Many-body expedition from semiconductors to atomic BECs

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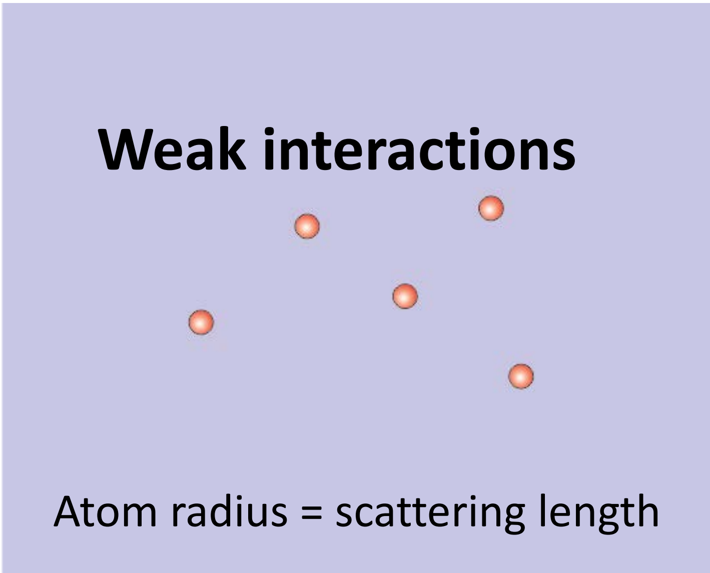
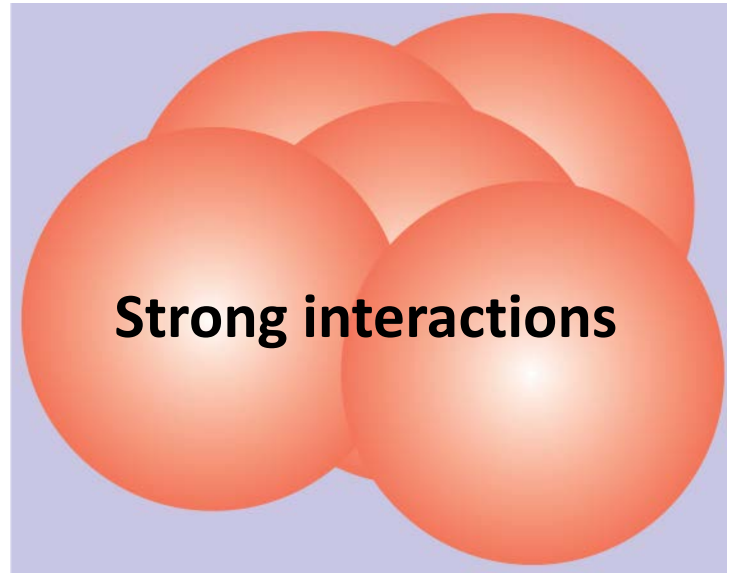
pedia [ancient Greek] = upbringing, a learning process
-pedia [Wiktionary] = something related to learning

Manybodypedia [MK] = learning process of many-body physics, synergy from quantum processes within seemingly different systems.

BECs with strong interactions

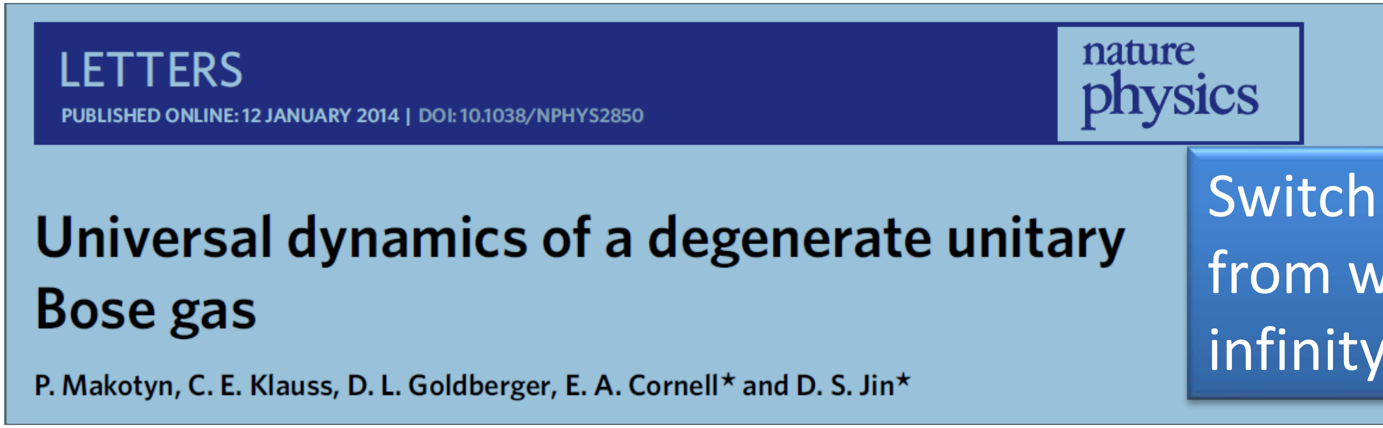




BEC experiments routine	BEC Experiments tough
Perturbative approaches work: e.g. Gross-Pitaevskii equation, Hartree-Fock Bogoliubov,...	**Perturbative approaches fail...**

Nonperturbative, strong Coulomb interaction is already the norm in semiconductor experiments/theory

BEC-quench experiment

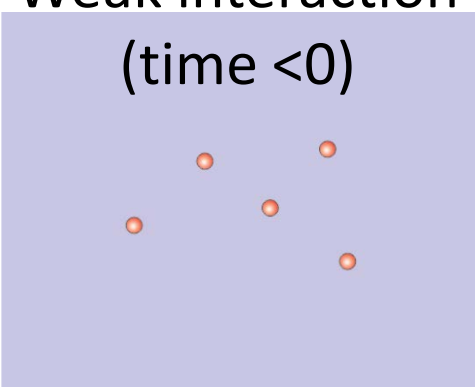

LETTERS
PUBLISHED ONLINE: 12 JANUARY 2014 | DOI: 10.1038/NPHYS2850
nature physics

Universal dynamics of a degenerate unitary Bose gas

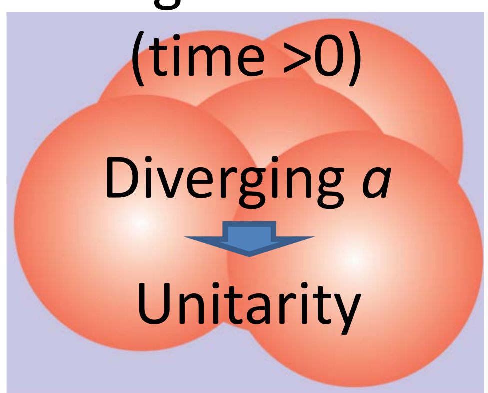
P. Makotyn, C. E. Klauss, D. L. Goldberger, E. A. Cornell* and D. S. Jin*

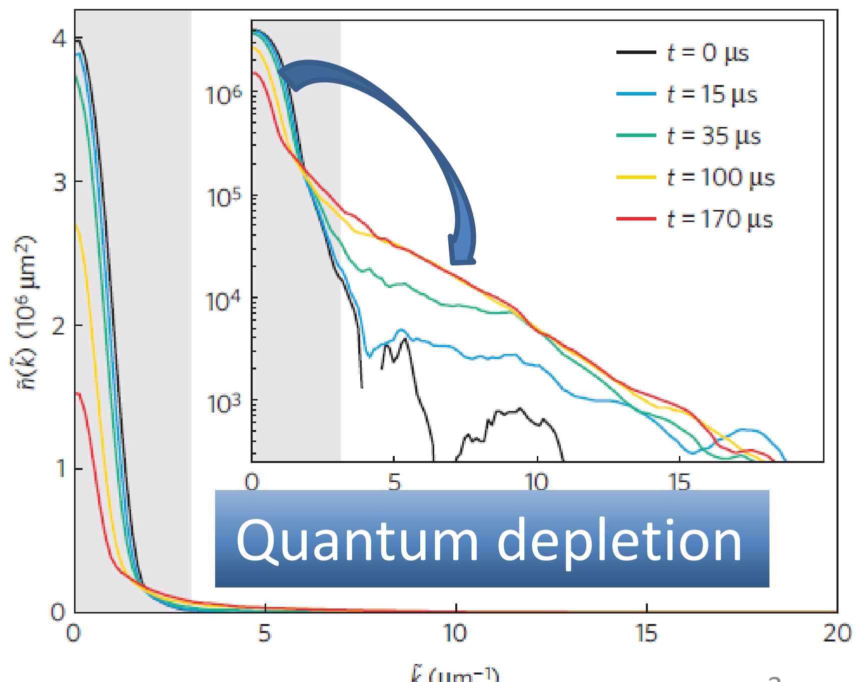
Switch a BEC very fast from weak to infinity interaction

Weak interaction (time <0)



Strong interaction (time >0)

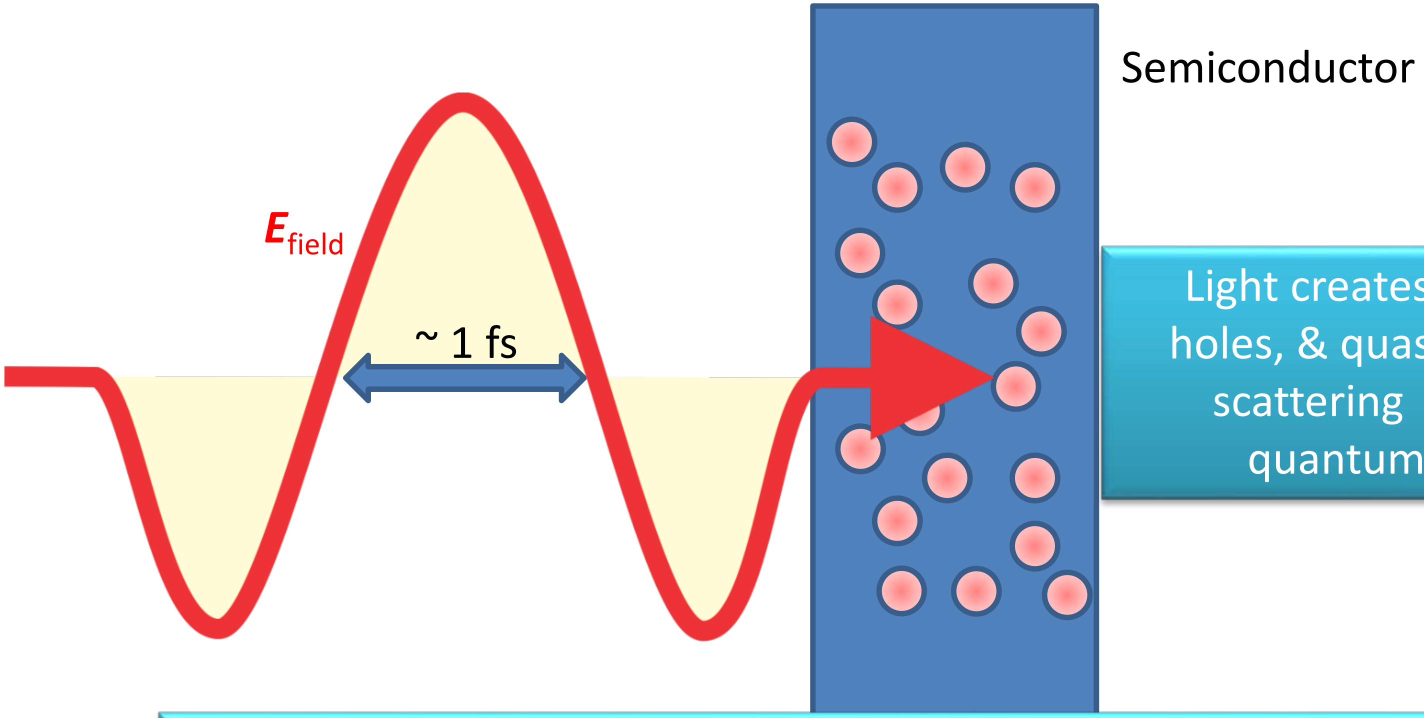




Need for a new approach!!!
Could semiconductor theory work?

Makotyn *et al.* Nat. Phys. 10, 116 (2014)

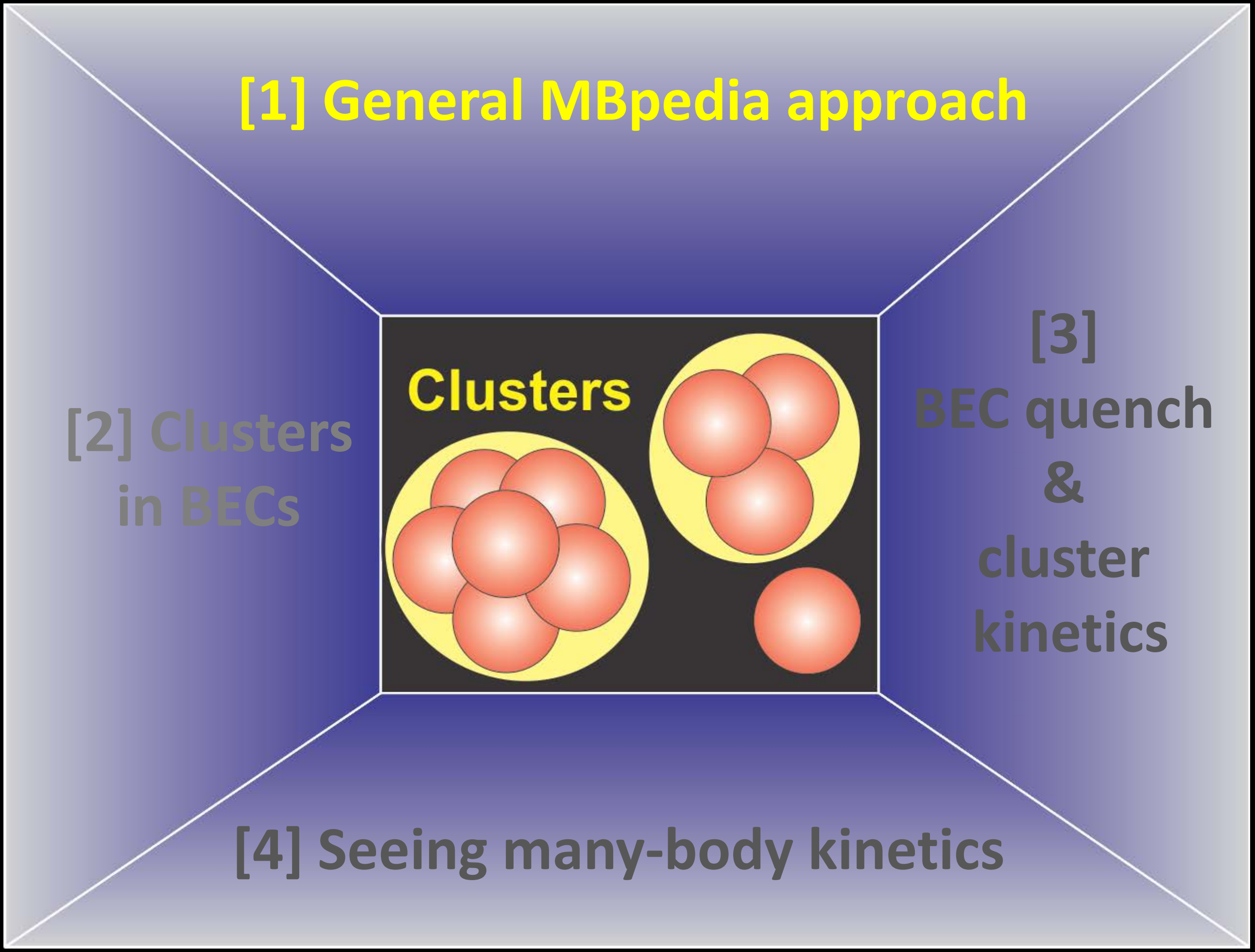
Ultrafast lightwave electronics



Light creates & moves electrons, holes, & quasiparticles faster than scattering ⬇ quantum coherent effects

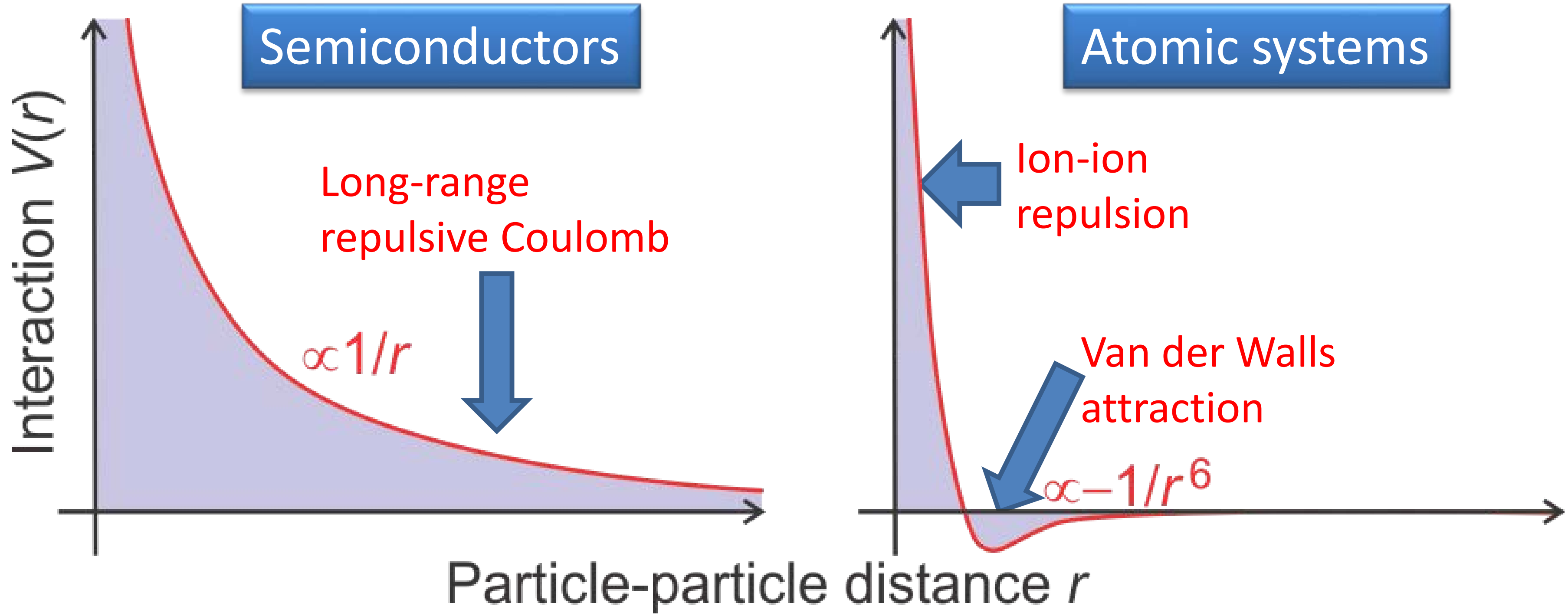
Qubit interests: 1) Is lightwave-driven quantum optoelectronics possible?
2) Can one single out quantum states/processes in a many-body system?

Topic [1 of 4]



[1] General MBpedia approach
[2] Clusters in BECs
Clusters
[3] BEC quench & cluster kinetics
[4] Seeing many-body kinetics

MBpedia [embedia] interactions



Electrons repel each other over long distances

- Close atoms attract/repel each other
- Contact-potential approximation possible

+ Photon-, phonon- or RF-matter = small particle + long-wave dipole interaction

Cluster-expansion approach

1) System Hamiltonian via $\hat{\Psi}(\mathbf{r})$ for fermionic/bosonic particles

$$\hat{H}_{\mathrm{sys}} = \int d^3r\, \hat{\Psi}^\dagger(\mathbf{r}) \left[\frac{\hat{\mathbf{p}}^2}{2m} + U(\mathbf{r})\right] \hat{\Psi}(\mathbf{r})$$

Single-particle problems

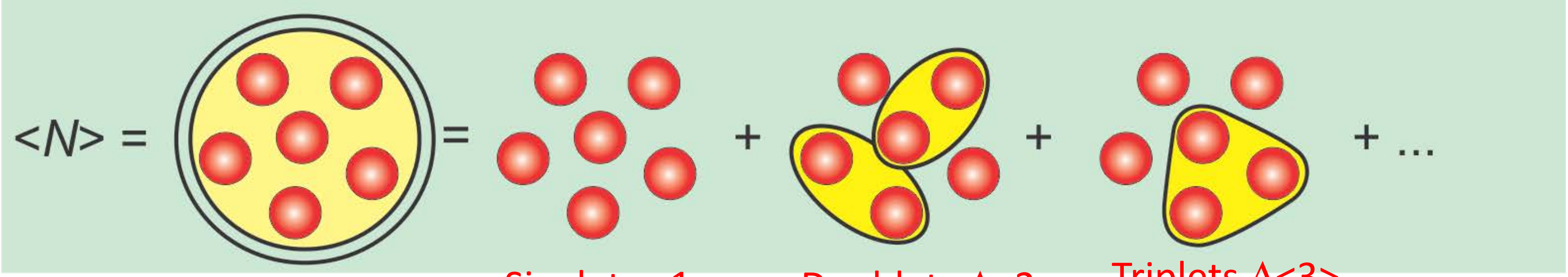
MB dynamics

Particle-particle interaction

$$+\frac{1}{2}\int d^3r_1\, d^3r_2\, \hat{\Psi}^\dagger(\mathbf{r}_1)\hat{\Psi}^\dagger(\mathbf{r}_2)V(\mathbf{r}_1-\mathbf{r}_2)\hat{\Psi}(\mathbf{r}_2)\hat{\Psi}(\mathbf{r}_1)$$

+ light-matter, matter-lattice vibration (phonon), RF field-matter interactions

Exact **identification of connected particle *clusters*:**



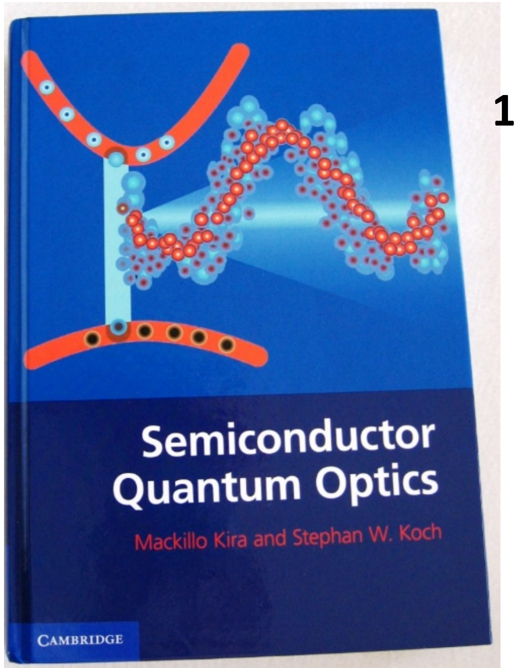
Singlets <1> Doublets Δ<2> Triplets Δ<3>

Cluster *C* = set of *C* correlated particles = **quasiparticle**

Quasiparticles drive all system properties

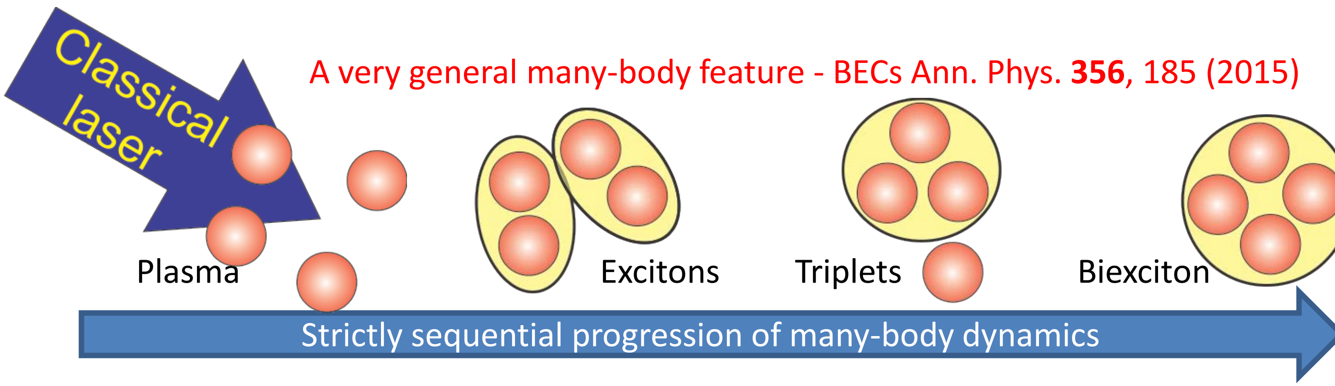
(from dimers/excitons and trimers/dropletons…. to entanglement).

MBpedia language: Quantum-dynamic cluster expansion (QDCE)



1) Matrix elements define $\hat{H} = \hat{H}_{\mathrm{sp}} + \hat{H}_{\mathrm{Coul}} + \hat{H}_{\mathrm{phon}} + \hat{H}_{\mathrm{l-m}} + \cdots$

A very general many-body feature - BECs Ann. Phys. **356**, 185 (2015)



Sequential source

2) Cluster kinetics $i\hbar \frac{\partial}{\partial t}\Delta\langle C\rangle = \mathrm{F}_{\mathrm{1ple}}\left[\Delta\langle C\rangle\right] + S\left[\langle 1\rangle, \Delta\langle 2\rangle, \cdots, \Delta\langle C-1\rangle\right] + \mathrm{Hi}\left[\Delta\langle C+1\rangle\right]$

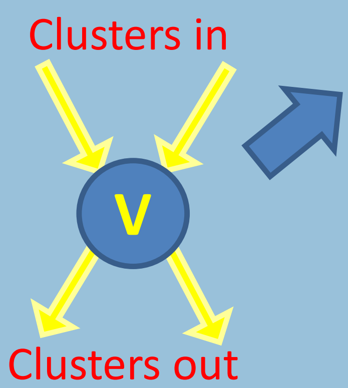
- exactly solvable & nonperturbative until Δ<C+1> cluster is formed
  ➡ 1st-principles approach to solve quantum kinetics

3) QDCE Ideal approach for determining how many-body effects evolve after a quench or ultrafast excitation

Typical 1st-principles cluster dynamics

$$S_{0,3}^{\mathbf{k}',\mathbf{k}} \equiv \frac{V_{\mathbf{k}}}{2}\left(\sqrt{N_C}\,(1+f_{\mathbf{k}}+s_{\mathbf{k}})(s_{\mathbf{k}'}+s_{\mathbf{k}+\mathbf{k}'}) + \sqrt{N_C}\,f_{\mathbf{k}'}\,s^{\star}_{\mathbf{k}+\mathbf{k}'} + \sqrt{N_C}\,f_{\mathbf{k}+\mathbf{k}'}\,s^{\star}_{\mathbf{k}'}\right). \qquad (C.5)$$

The full correlation dynamics becomes



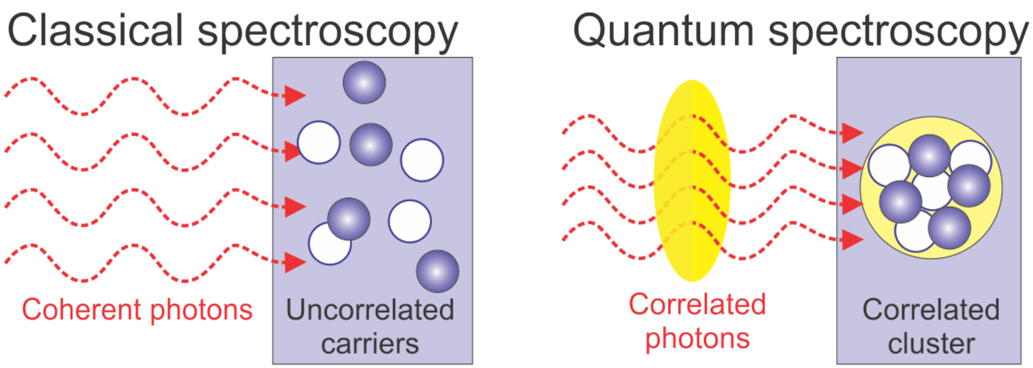
$$\begin{aligned} i\hbar\frac{\partial}{\partial t}T_{0,3}^{\mathbf{k}',\mathbf{k}} = {} & \left(E_{\mathbf{k}}^{\mathrm{ren}}+E_{\mathbf{k}'}^{\mathrm{ren}}+E_{\mathbf{k}+\mathbf{k}'}^{\mathrm{ren}}\right)T_{0,3}^{\mathbf{k}',\mathbf{k}} + \Delta_{\mathbf{k}}^{\mathrm{ren}}\,T_{2,1}^{\mathbf{k}',-\mathbf{k}-\mathbf{k}'} + \Delta_{\mathbf{k}'}^{\mathrm{ren}}\,T_{2,1}^{-\mathbf{k}-\mathbf{k}',\mathbf{k}} - \Delta_{\mathbf{k}+\mathbf{k}'}^{\mathrm{ren}}\,T_{2,1}^{\mathbf{k}',\mathbf{k}} \\ & + S_{0,3}^{\mathbf{k}',\mathbf{k}} + S_{0,3}^{\mathbf{k}',\mathbf{k}} + S_{0,3}^{\mathbf{k}',-\mathbf{k}-\mathbf{k}'} + (1+f_{\mathbf{k}}+f_{\mathbf{k}+\mathbf{k}'})\sum_{\mathbf{l}} V_{\mathbf{l}-\mathbf{k}}\,T_{0,3}^{\mathbf{k}',\mathbf{l}} \\ & + (1+f_{\mathbf{k}'}+f_{\mathbf{k}+\mathbf{k}'})\sum_{\mathbf{l}} V_{\mathbf{l}-\mathbf{k}'}\,T_{0,3}^{\mathbf{l},\mathbf{k}'} + (1+f_{\mathbf{k}}+f_{\mathbf{k}'})\sum_{\mathbf{l}} V_{\mathbf{l}}\,T_{0,3}^{\mathbf{k}'+\mathbf{l},\mathbf{k}-\mathbf{l}} \\ & + s_{\mathbf{k}}\sum_{\mathbf{l}}\left[V_{\mathbf{l}+\mathbf{k}+\mathbf{k}'}\,T_{1,2}^{\mathbf{k}',\mathbf{l}'} + V_{\mathbf{l}-\mathbf{k}'}\,T_{1,2}^{-\mathbf{k}-\mathbf{k}',\mathbf{l}}\right] + s_{\mathbf{k}'}\sum_{\mathbf{l}}\left[V_{\mathbf{l}+\mathbf{k}+\mathbf{k}'}\,T_{1,2}^{\mathbf{l},\mathbf{k}}\right. \\ & \left. + V_{\mathbf{l}-\mathbf{k}}\,T_{1,2}^{-\mathbf{k}-\mathbf{k}',\mathbf{l}}\right] + s_{\mathbf{k}+\mathbf{k}'}\sum_{\mathbf{l}}\left[V_{\mathbf{l}-\mathbf{k}}\,T_{1,2}^{\mathbf{k}',\mathbf{l}} + V_{\mathbf{l}-\mathbf{k}'}\,T_{1,2}^{\mathbf{l},\mathbf{k}}\right] \\ & + V_{\mathbf{k}}\,(s_{\mathbf{k}'}+s_{\mathbf{k}+\mathbf{k}'})\sum_{\mathbf{l}} T_{1,2}^{\mathbf{l},\mathbf{k}} + V_{\mathbf{k}'}\,(s_{\mathbf{k}}+s_{\mathbf{k}+\mathbf{k}'})\sum_{\mathbf{l}} T_{1,2}^{\mathbf{k}',\mathbf{l}} \\ & + V_{\mathbf{k}+\mathbf{k}'}\,(s_{\mathbf{k}}+s_{\mathbf{k}'})\sum_{\mathbf{l}} T_{1,2}^{-\mathbf{k}-\mathbf{k}',\mathbf{l}} + \mathrm{Hi}_{0,3}^{\mathbf{k}',\mathbf{k}} + \mathrm{Hi}_{0,3}^{\mathbf{k},\mathbf{k}'} + \mathrm{Hi}_{0,3}^{\mathbf{k}',-\mathbf{k}-\mathbf{k}'}, \end{aligned} \qquad (C.6)$$

Integro,
Non-perturbative

Ann. Phys. **356**, 185 (2015)

Some semiconductor examples

[1] Create entanglement directly with quantum spectroscopy:

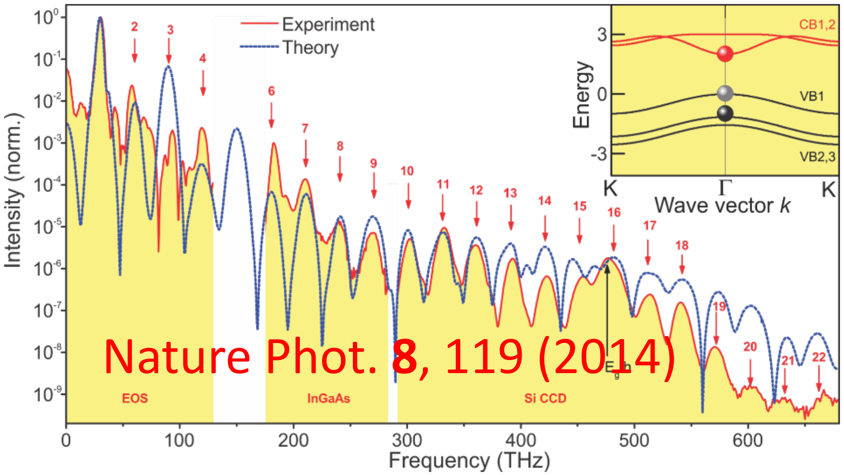


Kira et al. PRA **73**, 013813 (2006), Nature Phys. **7**, 799 (2011)

Dropletons

A GaAs study, Kira et al., Nature **506**, 471 (2014)

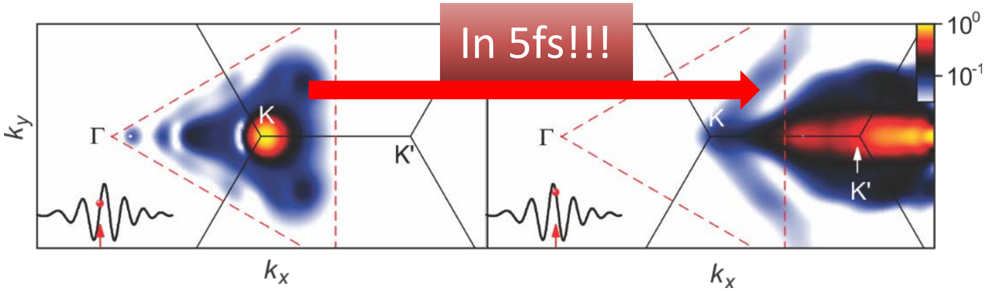
[3] High-harmonic generation in solids



Nature Phot. **8**, 119 (2014)

[2] Transport excitons with light:

A TMDC study, *Kira et al.* Nature **557**, 76 (2018)

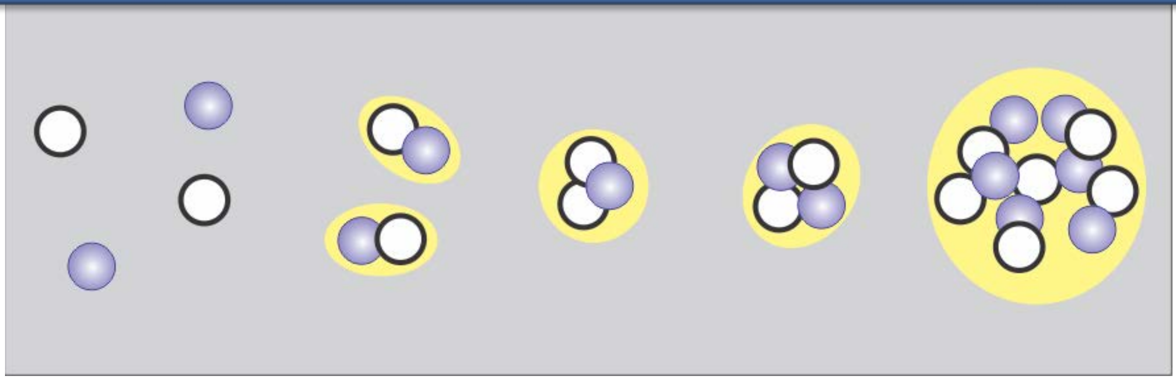


Quantum optoelectronics → PHz clock rates.

QDCE extremely efficient and accurate in describing relevant entanglement dynamics

MBpedia quasiparticles

Stable configurations of interacting particles = quasi-particles (clusters)

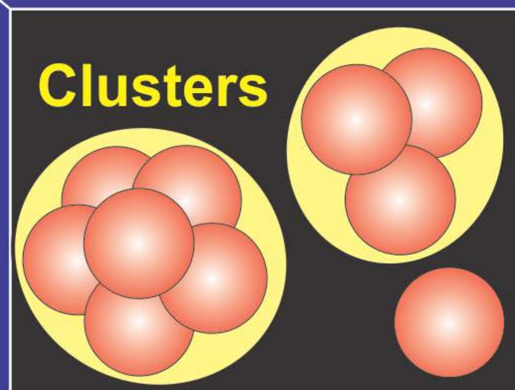


Clusters	semiconductors	atomic BECs	Quantum optics
singlets	Plasma = density & polarization	Not needed	Coherent state
doublets	*Excitons # Biexciton coherence	*Dimer # Bogoliubov excitations	* Thermal state # squeezed state, ~ Schrödinger's cat
triplets	* Dropletons Trion	*Efimov trimer	* Slanted cat
> 3	* Dropletons, polyexcitons	* tetramer	* Cubic cat, etc.

Topic [2 of 4]

[1] General MBpedia approach

[2] Clusters in BECs

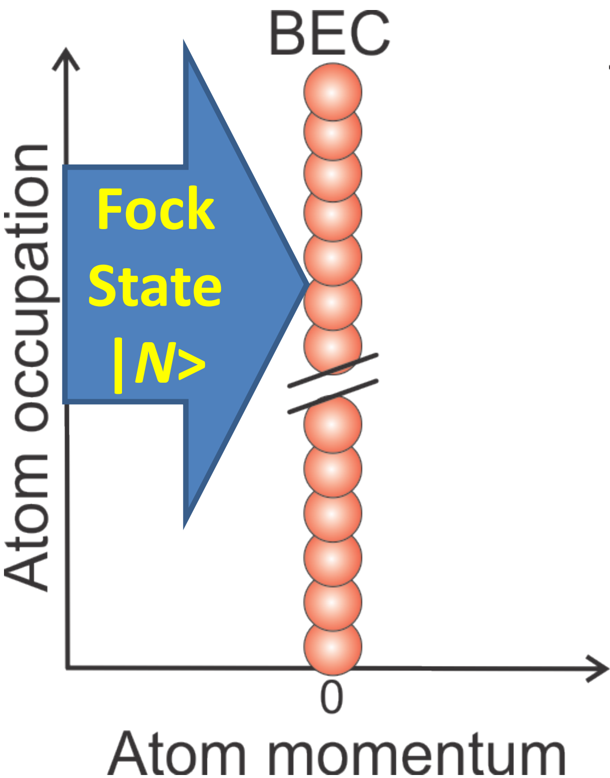


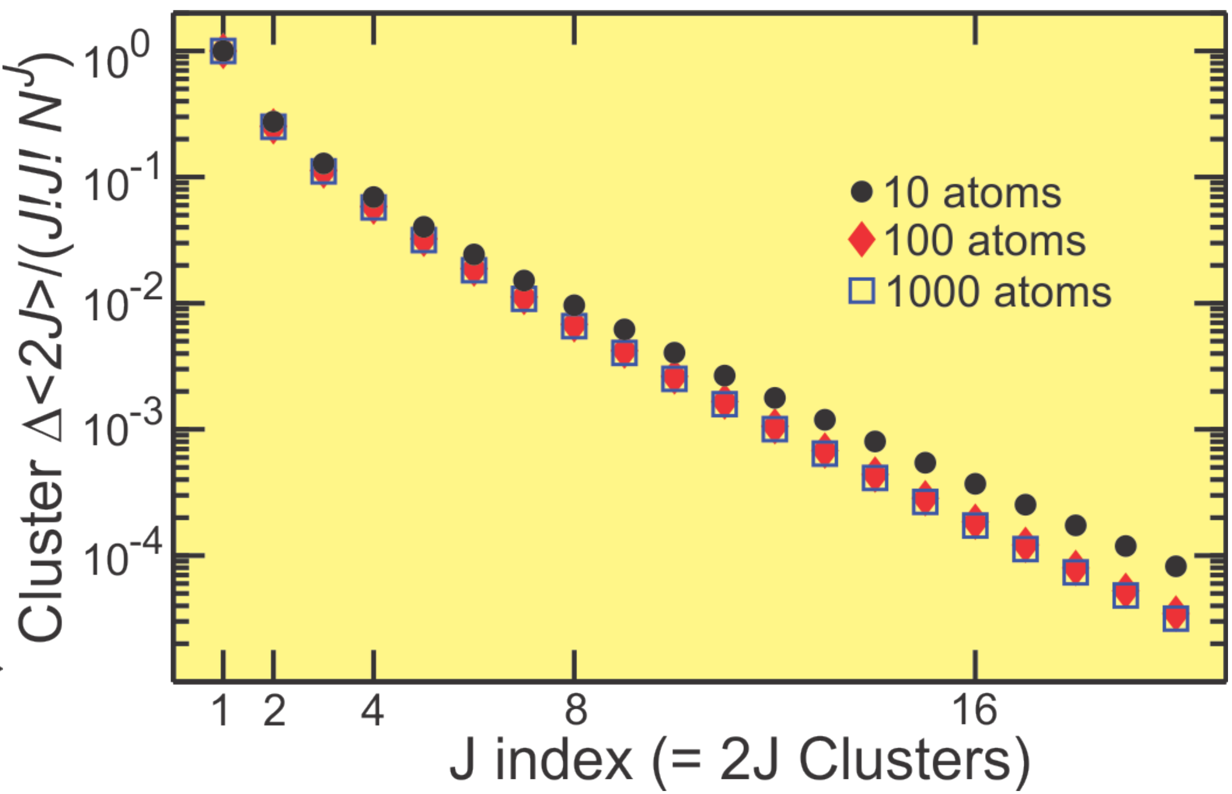
[3] BEC quench & cluster kinetics

[4] Seeing many-body kinetics

Clusters in an atomic BEC

- BEC @ 0K, no interactions, particle number is conserved

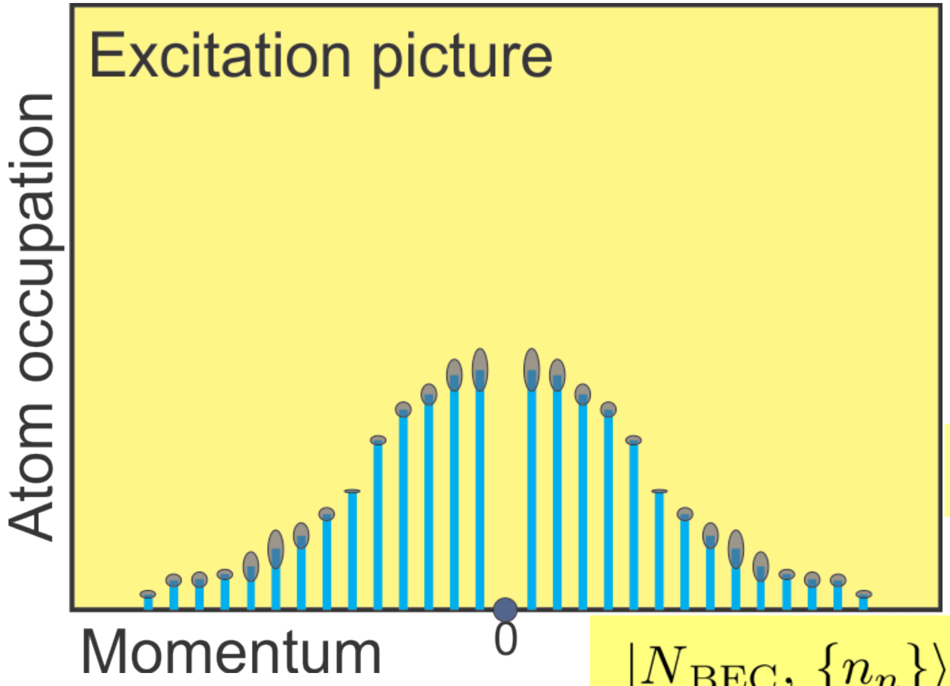




A BEC contains initially particle clusters to all orders → trouble for cluster-expansion approach

Quantum depletion in excitation picture

Quantum depletion = interactions eject BEC to non-condensed atoms



Total number $\mathcal{N}$ conserved

Rising BEC Operator: $\hat{L}^\dagger |N_{\rm BEC}-1\rangle = |N_{\rm BEC}\rangle$

Number operator for non-condensed atoms:

$$\hat{n}_n \,|\{n_n\}\rangle = n_n \,|\{n_n\}\rangle$$

Nonunitary transformation for basis states:

$$|N_{\rm BEC},\,\{n_n\}\rangle = [\hat{L}^\dagger]^{\mathcal{N}-\hat{n}_n}\,|0,\,\{n_n\}\rangle \equiv \hat{T}^\dagger_{\rm ex}\,|0,\,\{n_n\}\rangle$$

BEC converted to vacuum!!!

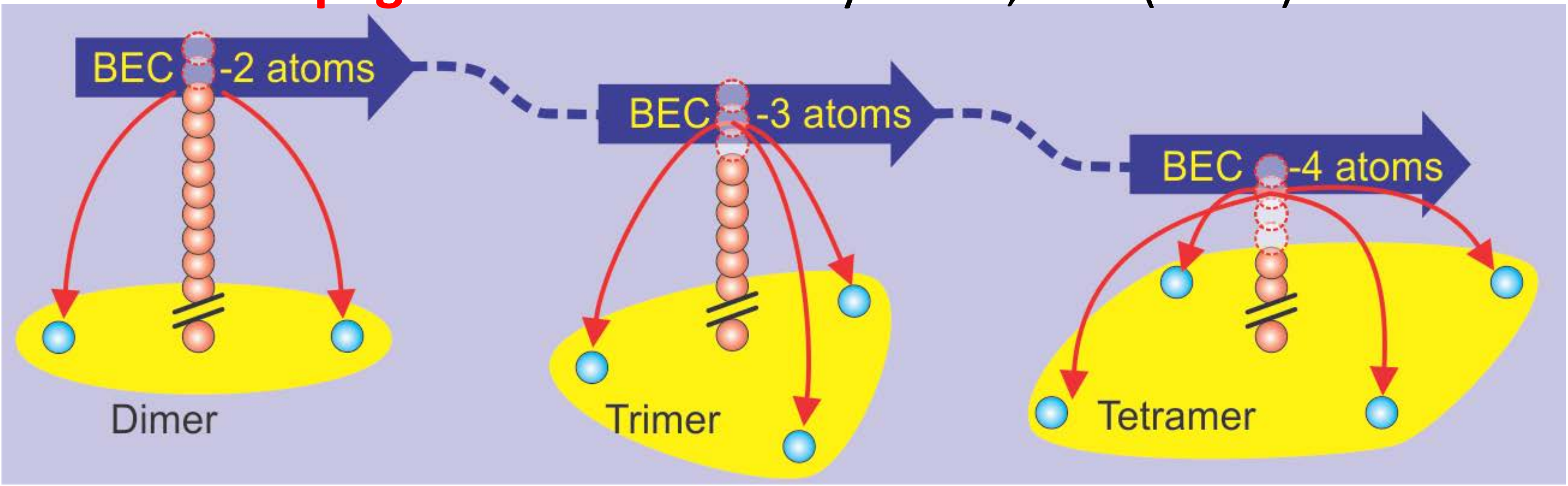
Excitation Picture: Ann. Phys. **351**, 200 (2014) [50 pages of fun!!!]

$$\hat{O}_{\rm ex} \equiv \hat{T}_{\rm ex}\,\hat{O}\,\hat{T}^\dagger_{\rm ex}$$

$$i\hbar\frac{\partial}{\partial t}\langle\hat{O}_{\rm ex}\rangle_{\rm ex} = \langle\left[\hat{O}_{\rm ex},\,\hat{H}_{\rm ex}\right]_-\rangle_{\rm ex}$$

Cluster dynamics in excitation picture

58-pages of fun: Ann. Phys. **356**, 185 (2015)



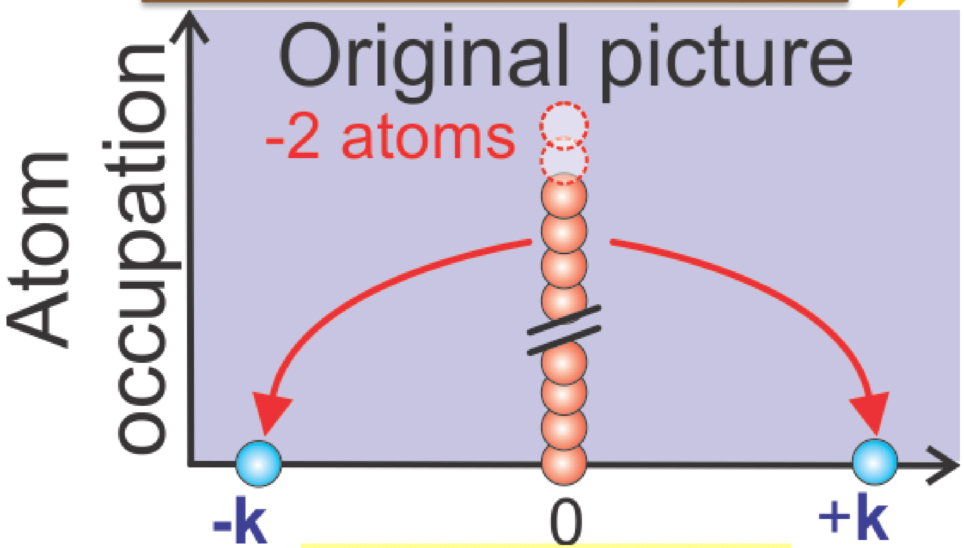
Quantum depletion creates non-condensed clusters sequentially

Nonperturbative truncation in terms of cluster

an "exact" description strongly interacting BECs

Elementary process of quantum depletion

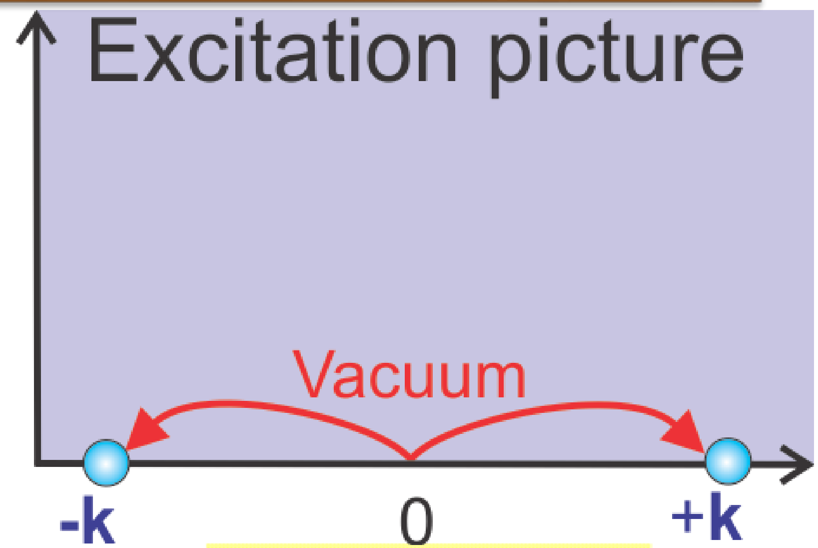
Momentum conservation ⇒ 2 BEC atoms ejected to +k & -k



$$s^{\star}_{\mathbf{k}} = \langle \hat{L}\,\hat{L}\,B^{\dagger}_{\mathbf{k}}B^{\dagger}_{-\mathbf{k}}\rangle$$

Transition amplitude contains all cluster orders $\hat{L} = \frac{1}{\sqrt{1+B^{\dagger}_{0}B_{0}}}B_{0}$

No simple description

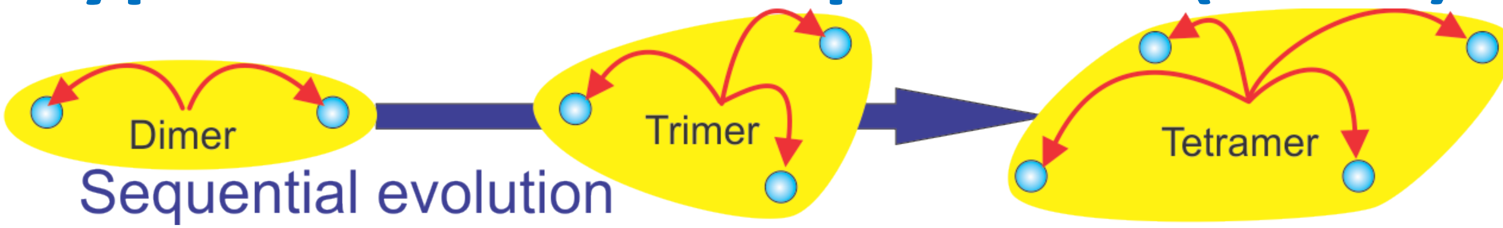


$$s^{\star}_{\mathbf{k}} = \langle B^{\dagger}_{\mathbf{k}}B^{\dagger}_{-\mathbf{k}}\rangle_{\mathrm{ex}}$$

Transition amplitude = doublet

BEC excites doublets
+ sequential cluster formation,
as in semiconductors!!!

Hyperbolic Bloch equations (HBEs)



Tracked by HBEs: 58-pages of fun, Ann. Phys. **356**, 185 (2015)

$$i\hbar\frac{\partial}{\partial t}s_{\mathbf{k}} = 2E_{\mathbf{k}}\,s_{\mathbf{k}} + (1+2f_{\mathbf{k}})\,\Delta_{\mathbf{k}} - 2i\gamma_{\mathbf{k}}$$

Coupling to triplets

$2E_{\mathbf{k}}$: pair energy $+\sum_{\mathbf{k}'} V_{\mathbf{k}'-\mathbf{k}}\, f_{\mathbf{k}'} - \mu$ ← E renorm.

$f_{\mathbf{k}}$: Atom occup.@ **k**

$\Delta_{\mathbf{k}}$: BEC atom number $N_{\mathrm{BEC}}\,V_{\mathbf{k}} + \sum_{\mathbf{k}'} V_{\mathbf{k}'-\mathbf{k}}\, s_{\mathbf{k}'}$

Semiconductor Bloch equations: Lindberg & Koch, PRB **38**, 3342 (1988)

$$i\hbar\frac{\partial}{\partial t}P_{\mathbf{k}} = E_{\mathbf{k}}\,P_{\mathbf{k}} - (1-f^{e}_{\mathbf{k}}-f^{h}_{\mathbf{k}})\,\Delta_{\mathbf{k}} - 2i\gamma_{\mathbf{k}}$$

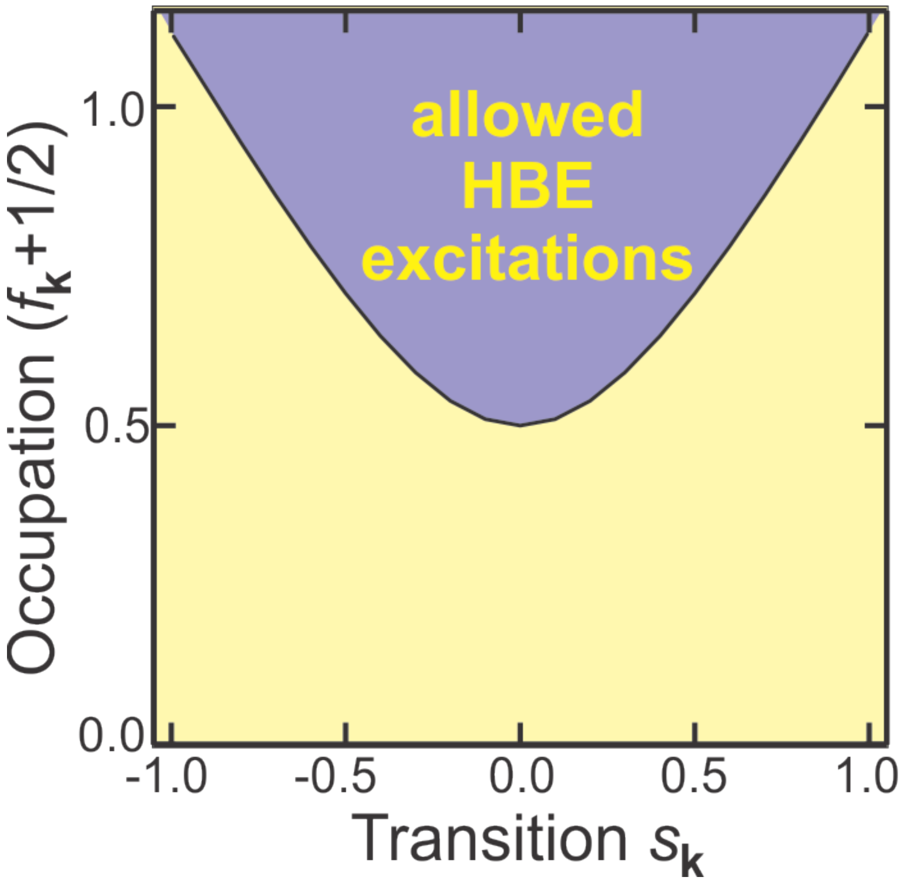
e-h occupations

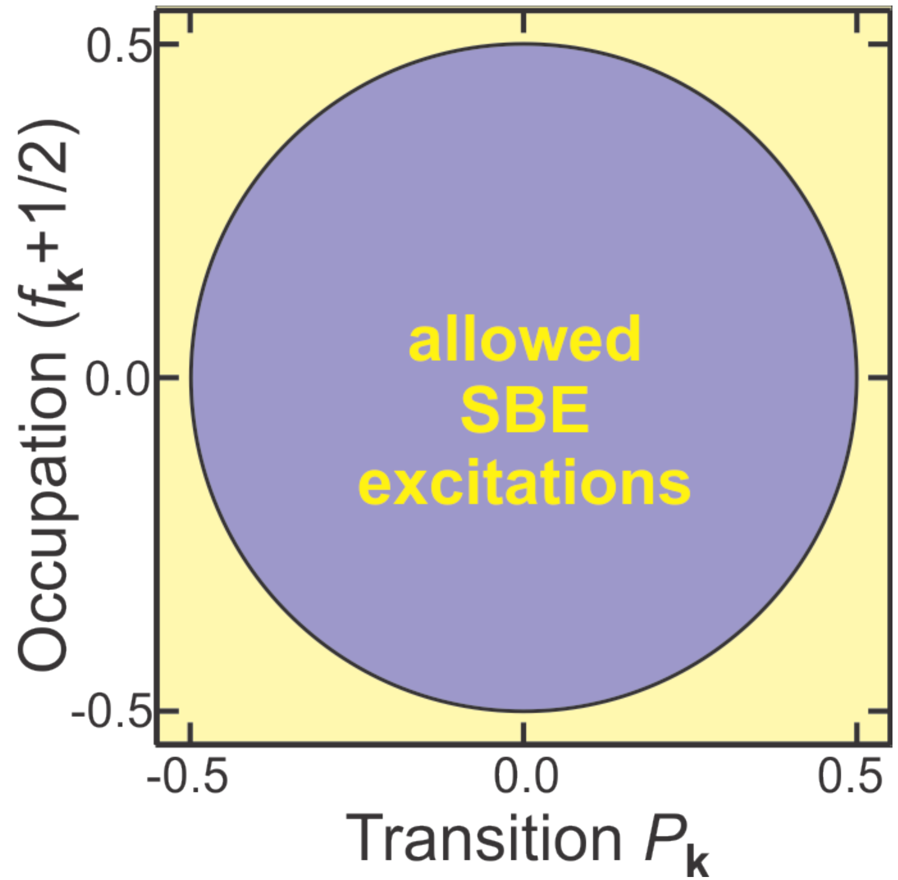
$\Delta_{\mathbf{k}}$: Optical field $d_{\mathrm{eh}}E(t) + \sum_{\mathbf{k}'} V_{\mathbf{k}'-\mathbf{k}}\, P_{\mathbf{k}'}$

HBE vs. SBE structure are analogous

N_{BEC} & $E(t)$ acts as analogous sources to a transition amplitude

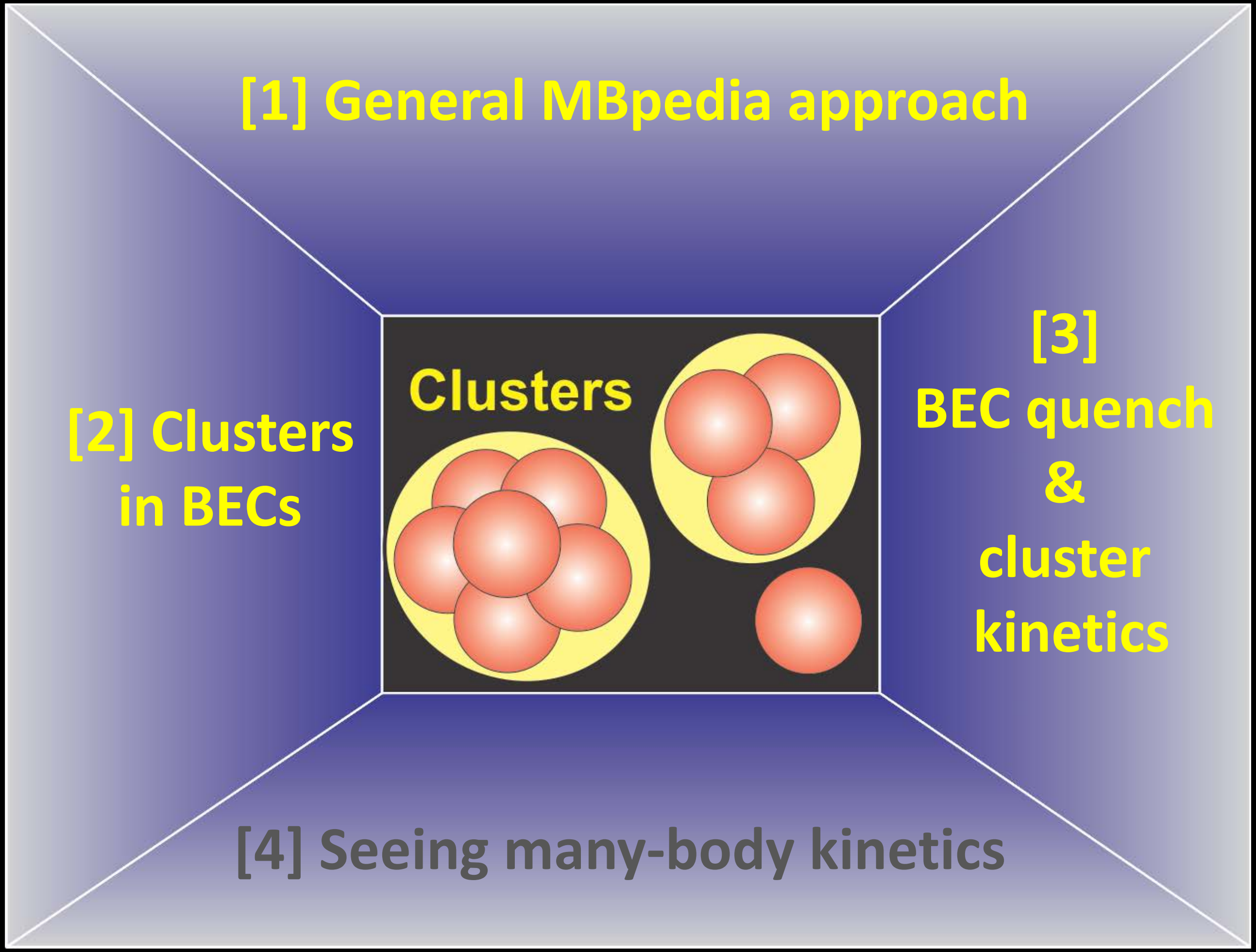
Geometry of HBE vs. SBE excitations





HBE excitations at outer rim of hyperbola
SBE excitations inside Bloch sphere

Topic [3 of 4]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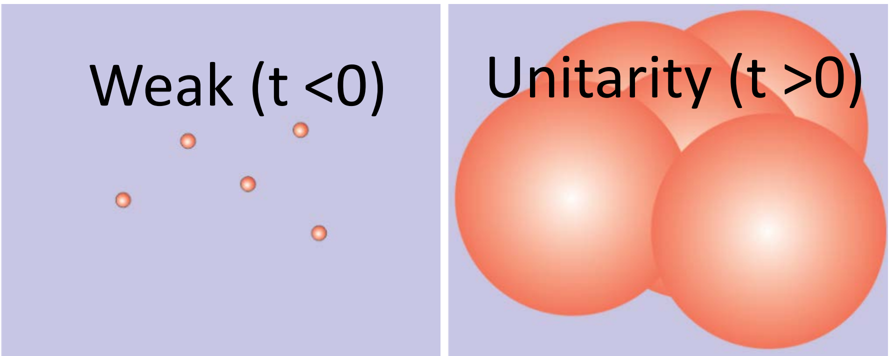
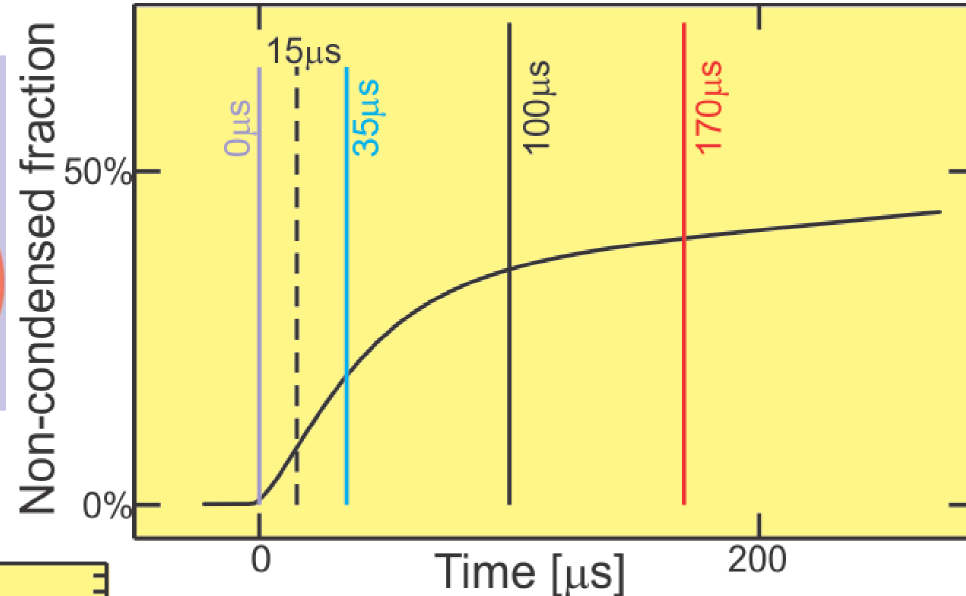
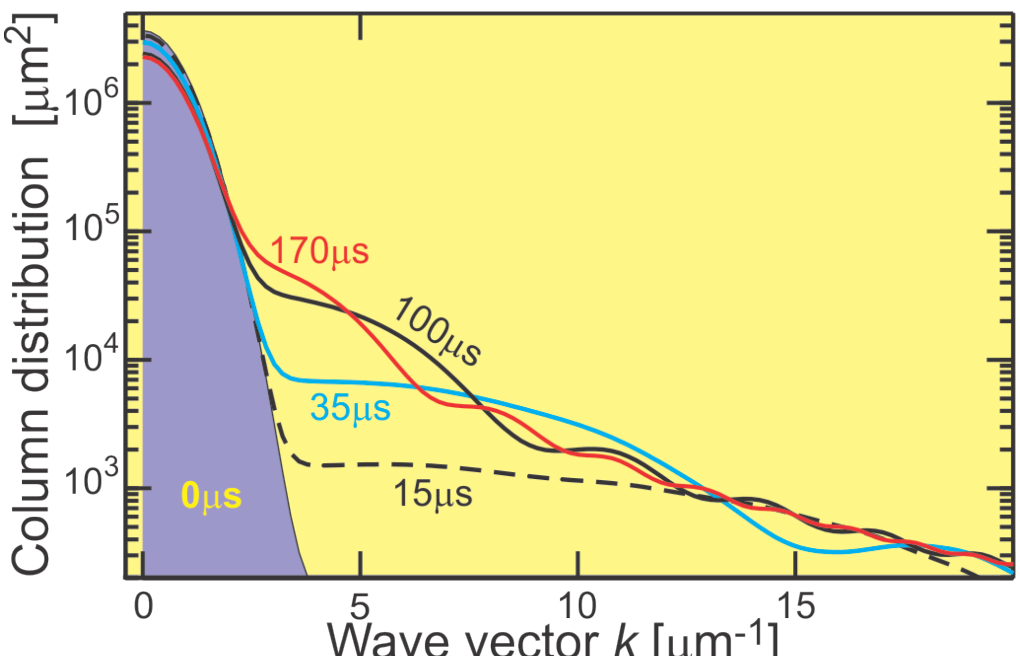
Explaining BEC quench with HBEs







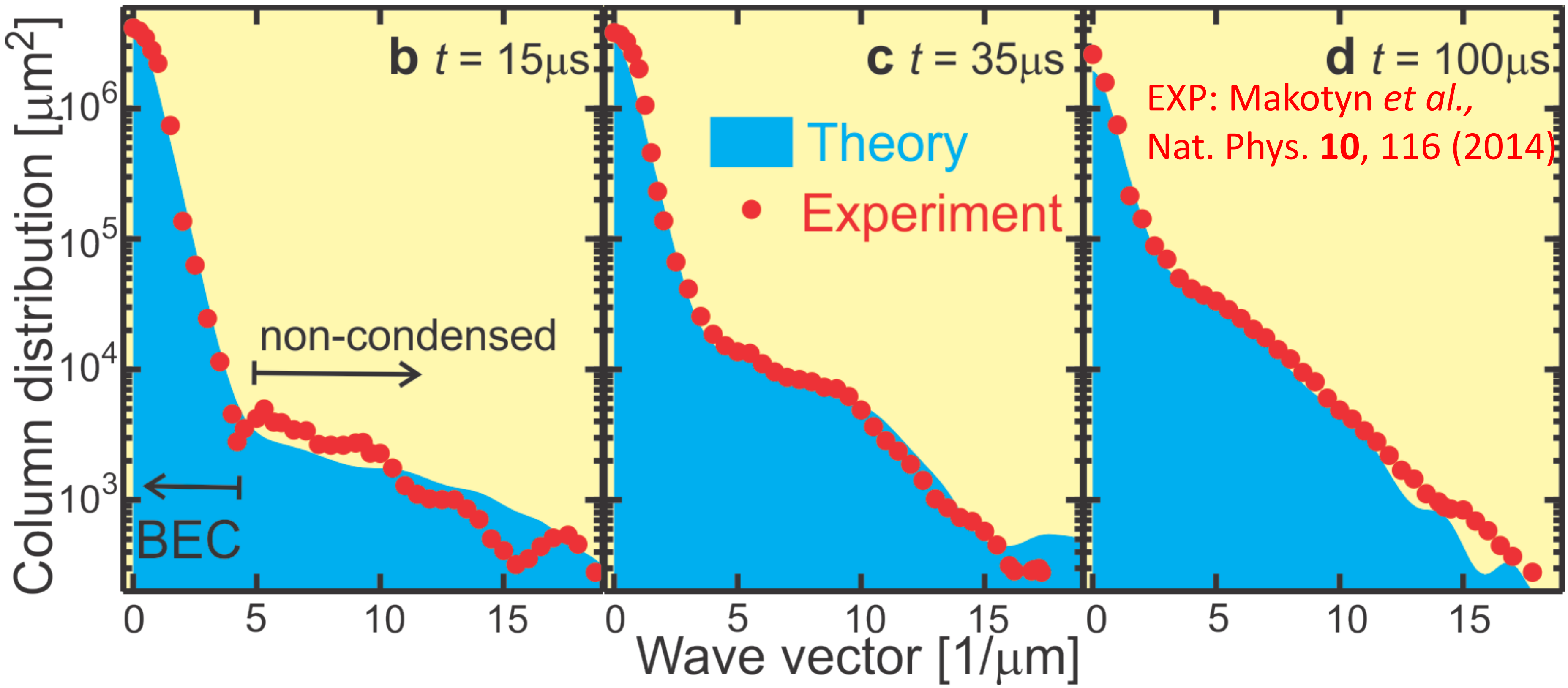
100μs scale quantum kinetics

BEC survives unitarity

Nat. Comm. **6**, 6624 (2015)

HBEs get quantitative

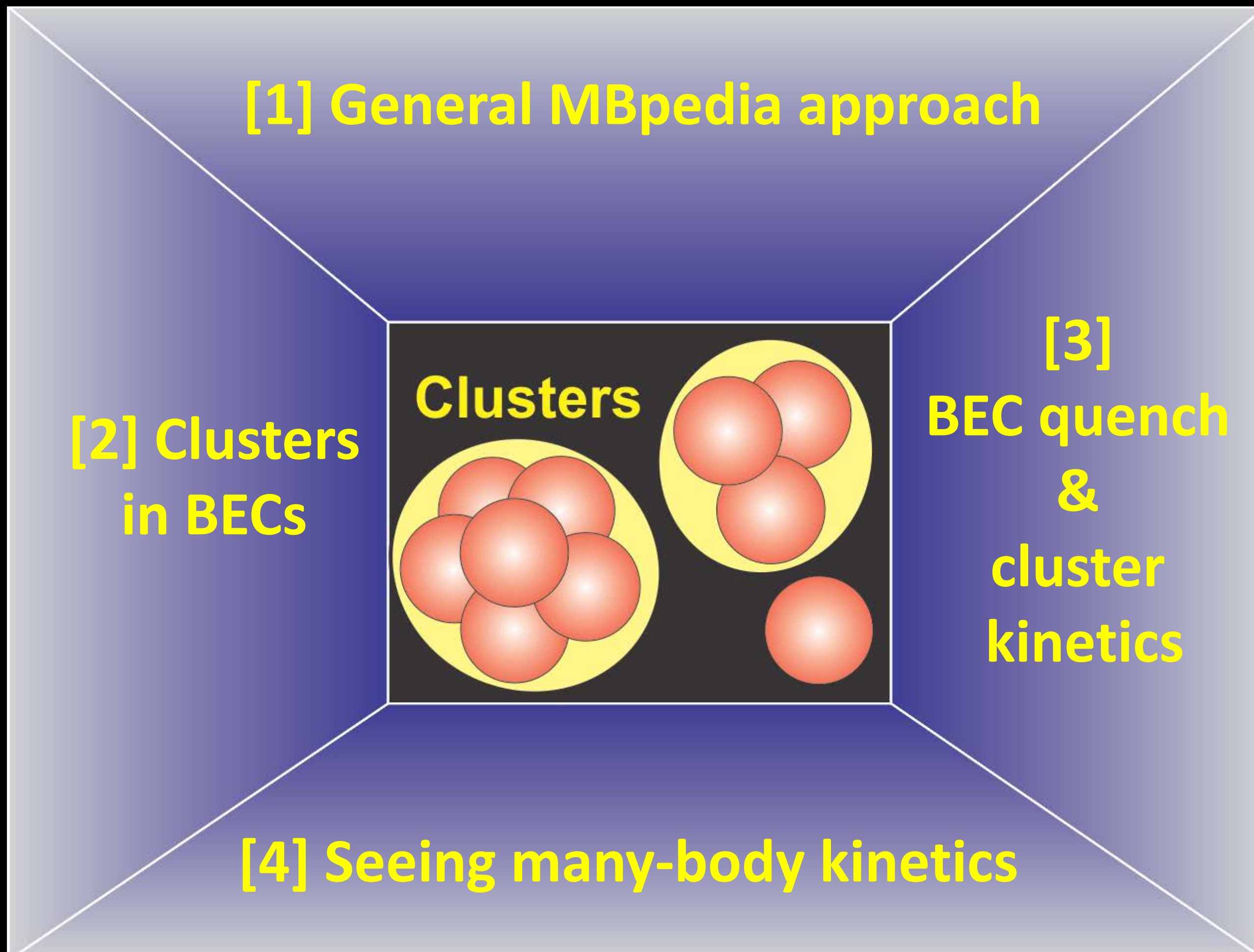
Experimental vs. HBE (column) distributions in same absolute units



Switch to unitarity explained quantitatively by doublet HBEs

Coherent s_k and f_k dominate until >100μs, Nat. Comm. **6**, 6624 (2015)

Topic [4 of 4]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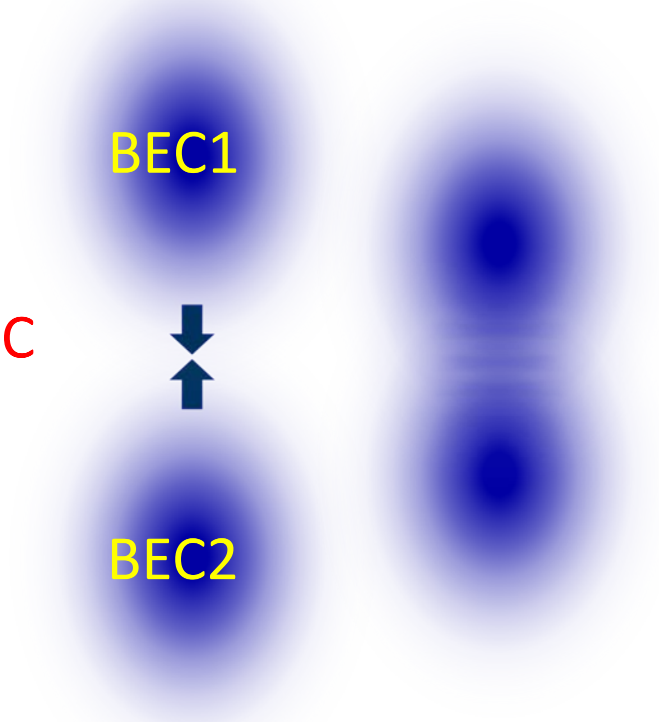
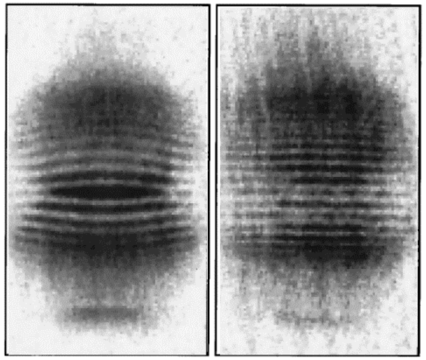
Macroscopic quantum classic, *Science* **275**, 637 (1997)

The final nail that convinced BEC skeptics:

Collide 2 BEC



Ketterle group,
Science **275**, 637 (1997)

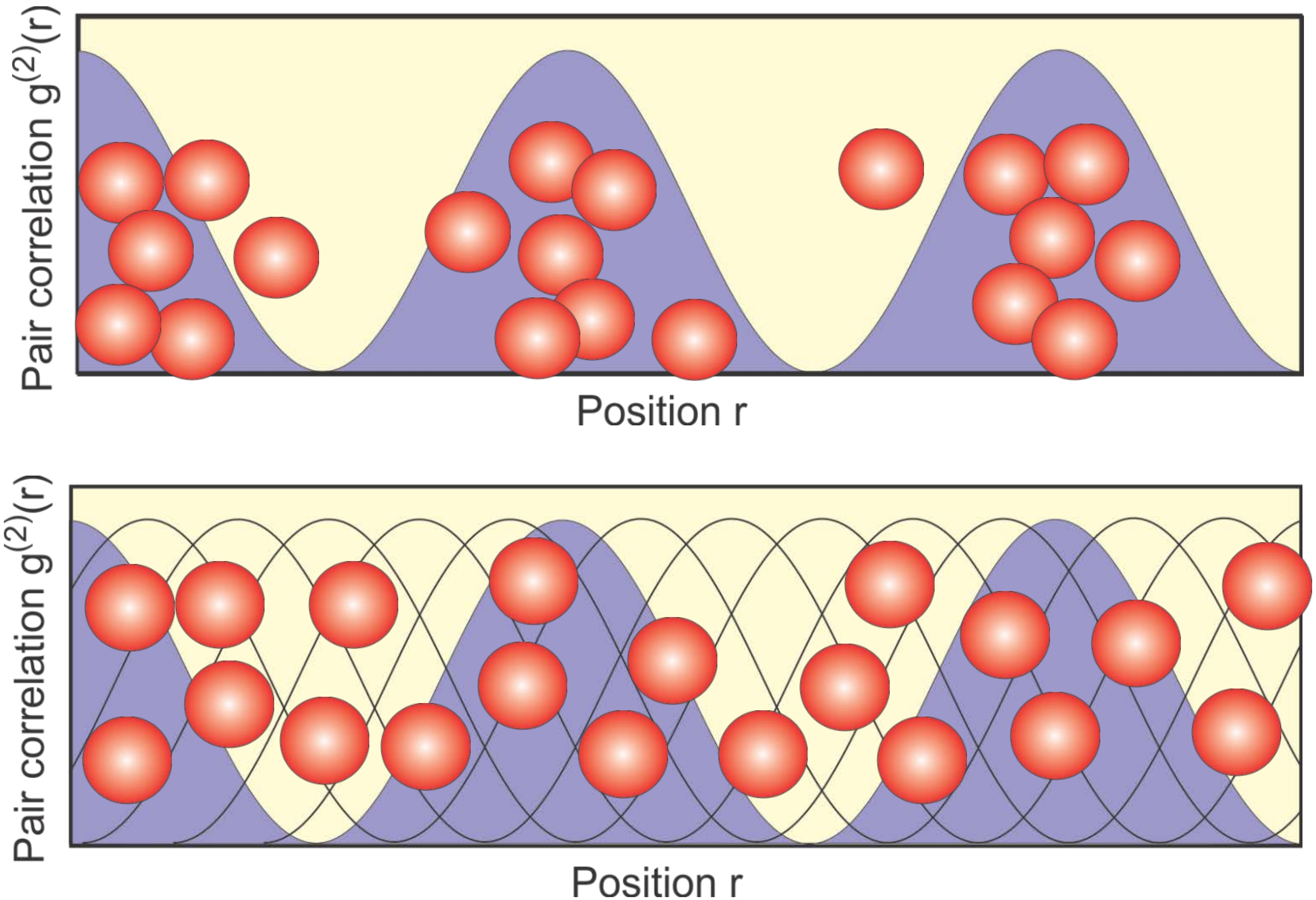


Macroscopic coherence creates particle interference
BUT only in a single-shot experiment!!!

Single-shot vs. averaged measurements

Single-shot experiment

Repetitive experiment



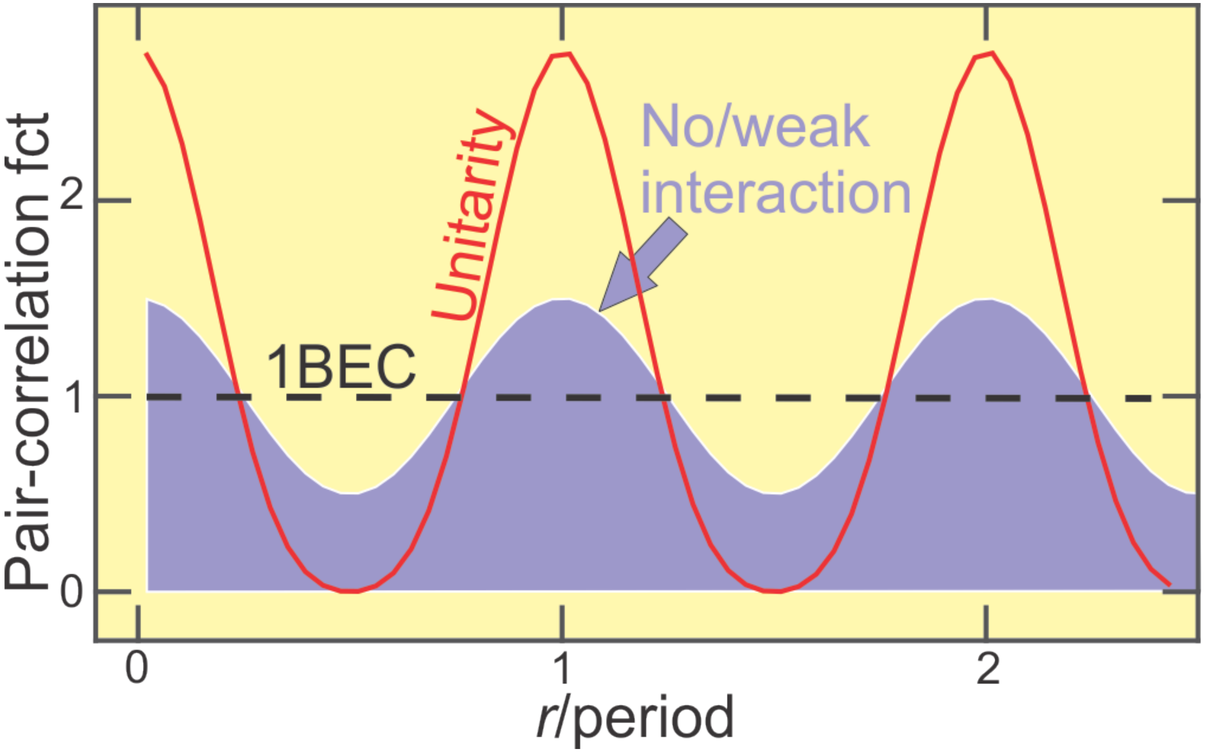
Single-shot measurement can resolve correlations

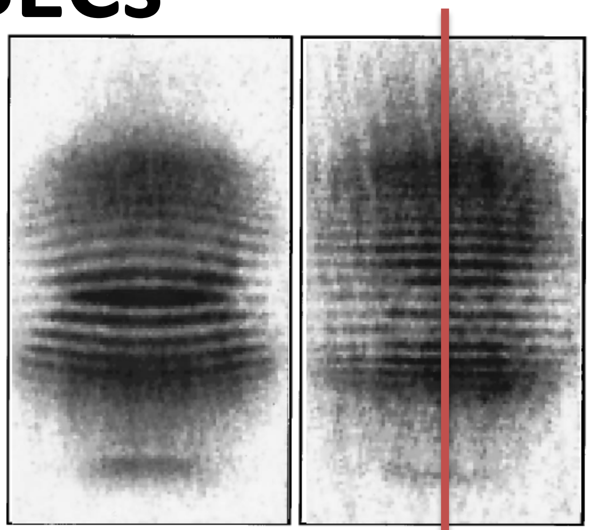
Quench of double BECs

+ħK BEC collides with -ħK BEC

+ unitary interactions

HBE computation: quench during collision



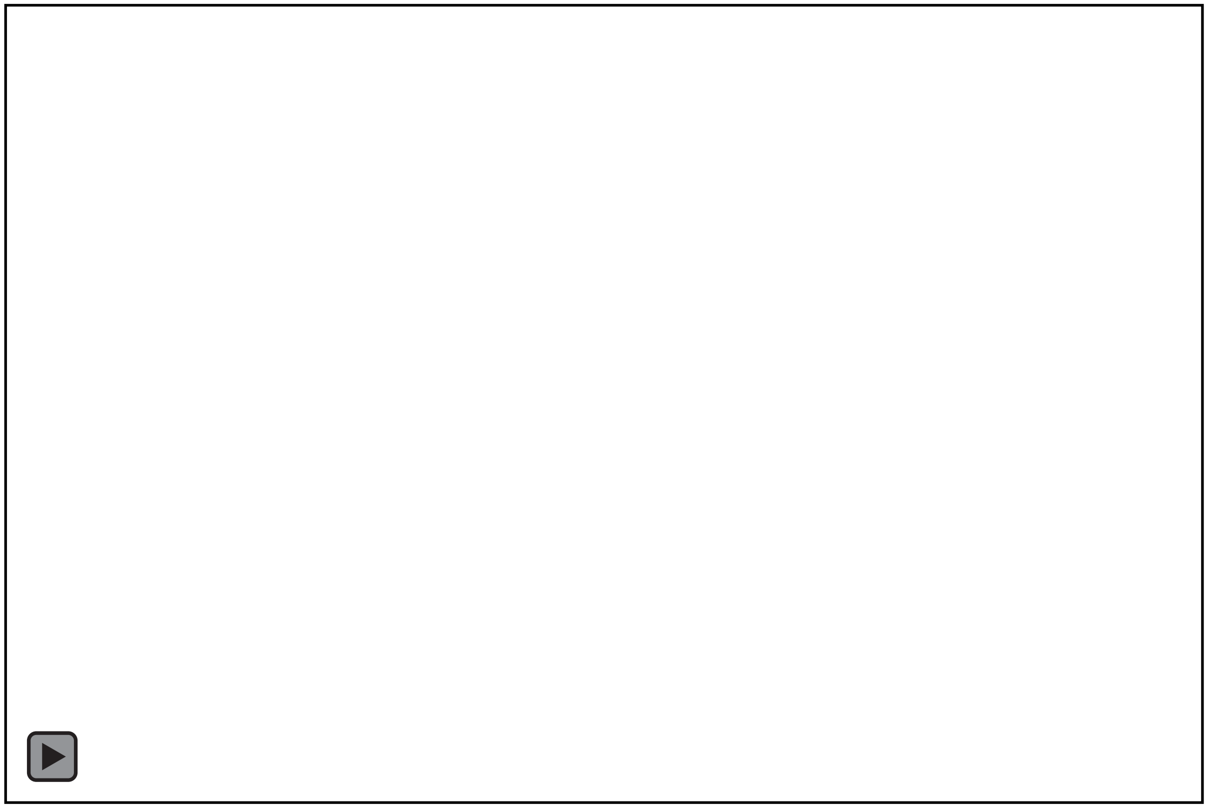


Follow pair-correlation "along a line"

Real-space signatures of interactions:
- Increased interference & shape changes

Correlations in semiconductors

Histogram separations of 2 electron-hole pairs: see $g^{(4)}$



Need:

- N-body resolving detectors
- Single-shot recording

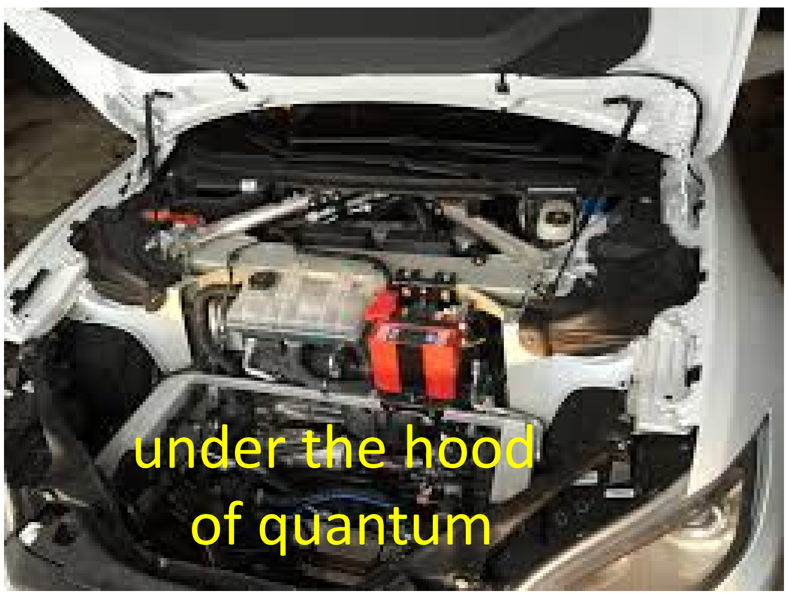
Access to entanglement information

New/Future entries to Manybodypedia

Unitarity BEC can be described with semiconductor methods:

- Need excitation picture for rigorous derivations
- Then, clusters build up truly sequentially
- HBEs quantitatively explain recent fast-switch experiments in unitary BECs
- **Future:** triplets, Efimov physics, screening, FWM, quantum spectroscopy,…

See many-body world directly via single-shot experiments:



Many-body state measurements

- seeing many-body dynamics,
- seeing entanglement.

↓

Big^{QST} data challenge

QST = Quantum Science & Technology

Thank You!