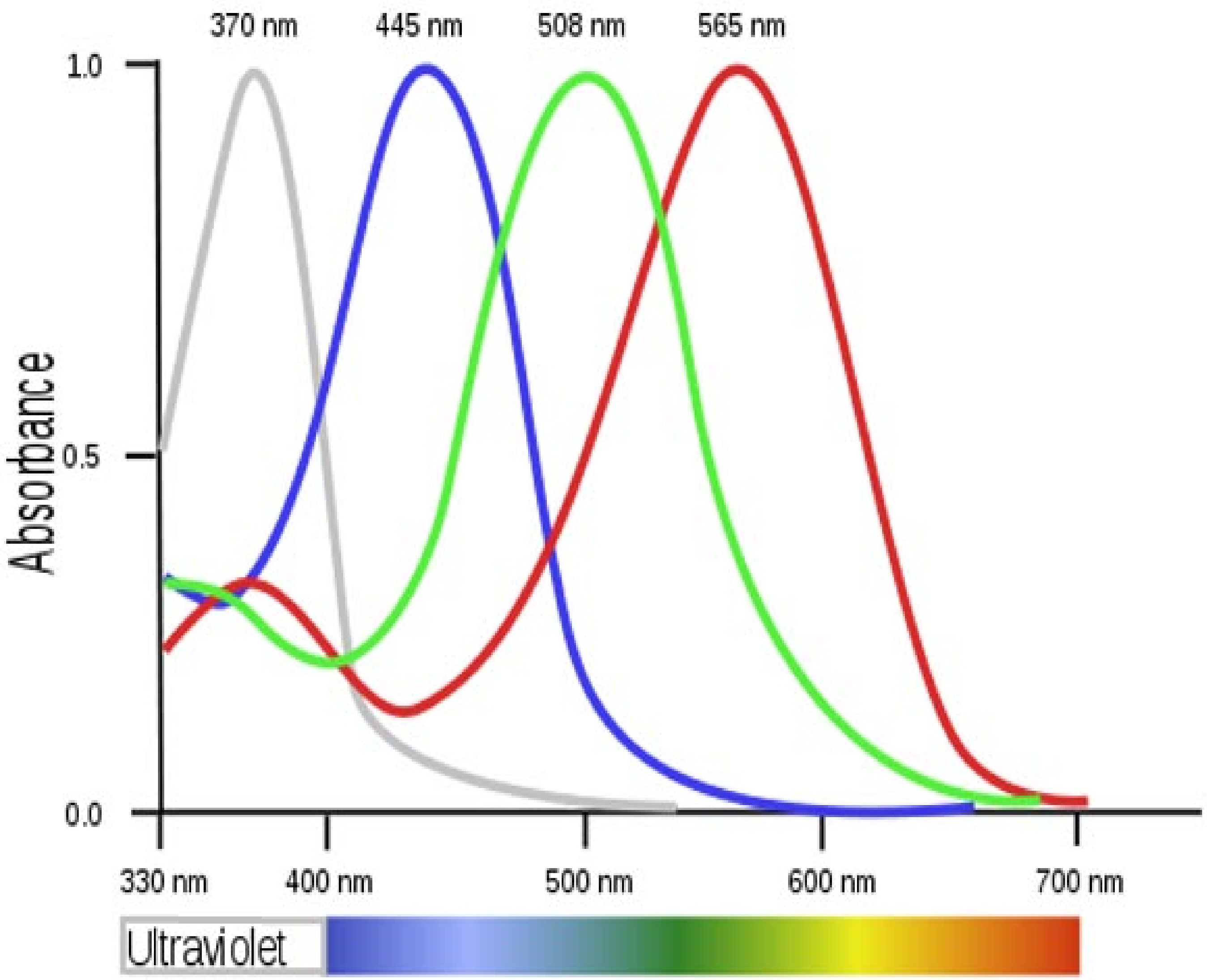
(U, B, G, R):

15 colours

1111

400 nm ≈ 3 eV, 600 nm ≈ 2 eV

Four Color Vision in Birds

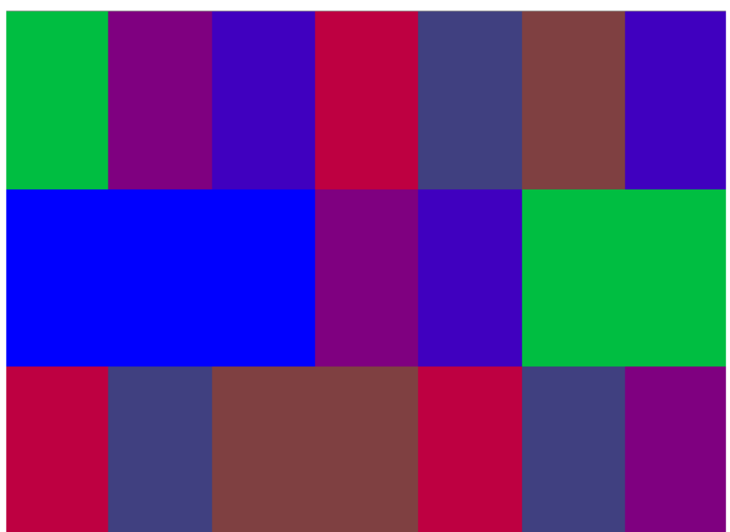


in white light in uv bird’s eye-view

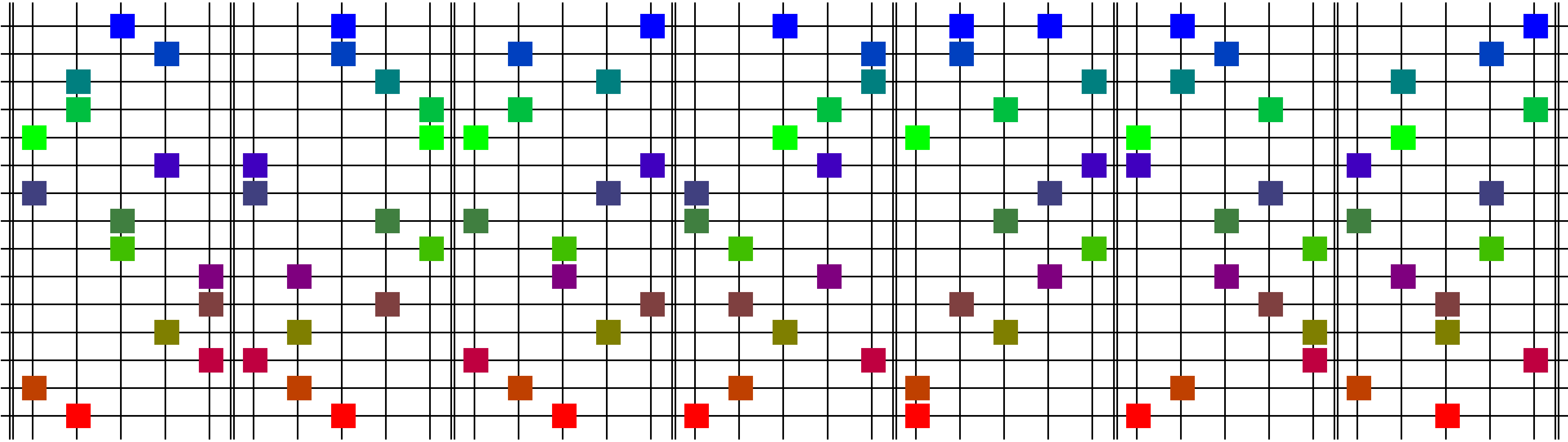
http://www.nature.com/scitable/blog/the-artful-brain/alternate_realities

Alternative (7, 3, 1) Designs with Alternative "Seeds"

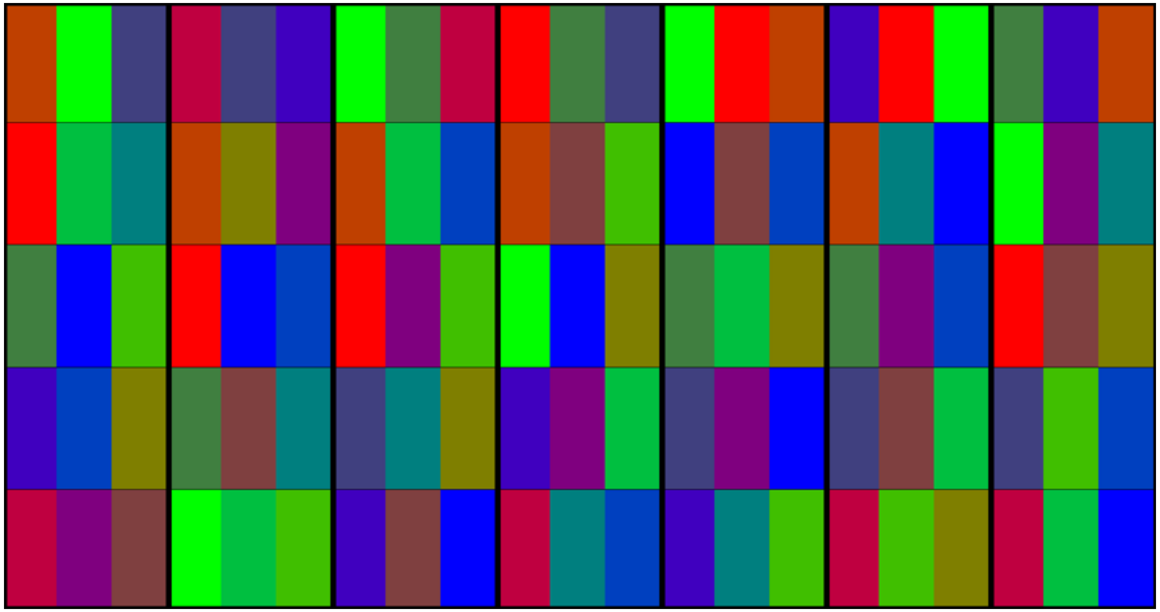
FIGURE 4.1: The color design of the system $D_c[7, 3, 1]|O_1, O_4, O_8\rangle$. This is a tiling produced from the design of Figure 3.1. The colors give the identities of the operators with the explicit equivalences given in the Dictionary A.1.

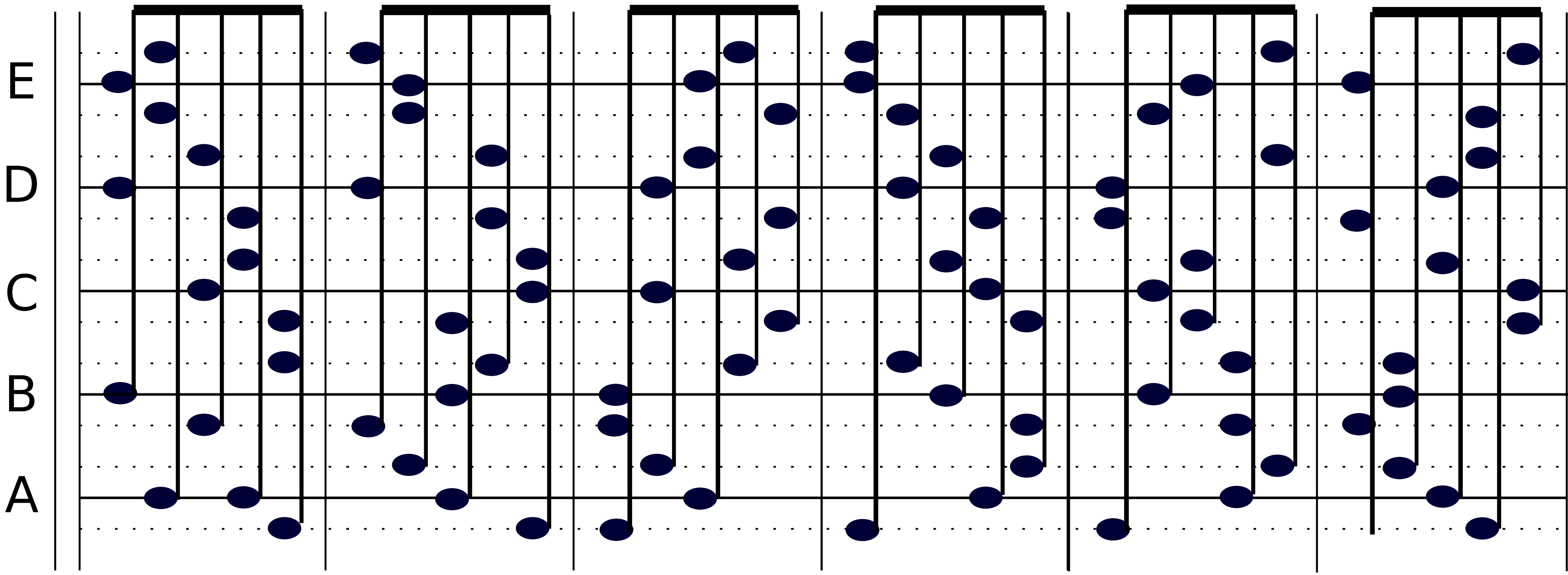


Acoustic and Colour Designs



Kirkman
(15, 3, 1)
Designs
3 X 5
7 blocks/
days





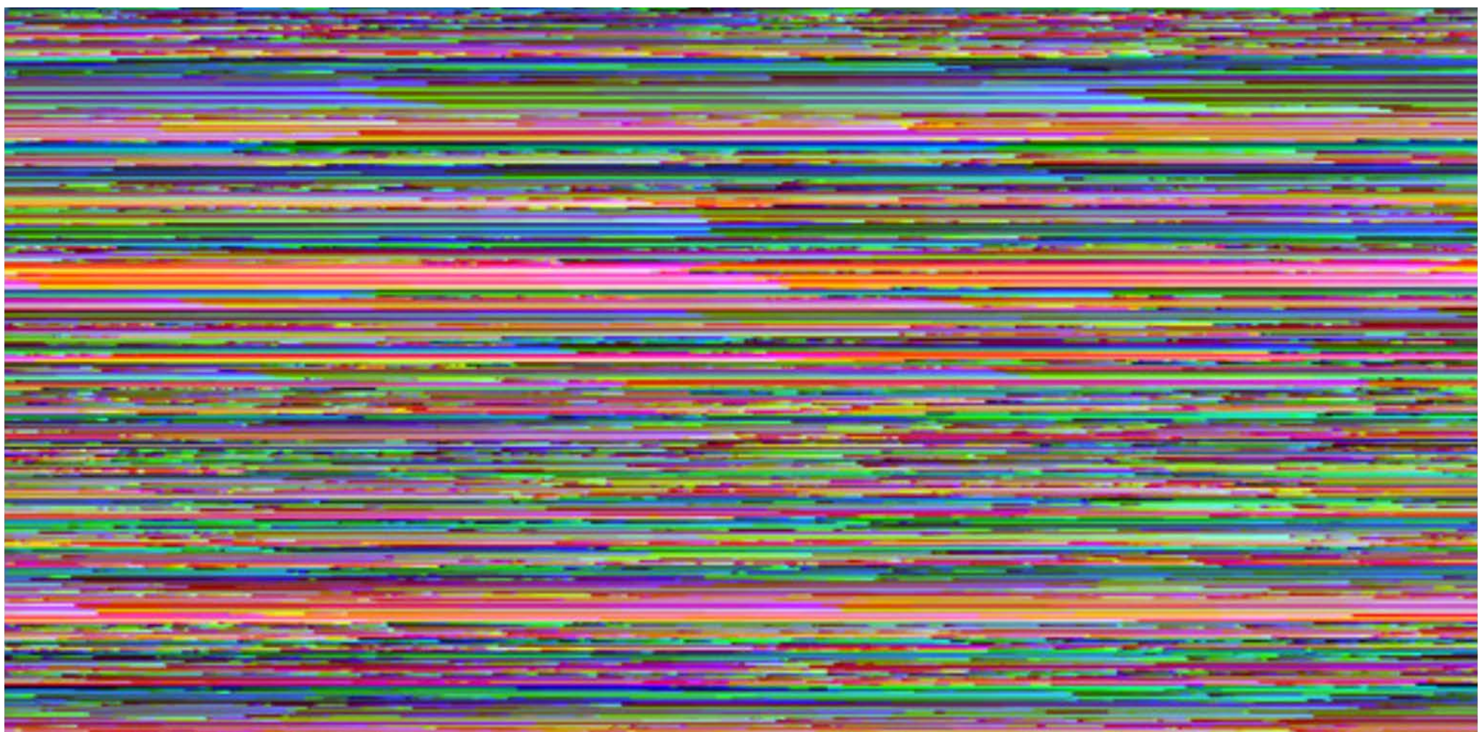
Western musical notation: Octave (A, B, C, D, E, F, G) and flat-natural-sharp.

For Kirkman, five letters and 3 signatures. 3 notes in each chord v = 15, 31, 63, 127, 255, … IP addresses

Conclusion

4 qubit, $2^8 - 1$, 11111111

Ensembles of quantum information exhibit a group structure that can be encoded into a design system. A representation of the design system gives a mapping of the overarching structure of the internal quantum state. The equivalence imposed between physical state and psychological ideal gives rise to methods of communication.



Lisa Jevbratt, 1:1 (*One to one*), 1999-01, digital media, the United States, http://jevbratt.com/1_to_1/.