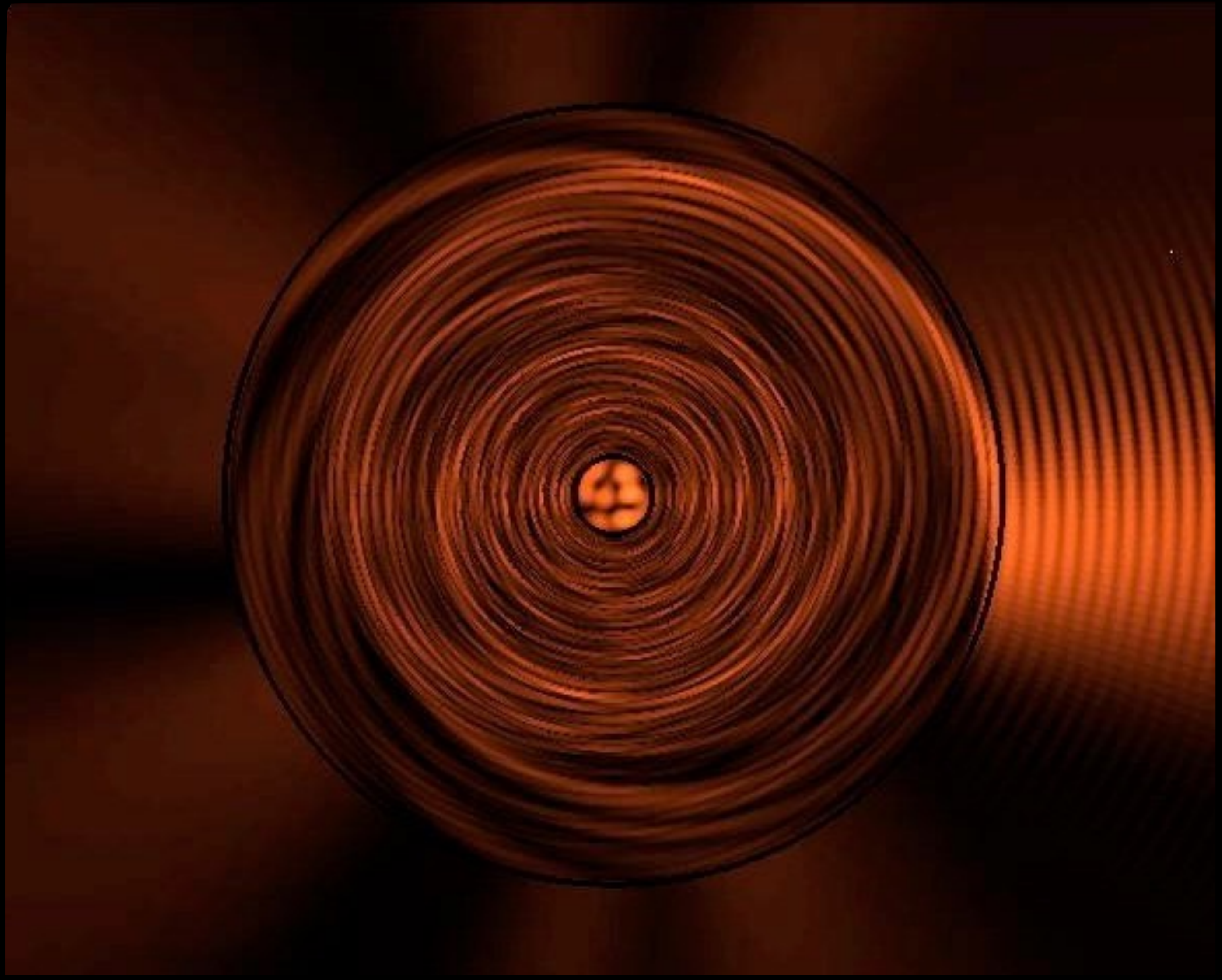
Optical Hyperspace:
light in hyperbolic materials

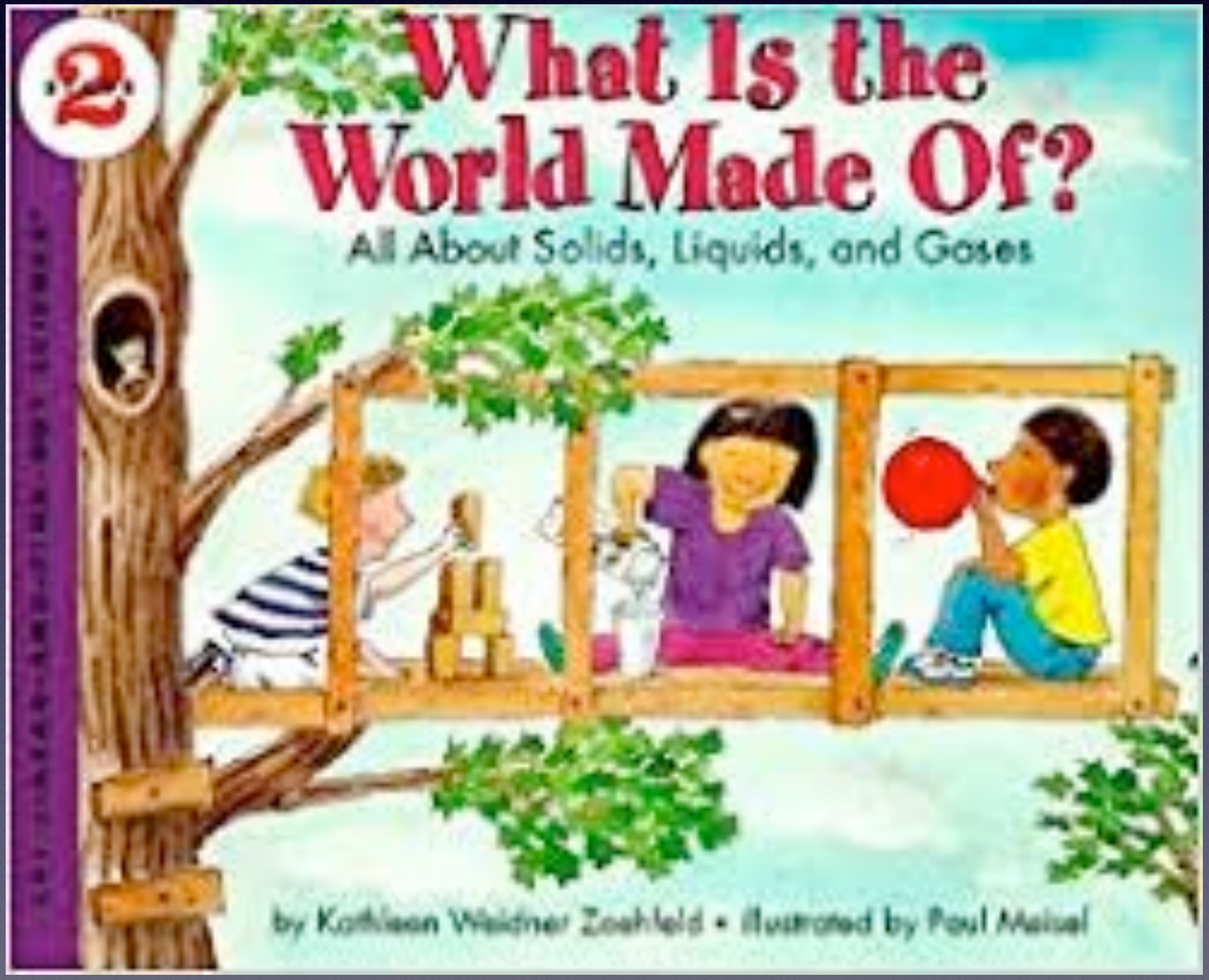
Evgenii Narimanov
ECE



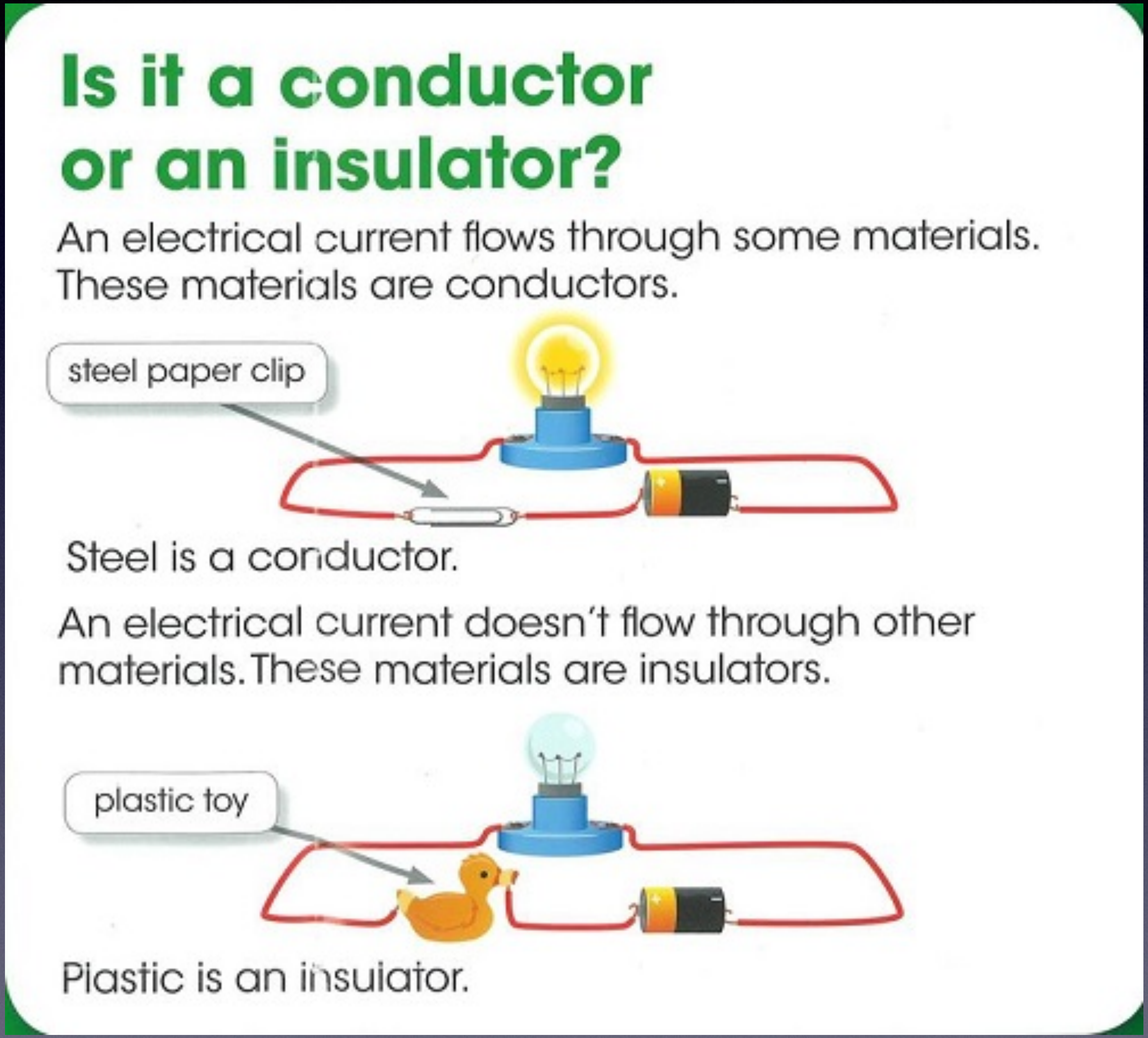
“... for my purpose holds to sail beyond the sunset ...”

Alfred Tennison, *Ulysses*

States of Matter

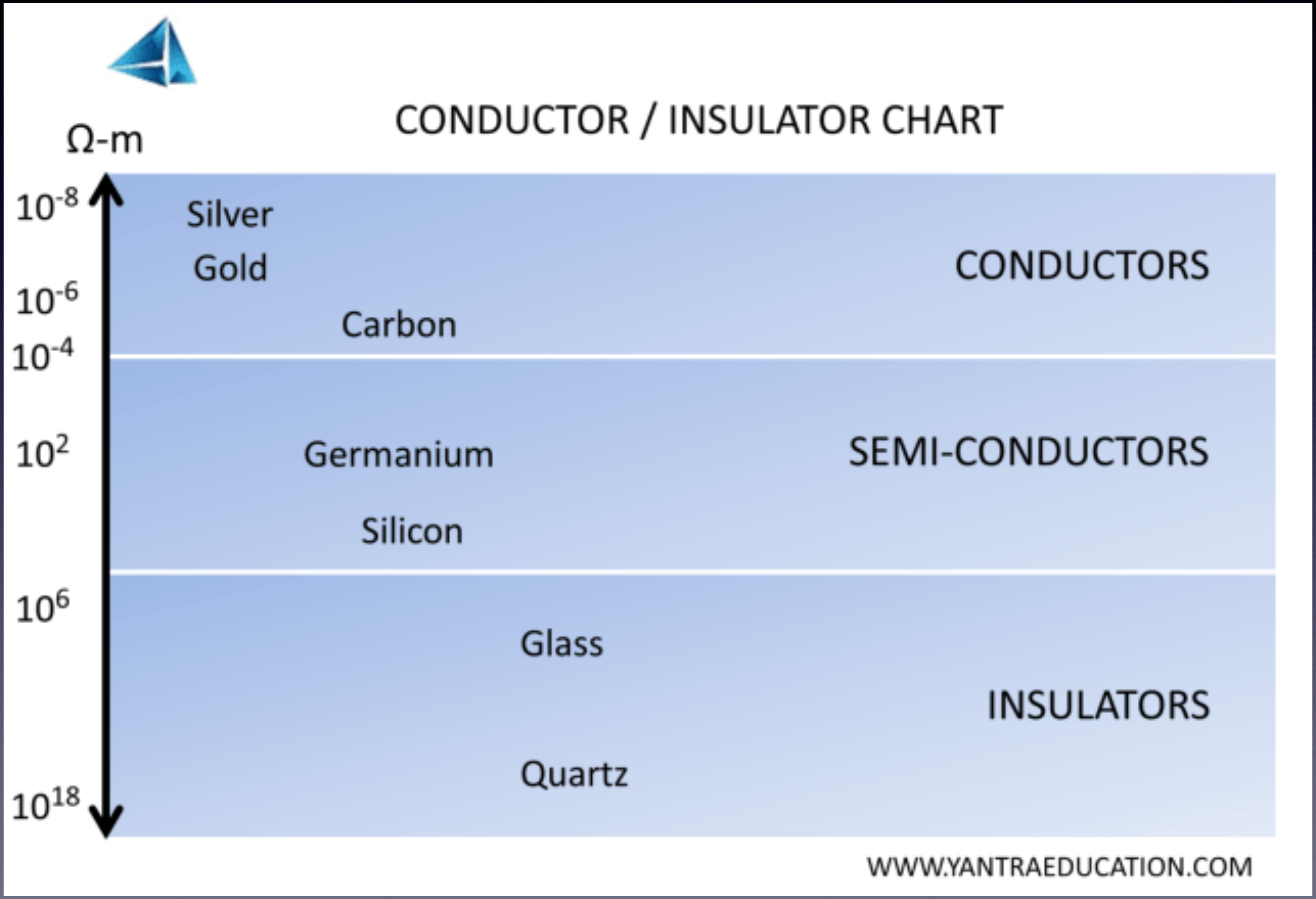


Conductors vs. Insulators

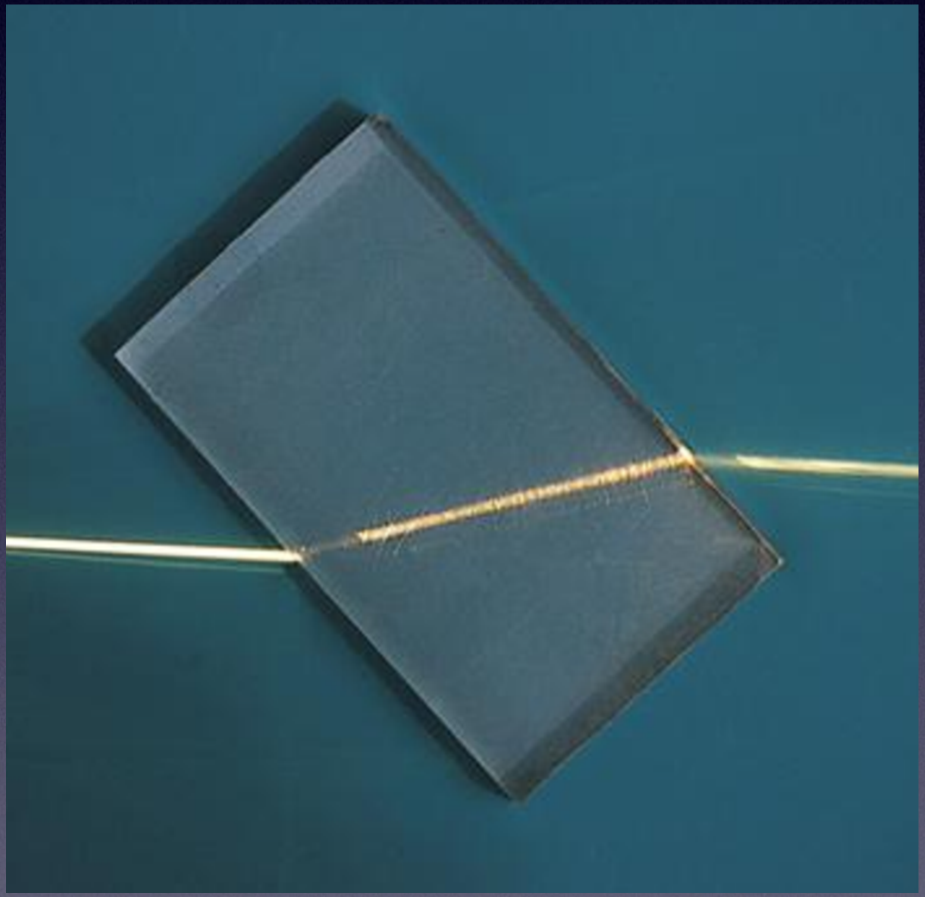


Sagar Khillar. "Difference between Conductors and Insulators." DifferenceBetween.net

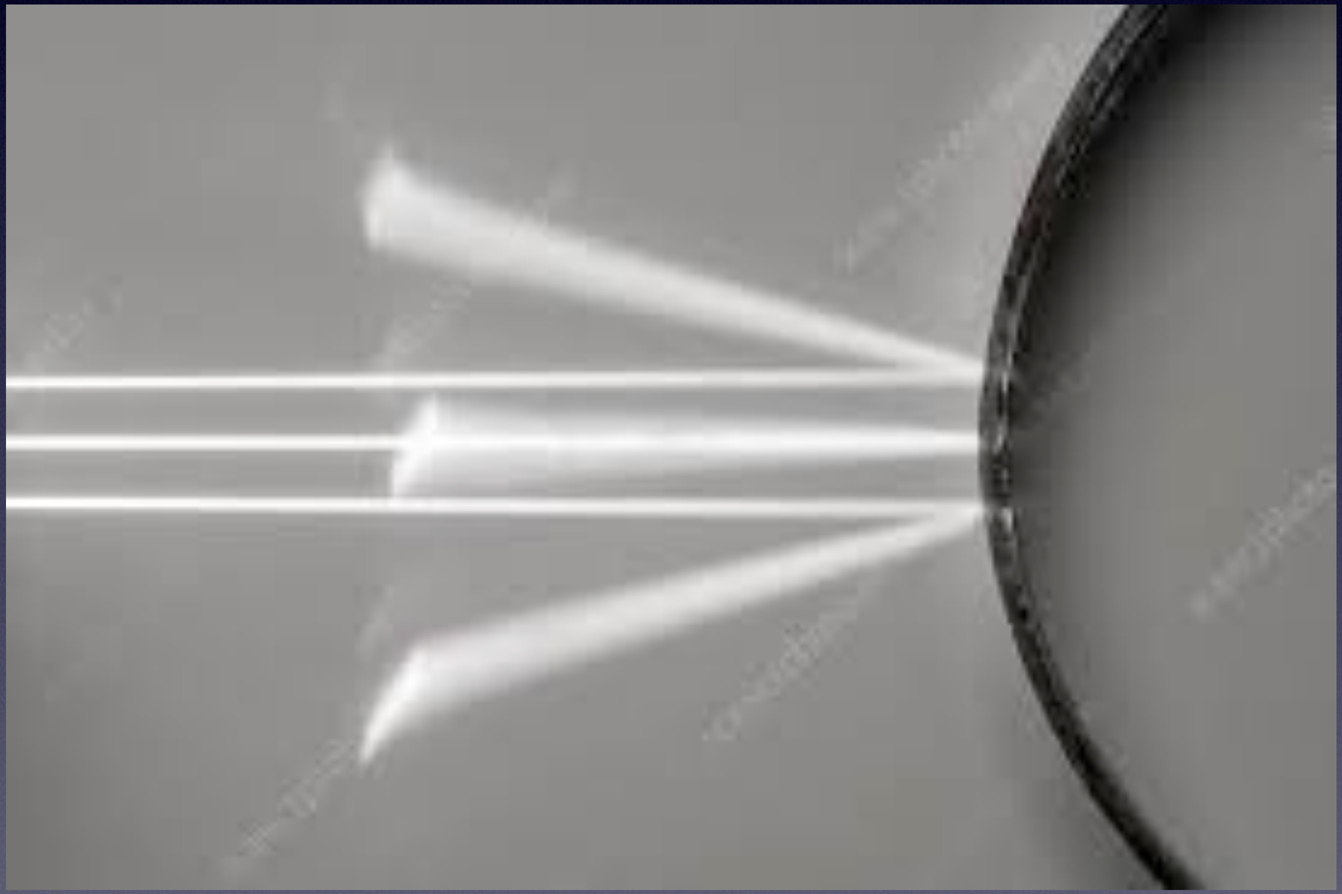
Semiconductors: “the Third Estate”



Optical materials: dielectric vs. metallic



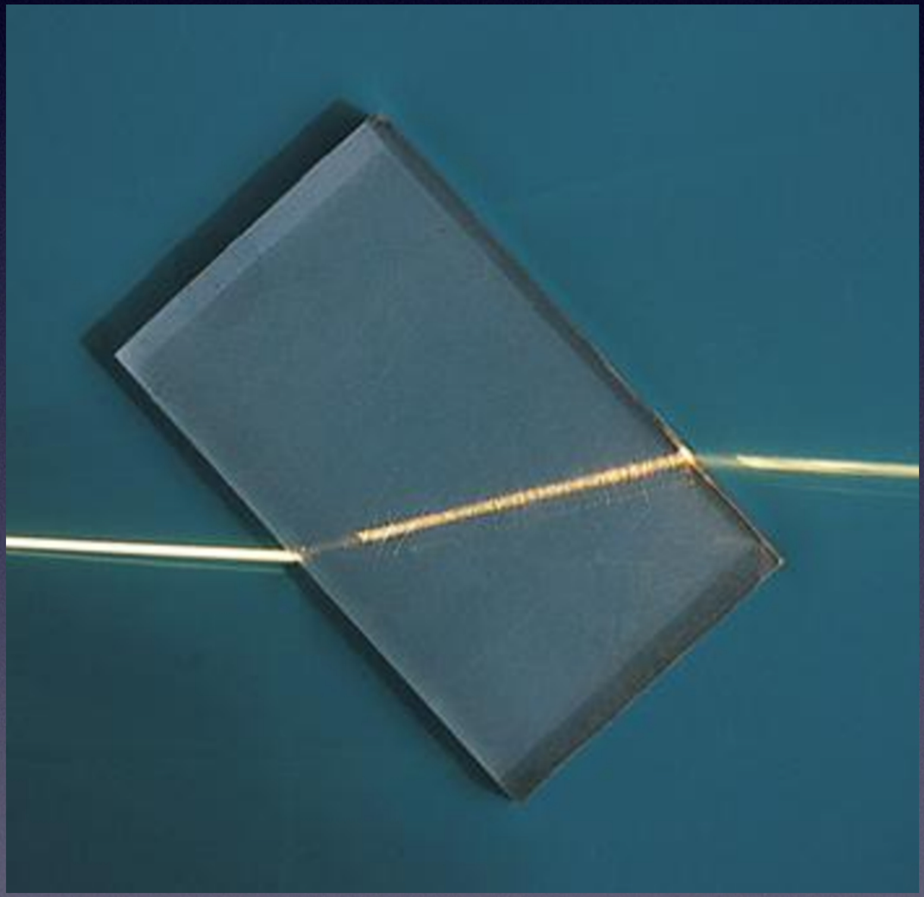
wikipedia.org



sciencephoto.com

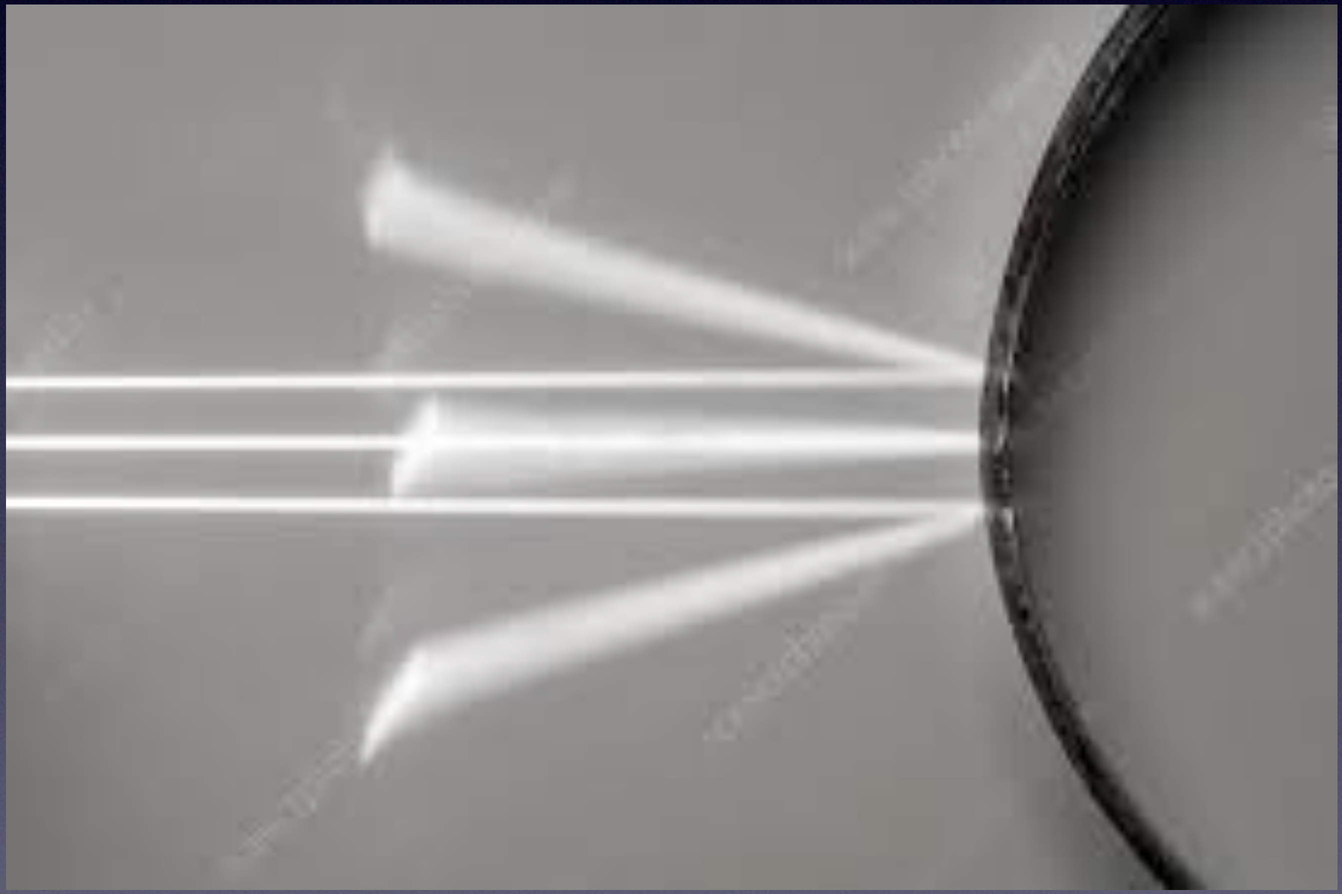
Optical materials: dielectric vs. metallic

Dielectric perittivity ϵ, refractive index $n = \sqrt{\epsilon}$



Dielectric :

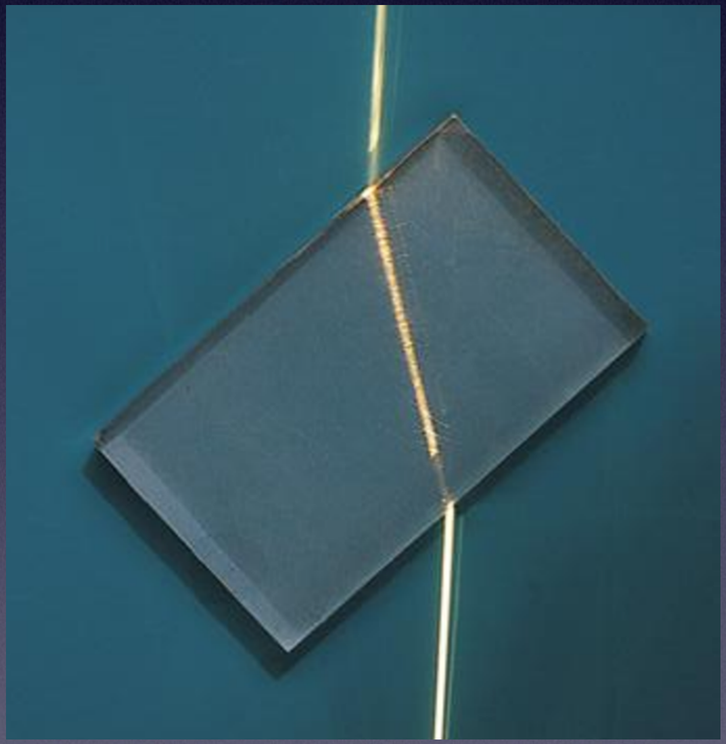
$\epsilon > 0 \Rightarrow$ real n

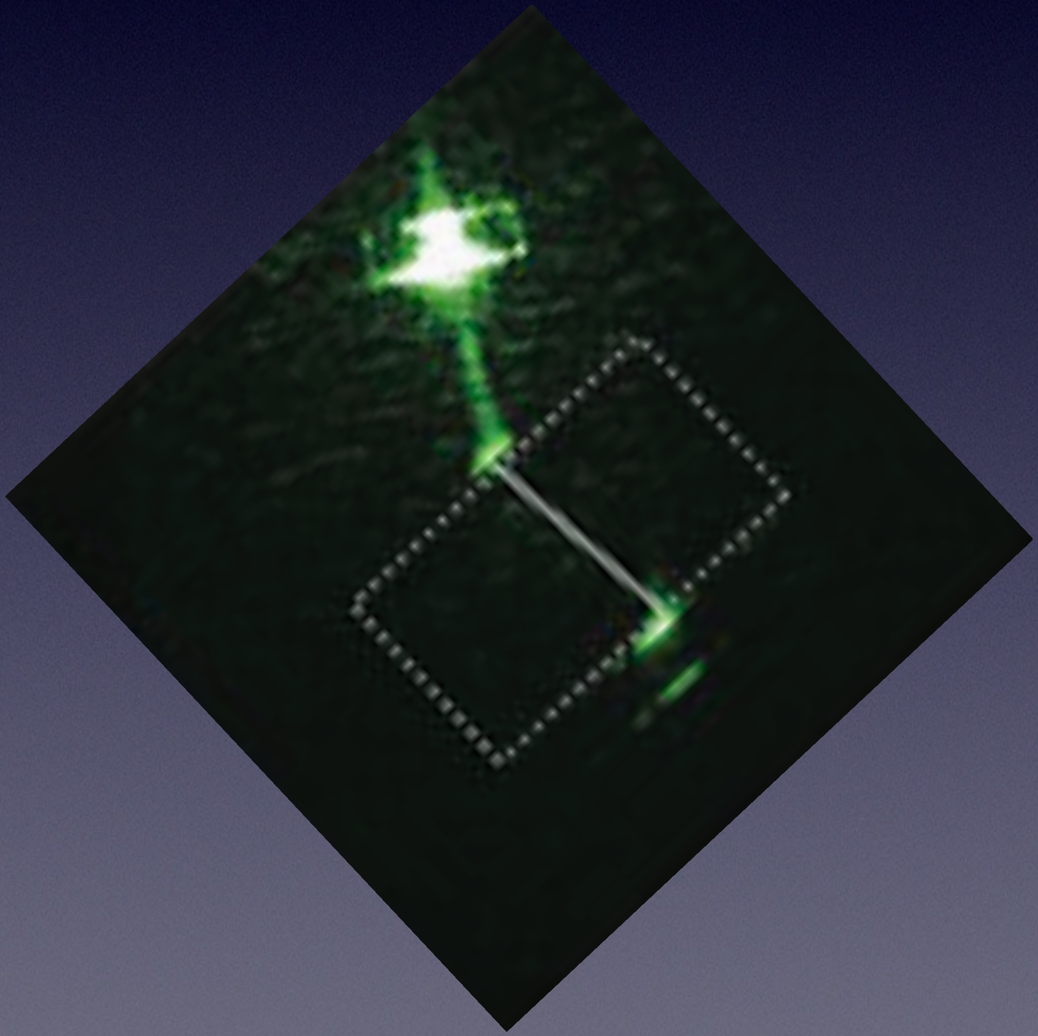


Metal :

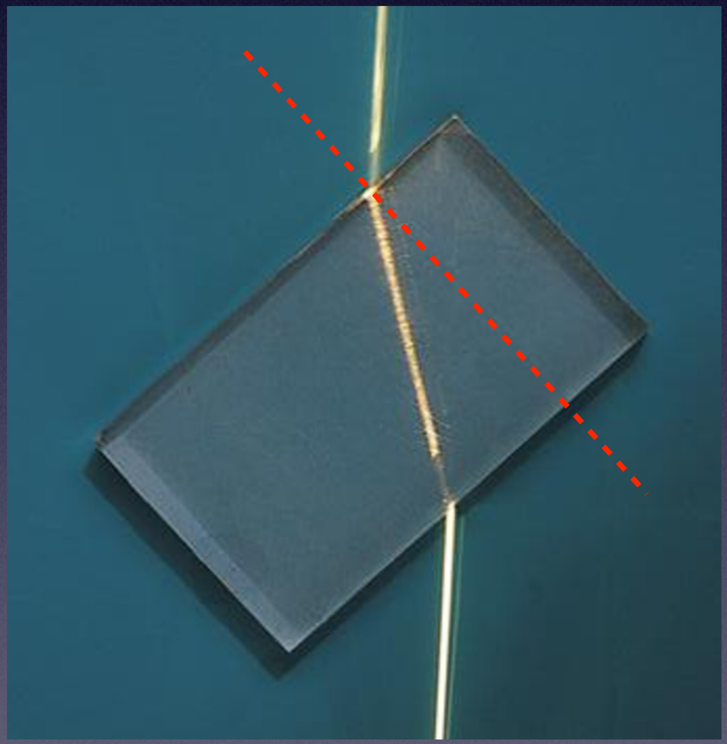
$\epsilon < 0 \Rightarrow$ imaginary n

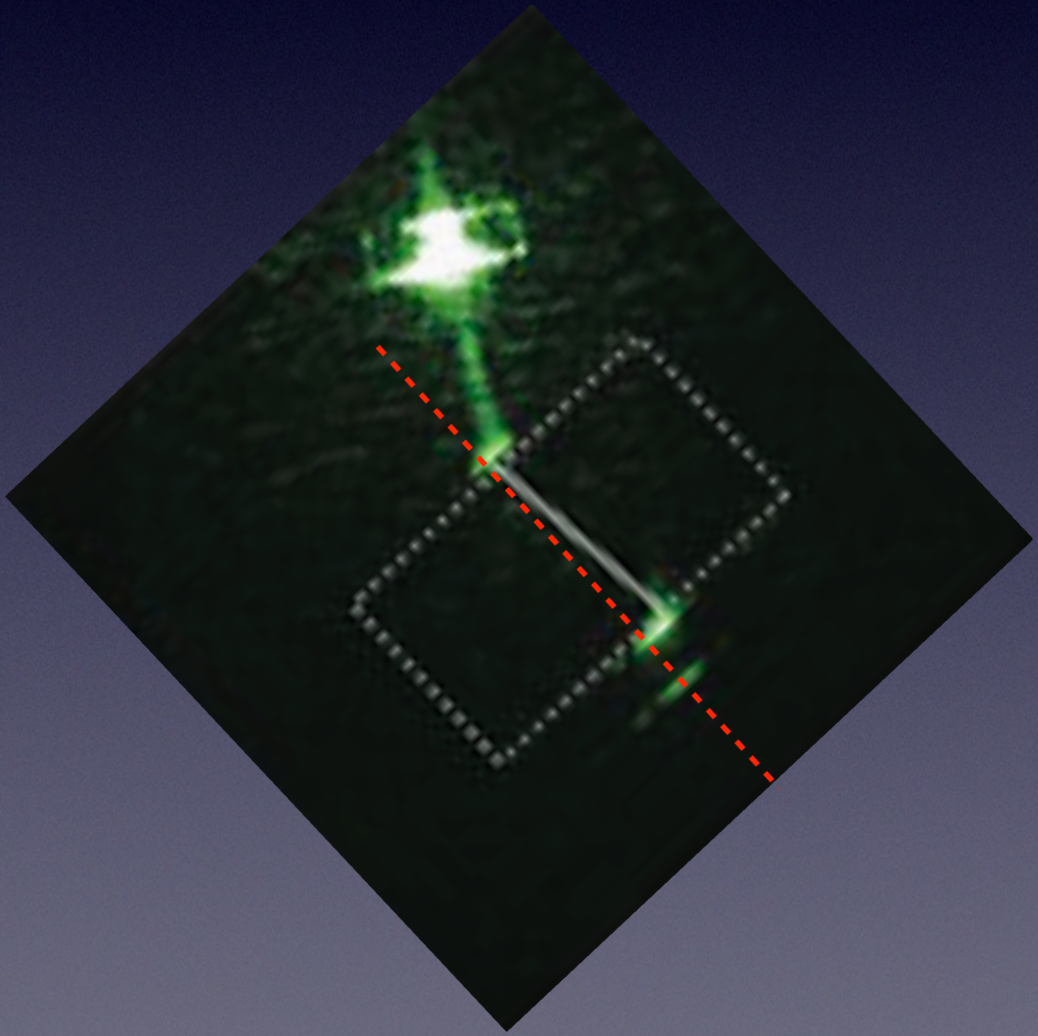
Optical Materials:
dielectric, hyperbolic and metallic





Optical Materials:
dielectric, hyperbolic and metallic





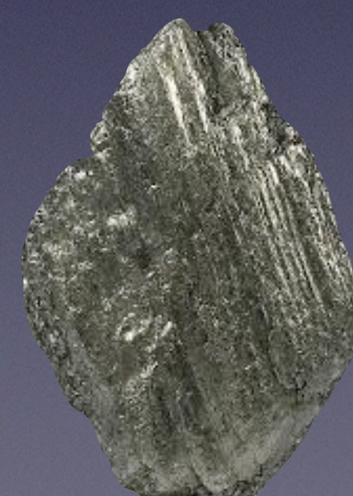
Hyperbolic Materials:
“the Third Estate” of optical media

nanowire metamaterial

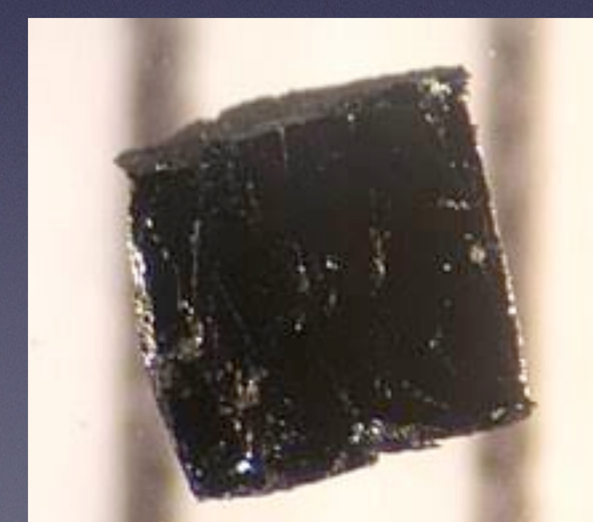
bismuth

sapphire

α - quartz



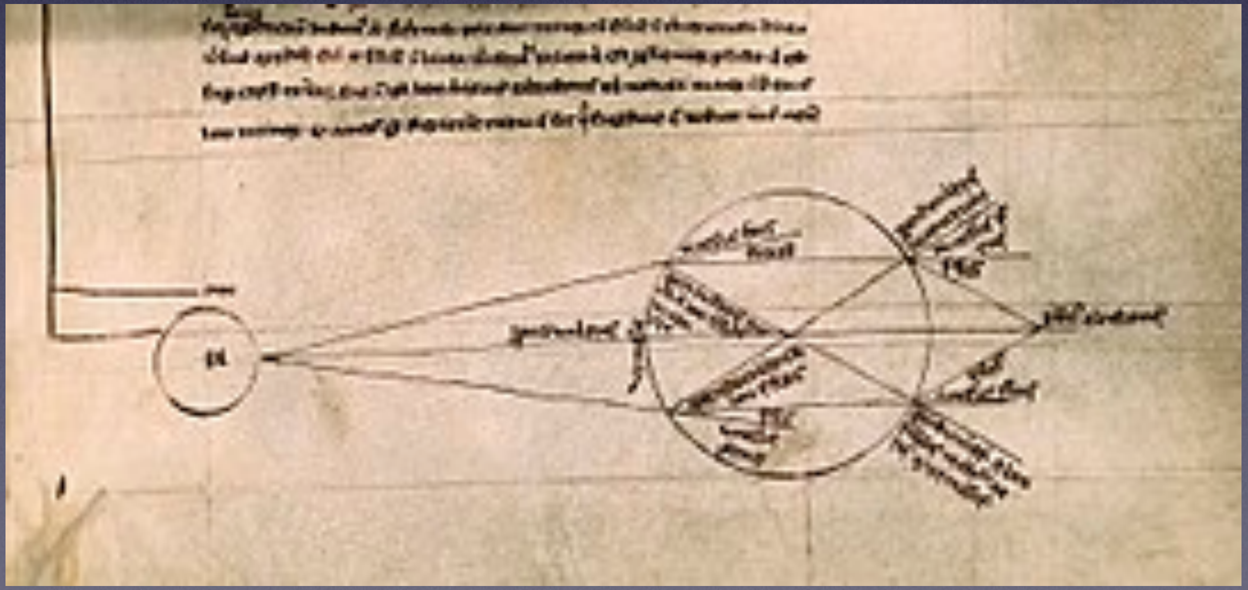
graphite



Bismuth strontium calcium copper oxide

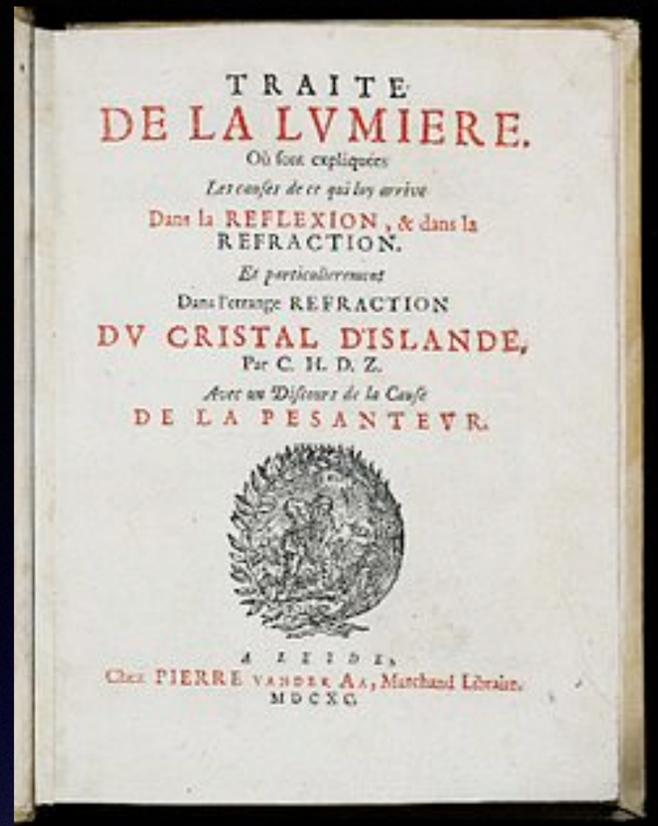
Light as “Rays”

LA
PROSPETTIVA
DI EVCLIDE,
Nella quale si tratta di quelle cose, che per raggi
ritti si veggono: & di quelle, che con raggi
reflessi nelli Specchi appariscono.
Tradotta dal R. P. M. Egnatio Danti Cosmografo
Seren. Gran Duca di Toscana. Con alcune sue
Annotationi de' luoghi più importanti.
Insieme Con La Prospettiva Di
Eliodoro Larisseo
Cavata della Libreria Vaticana, e tradotta dal
medesimo nuovamente data in luce.
IN FIORENZA.
Nella Stamperia de' Giunti.
MDLXXIII.
Con licenzia & Privilegio.



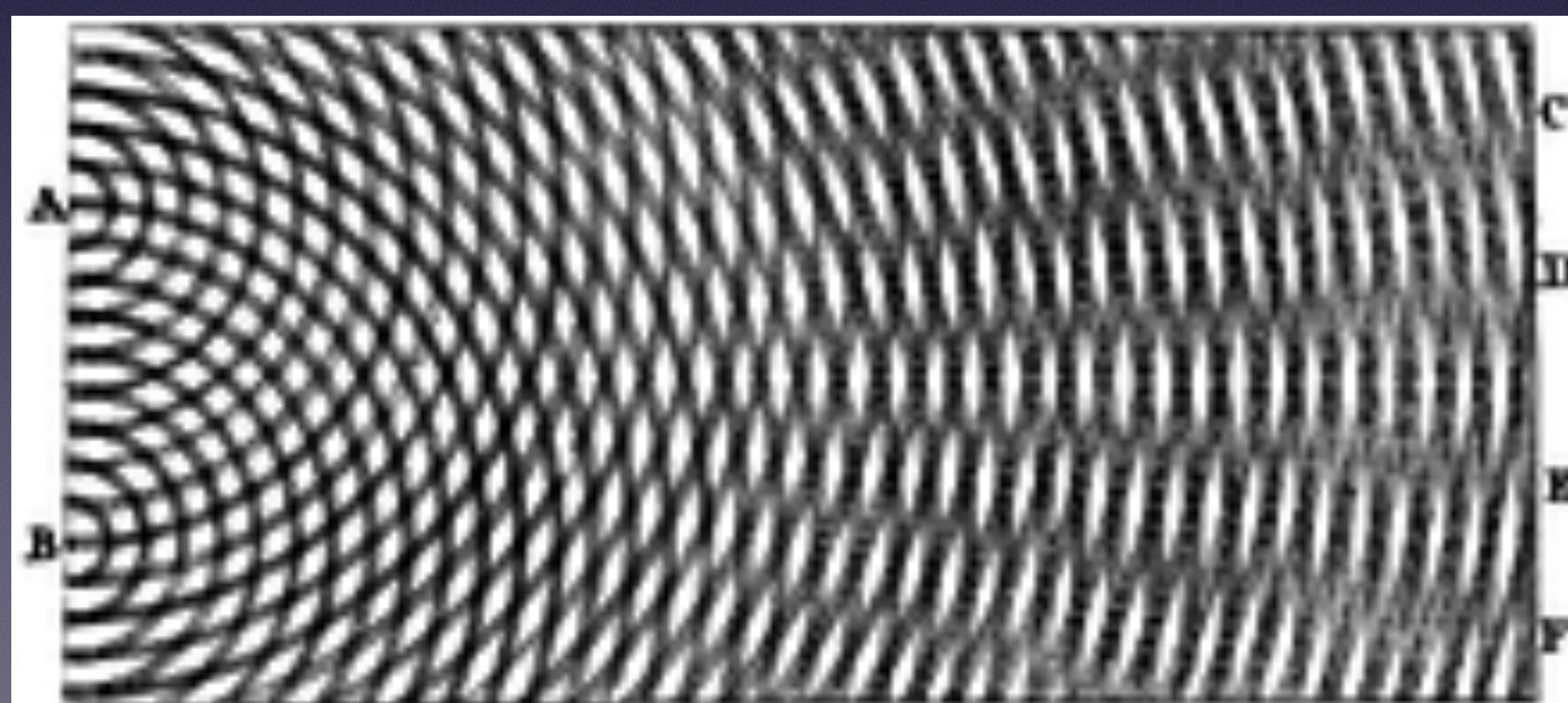
Roger Bacon, *De multiplicatione specierum*

Light as a wave



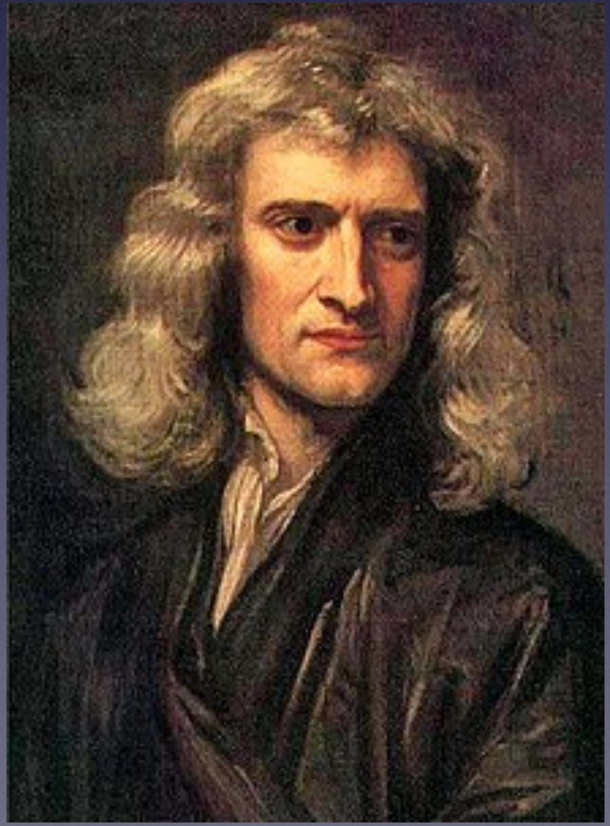
TRAITE
DE LA LVMIERE.
Où sont expliquées
Les causes de ce qui luy arrive
Dans la REFLEXION, & dans la REFRACTION.
Et particulierement
Dans l'estrange REFRACTION
DV CRISTAL D'ISLANDE,
Par C. H. D. Z.
Avec un Discours de la Cause
DE LA PESANTEVR.
A LEIDE,
Chez PIERRE VANDER AA, Marchand Libraire.
MDCXC.

Christiaan Huygens, *Treatise on Light*

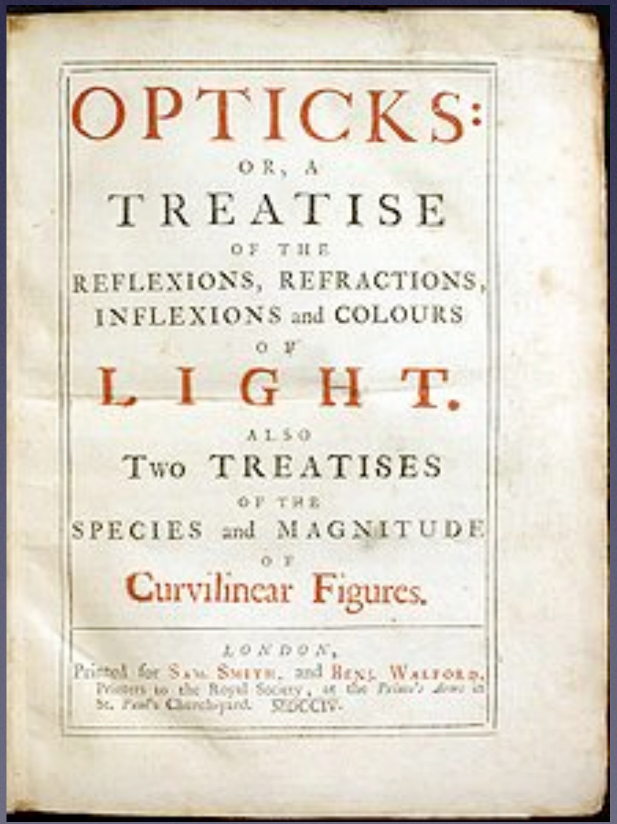


Thomas Young, *sketch of two-slit diffraction from the presnetation to the Royal Society*

Erasmus Bartholin



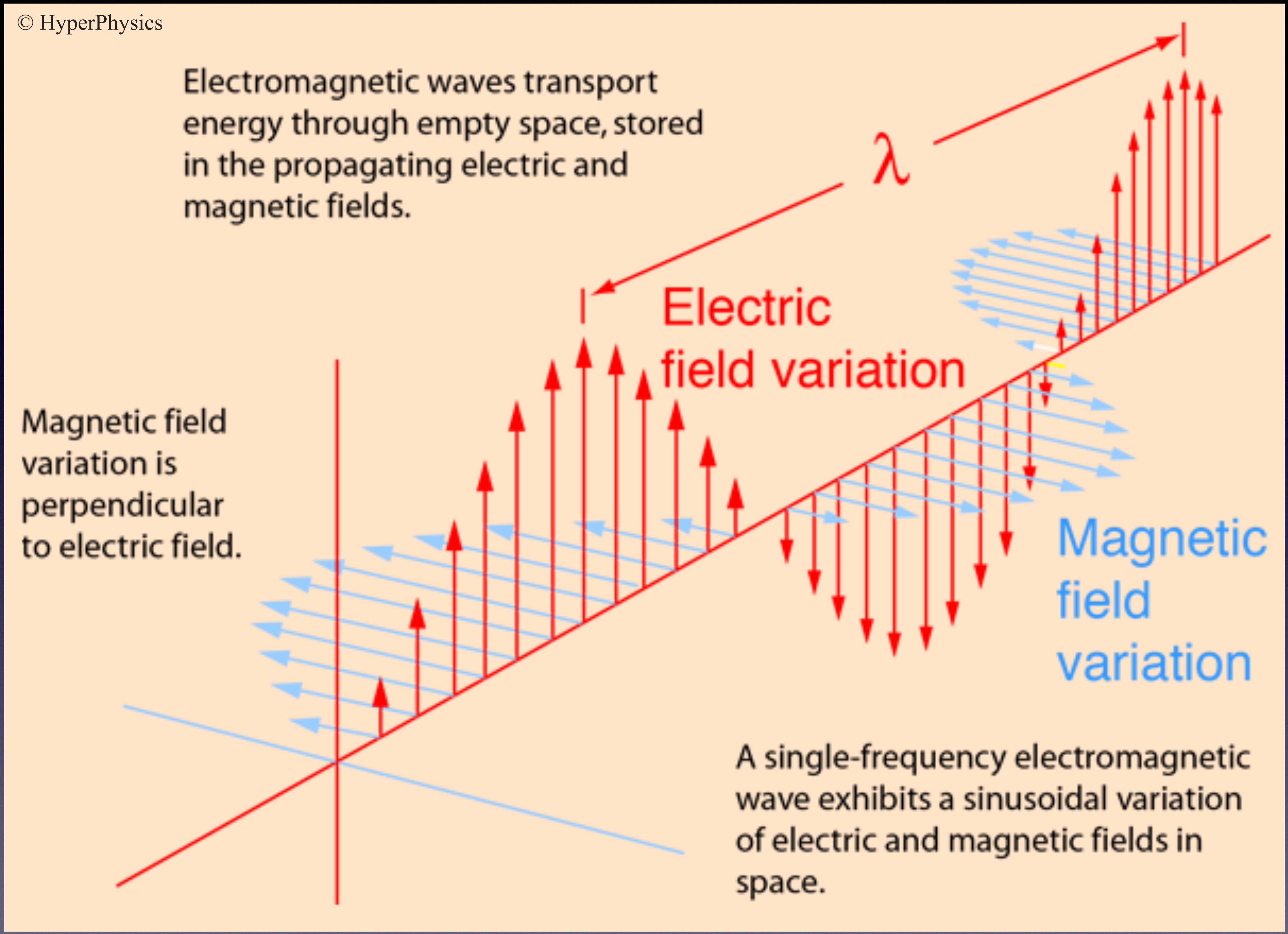
Isaak Newton



OPTICKS:
OR, A
TREATISE
OF THE
REFLEXIONS, REFRACTIONS, INFLEXIONS and COLOURS
OF
LIGHT.
ALSO
Two TREATISES
OF THE
SPECIES and MAGNITUDE
OF
Curvilinear Figures.

LONDON,
Printed for Sam. Smith, and Benj. Walford, Printers to the Royal Society, at the Prince's Arms in St. Paul's Church-yard. MDCCIV.

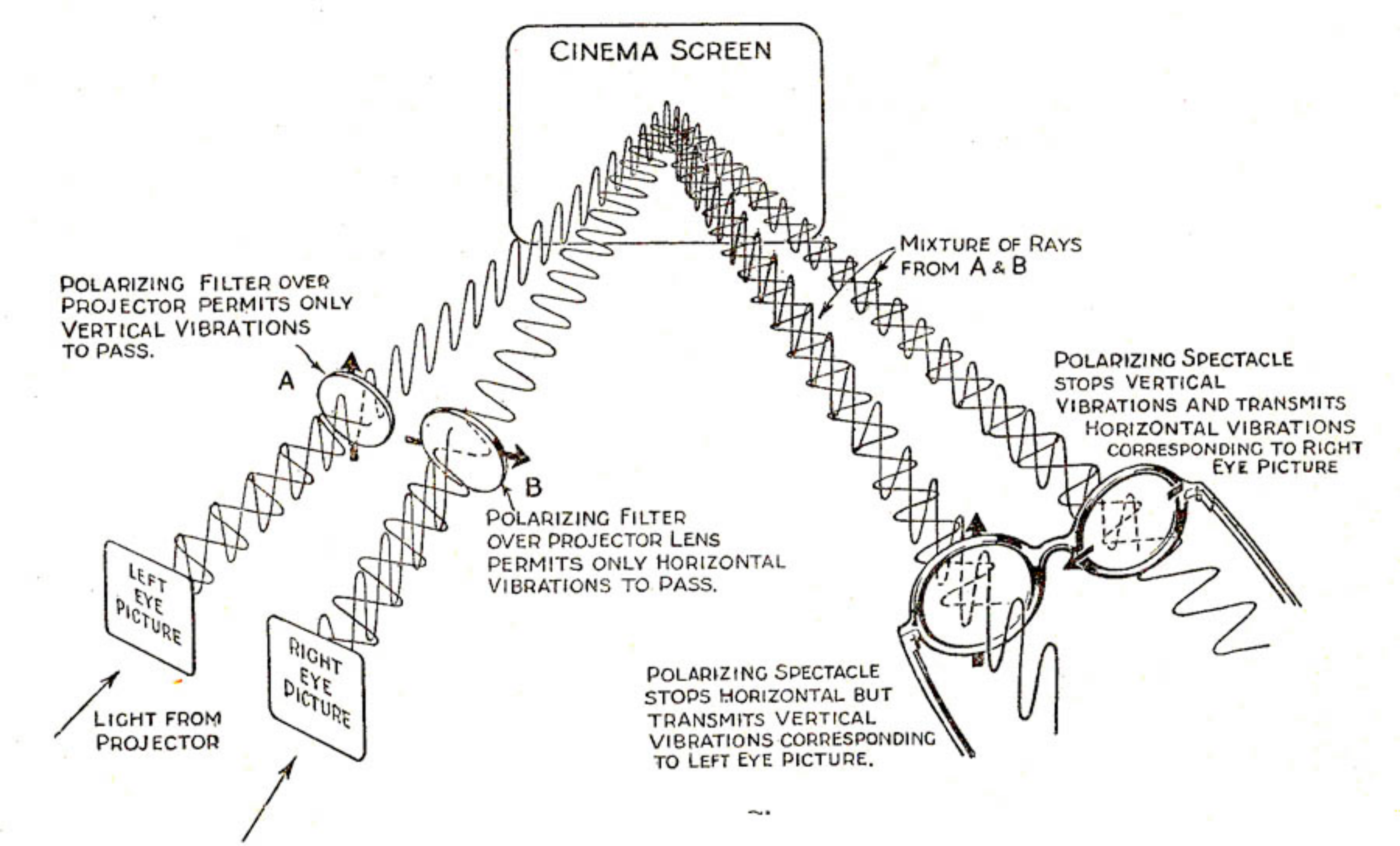
- particles of light create waves in the aether
- light has “sides”



Wavenumber $k \equiv \frac{2\pi}{\lambda}$, momentum $p = \hbar k$

Polaroid

Edwin Land

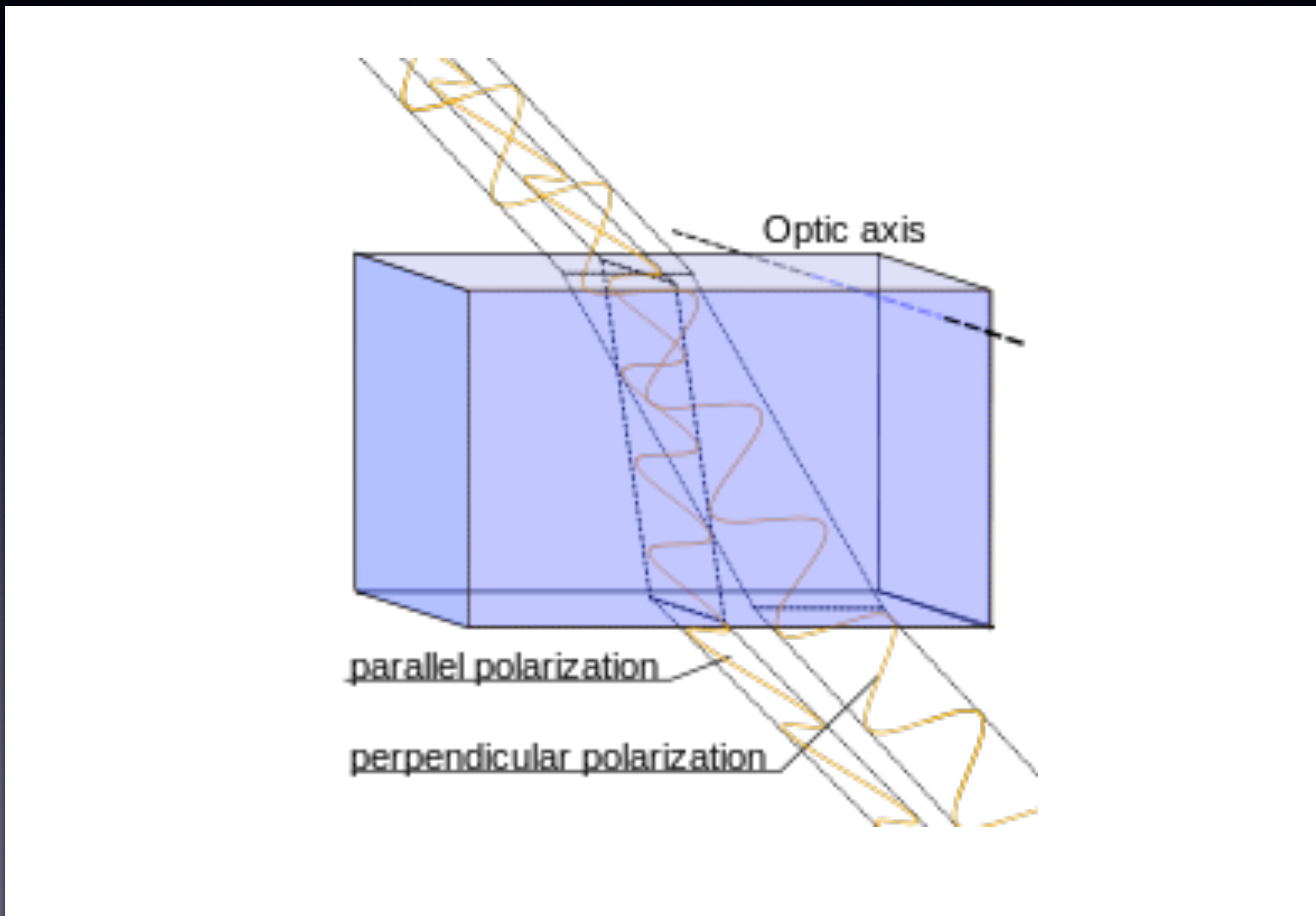


Ray Zone, The 3D Zone: Its Past & Its Future


DOUBLE REFRACTION



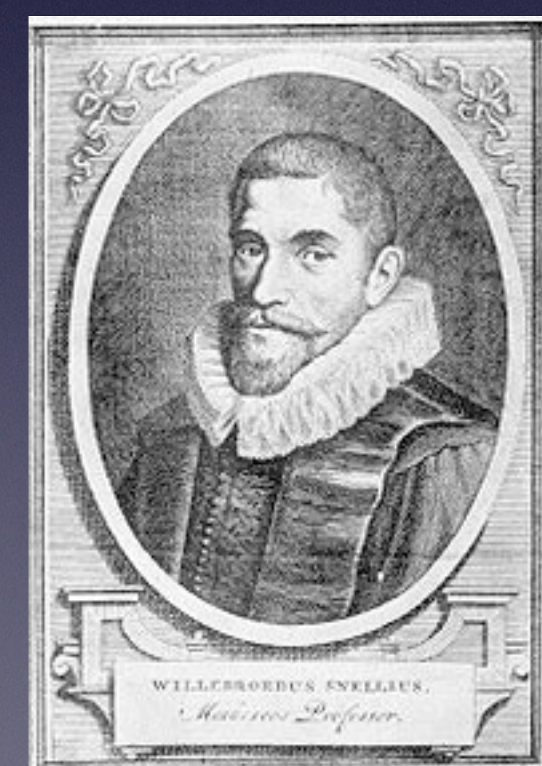
DOUBLE REFRACTION


Optic axis
parallel polarization
perpendicular polarization

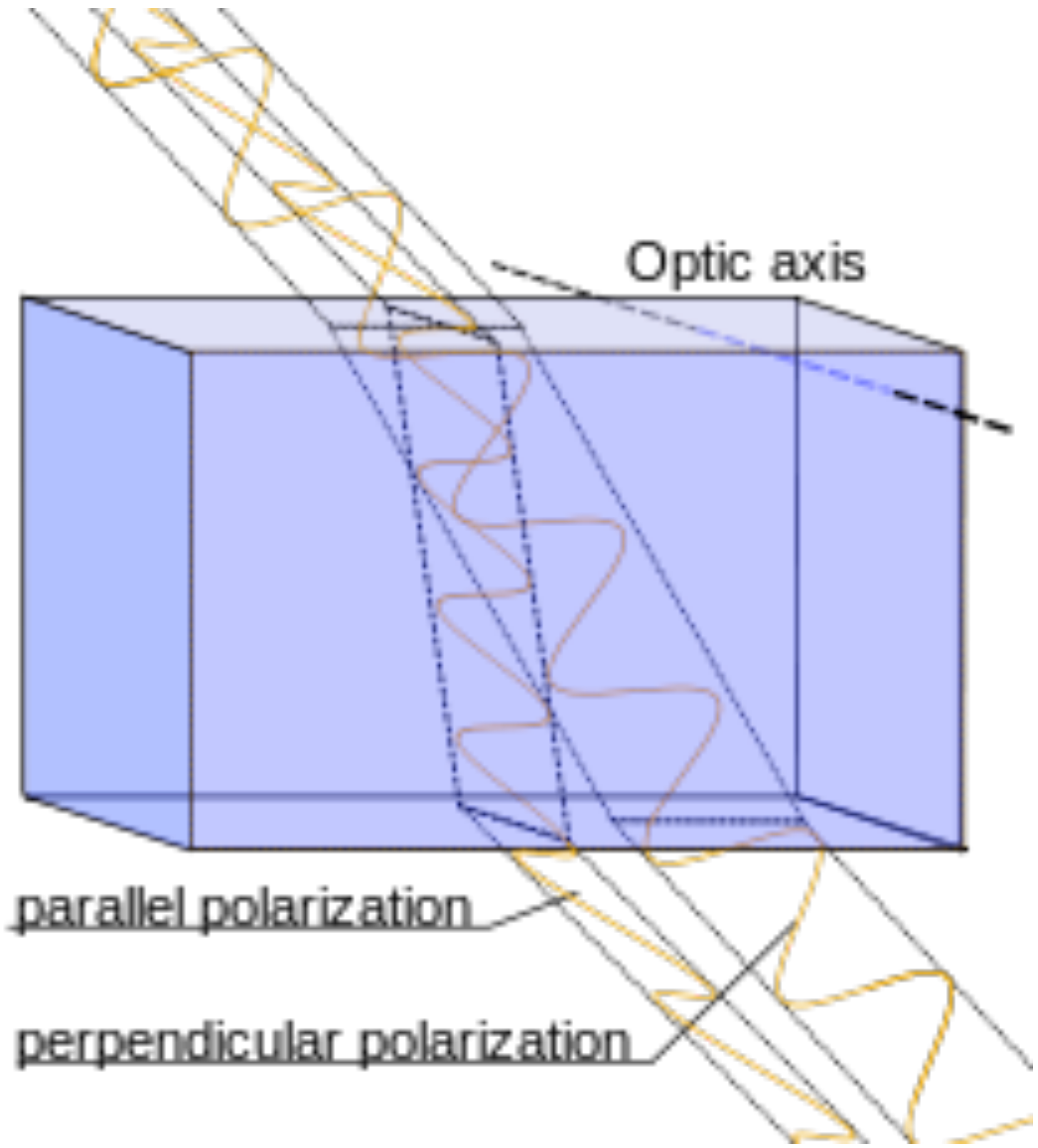
Snel’s (Snellius’) Law of Refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Corner of Hogewoerd and Sint Jorissteeg, Leiden

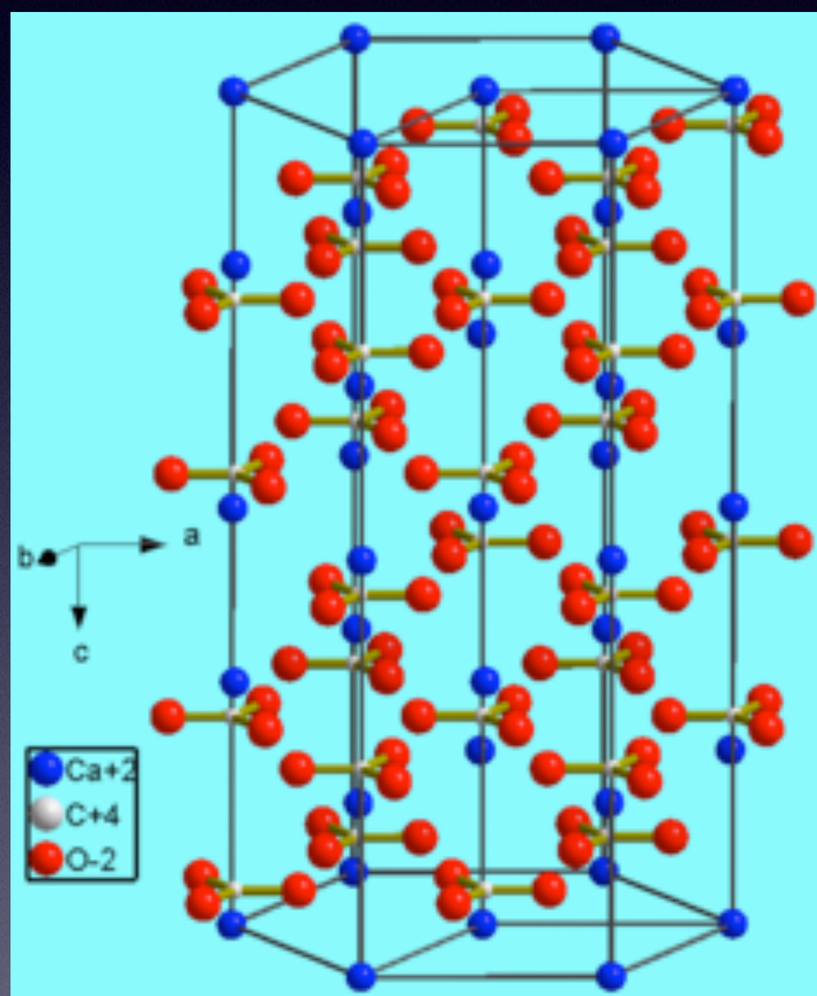


Willebrond Snel van Royen

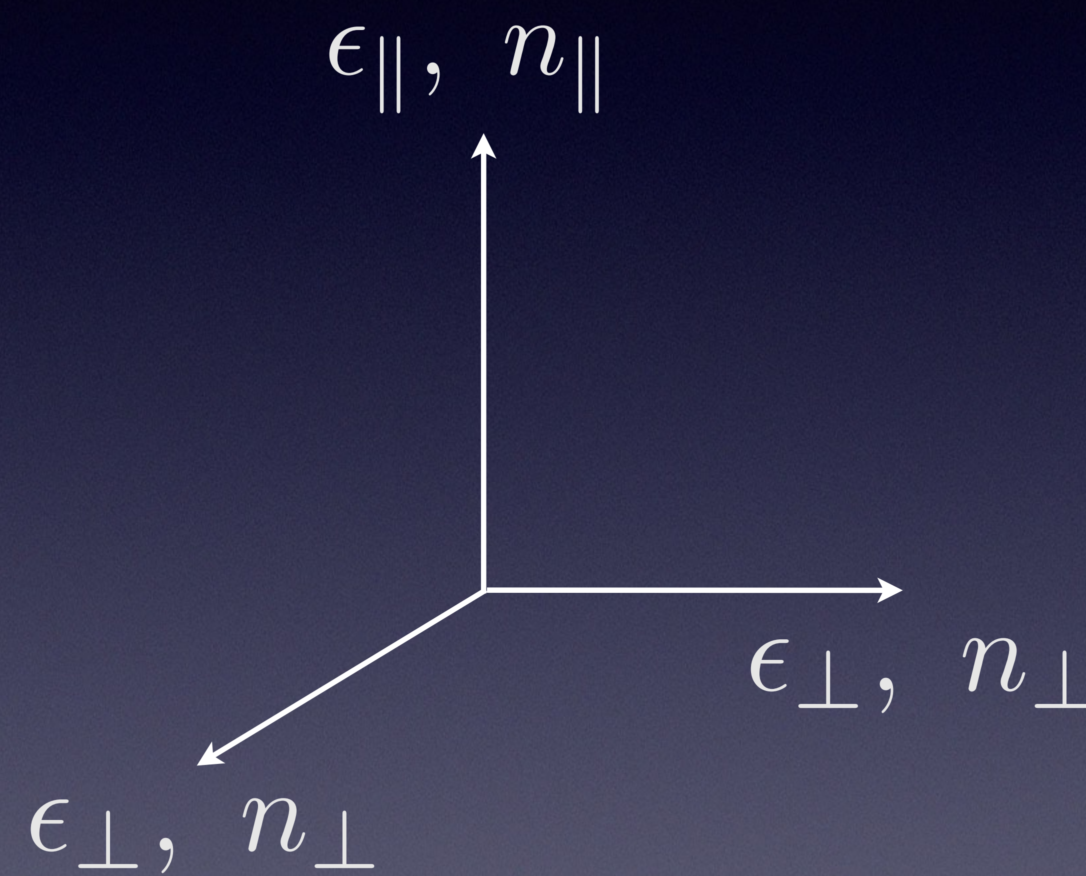


$\epsilon_{\parallel},\ n_{\parallel} \rightarrow$

$\epsilon_{\perp},\ n_{\perp} \rightarrow$



calcite $CaCO_3$



Isotropic Dielectric Materials:

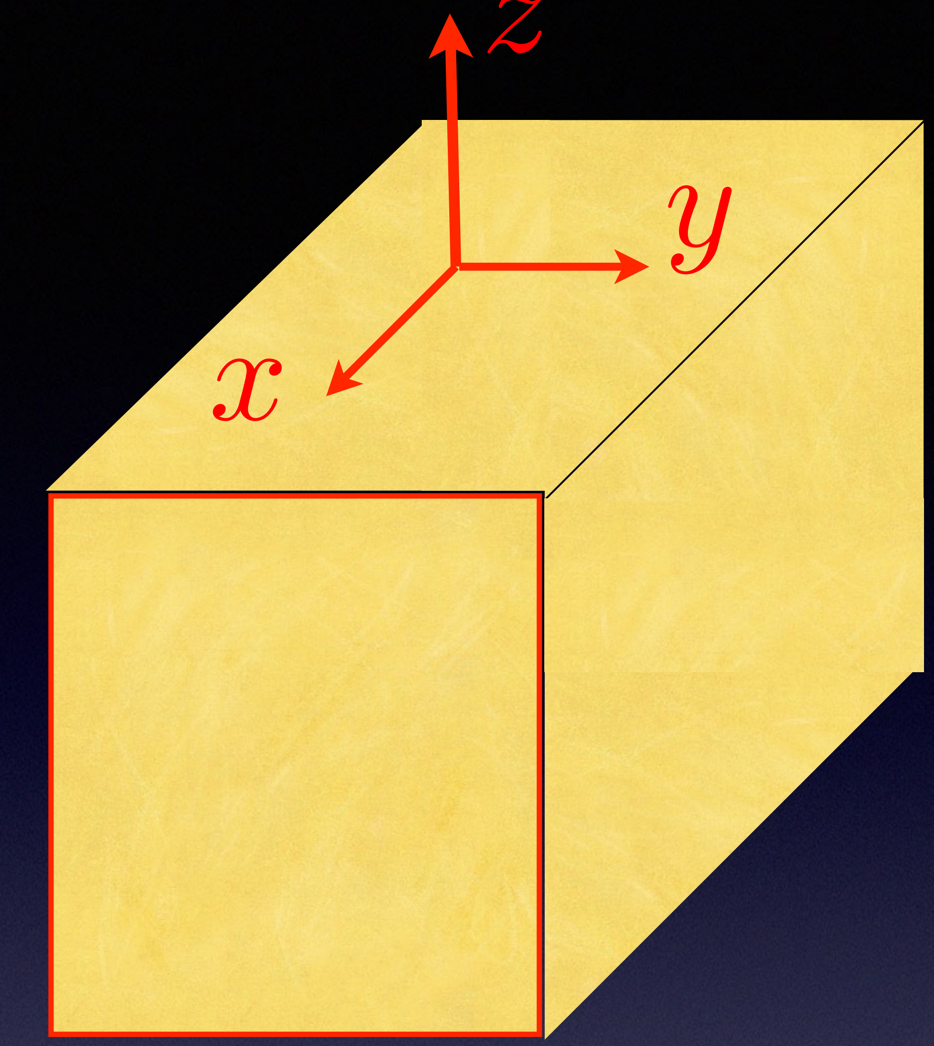
$$\lambda = \frac{\lambda_0}{n}$$

$$\Rightarrow \; k = \frac{2\pi}{\lambda} = n \, \frac{\omega}{c} = \sqrt{\epsilon} \, \frac{\omega}{c}$$

$$\Rightarrow \; \frac{k^2}{\epsilon} = \frac{\omega^2}{c^2}$$

Anisotropic Dielectric Materials ?

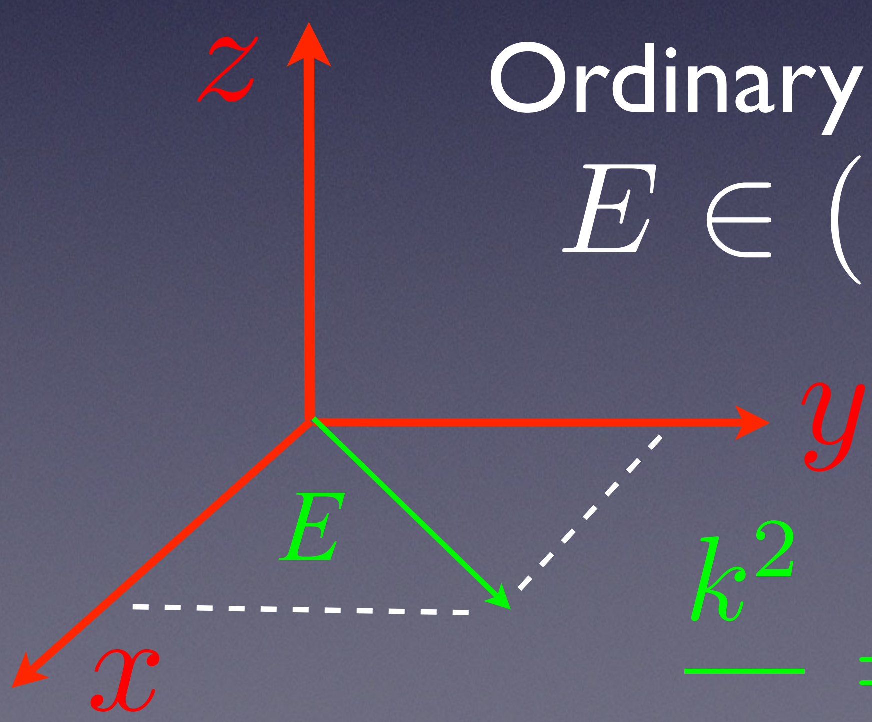
Uniaxial Anisotropic Materials



$$\epsilon_x = \epsilon_y \neq \epsilon_z$$

Ordinary Wave:

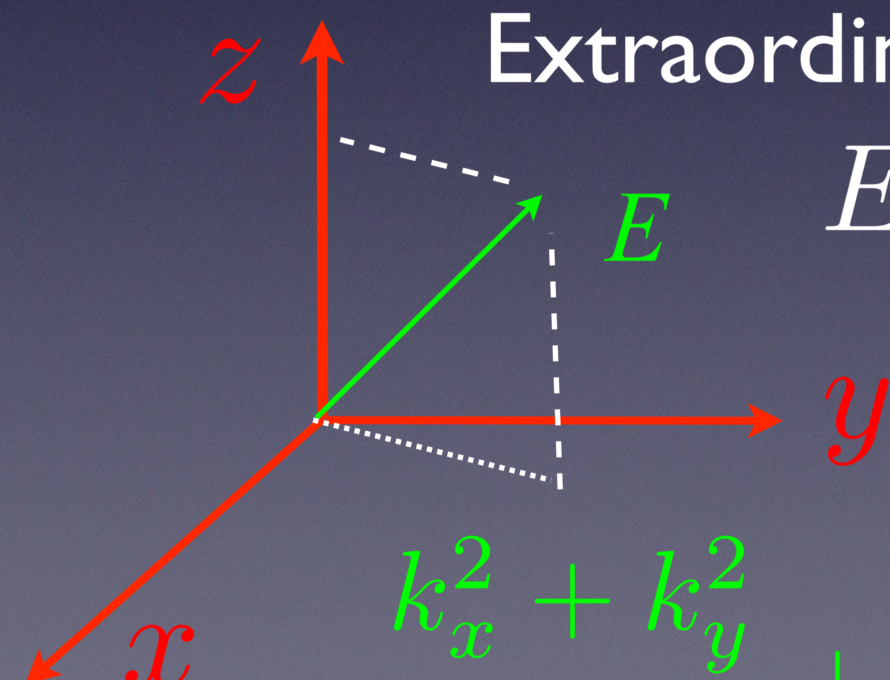
$$E \in (x, y)$$



$$\frac{k^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

Extraordinary Wave:

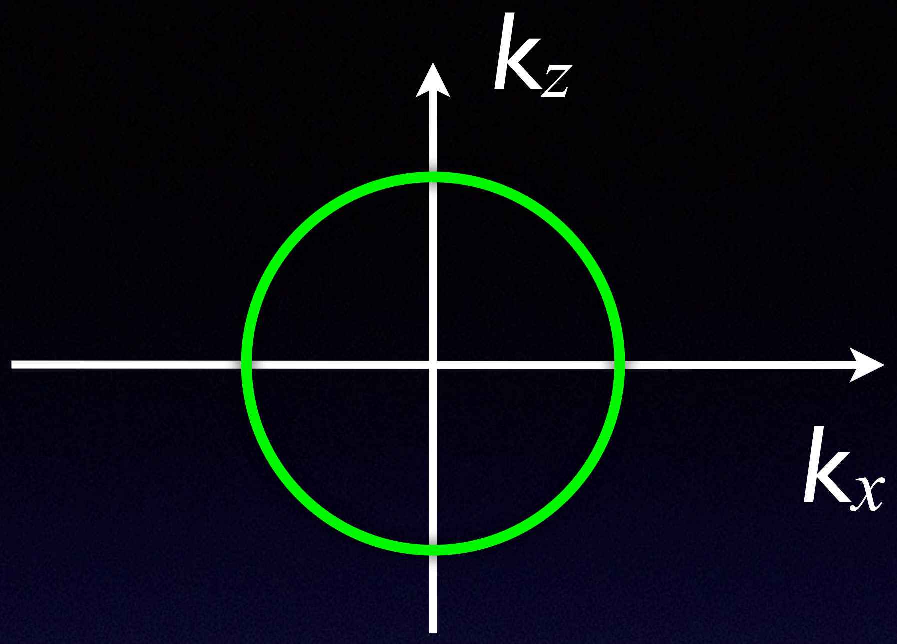
$$E \notin (x, y)$$



$$\frac{k_x^2 + k_y^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

Isotropic dielectric:

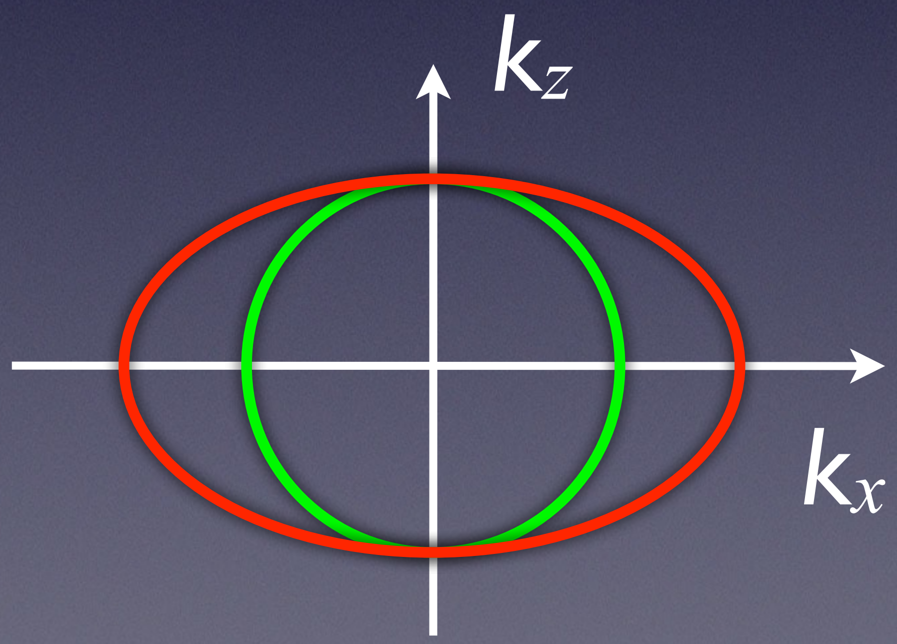
$$\frac{k_x^2}{\epsilon} + \frac{k_z^2}{\epsilon} = \frac{\omega^2}{c^2}$$



Anisotropic dielectric:

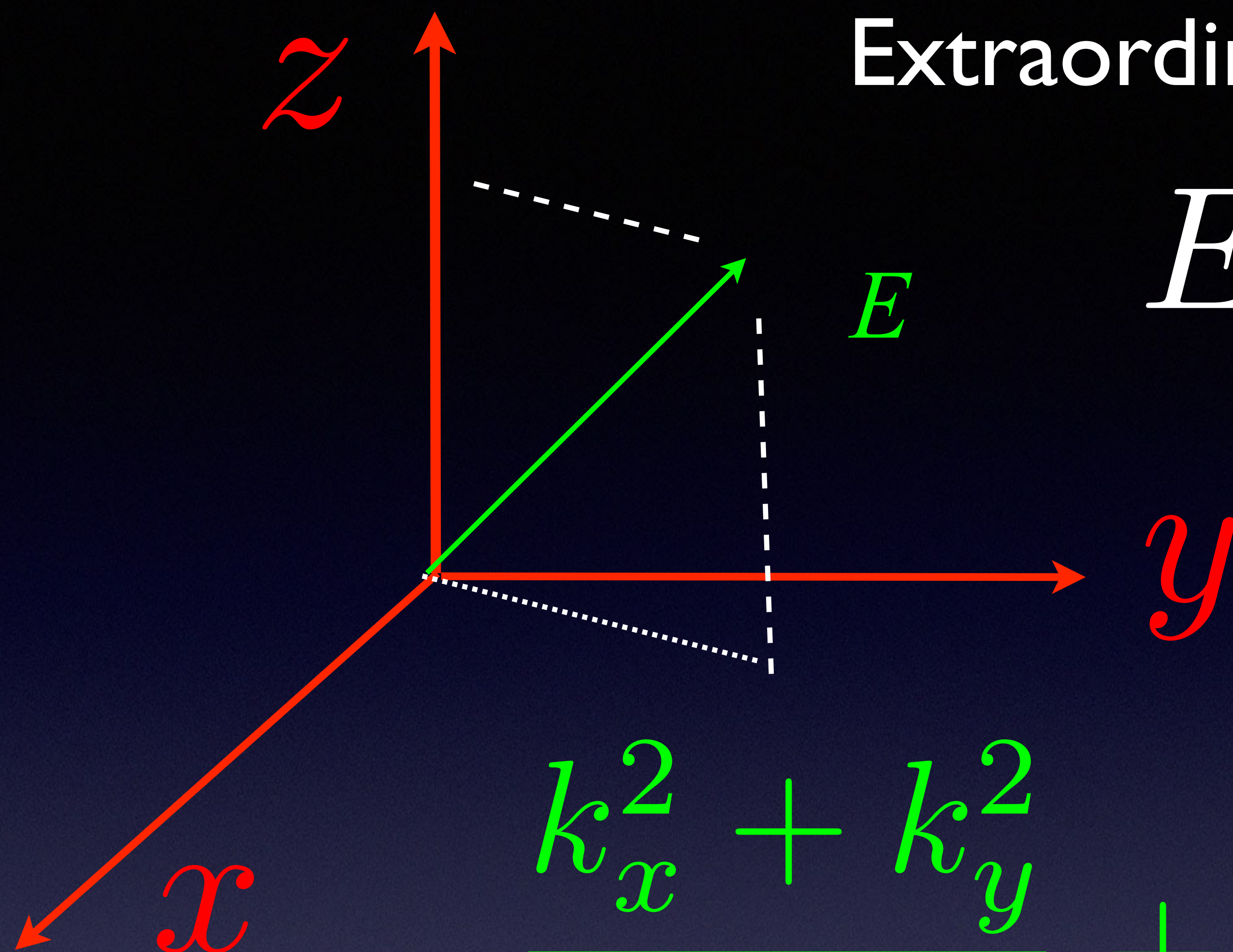
ordinary: $$\frac{k_x^2}{\epsilon_x} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

extraordinary: $$\frac{k_x^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$



Extraordinary Wave:

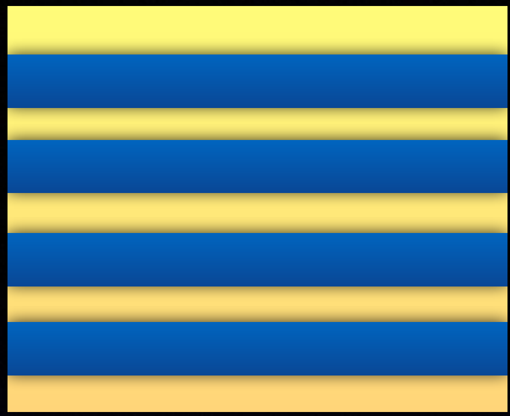
$$E \notin (x, y)$$



$$\frac{k_x^2 + k_y^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

$\varepsilon_x < 0, \ \varepsilon_z > 0$

What if the material is metallic in one direction and dielectric in the other ?



What if the material is metallic in one direction and dielectric in the other ?

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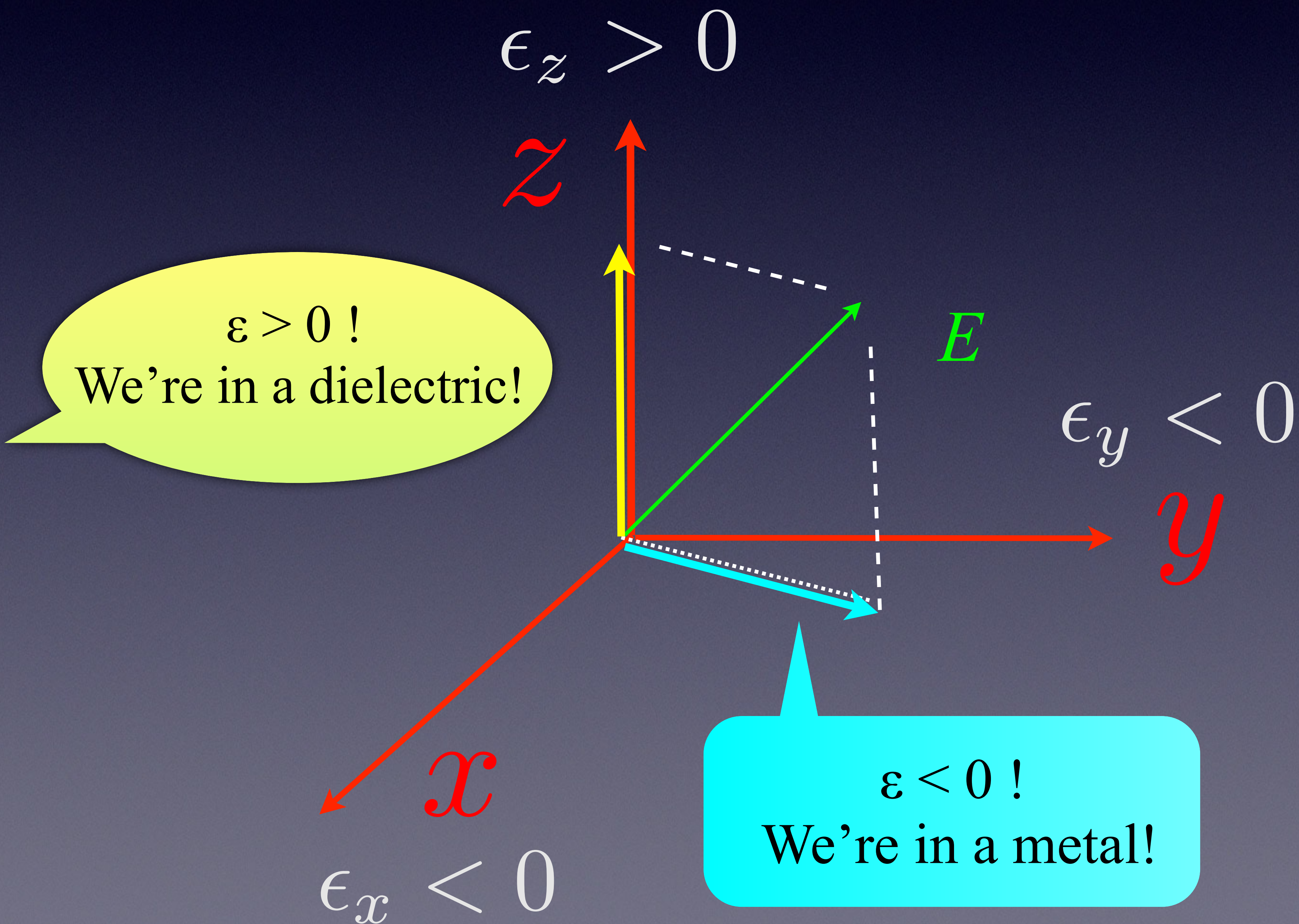
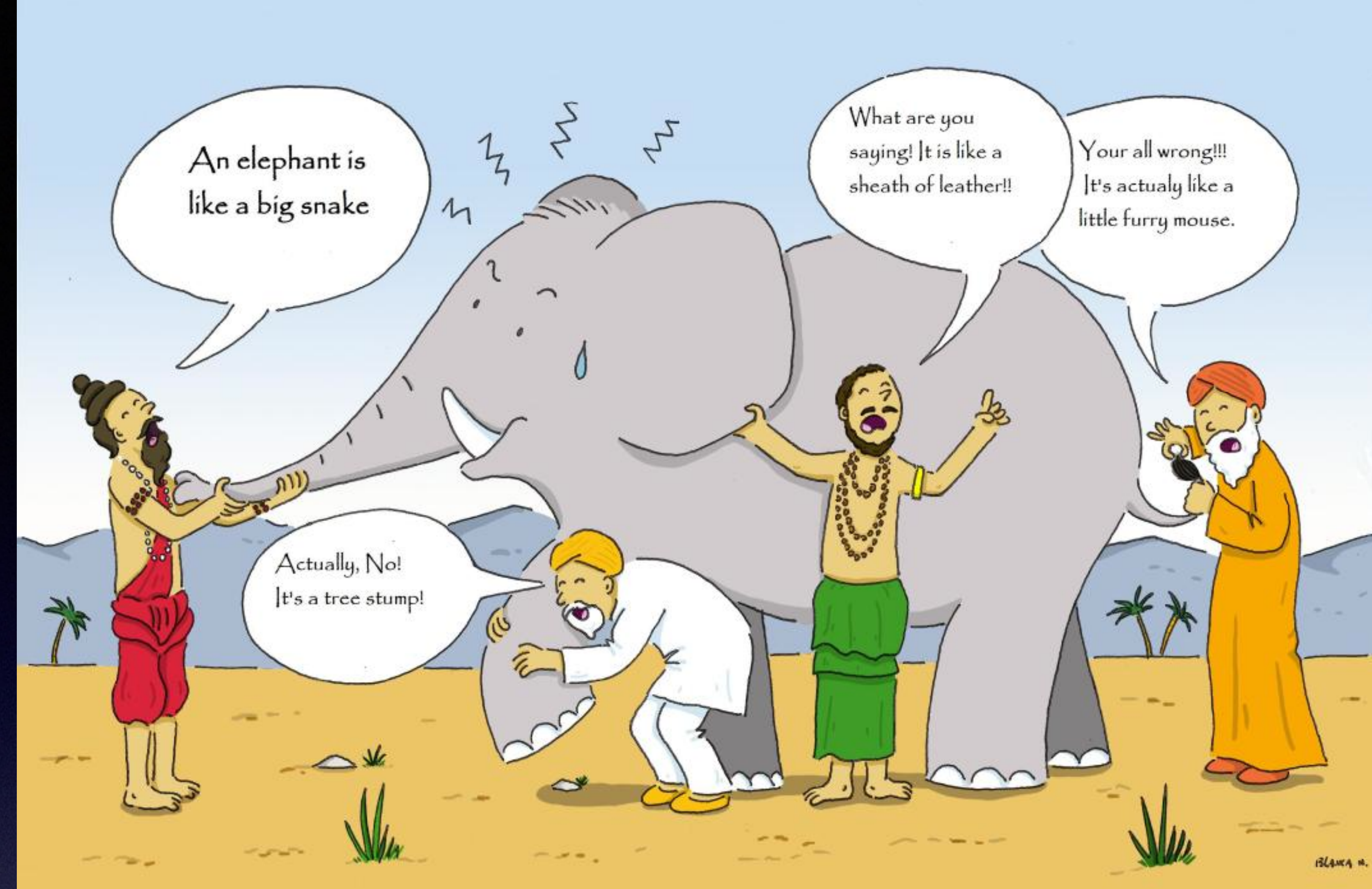
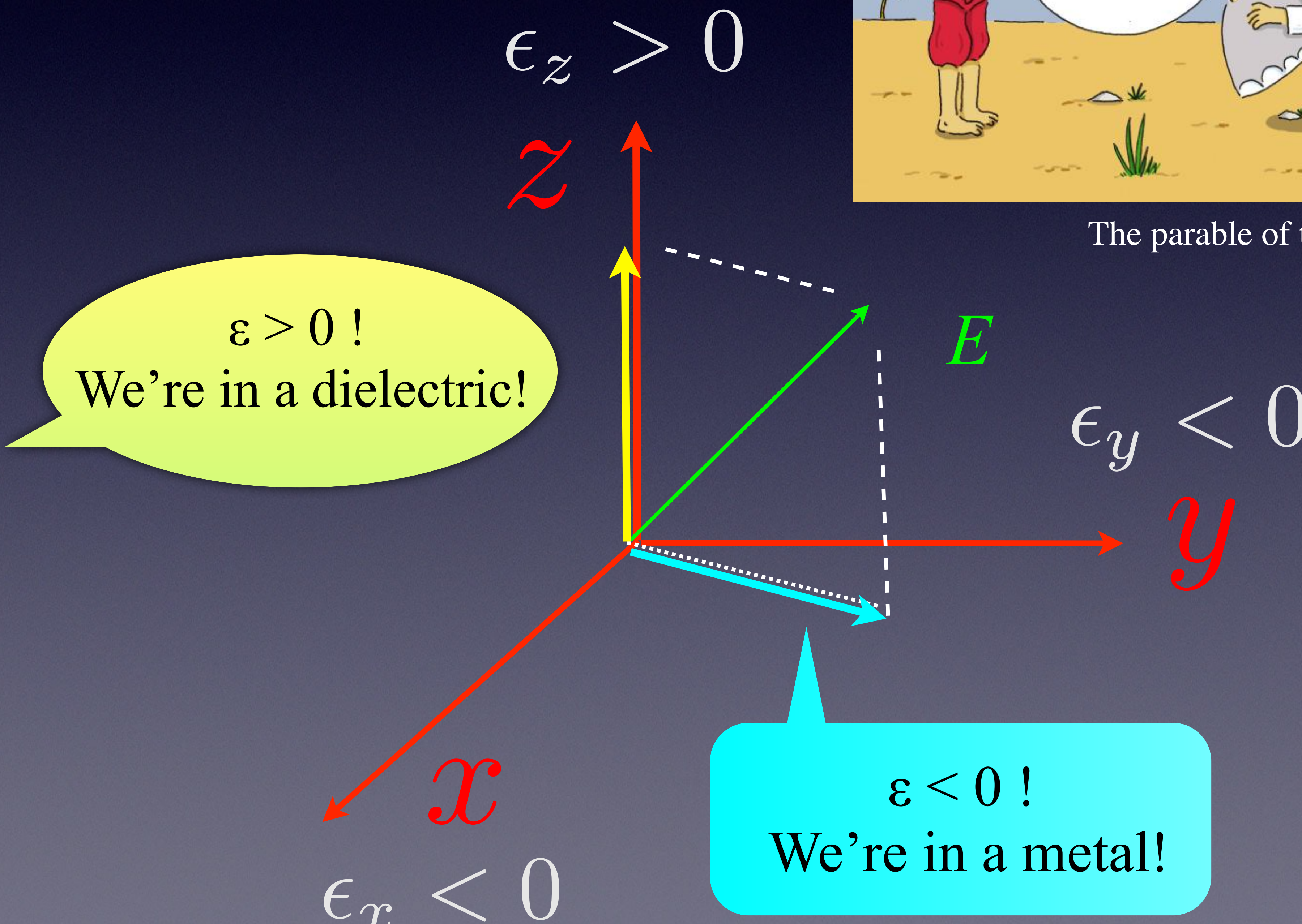


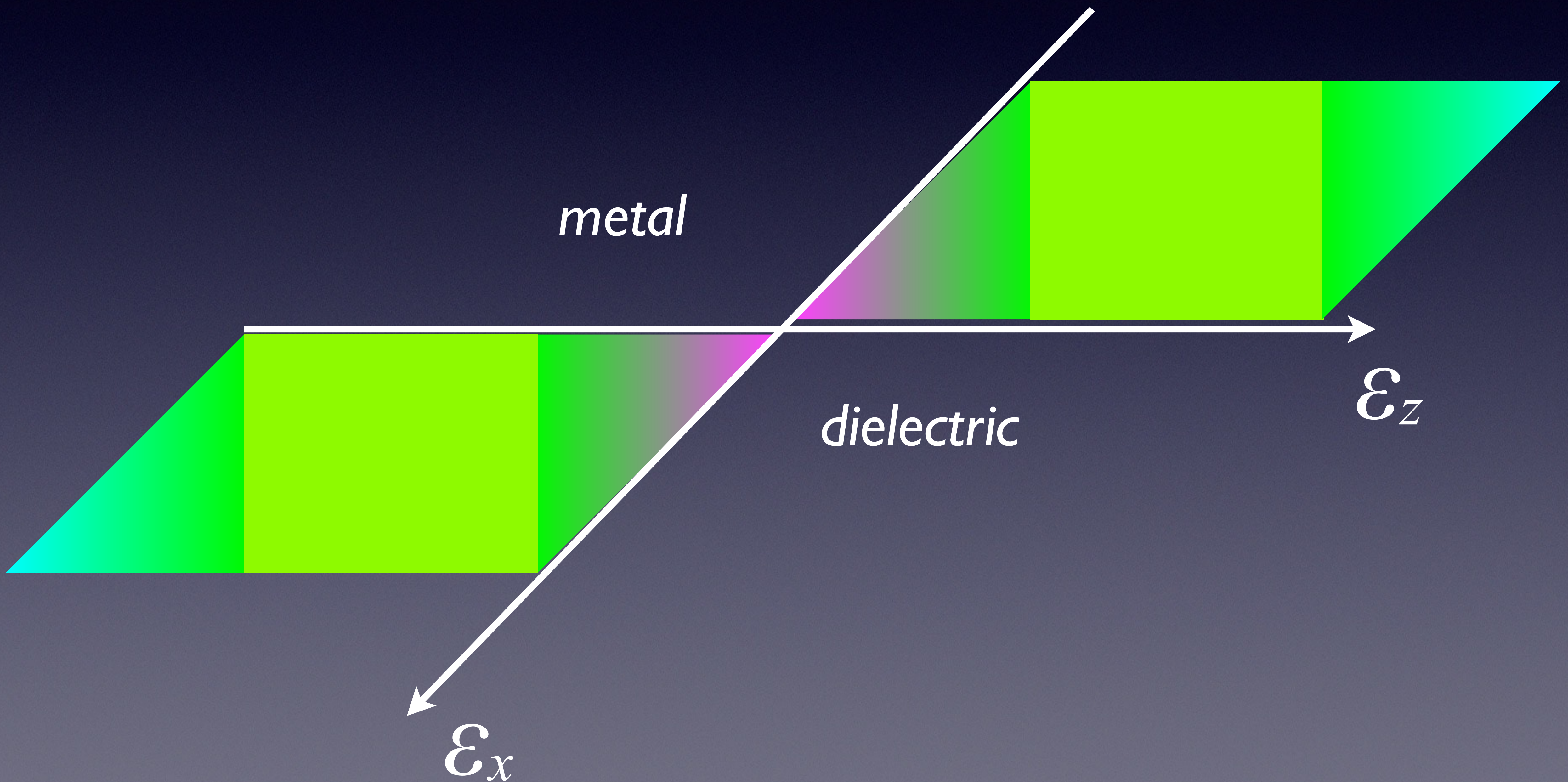
$\varepsilon_x < 0, \ \varepsilon_z > 0$

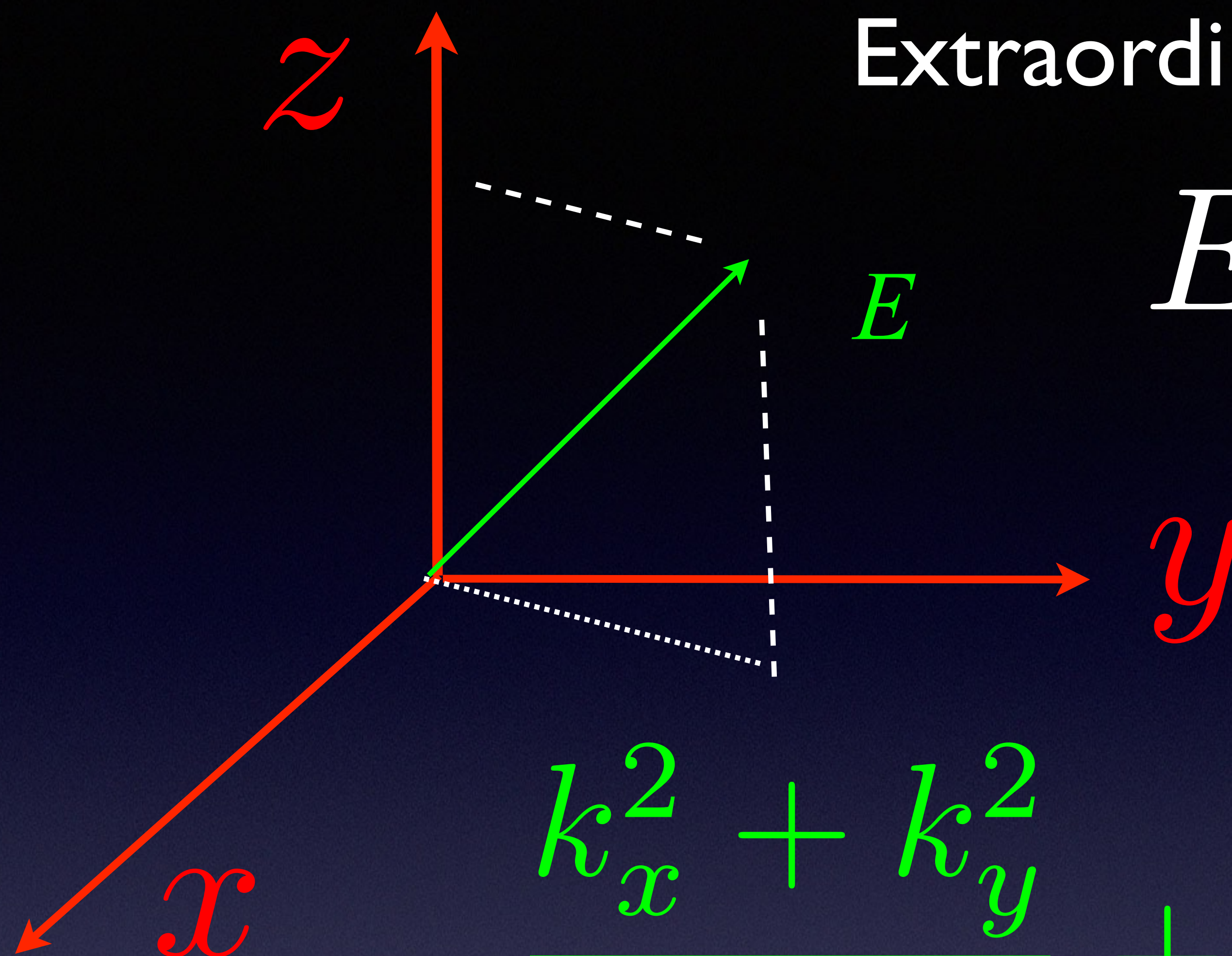


The parable of the blind men and an elephant



"... there be Dragons..."



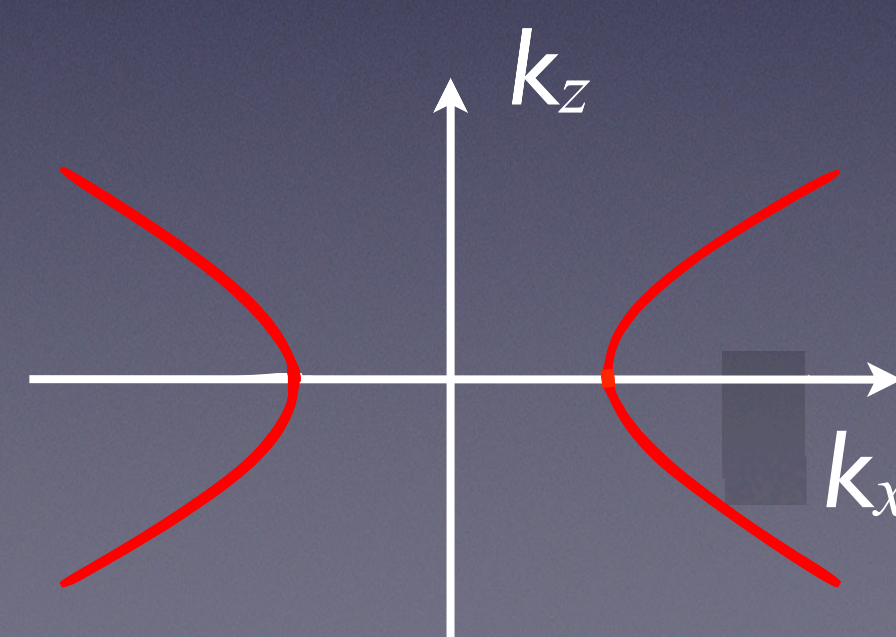


Extraordinary Wave:

$$E \notin (x, y)$$

$$\frac{k_x^2 + k_y^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

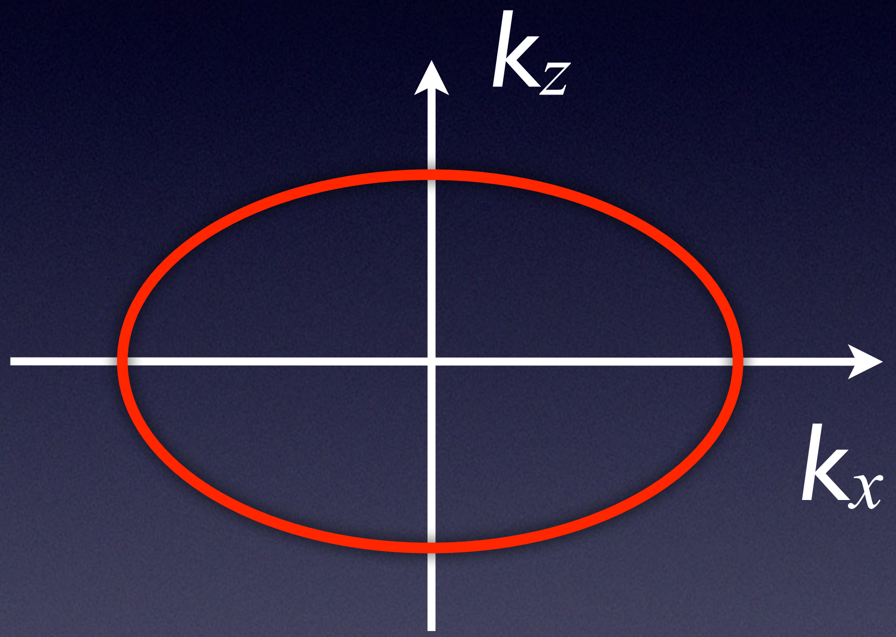
$\varepsilon_x < 0,\ \ \varepsilon_z > 0 \Rightarrow$



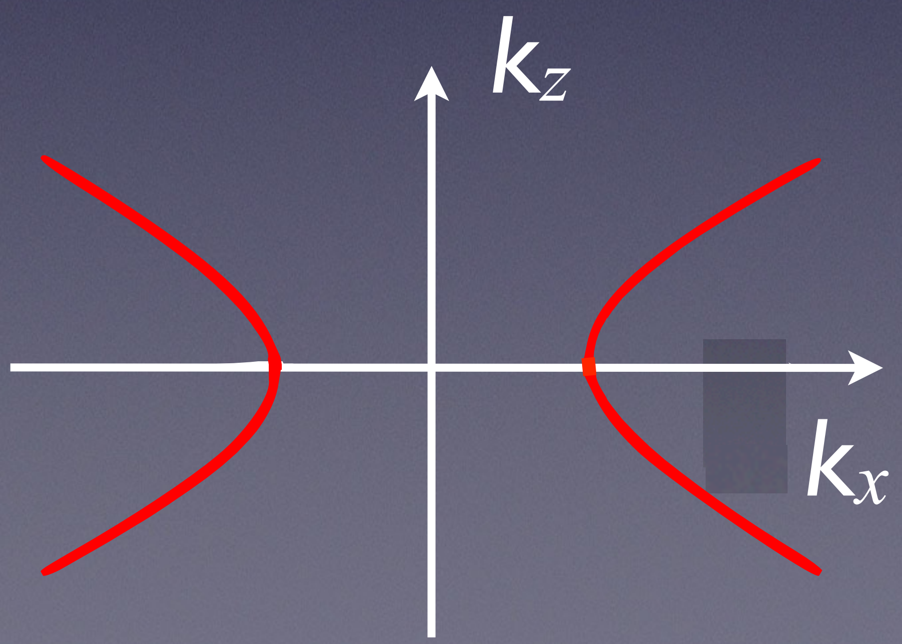
Anisotropic Materials

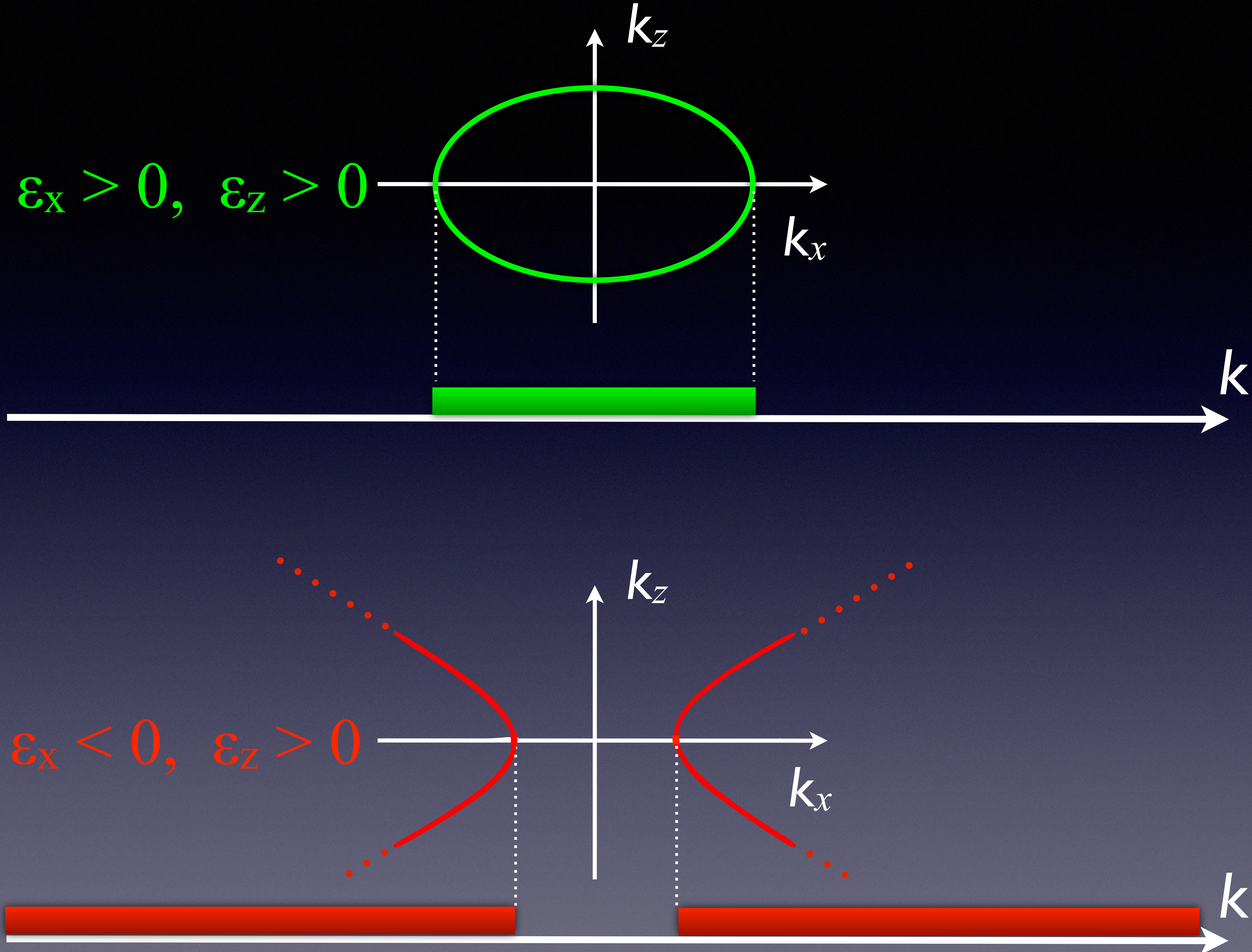
$$\frac{k_x^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

$\varepsilon_x > 0,\ \ \varepsilon_z > 0 \quad \Rightarrow$

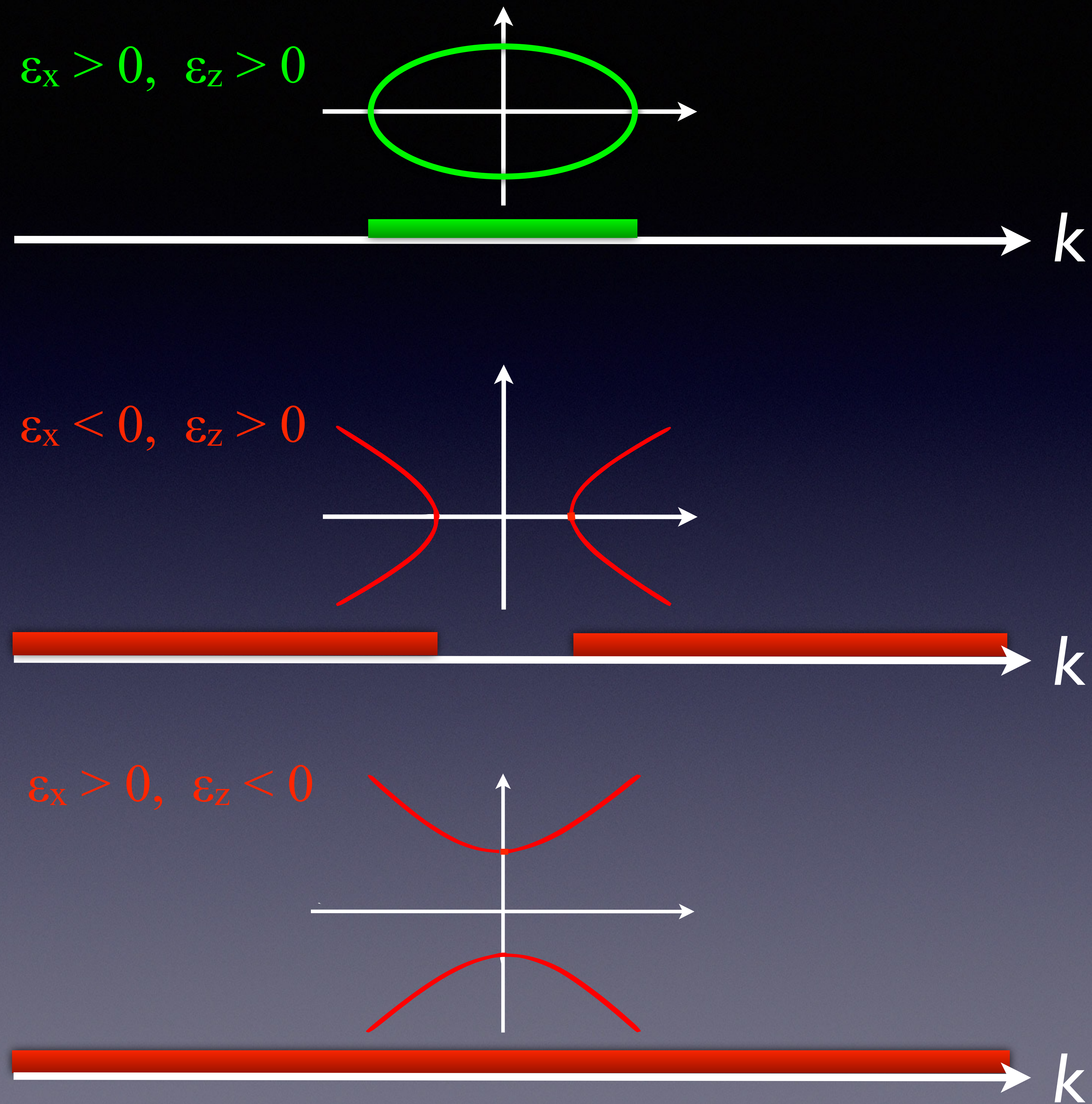


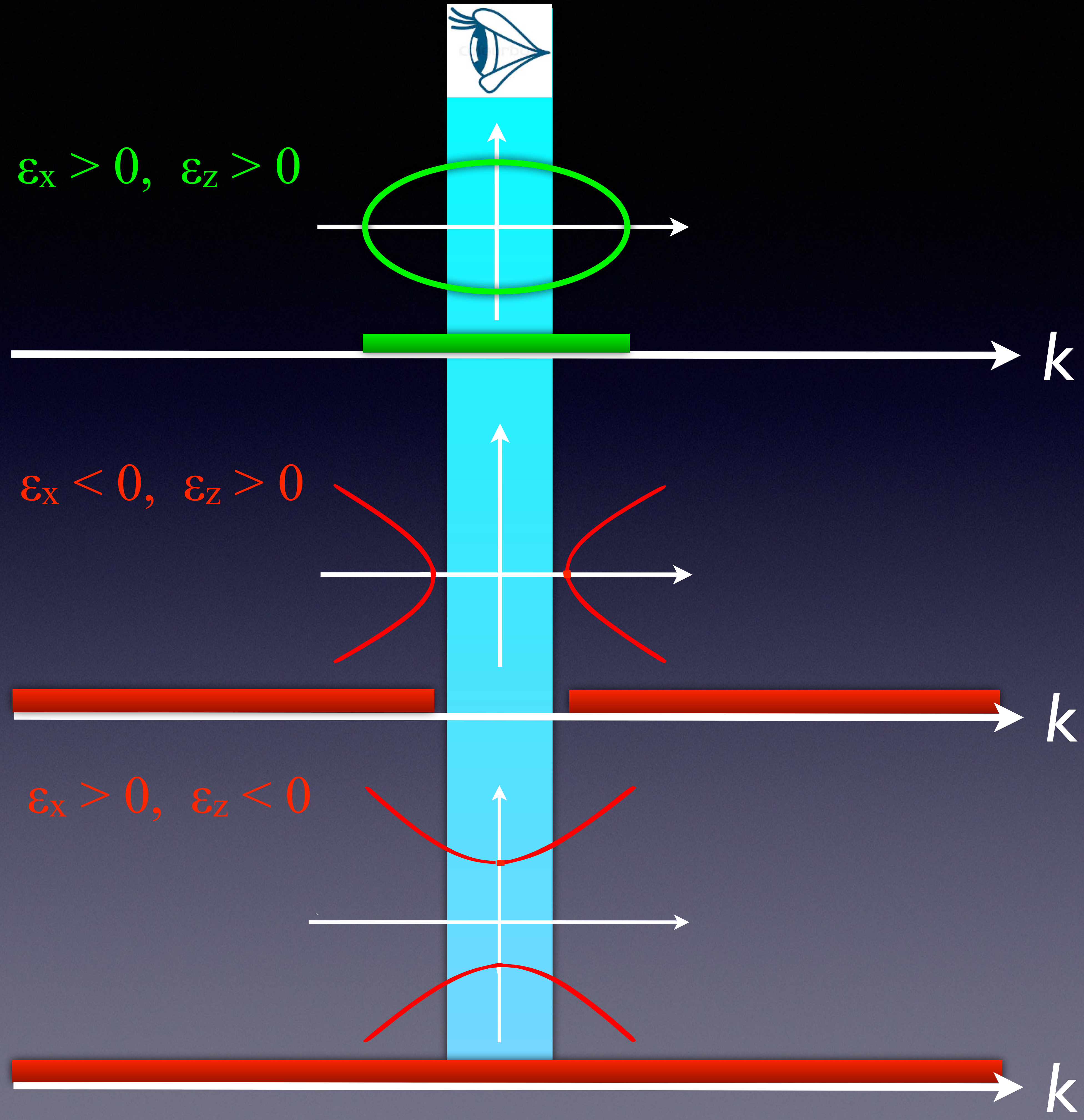
$\varepsilon_x < 0,\ \ \varepsilon_z > 0 \quad \Rightarrow$



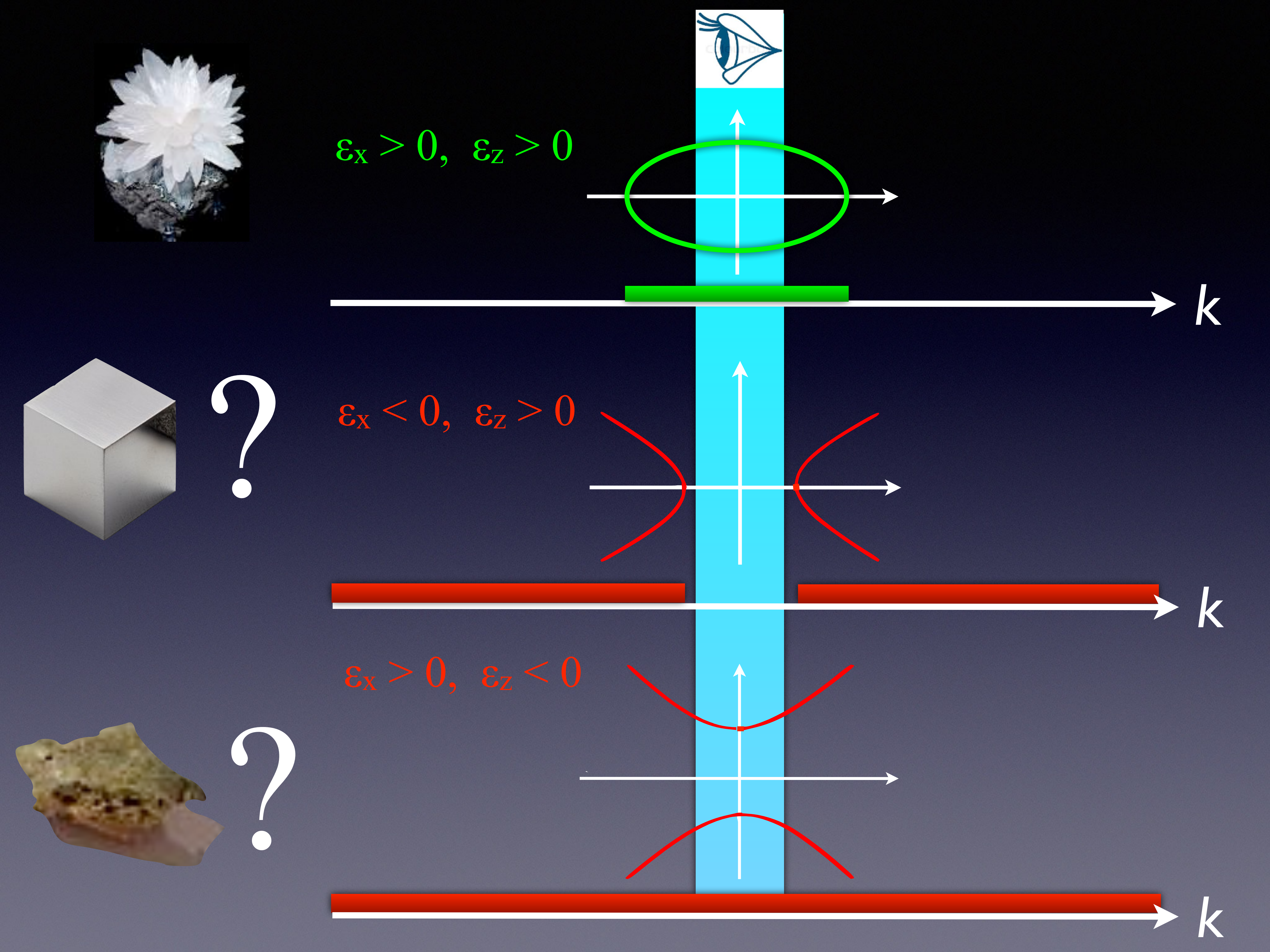


k_z
$\varepsilon_x > 0, \ \ \varepsilon_z > 0$
k_x
k
k_z
$\varepsilon_x < 0, \ \ \varepsilon_z > 0$
k_x
k

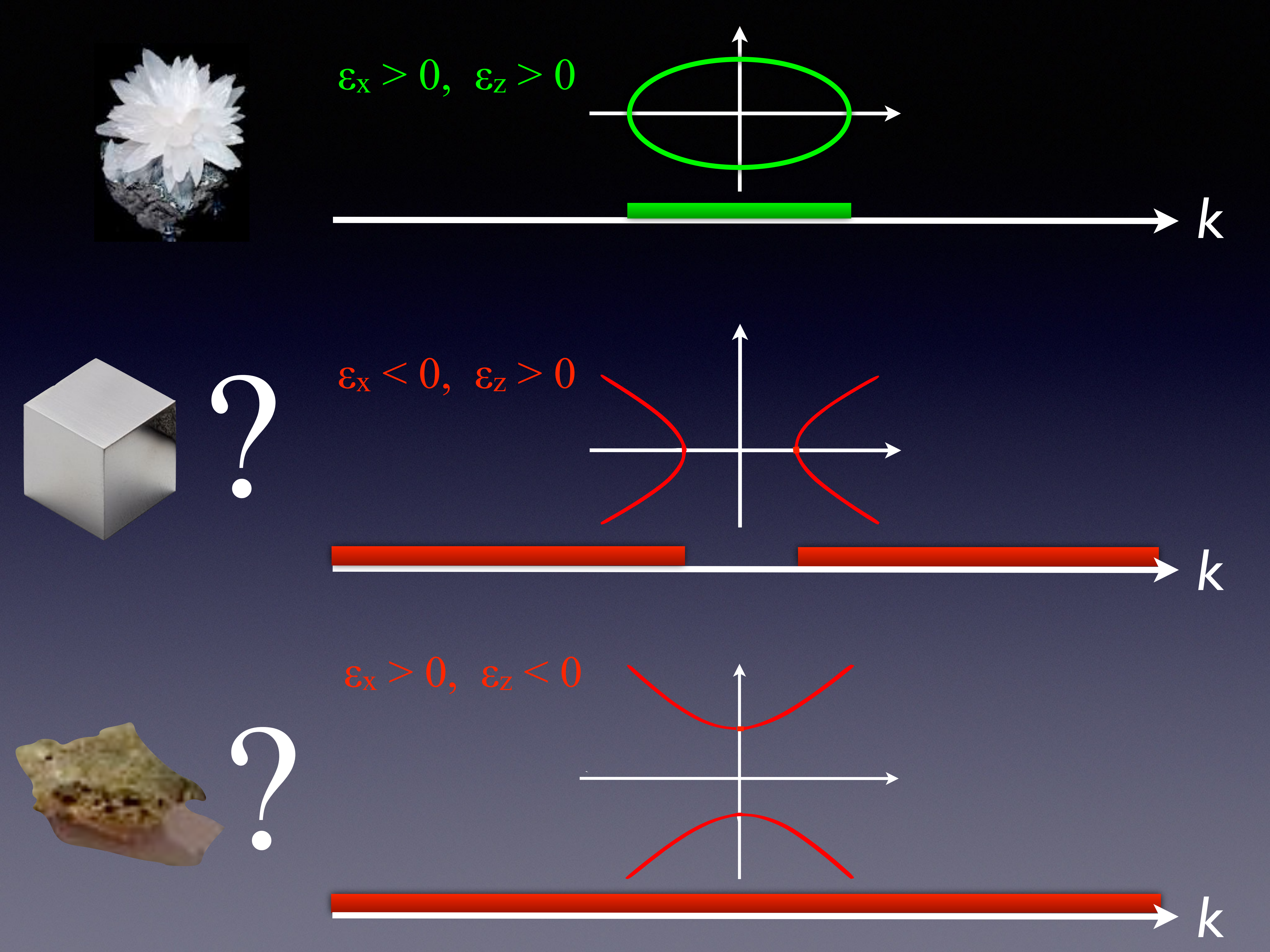

$\varepsilon_x > 0, \ \varepsilon_z > 0$
k
$\varepsilon_x < 0, \ \varepsilon_z > 0$
k
$\varepsilon_x > 0, \ \varepsilon_z < 0$
k



εx > 0, εz > 0
k
εx < 0, εz > 0
k
εx > 0, εz < 0
k

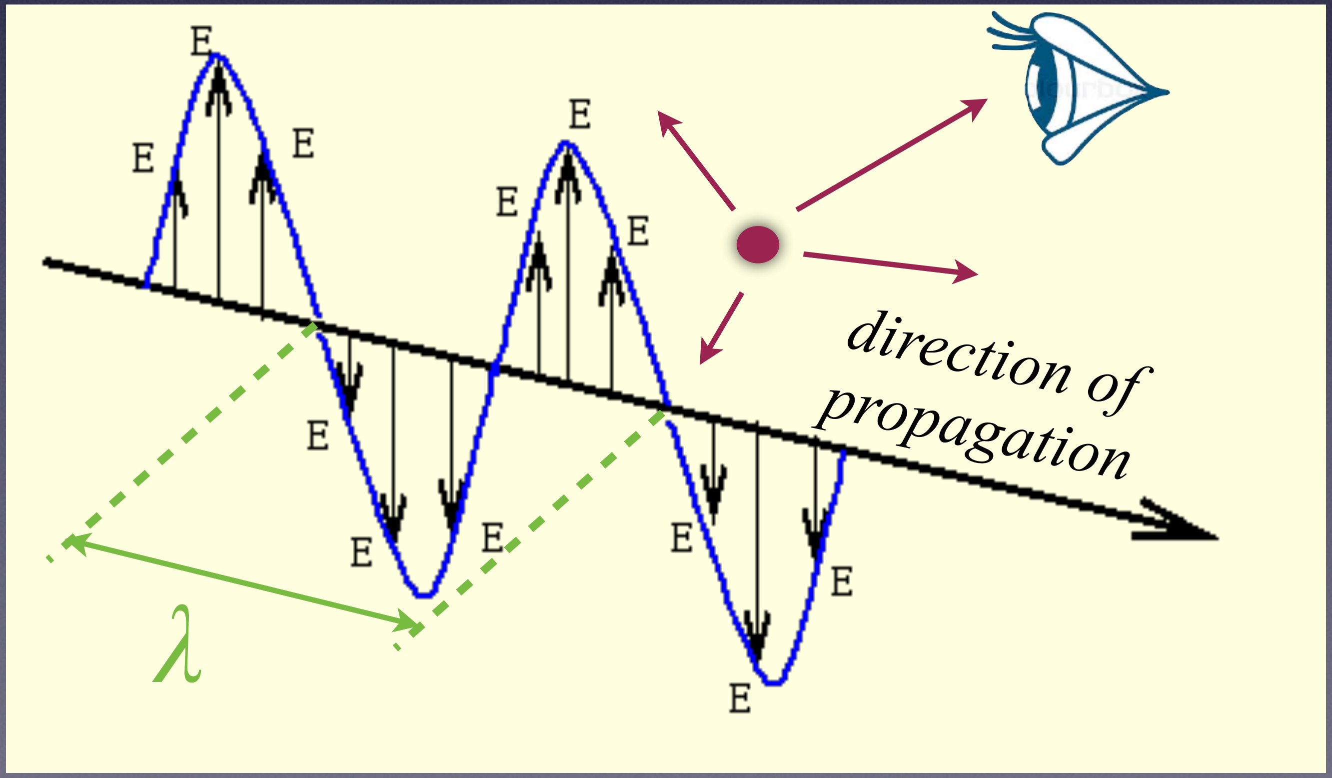


$\varepsilon_x > 0, \ \varepsilon_z > 0$
k
?
$\varepsilon_x < 0, \ \varepsilon_z > 0$
k
?
$\varepsilon_x > 0, \ \varepsilon_z < 0$
k

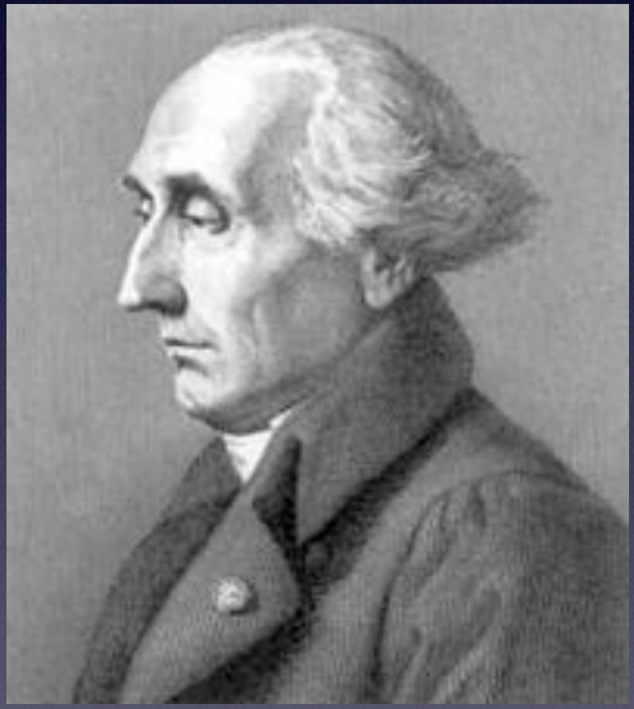

$\varepsilon_x > 0,\ \varepsilon_z > 0$
k
?
$\varepsilon_x < 0,\ \varepsilon_z > 0$
k
?
$\varepsilon_x > 0,\ \varepsilon_z < 0$
k

Optical Imaging

Wavelength λ gives characteristic scale of optical resolution, focusing, etc.

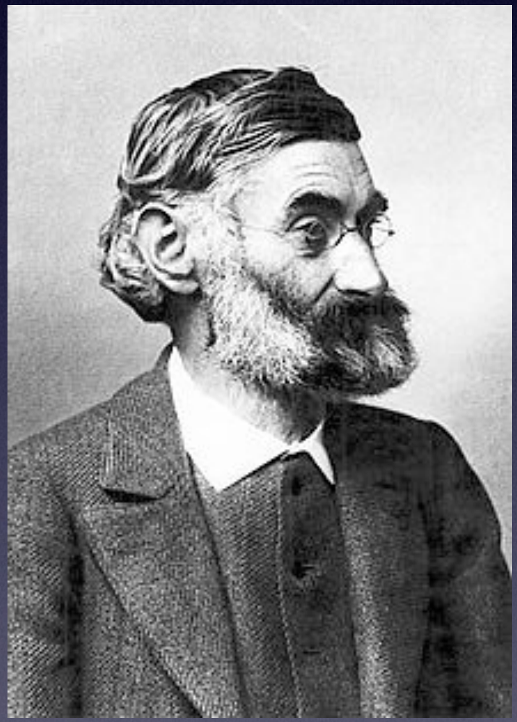


" Abbe's " Resolution Limit : $\Delta = \dfrac{\lambda}{2\ \mathrm{NA}}$



Joseph-Louis Lagrange
(1736 - 1813)

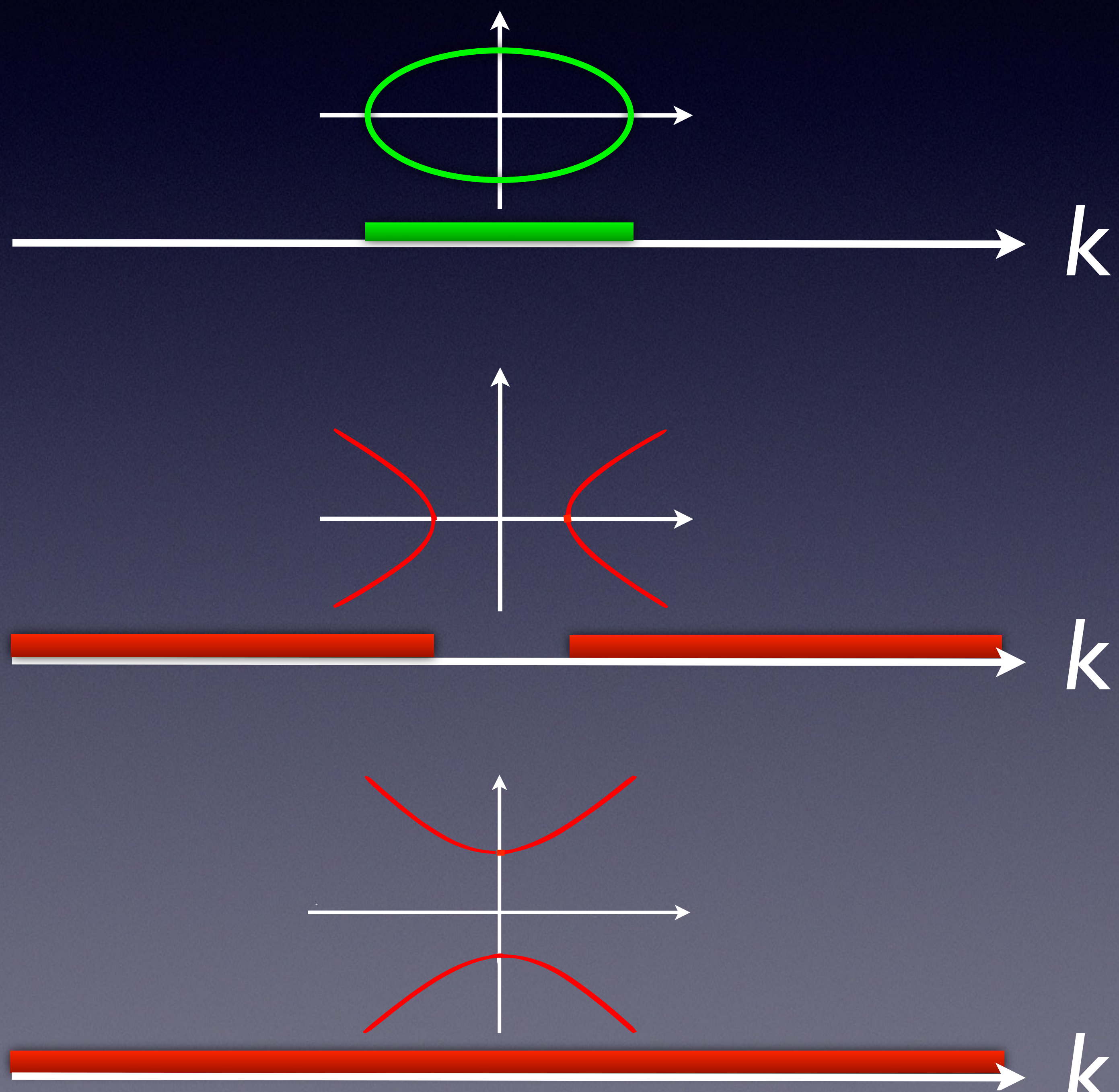
Hermann von Helmholz
(1821 - 1894)



Ernst Karl Abbe
(1840 - 1905)

“ Abbe’s ” Resolution Limit : $\Delta = \dfrac{\lambda}{2\ \mathrm{NA}}$

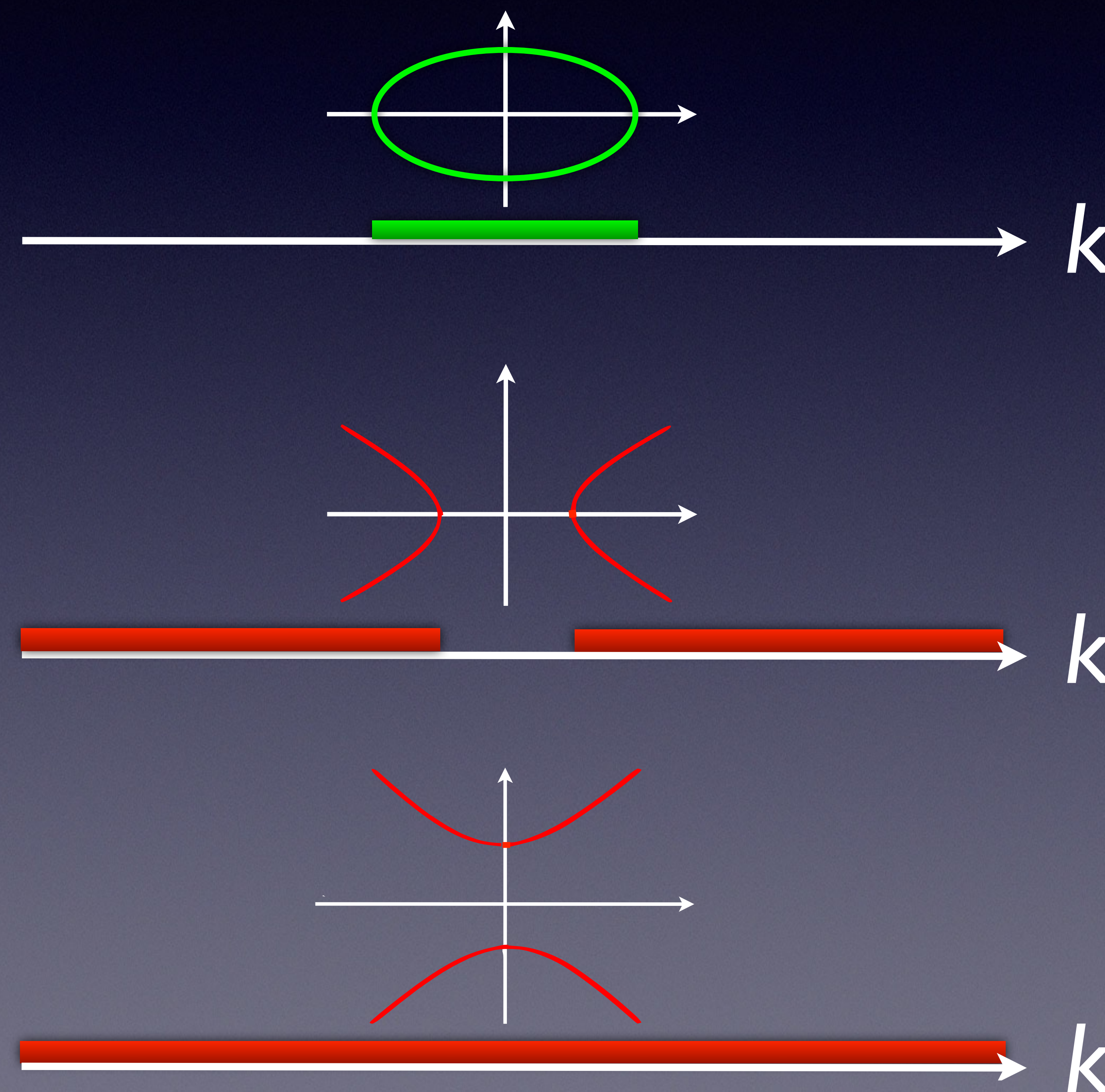
$$\lambda = \frac{2\pi}{k}$$



“ Abbe’s ” Resolution Limit : $\Delta = \dfrac{\lambda}{2\ \mathrm{NA}}$

$$\lambda = \frac{2\pi}{k}$$

No Limit !



What it means :

- resolution of a “hyperbolic” microscope is *not* limited by the (free-space) wavelength
- (e.g. visible light microscope with X-ray resolution)

- light (in a hyperbolic medium) *can* be focused to arbitrarily* small spot

- forbidden optical transitions are *not* forbidden

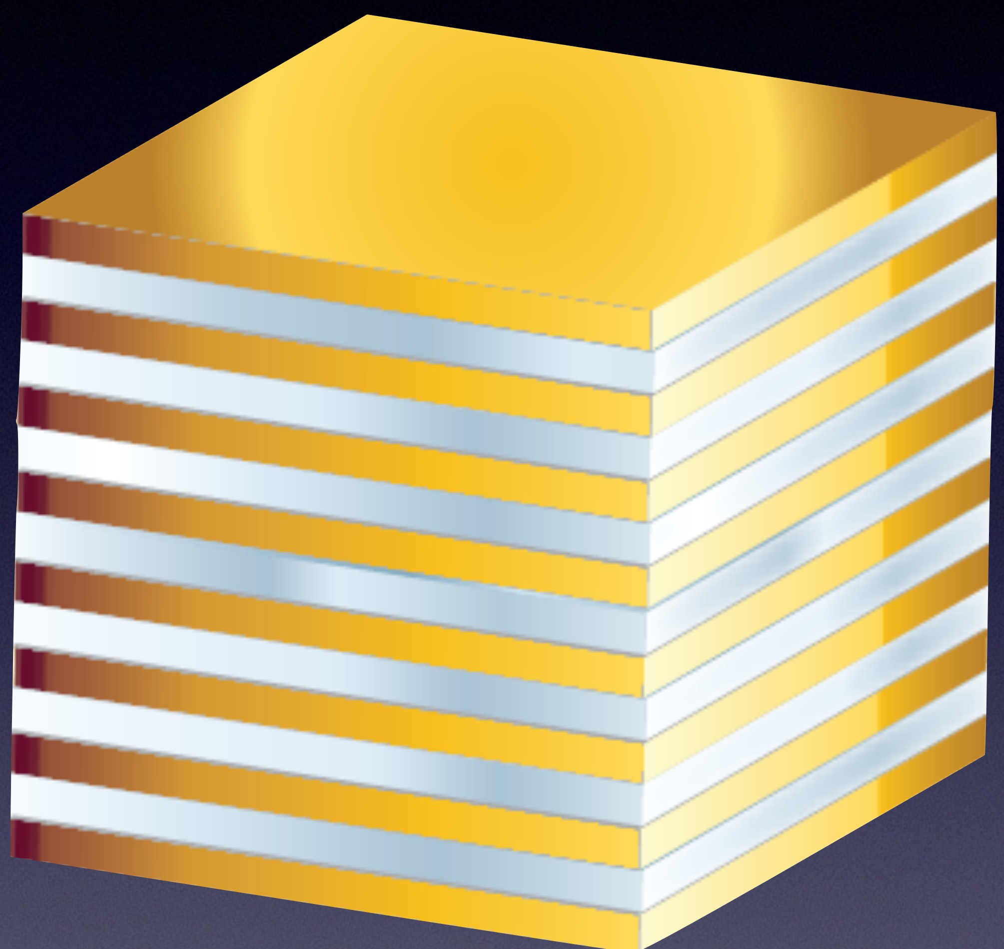
- LED modulation speed is *not* limited by the spontaneous emission rate of its active medium

- and a lot more

* subject to the atom size constraint

Hyperbolic Materials: practical realization

Hyperbolic Metamaterials: practical realization



- Low loss
- Broadband
- Bulk size

X. Zhang (Berkeley)
Ag/Al_2O_3 system
ultraviolet and visible

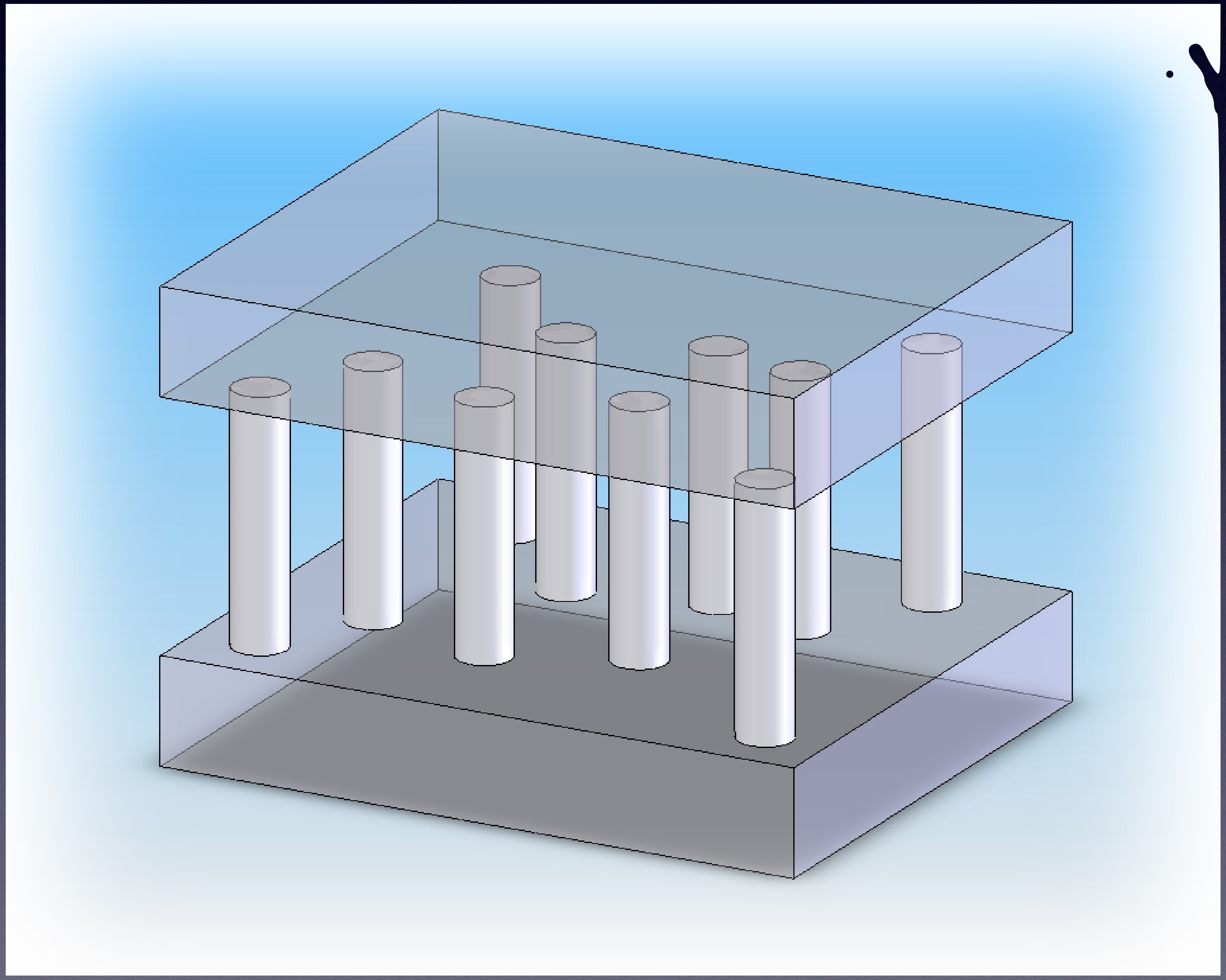
V. Menon (QUNY)
Visible

A. Boltasseva and
V. Shalaev (Purdue)
Visible and 1.55 μm

C. Gmachl (Princeton)
Semiconductor system
Mid-infrared

Hyperbolic Metamaterials: practical realization

Nanowires in dielectric membrane

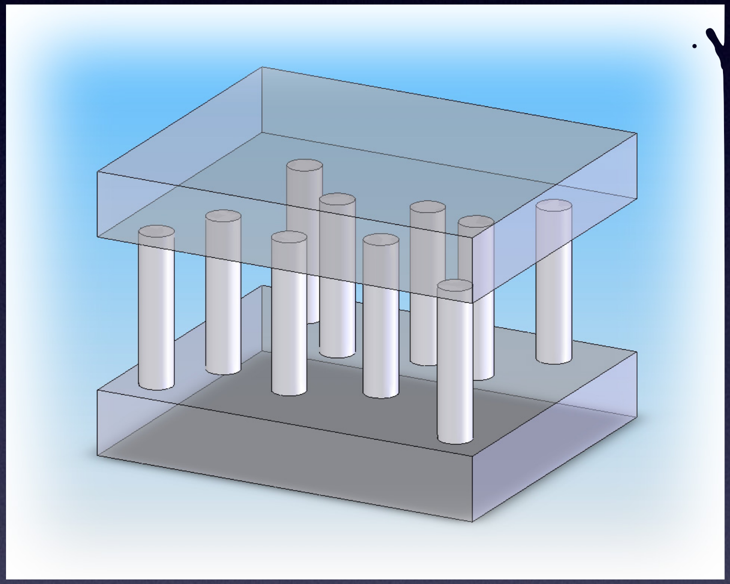


M. Noginov (NSU)
Visible

X. Zhang (Berkeley)
Visible

Hyperbolic Metamaterials: practical realization

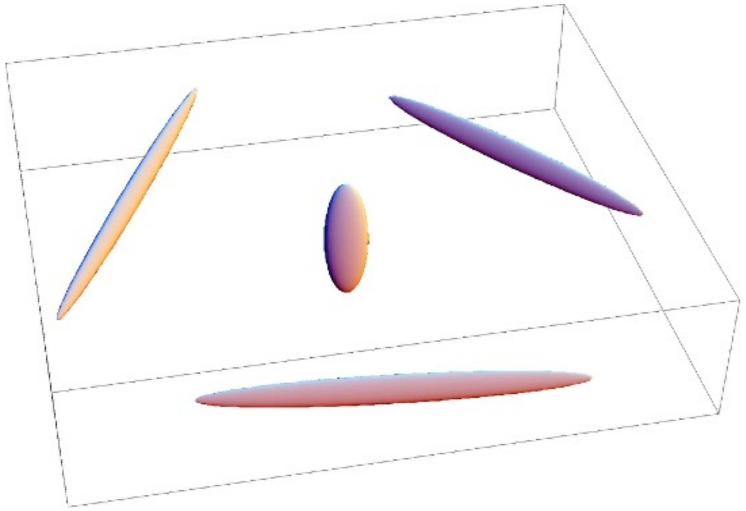
Nanowires in a dielectric membrane



- Low loss
- Broadband
- Bulk size

THz: Bismuth-ology

- most solid-state "quantum effects" (e.g. Shubnikov-de Haas & de Hass – van Alphen effects) were first discovered in bismuth – why?
- Ultra-low losses: carrier mean free path (at helium temperatures) on the order of millimeters
- Highly anisotropic Fermi surface
- Plasma frequency in the THz



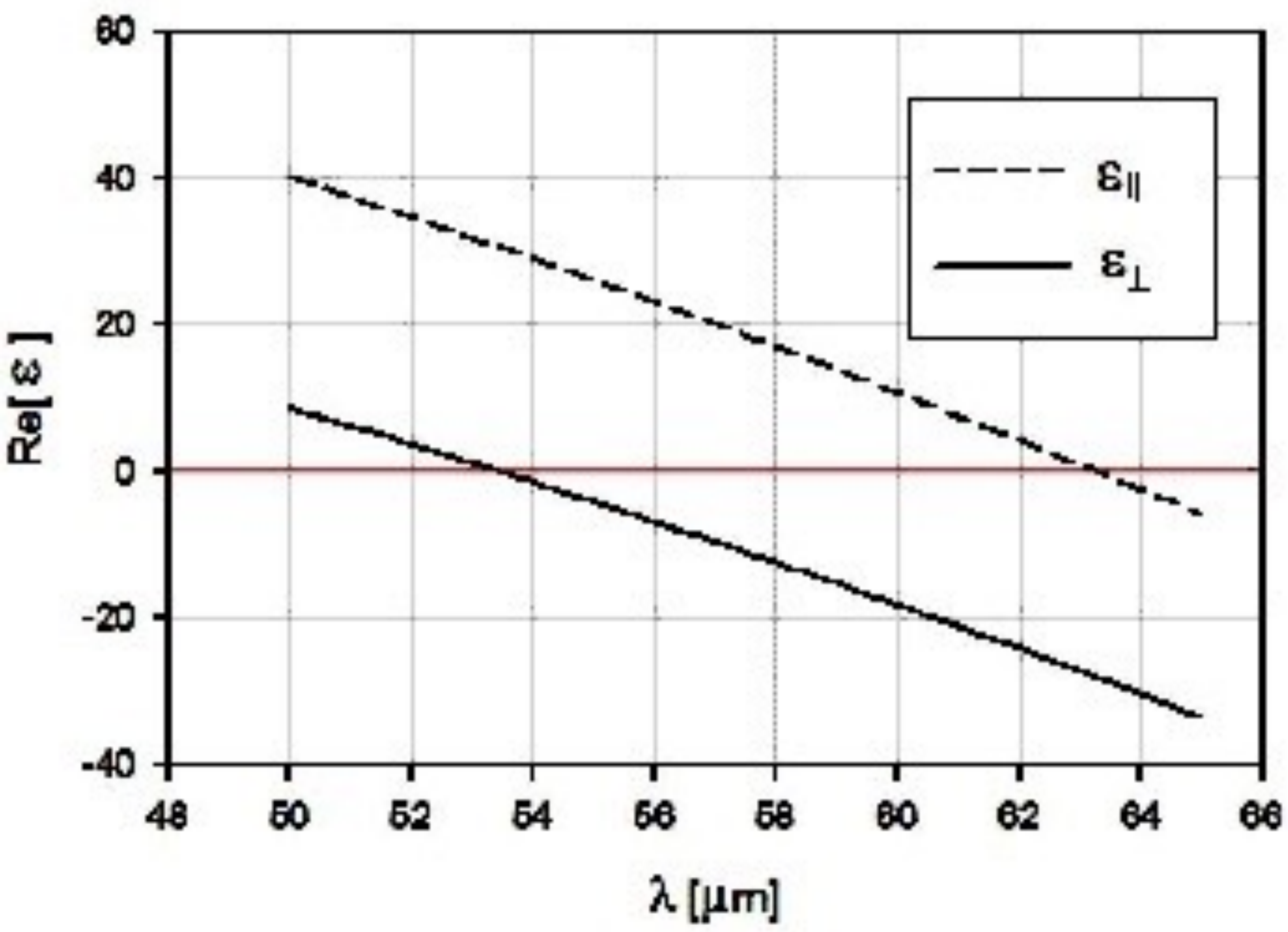
High-quality Bi monocrystals available since 1970s, high-quality monocrystalline Bi films – since 1990s

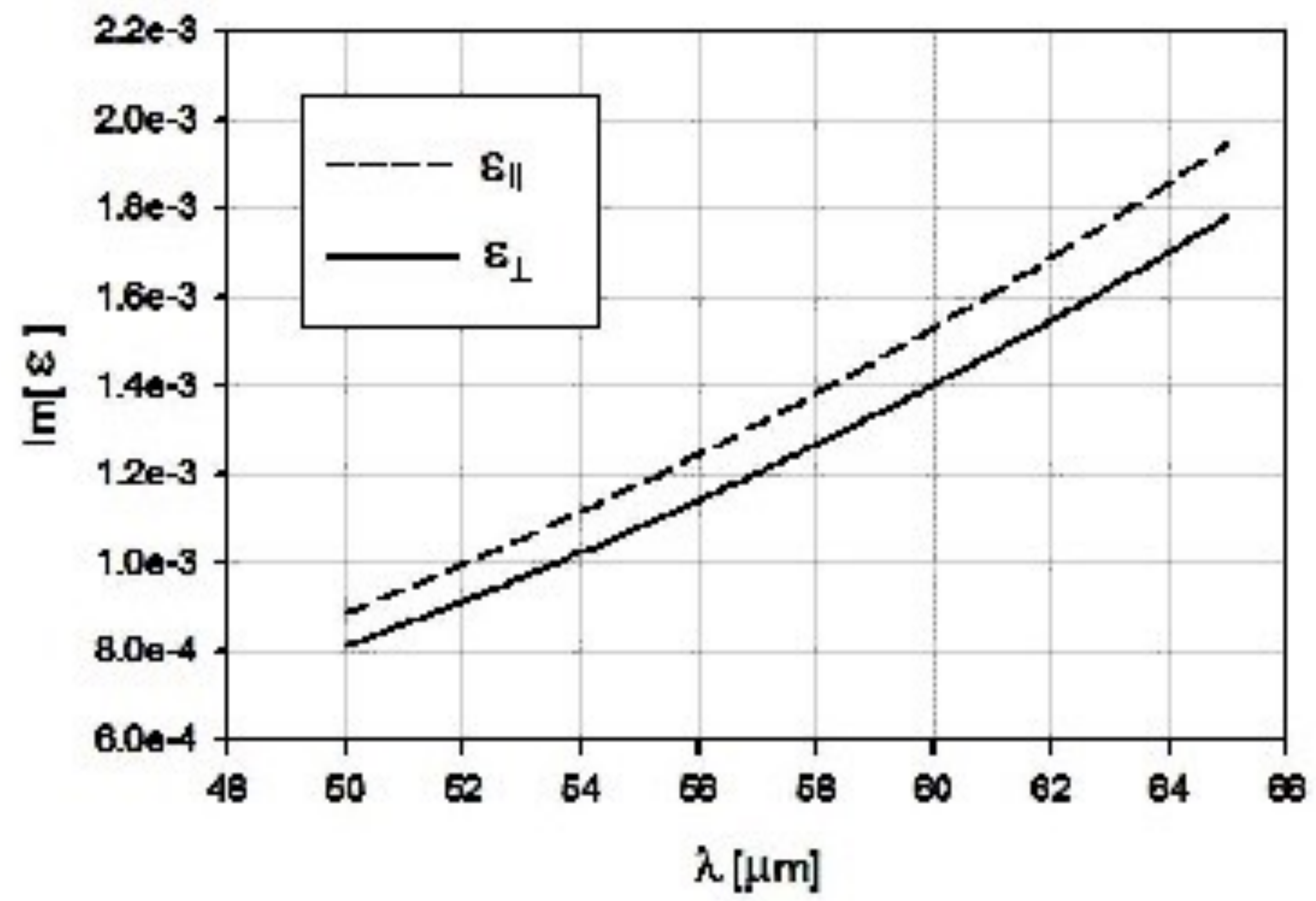
Strong effective mass anisotropy

Anisotropy in plasma frequencies

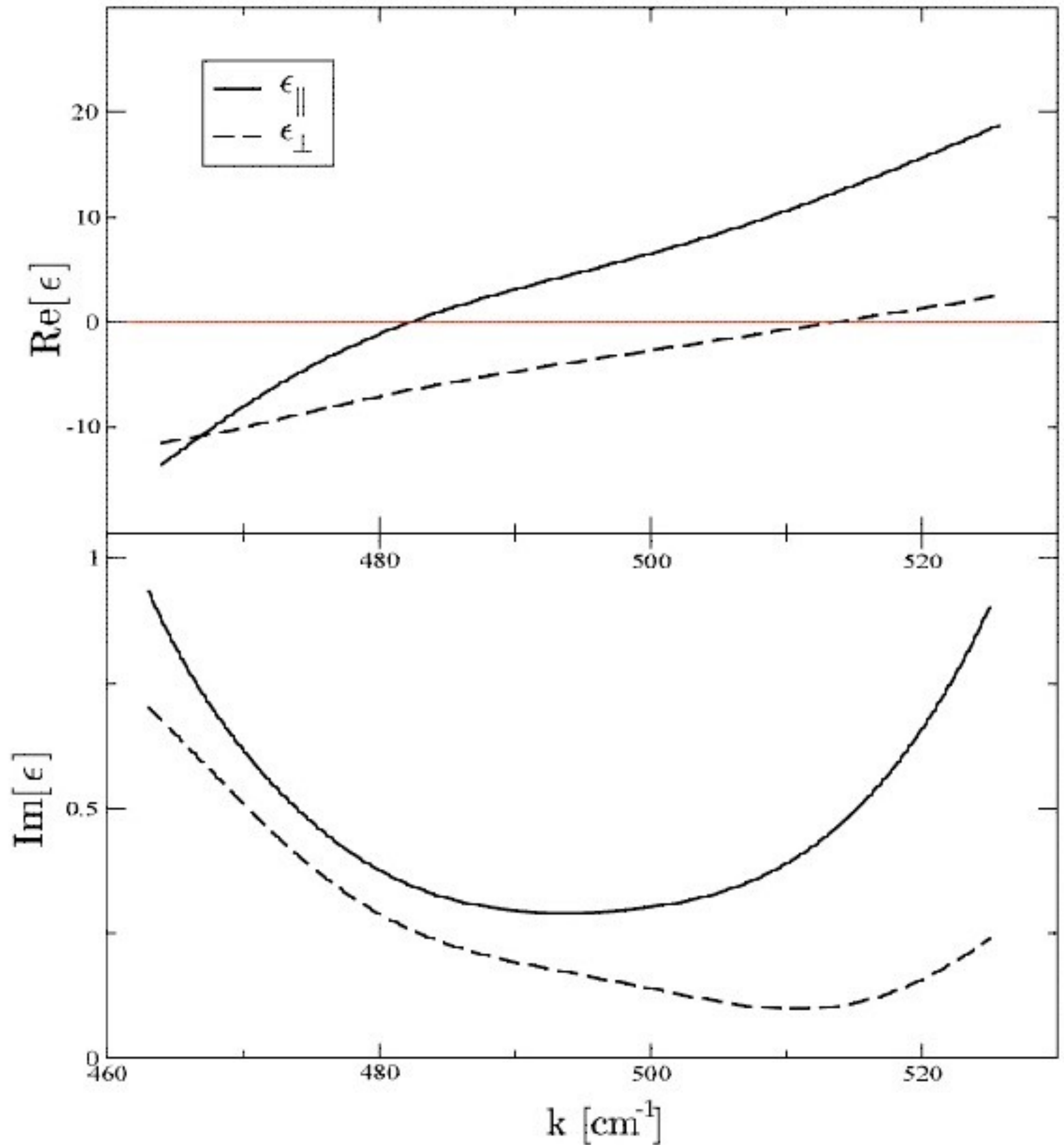
- observed in experiments since 1960s!

$\omega_{pl} = 187\,cm^{-1}$ for E $\parallel$ C_3 & $\omega_{pl} = 158\,cm^{-1}$ for E $\perp$ C_3

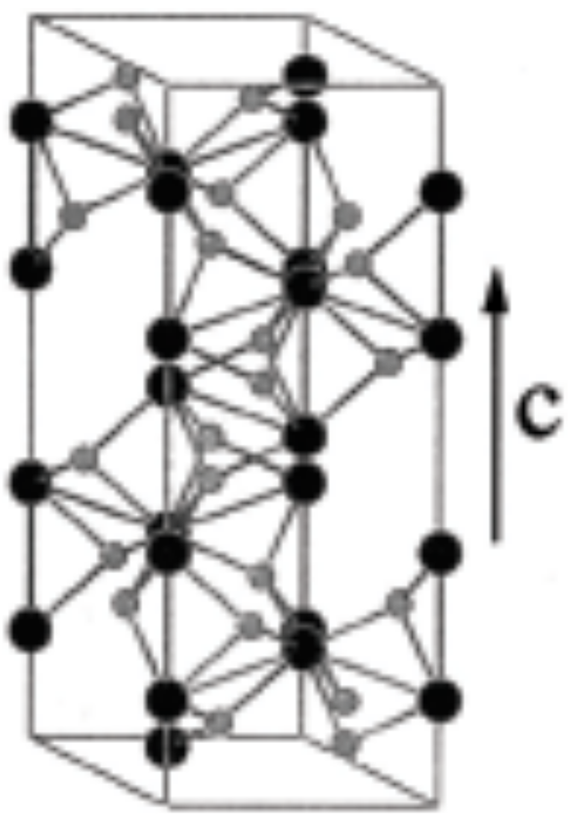




Far IR: Sapphire (Al_2O_3)



Strong phonon anisotropy



M.Schubert, T.E.Tiwald, C.M.Herzinger, "Infrared dielectric anisotropy and phonon modes of sapphire", PRB **61**, 8187 (2000)

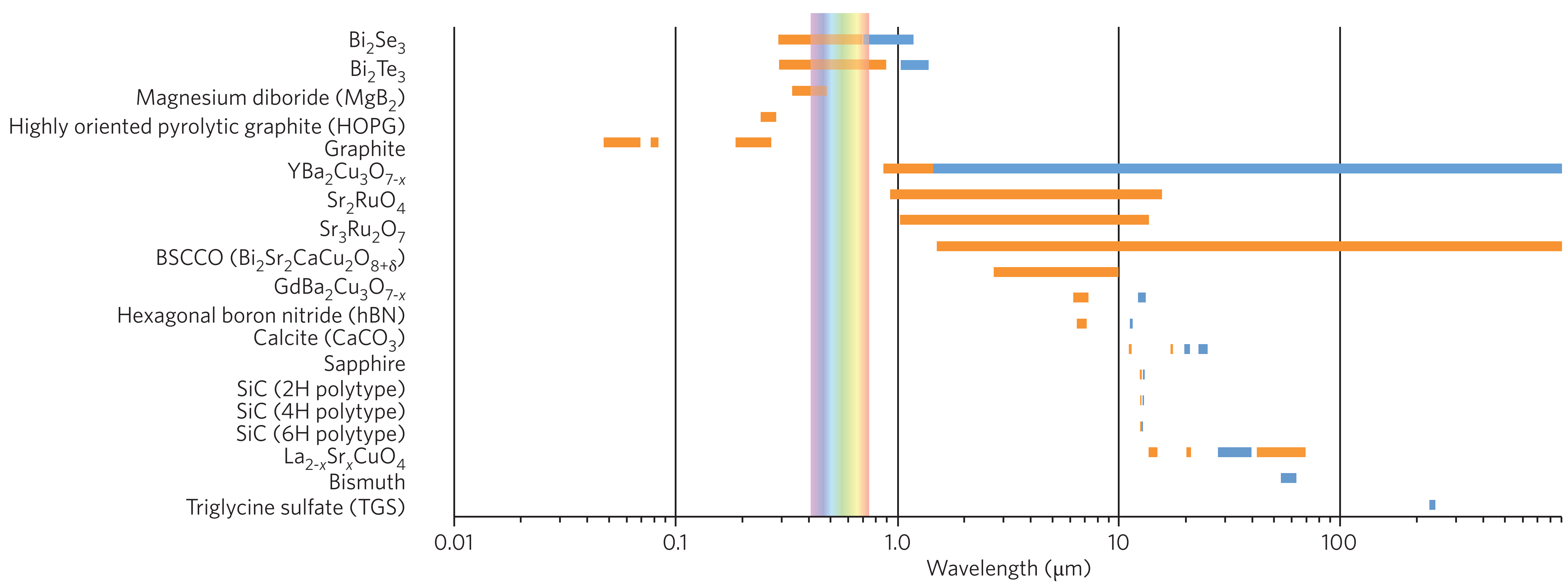
NATURE PHOTONICS | VOL 9 | APRIL 2015 | www.nature.com/naturephotonics

METAMATERIALS

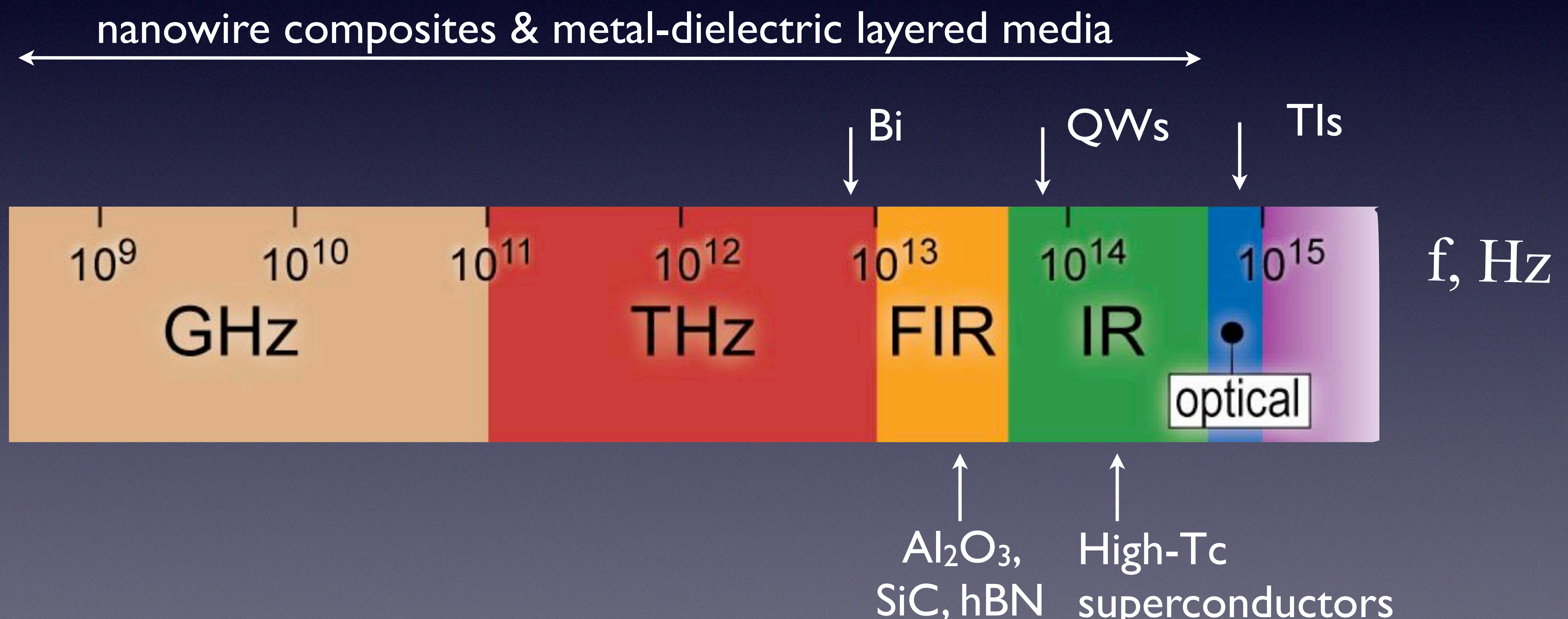
Naturally hyperbolic

Natural hyperbolic materials hold the key to unlocking the full potential of hyperbolic media in nanophotonics. Until now no such materials were available for visible light but recent work finally brings down this roadblock.

Evgenii E. Narimanov and Alexander V. Kildishev



Hyperbolic (meta)materials:



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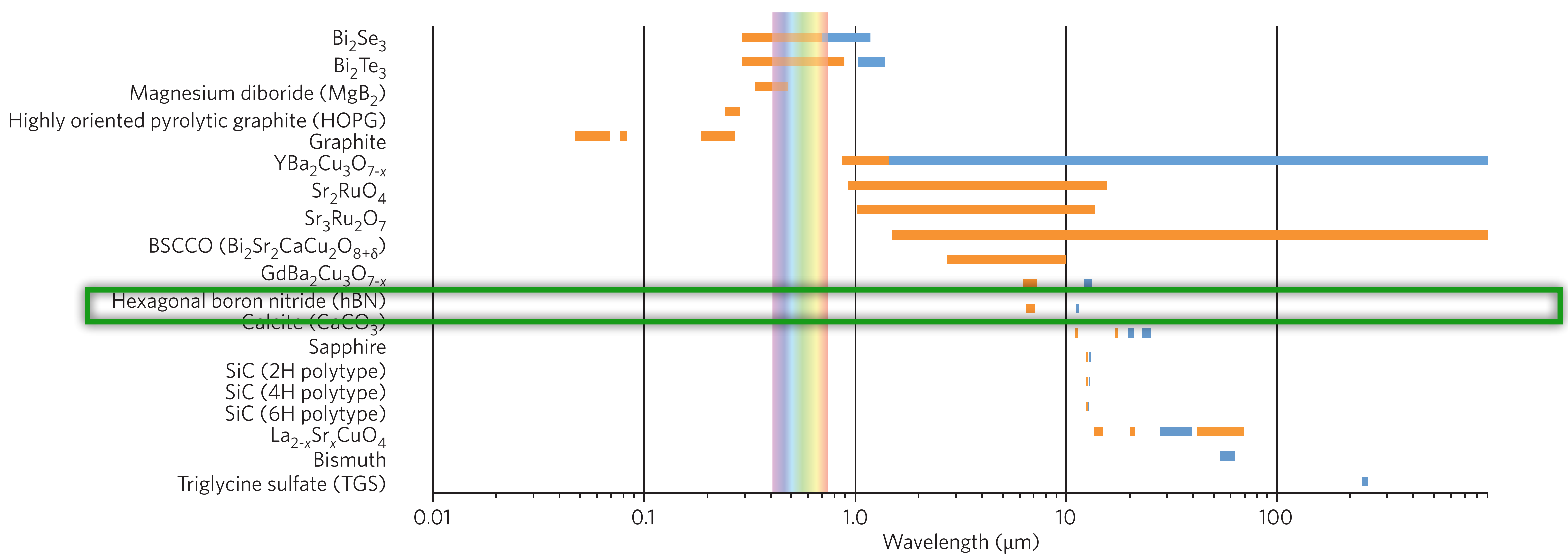
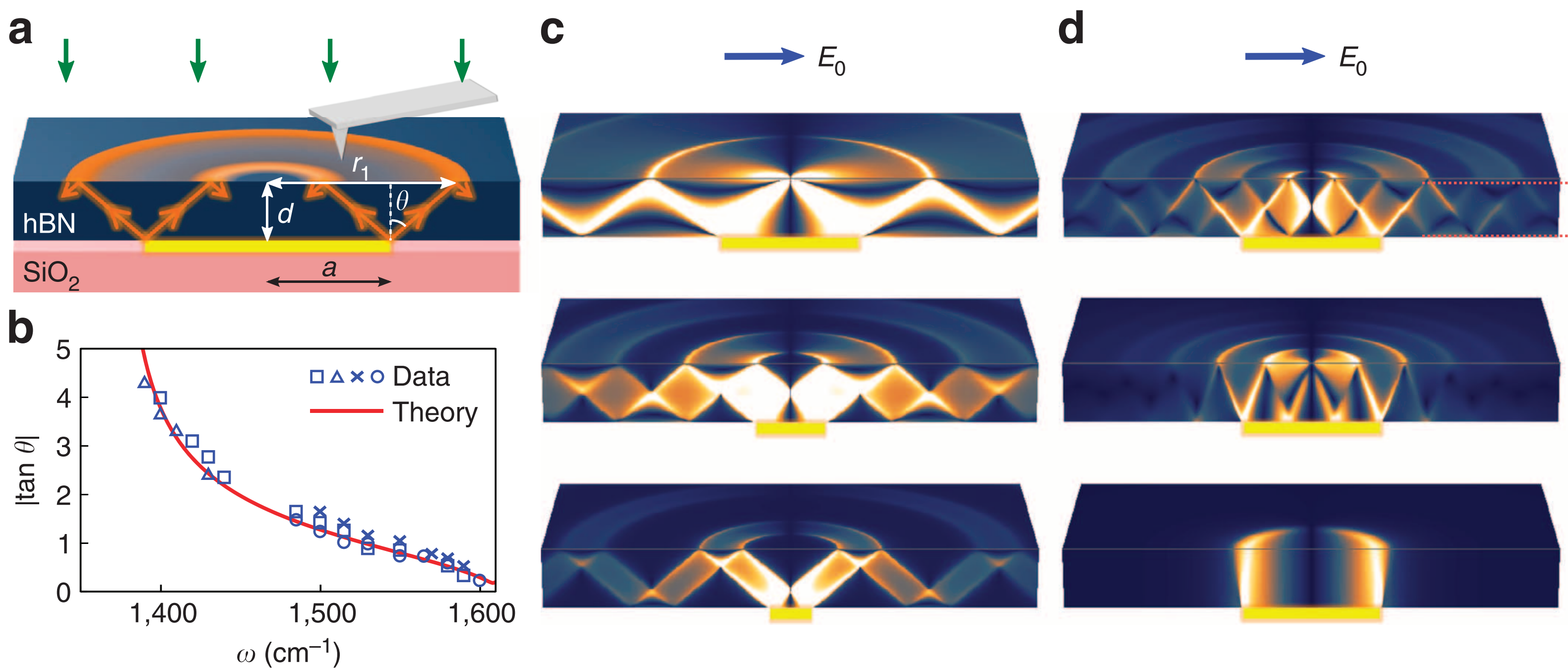


ARTICLE

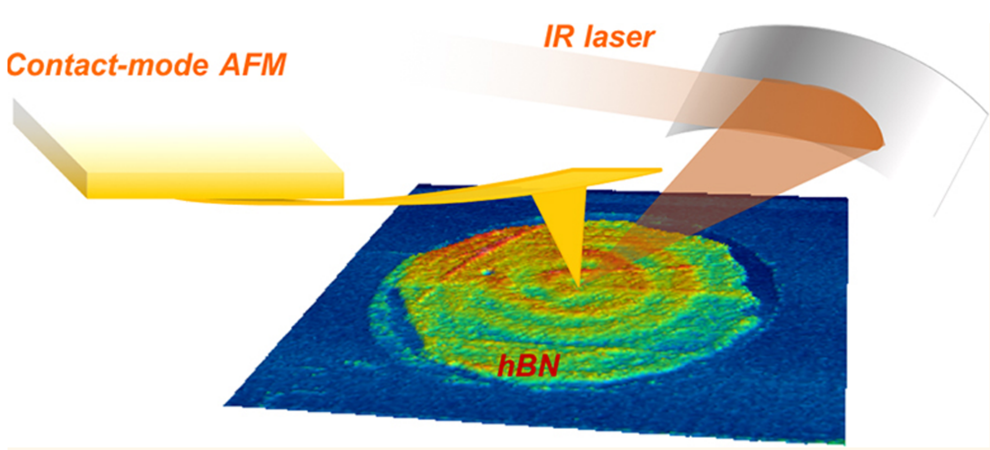
NATURE COMMUNICATIONS | DOI: 10.1038/ncomms7963



270 nm

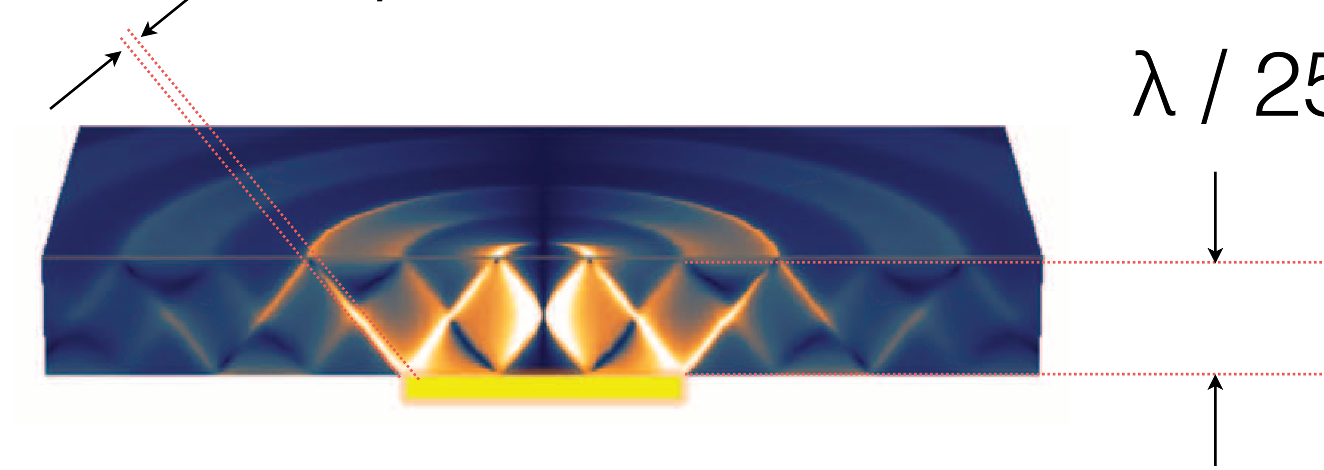
λ~ 7000 nm

S. Dai *et al*, Nature Comm. 2015



~ λ / 500 ~ 10 nm

λ / 25



A. Ambrosio *et al*, ACS nano 2016

- resolution of a “hyperbolic” microscope is *not* limited by the (free-space) wavelength
(e.g. visible light microscope with X-ray resolution)

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- and a lot more (negative refraction, “darker-than-black” materials, etc.)

* subject to the atom size constraint

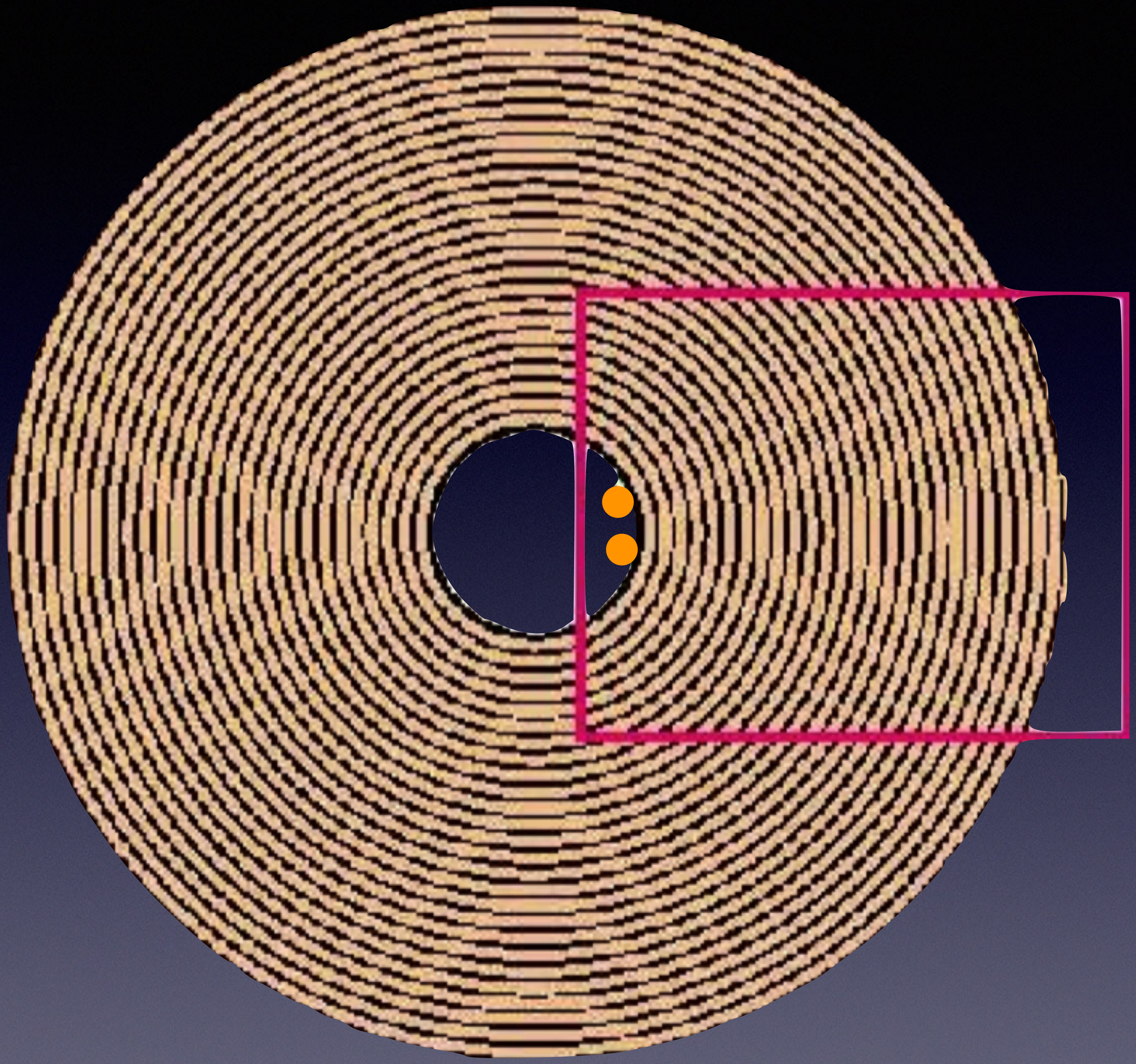
The Hyperlens

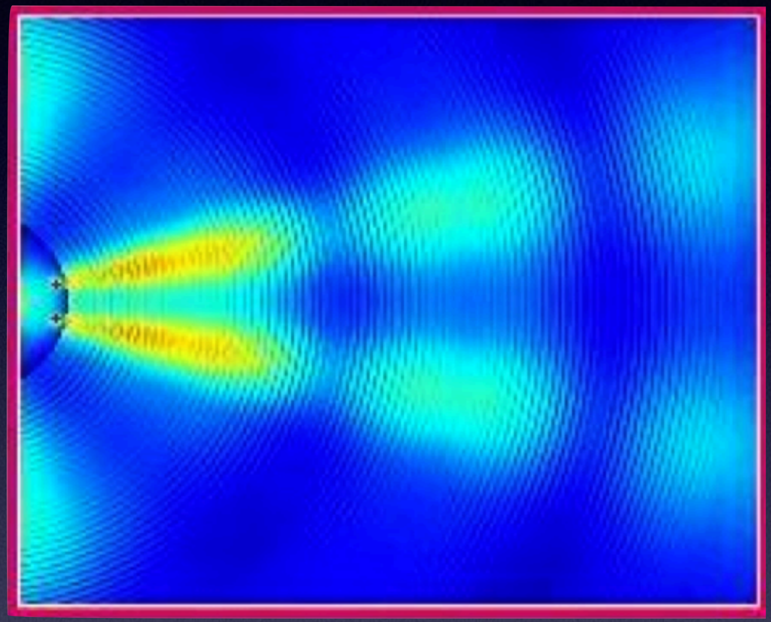
Super-resolution $\quad + \quad$ Magnification

$\Uparrow \qquad\qquad \Uparrow$

Hyperbolic Metamaterial $\qquad$ Curvature

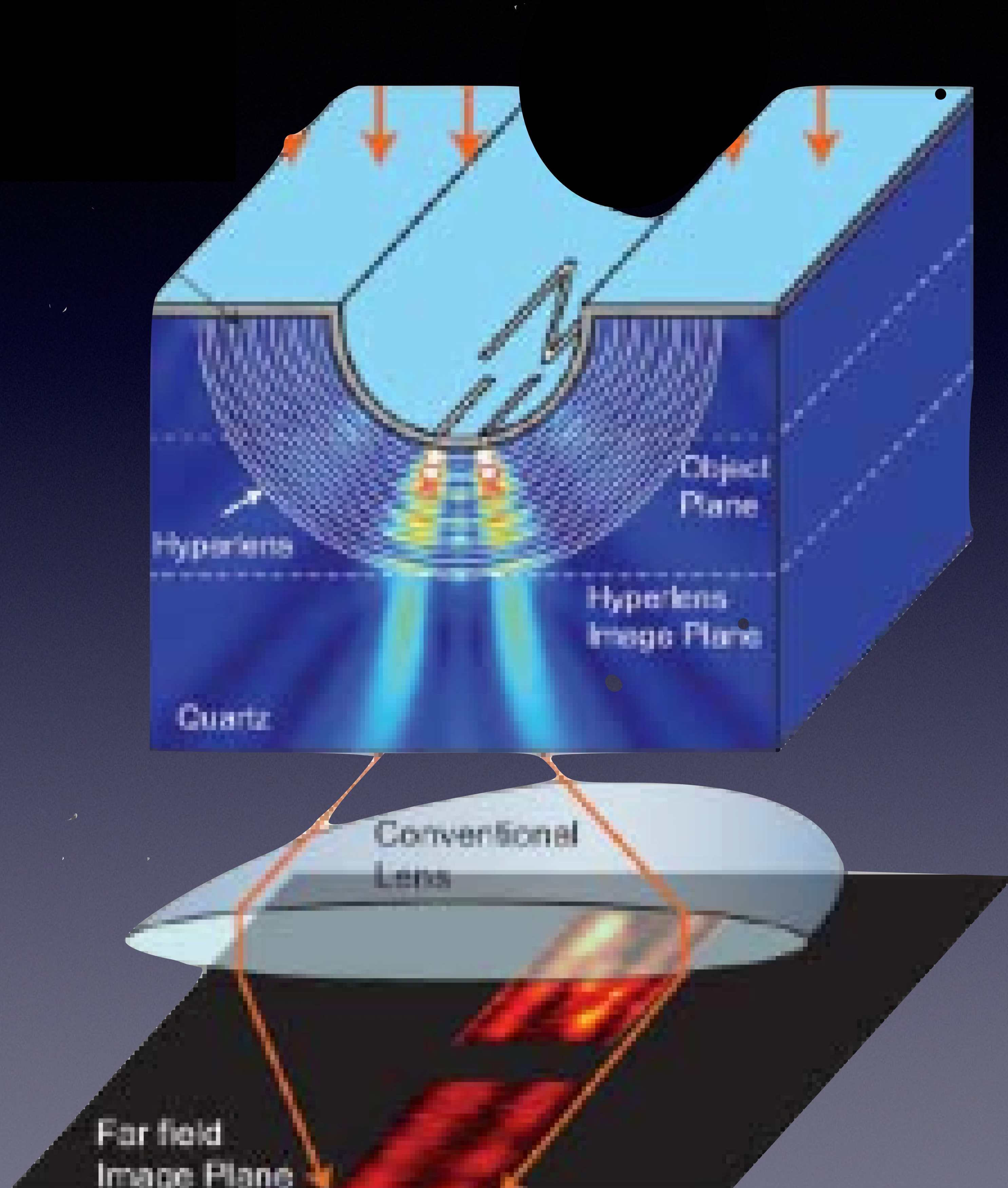
Inner radius ~ λ, outer radius ~ 5 λ



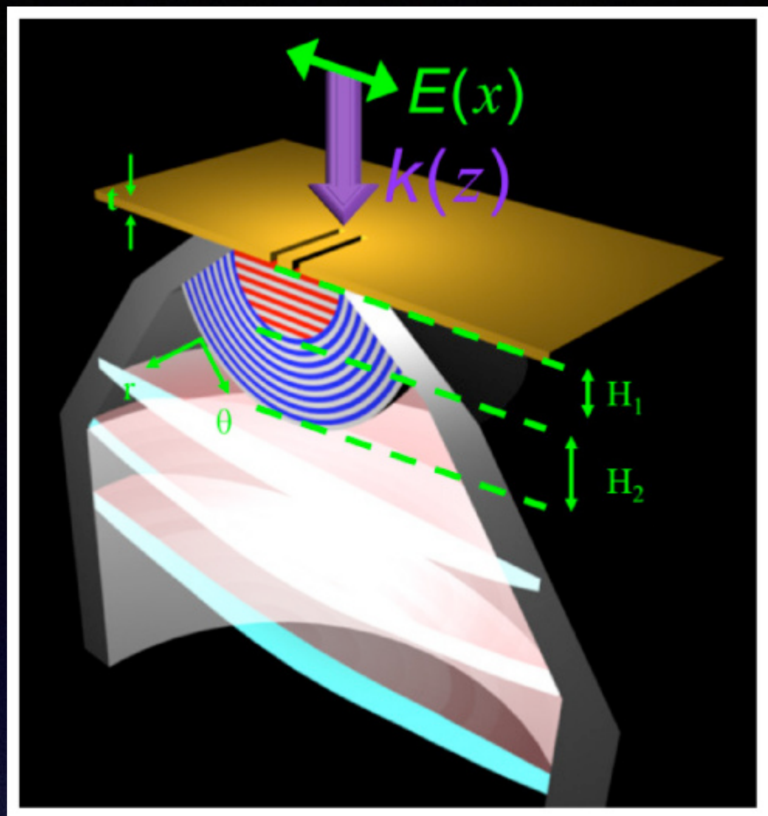


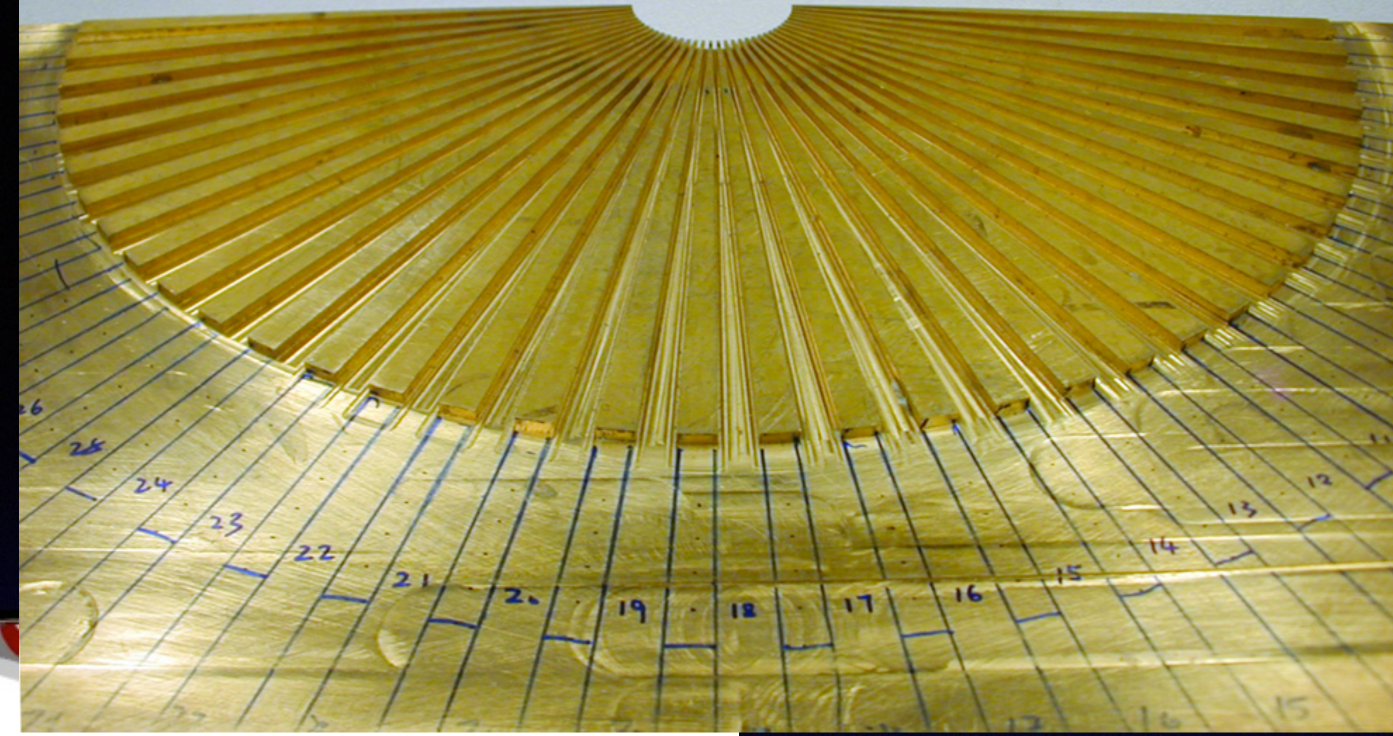
Z. Jacob, L. Alekseyev, EN, Optics Express 2006
A. Salandrino, N. Engheta, PRB 2006

The Hyperlens : experiment



X. Zhang et al, Science 2007


E(x)
k(z)
r
θ
H1
H2


26
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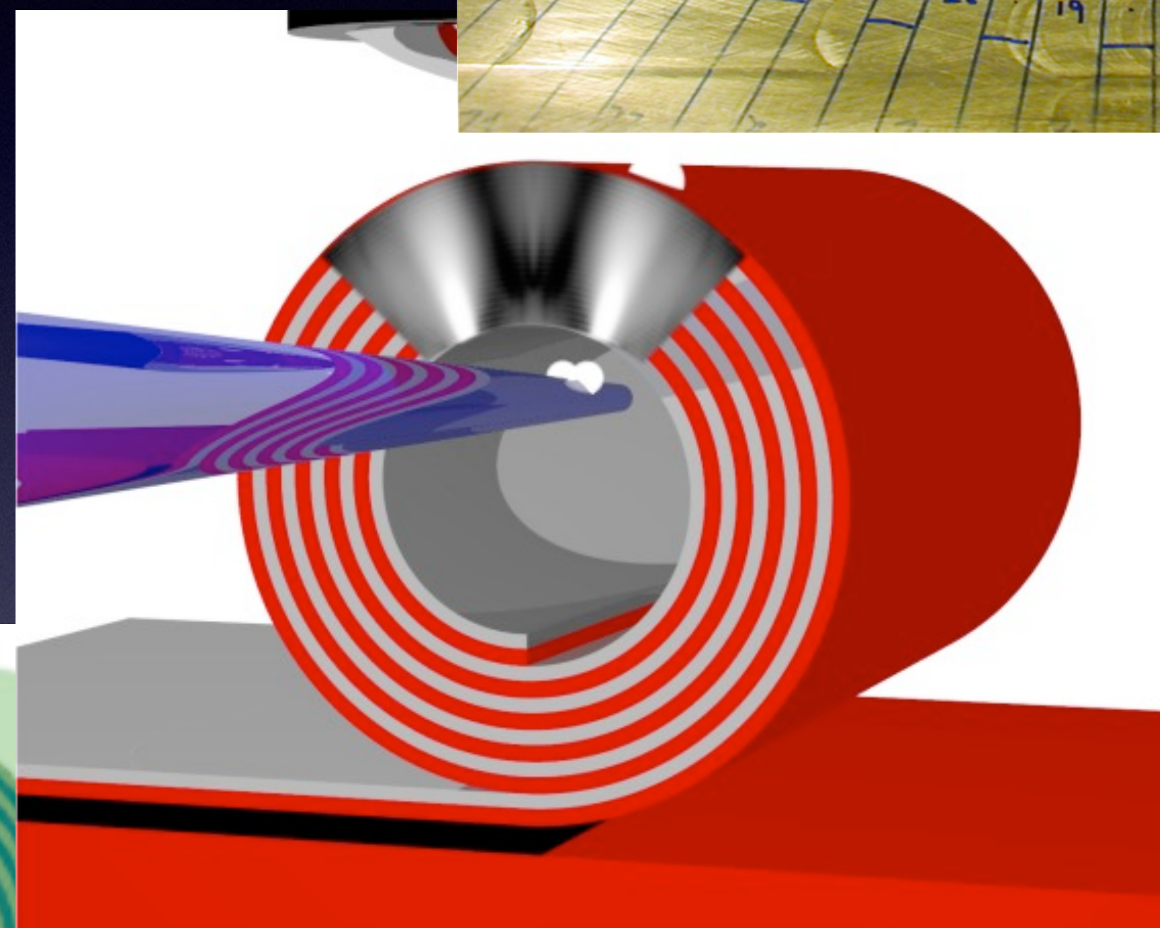
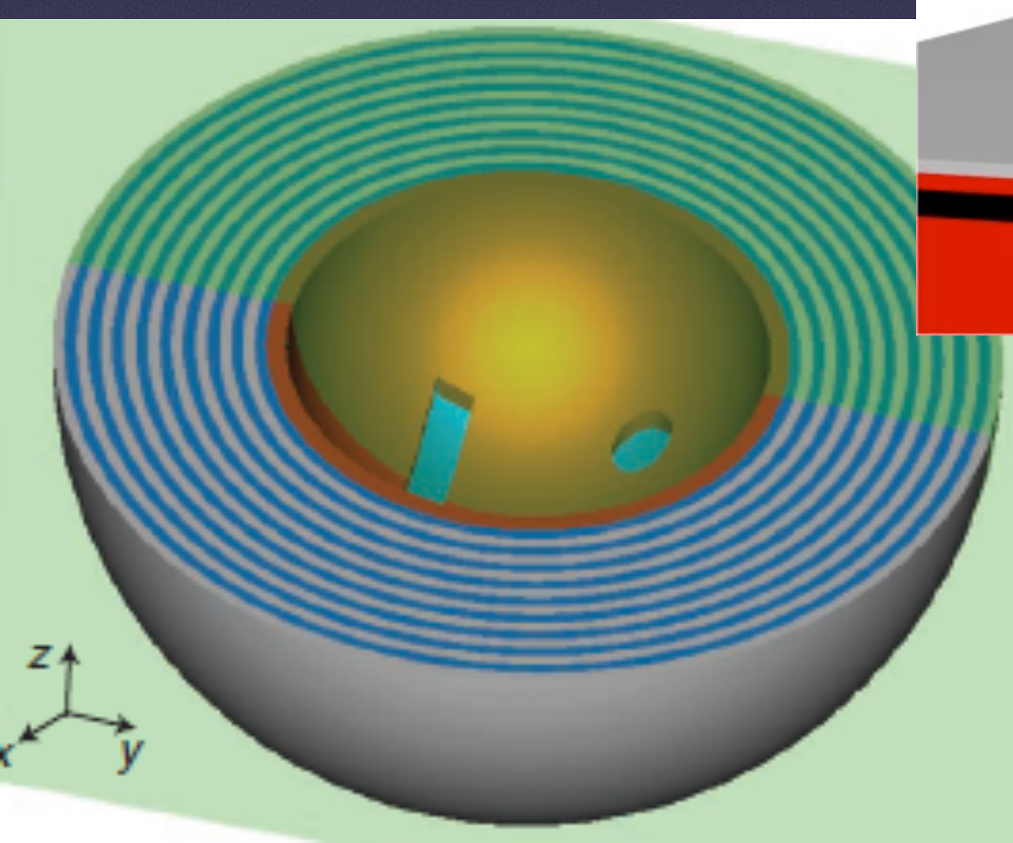
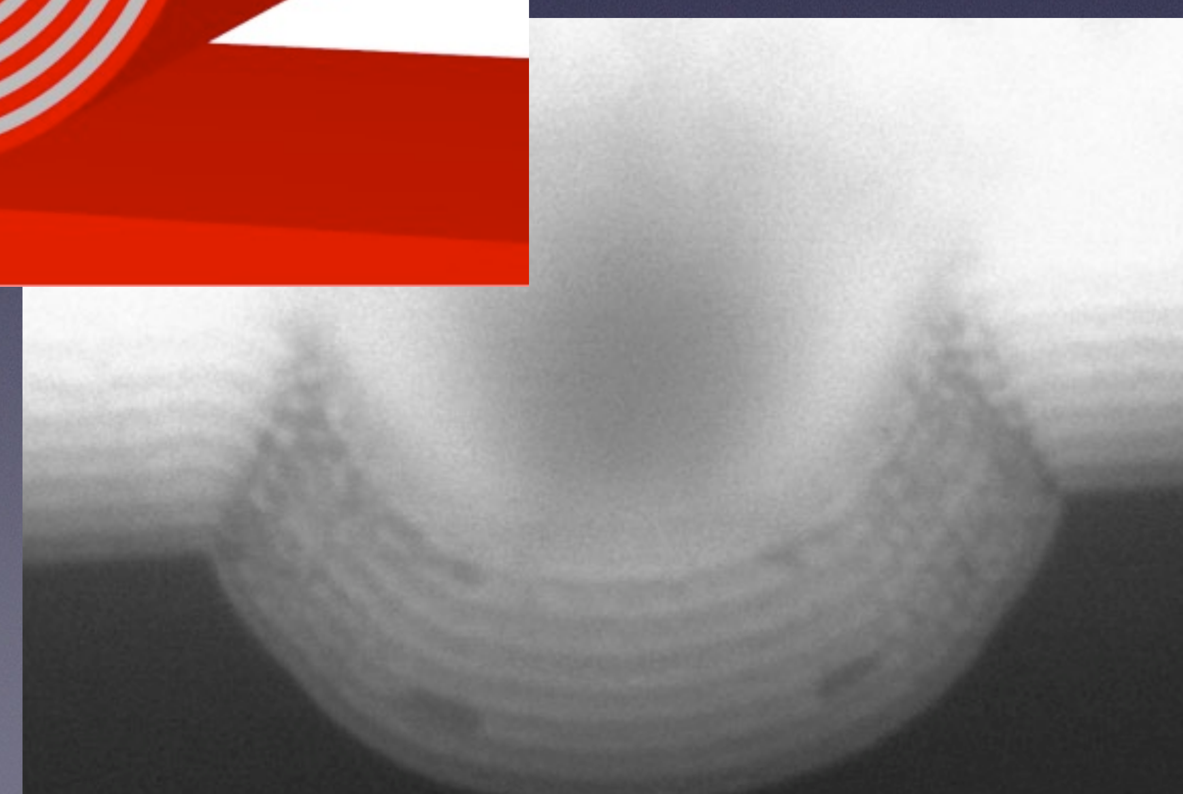
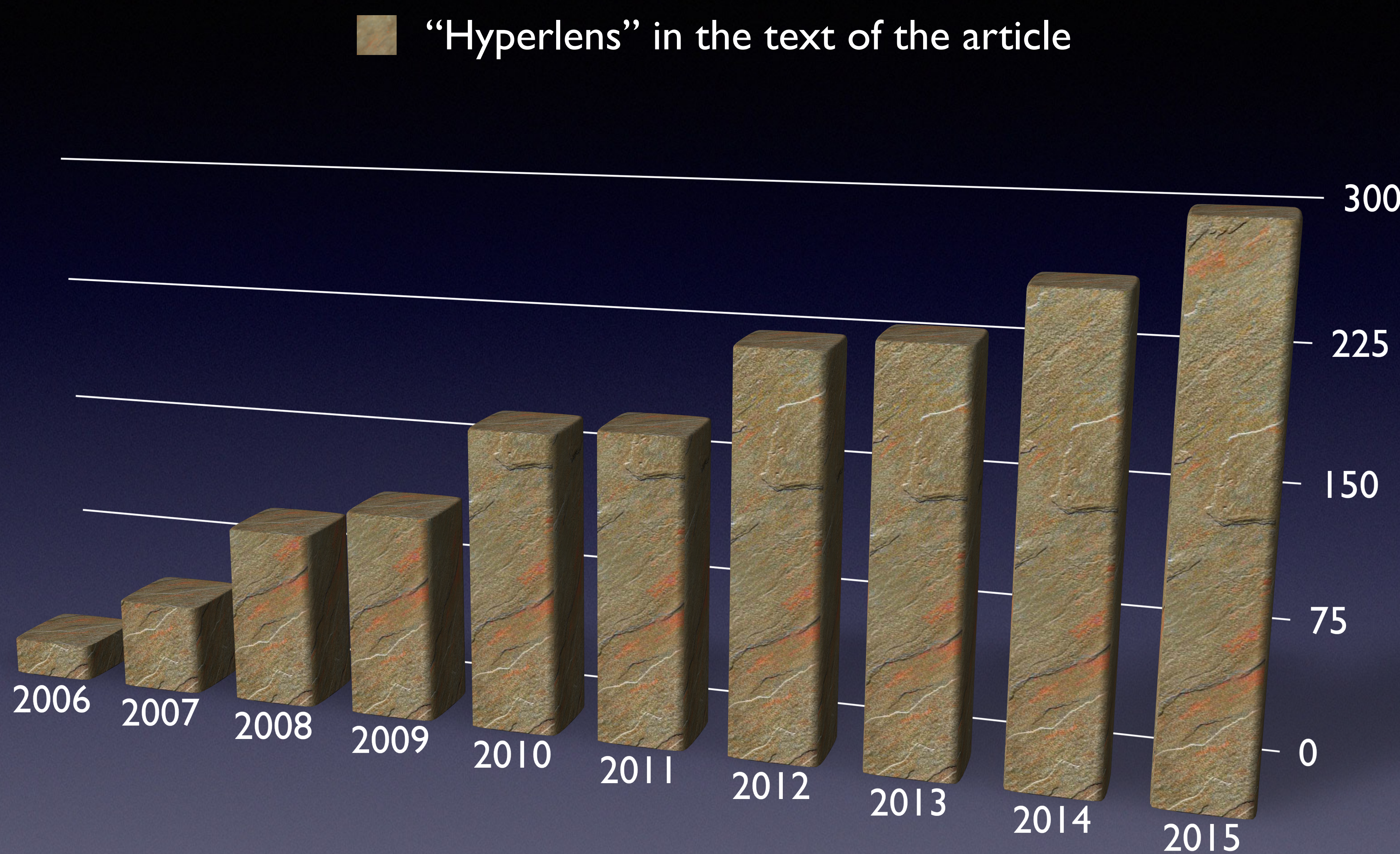
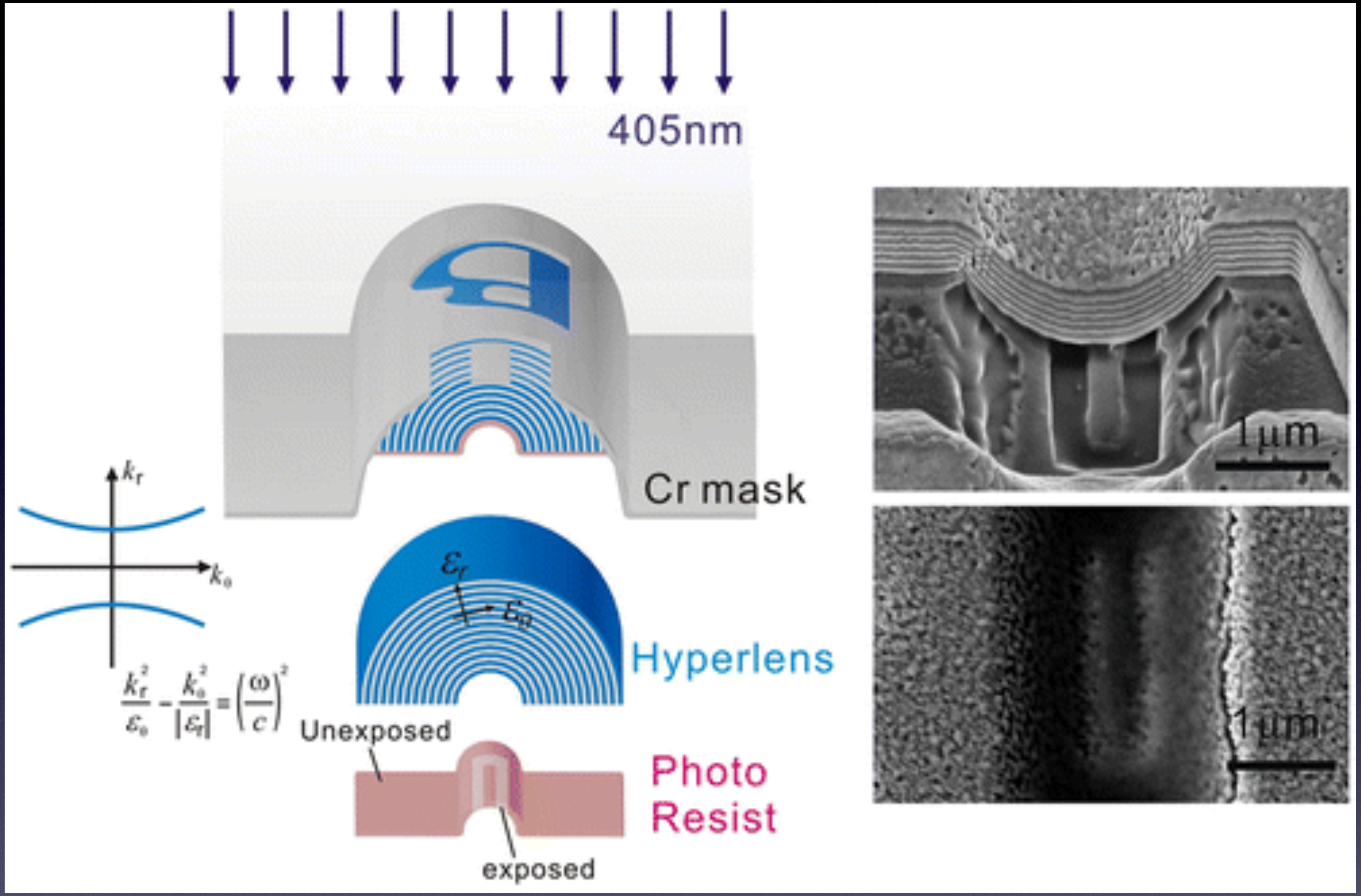
12




z
x
y




"Hyperlens" in the text of the article
300
225
150
75
0
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015



J. Sun, T. Xu, and N. M. Litchinitser, NanoLetters 16 (12), 7905 (2016)

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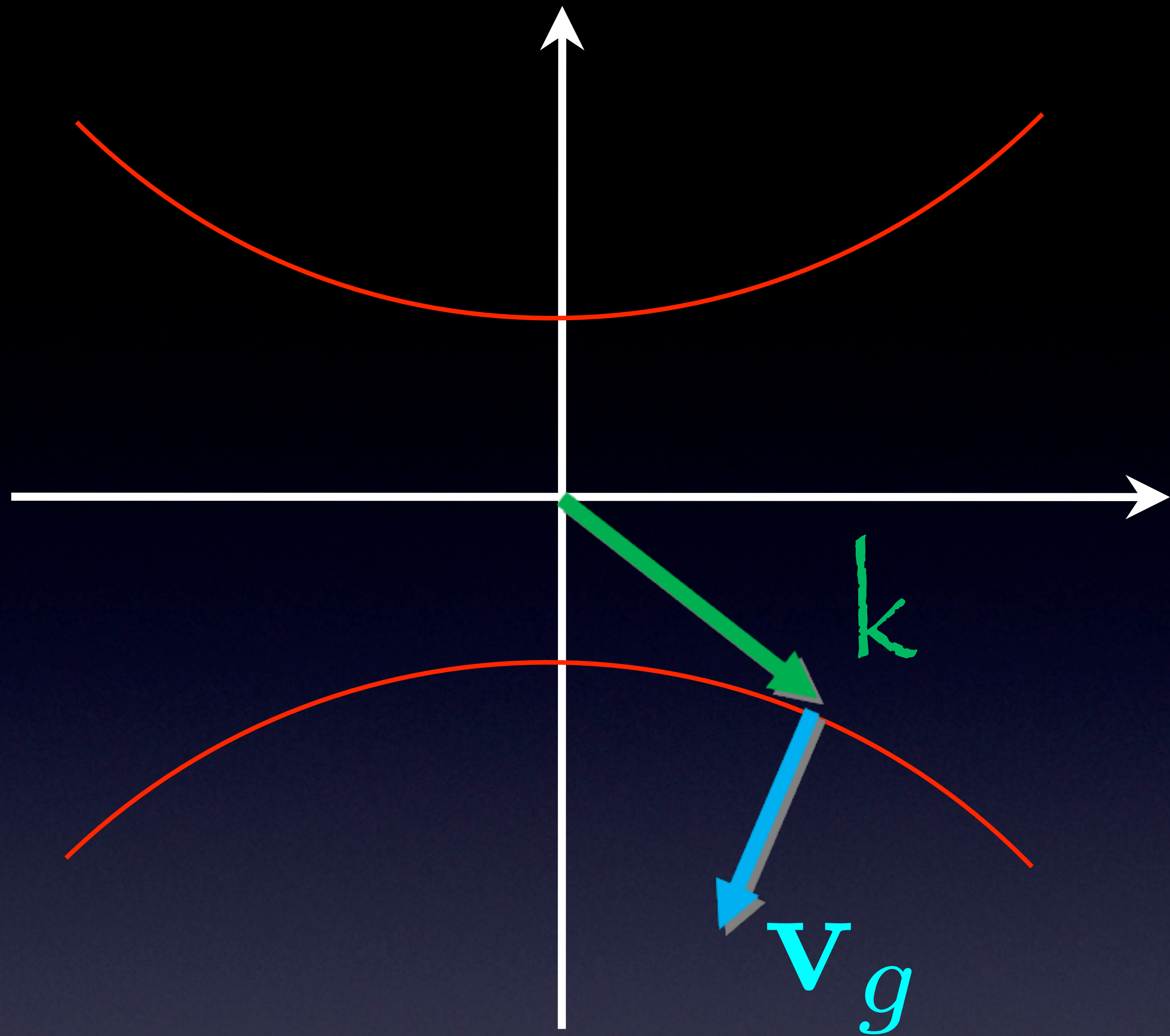
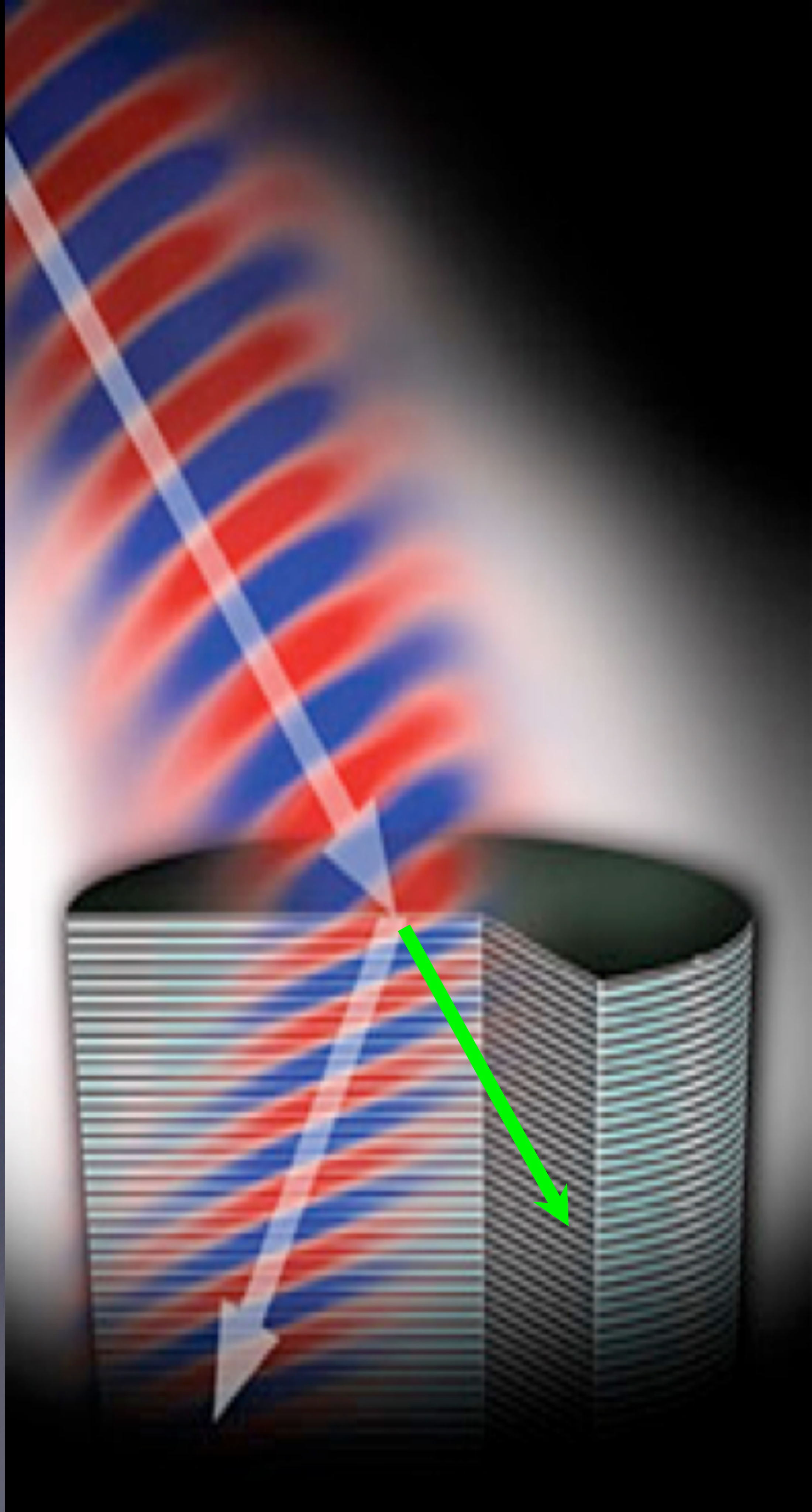


$$\mathbf{v}_g = \frac{\partial \omega}{\partial \mathbf{k}}$$



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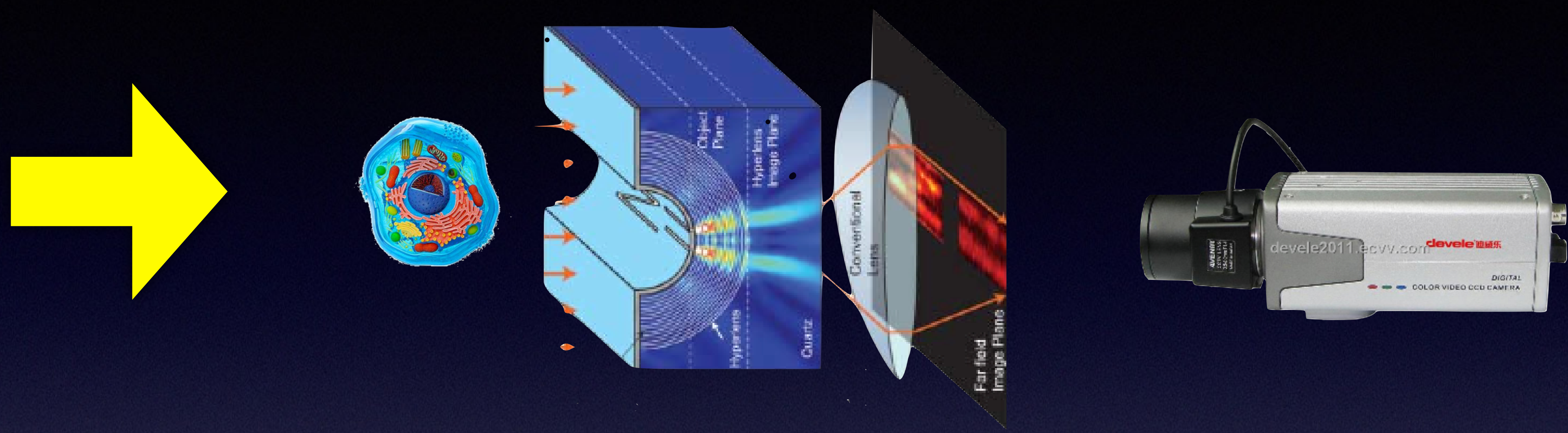
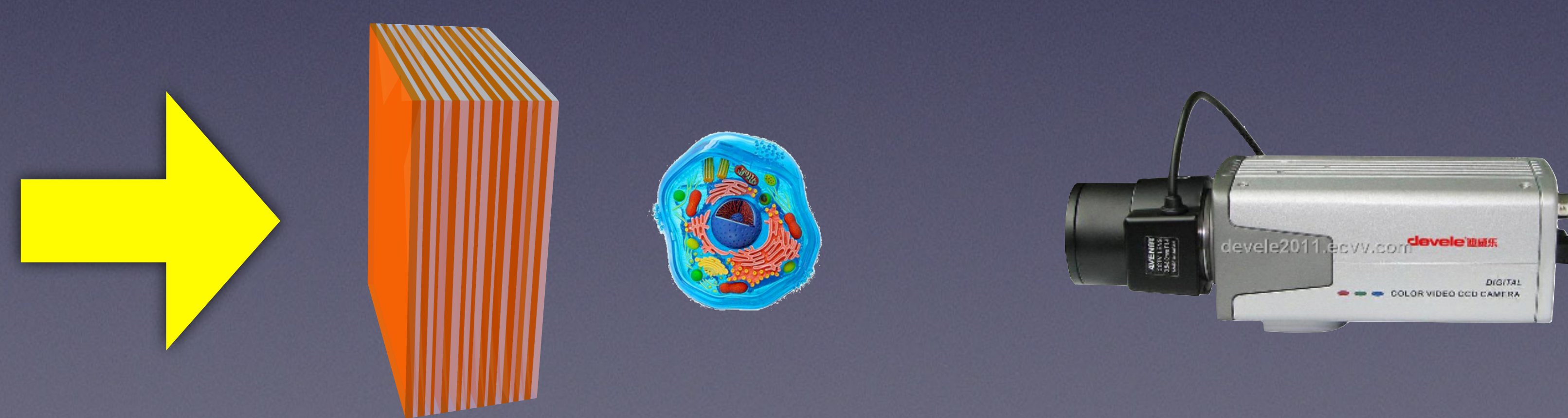
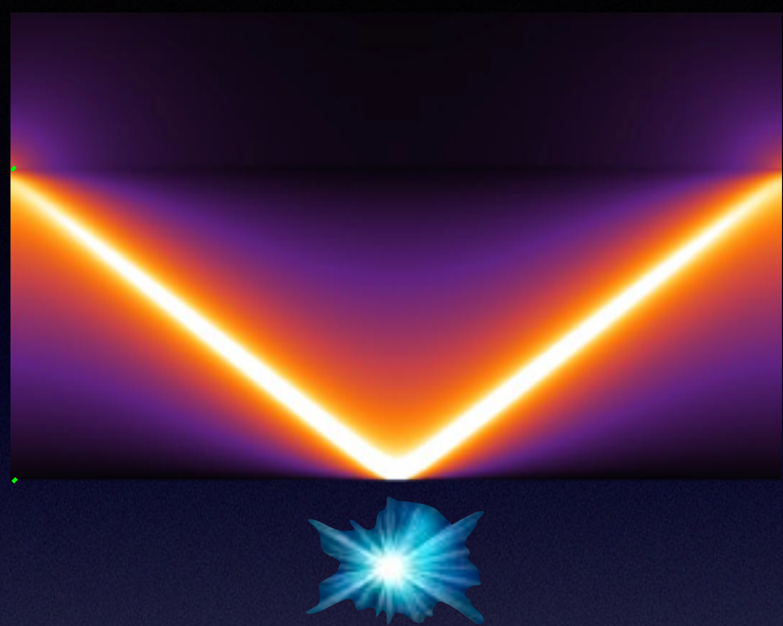
Direct Imaging vs. Structured Illumination



vs.

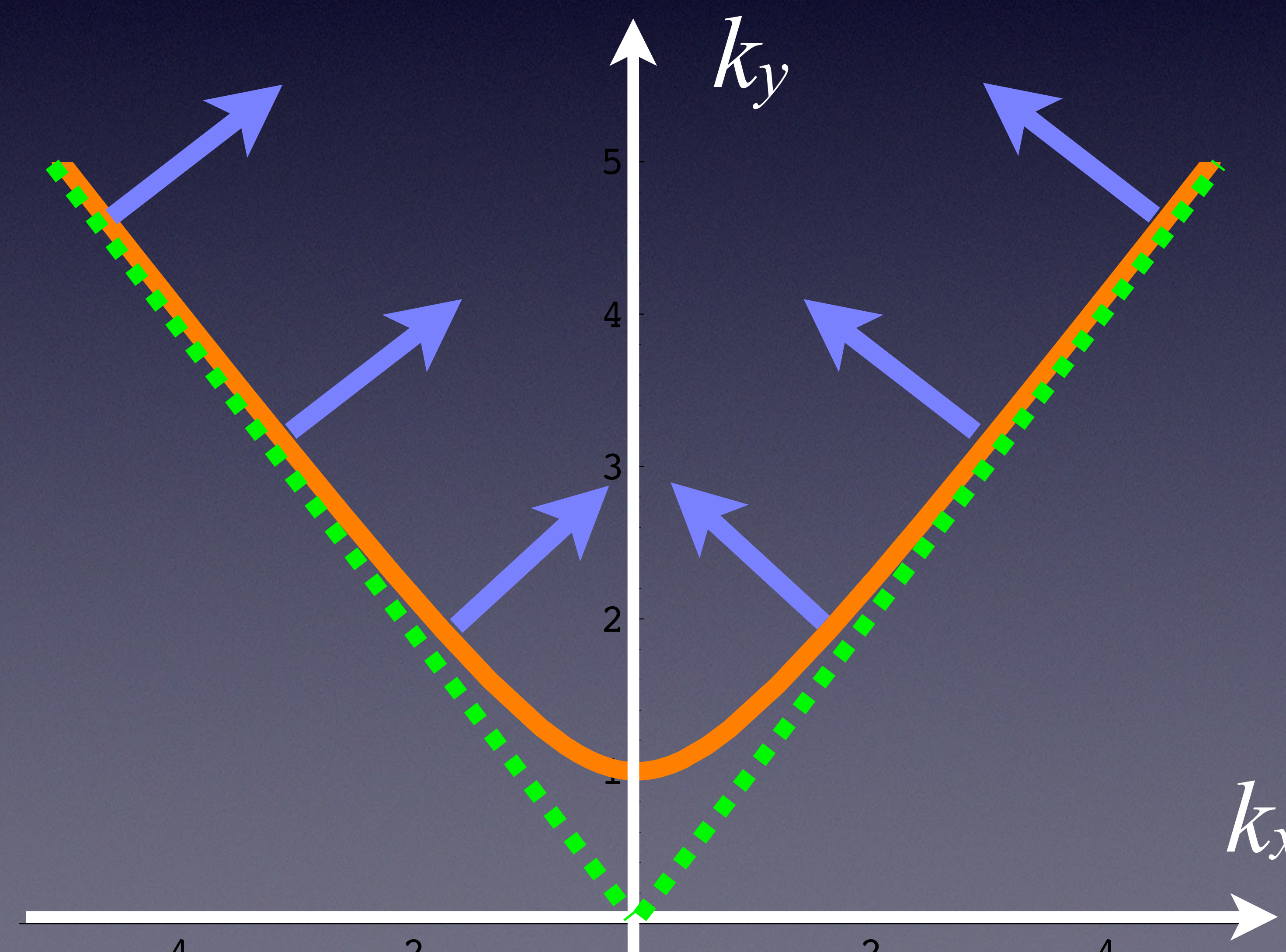


Point Source at the surface of a Hyperbolic Material

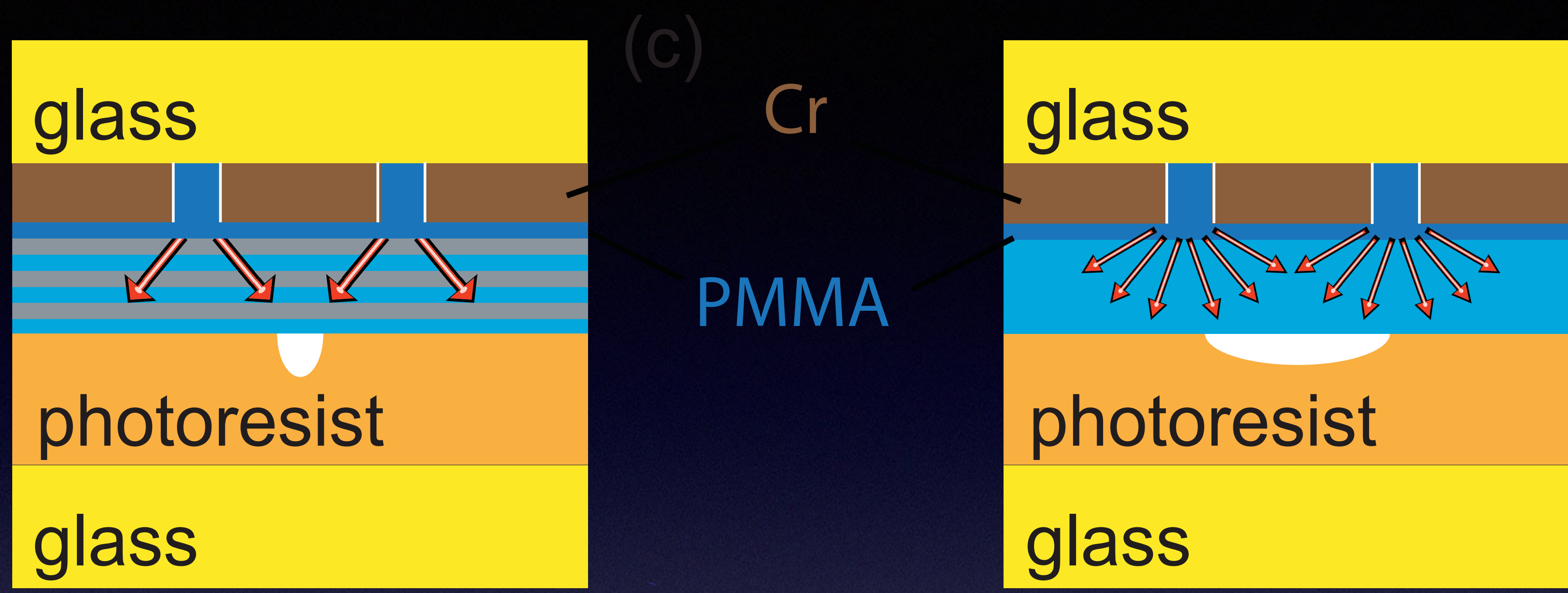


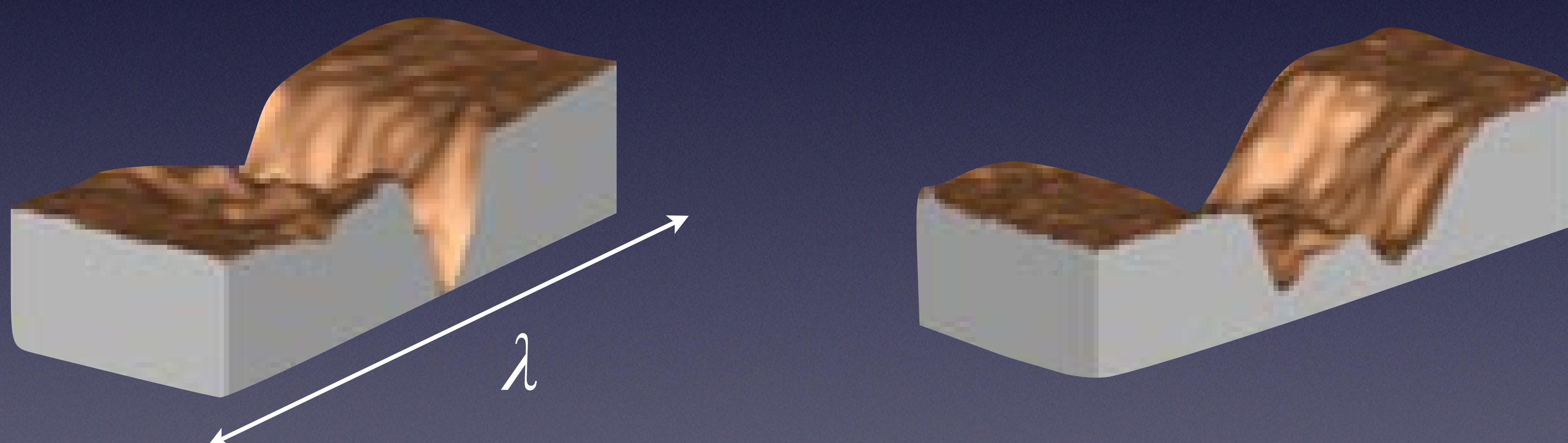
$$\frac{k_x^2}{\epsilon_y} + \frac{k_y^2}{\epsilon_x} = \frac{\omega^2}{c^2} \xrightarrow{k \to \infty} k_y \simeq \pm\sqrt{-\frac{\epsilon_x}{\epsilon_y}} \cdot k_x$$

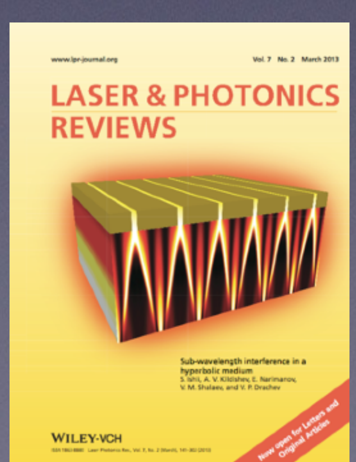
$$\vec{\mathbf{v}}_g = \frac{\partial \omega}{\partial \vec{\mathbf{k}}}$$



Experiment :

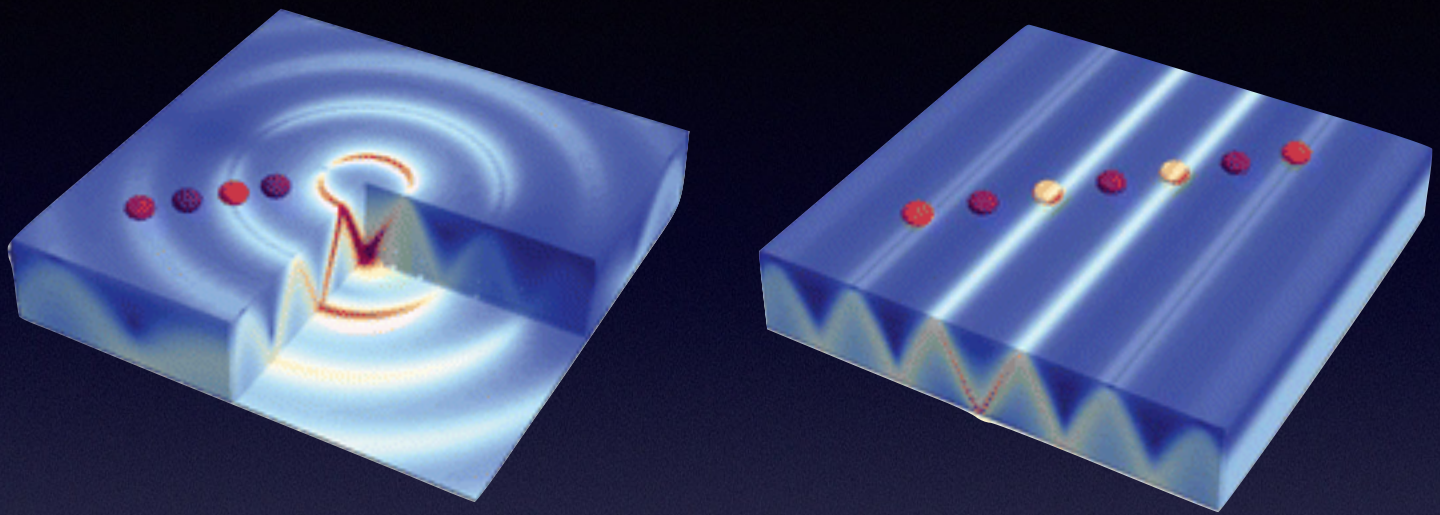


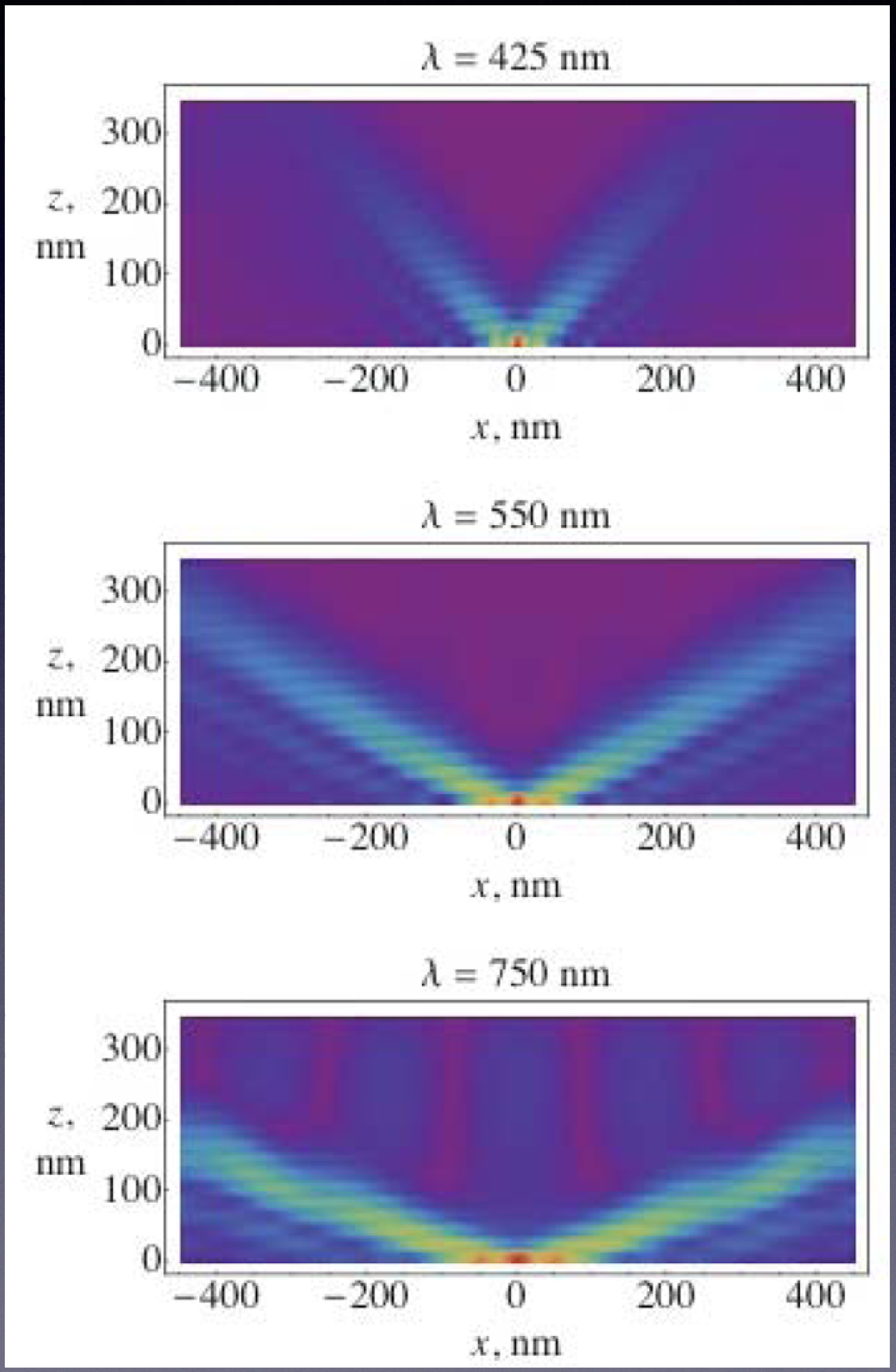


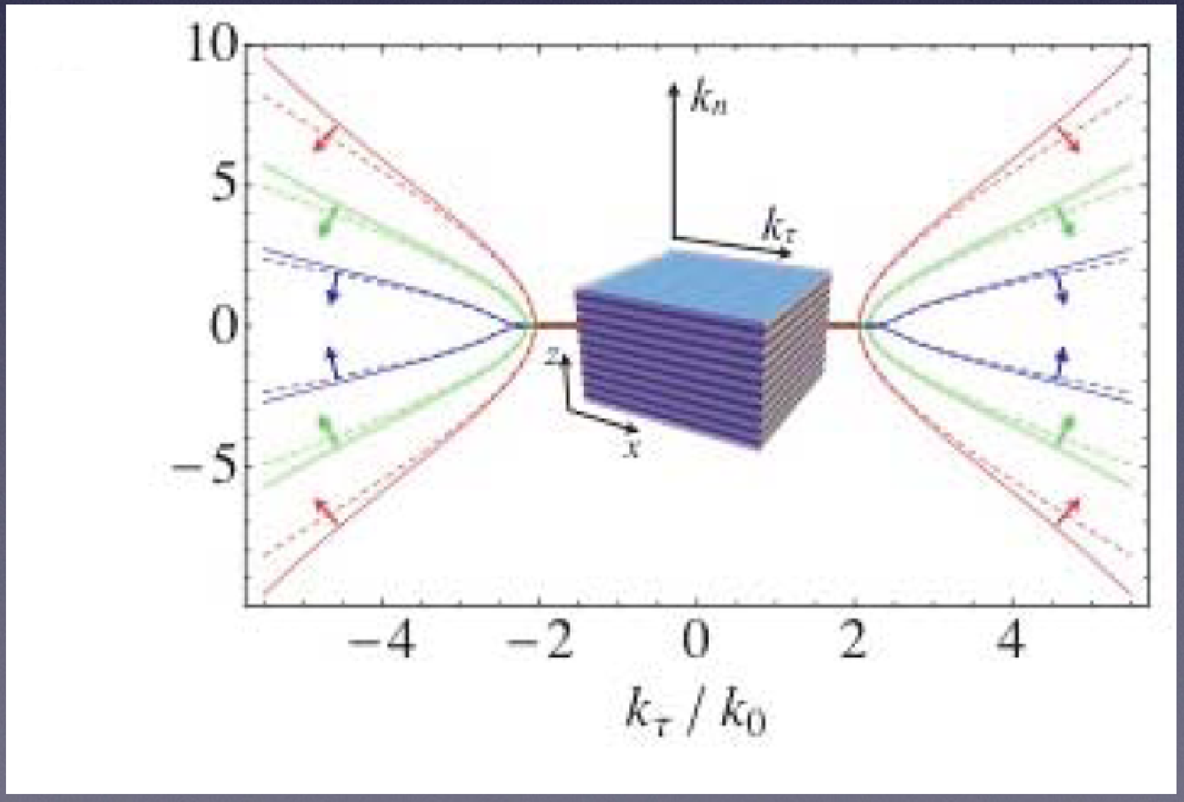


S. Ishii *et al*, Laser & Photonics Reviews 2013

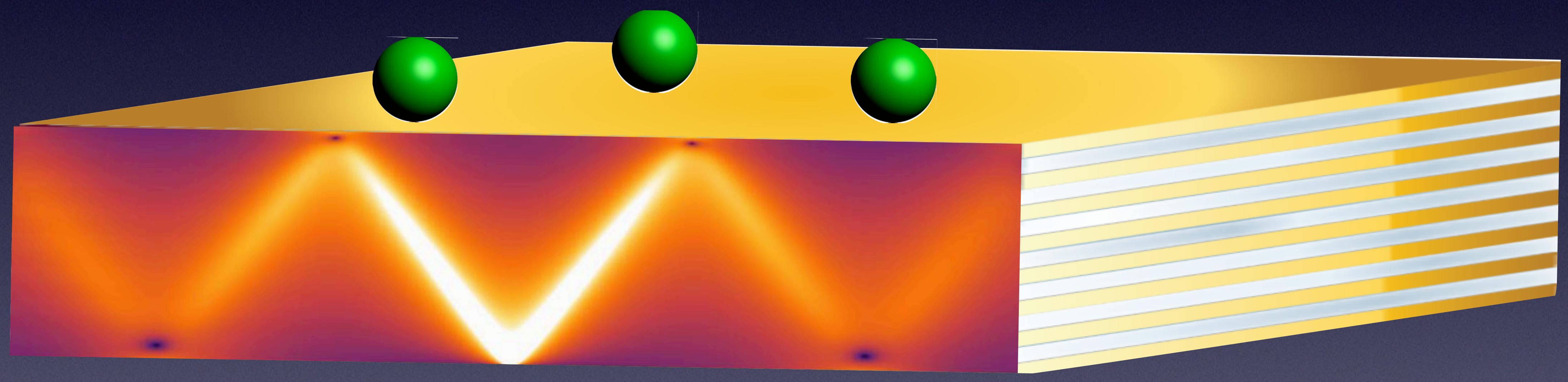
Hyperbolic Metamaterials: strongly dispersive



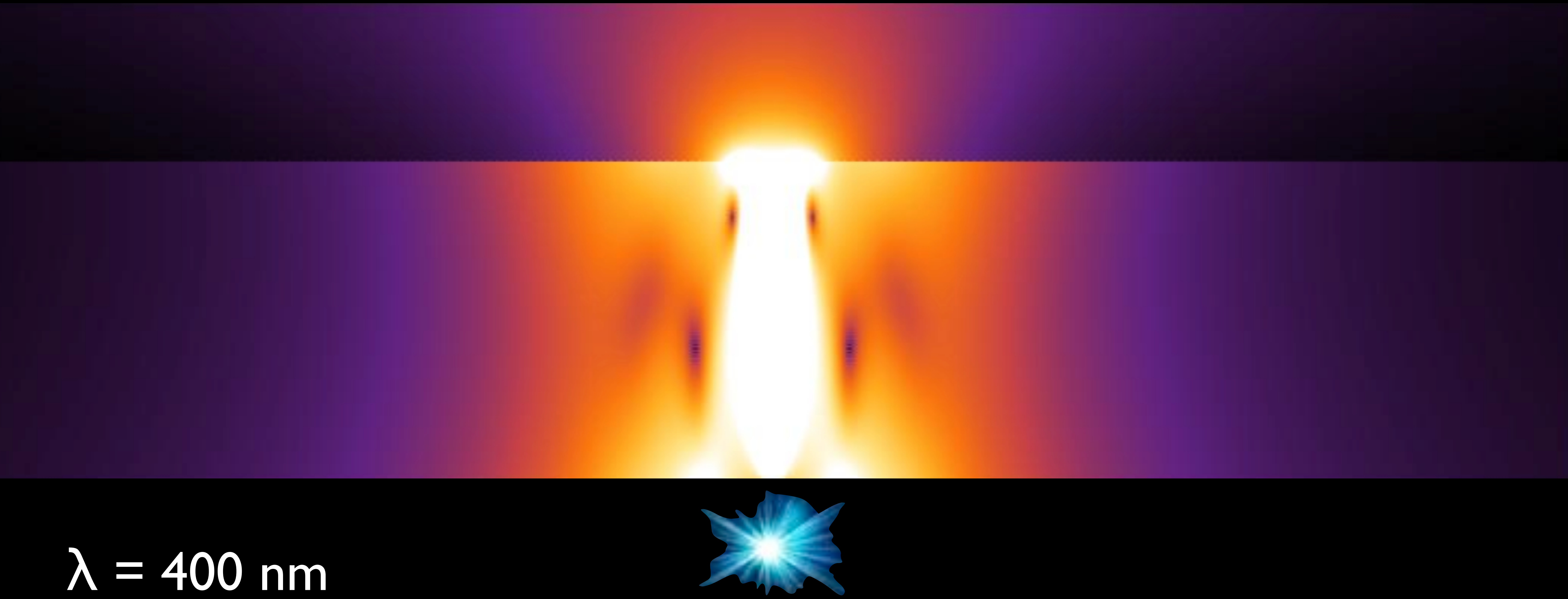




“Steer” the “beams” with wavelength!

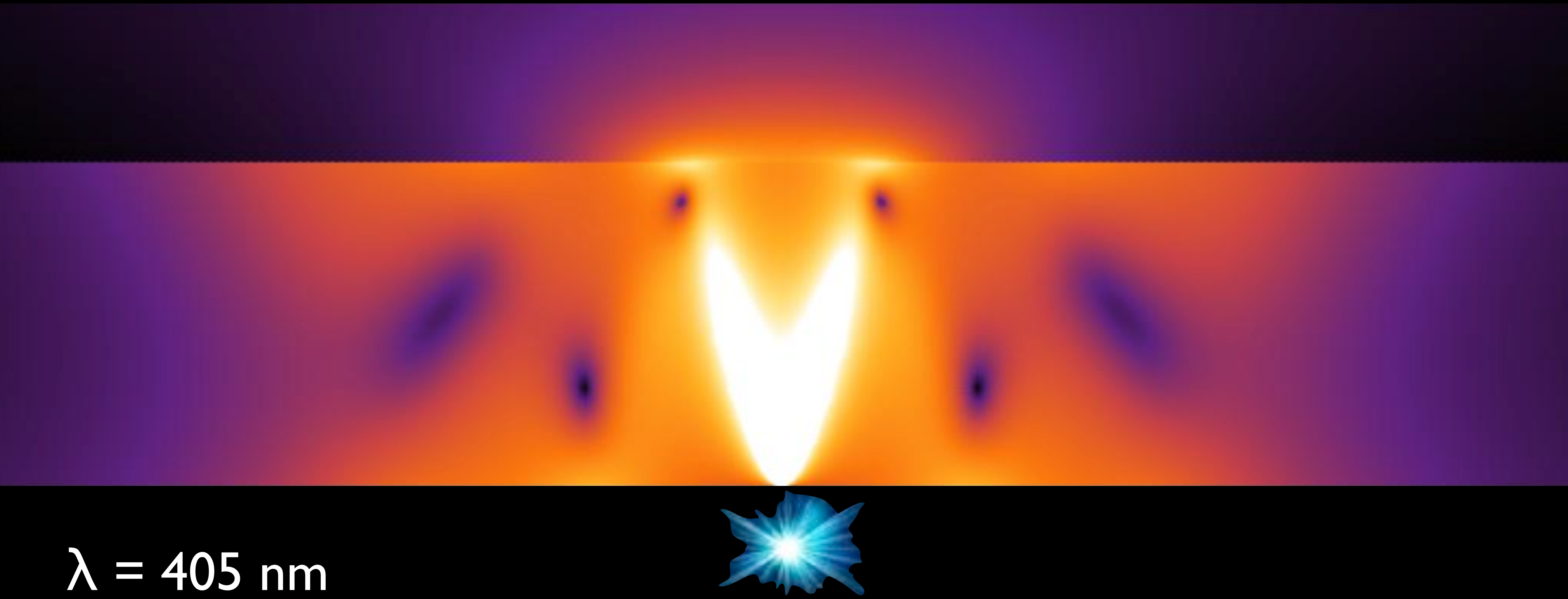


Hyper-Structured Illumination



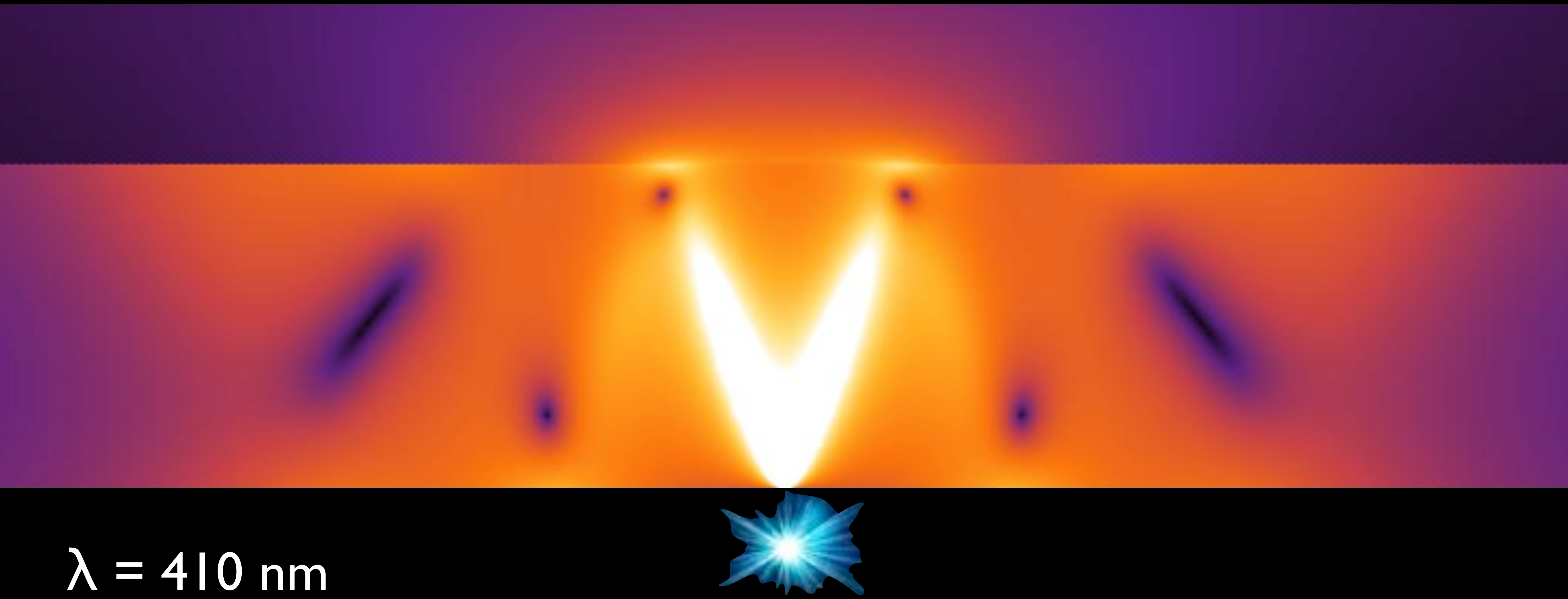
λ = 400 nm

Hyper-Structured Illumination



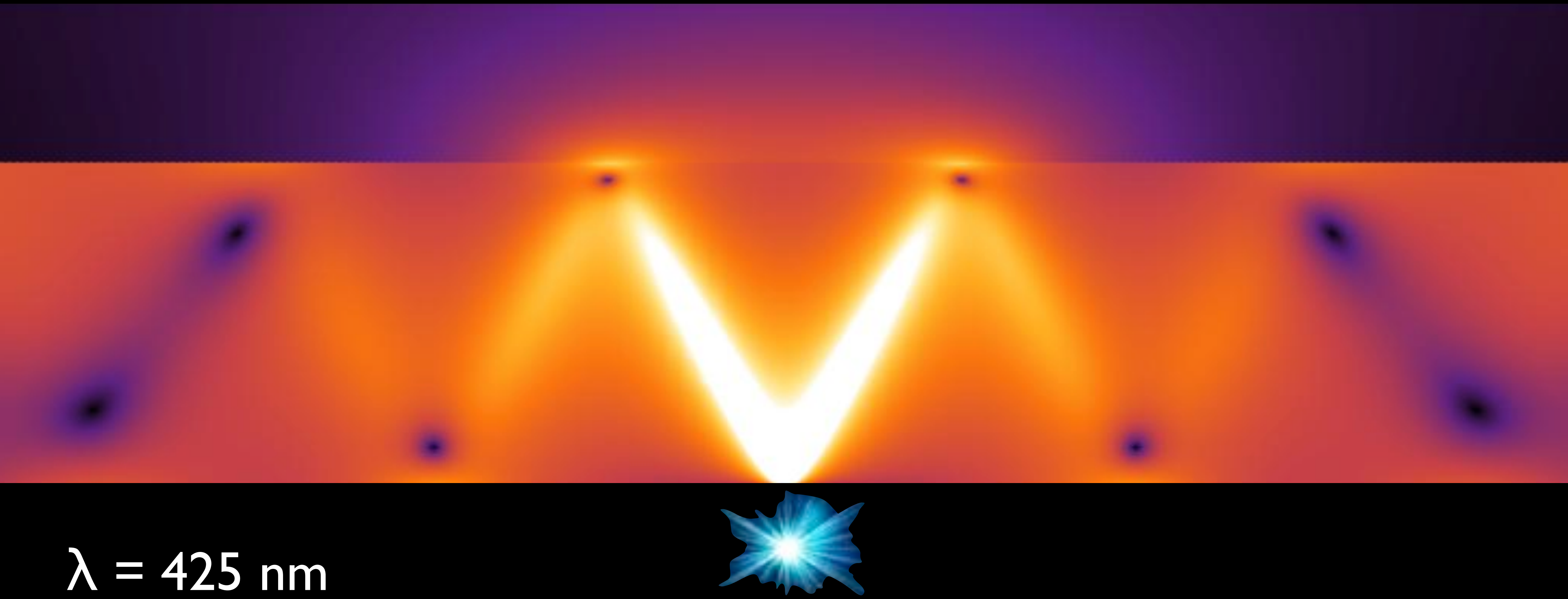
λ = 405 nm

Hyper-Structured Illumination



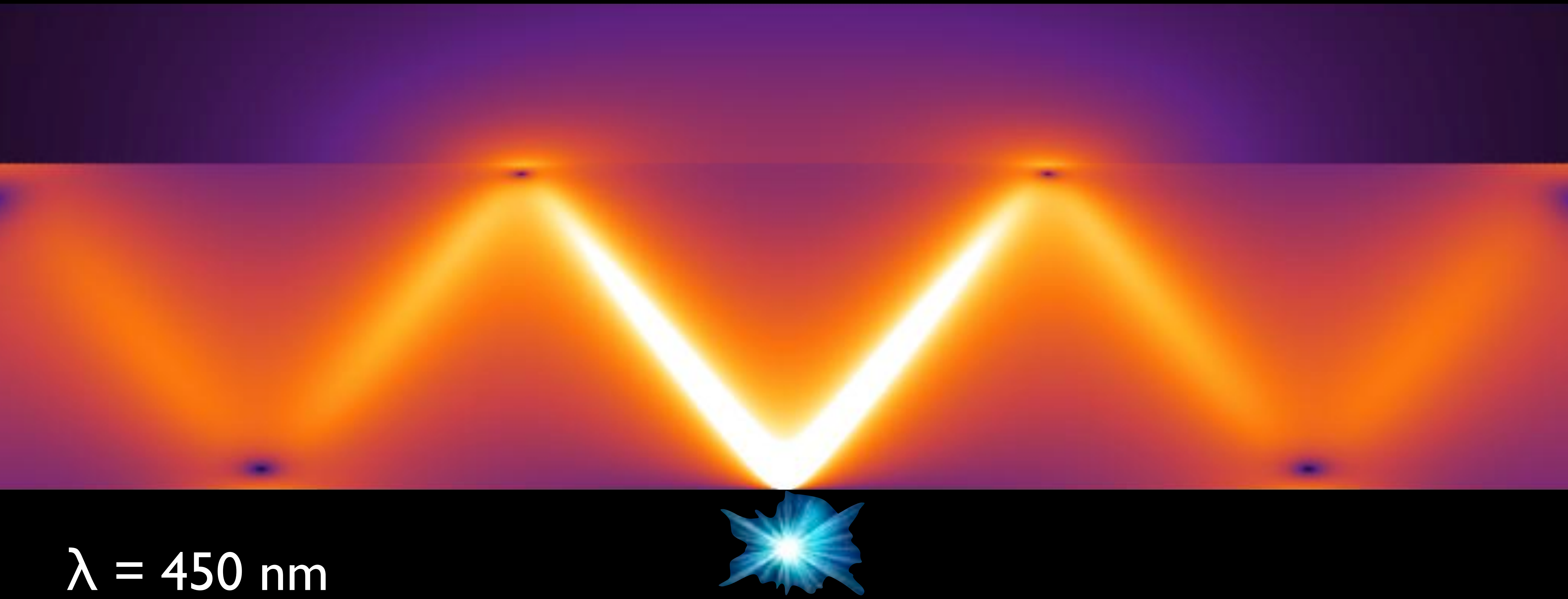
λ = 410 nm

Hyper-Structured Illumination



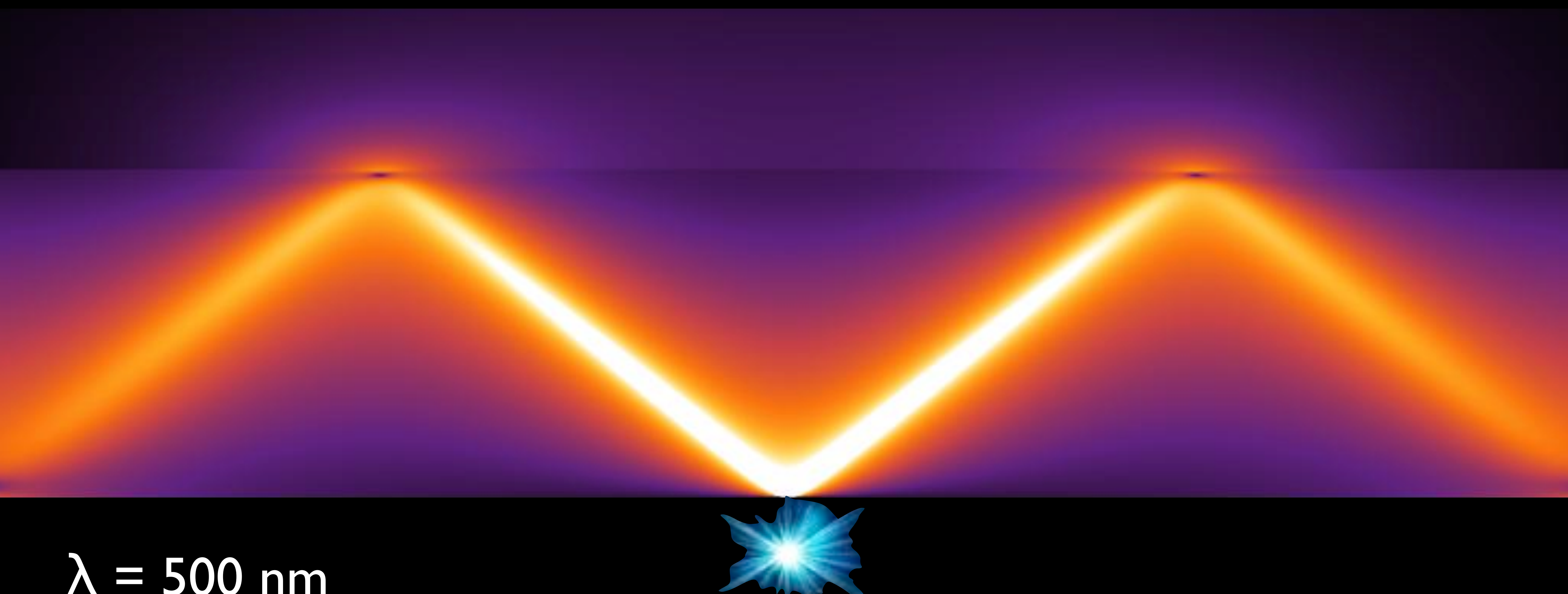
$\lambda = 425$ nm

Hyper-Structured Illumination



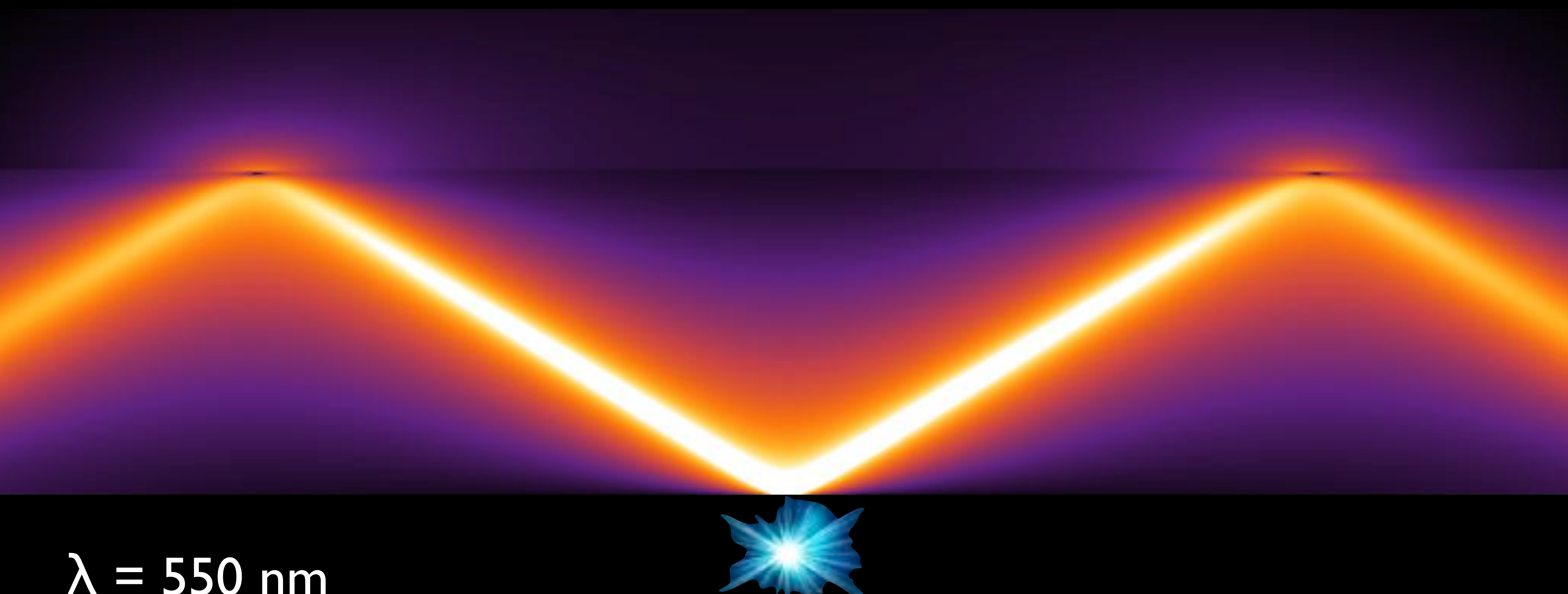
λ = 450 nm

Hyper-Structured Illumination



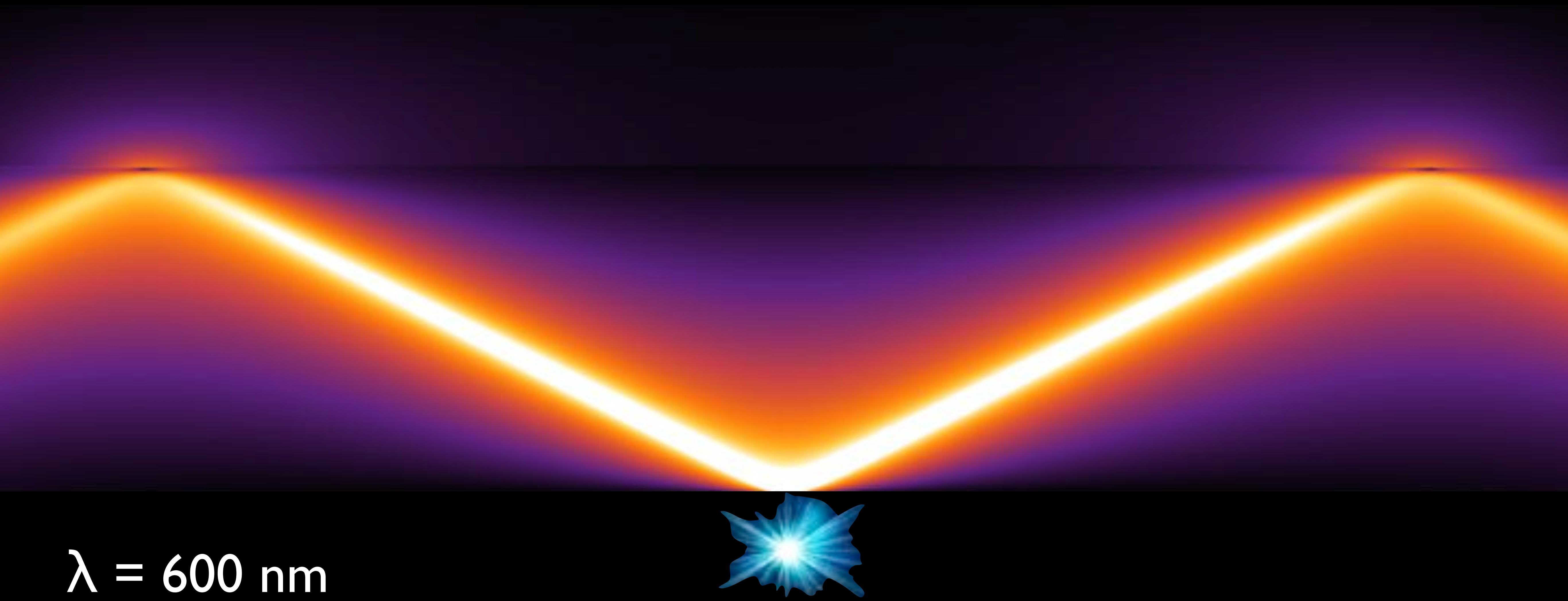
λ = 500 nm

Hyper-Structured Illumination



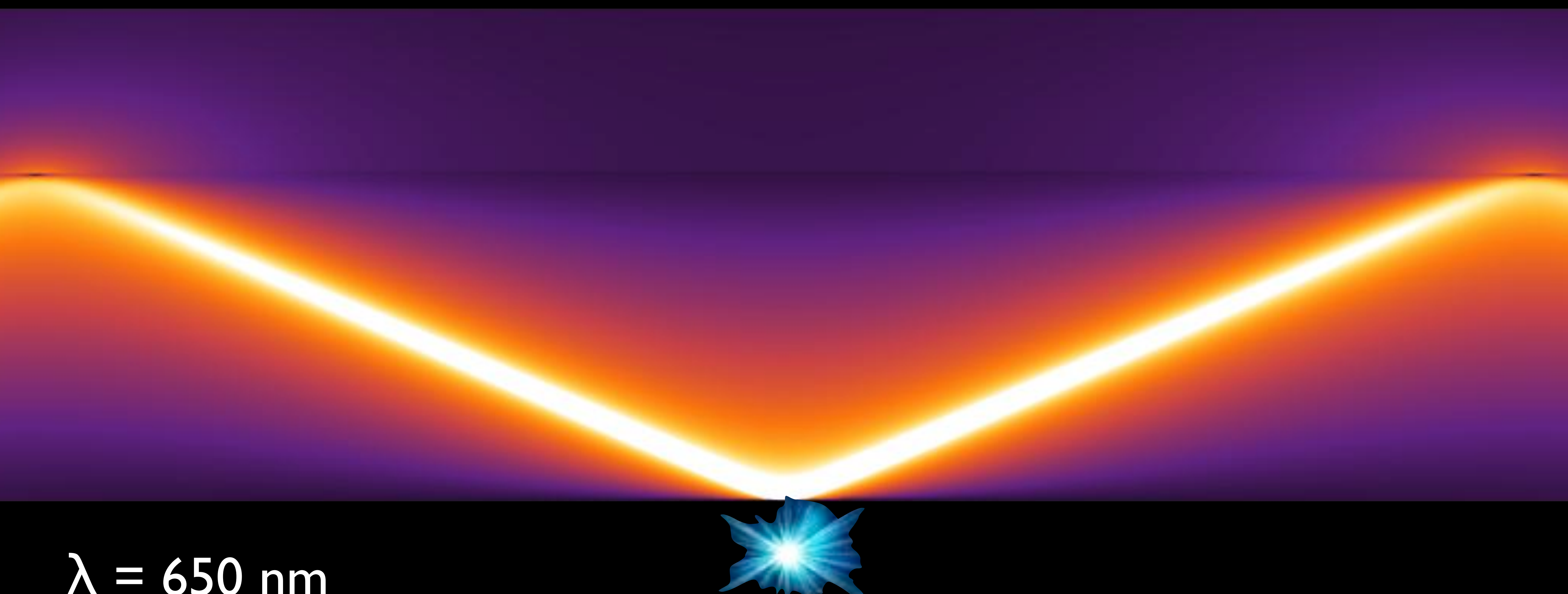
λ = 550 nm

Hyper-Structured Illumination



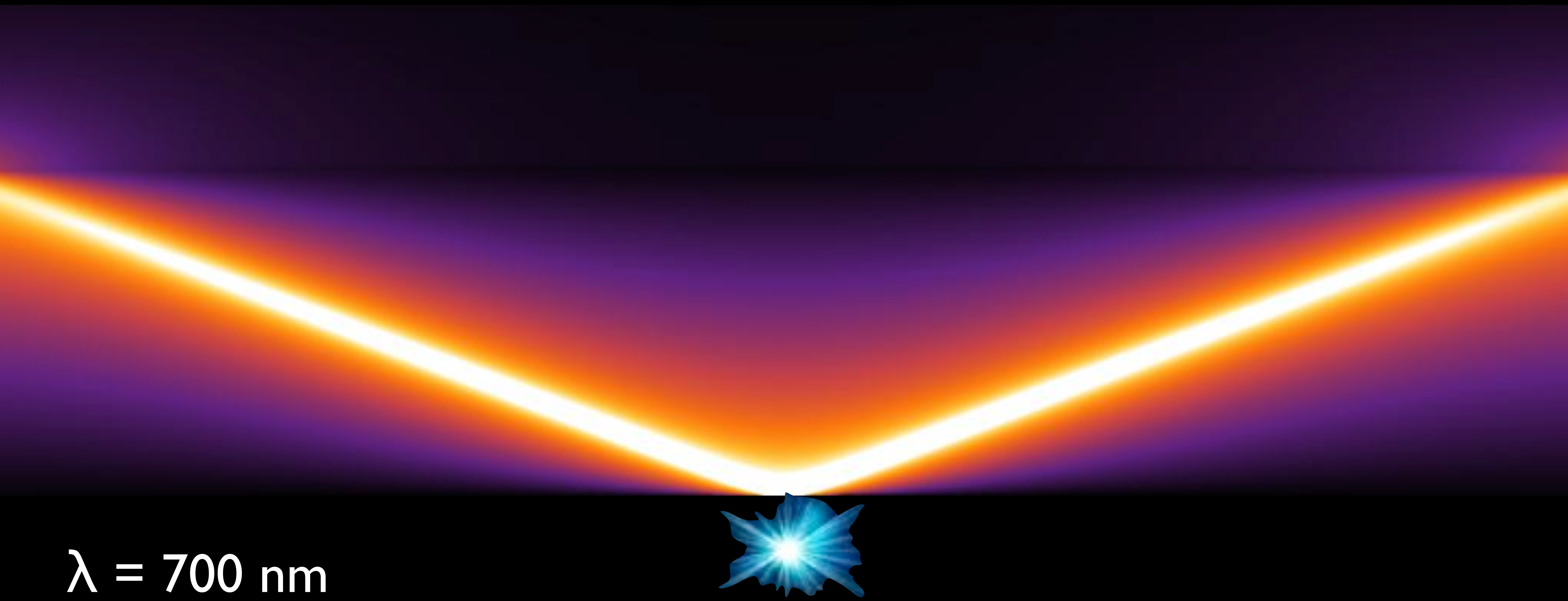
λ = 600 nm

Hyper-Structured Illumination



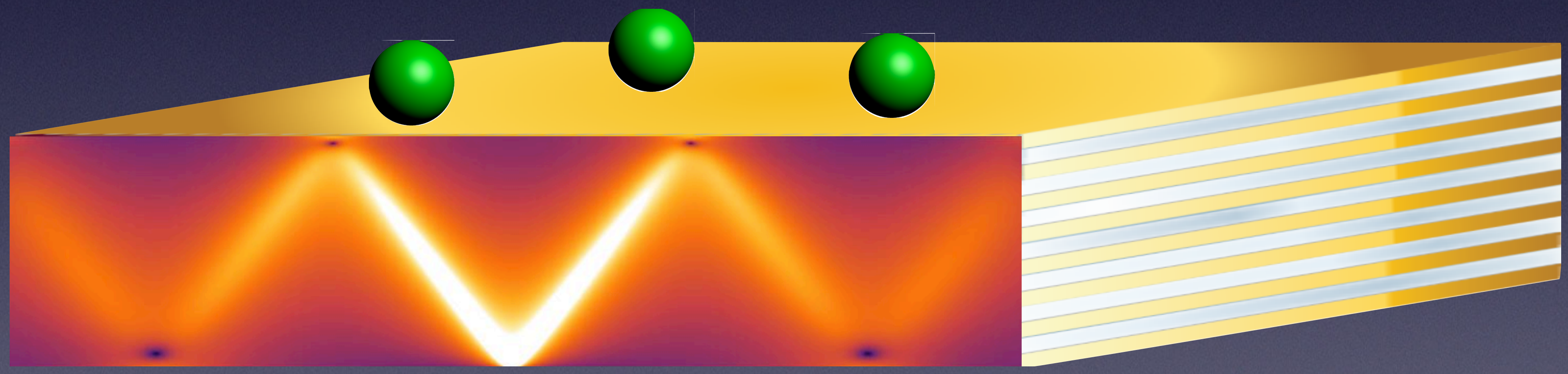
λ = 650 nm

Hyper-Structured Illumination



λ = 700 nm

“Steer” the “beams” with wavelength!

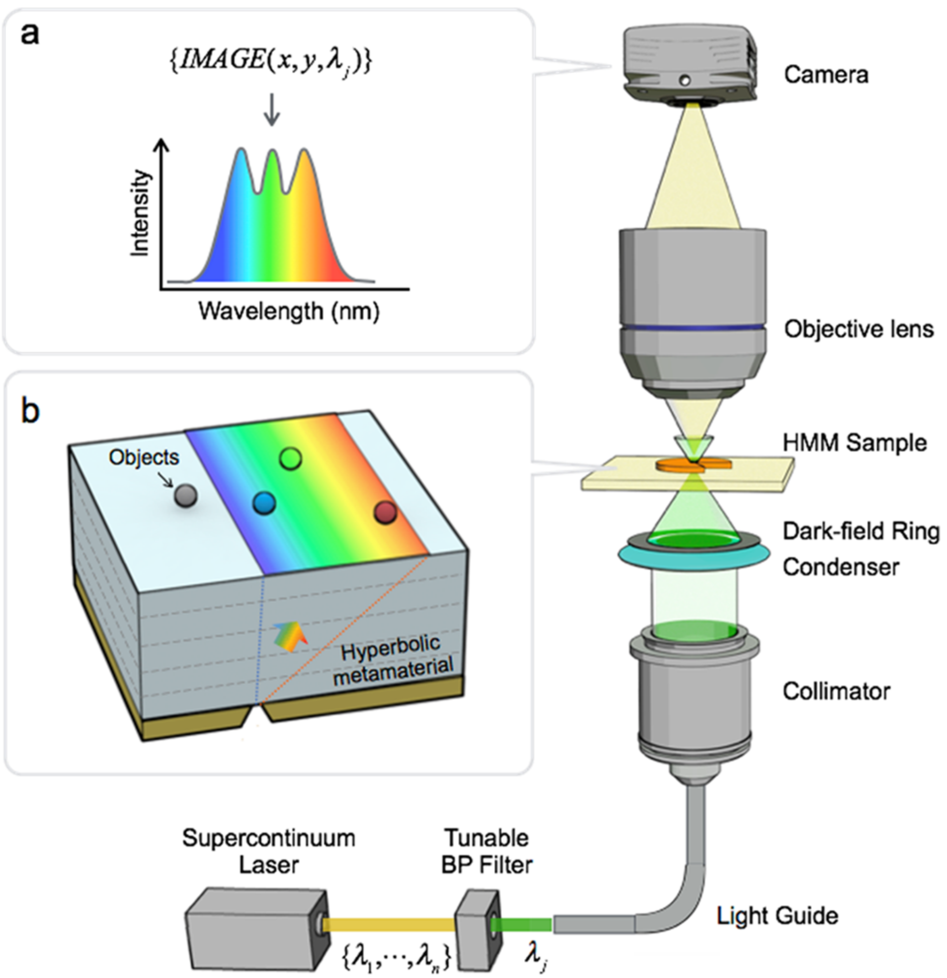


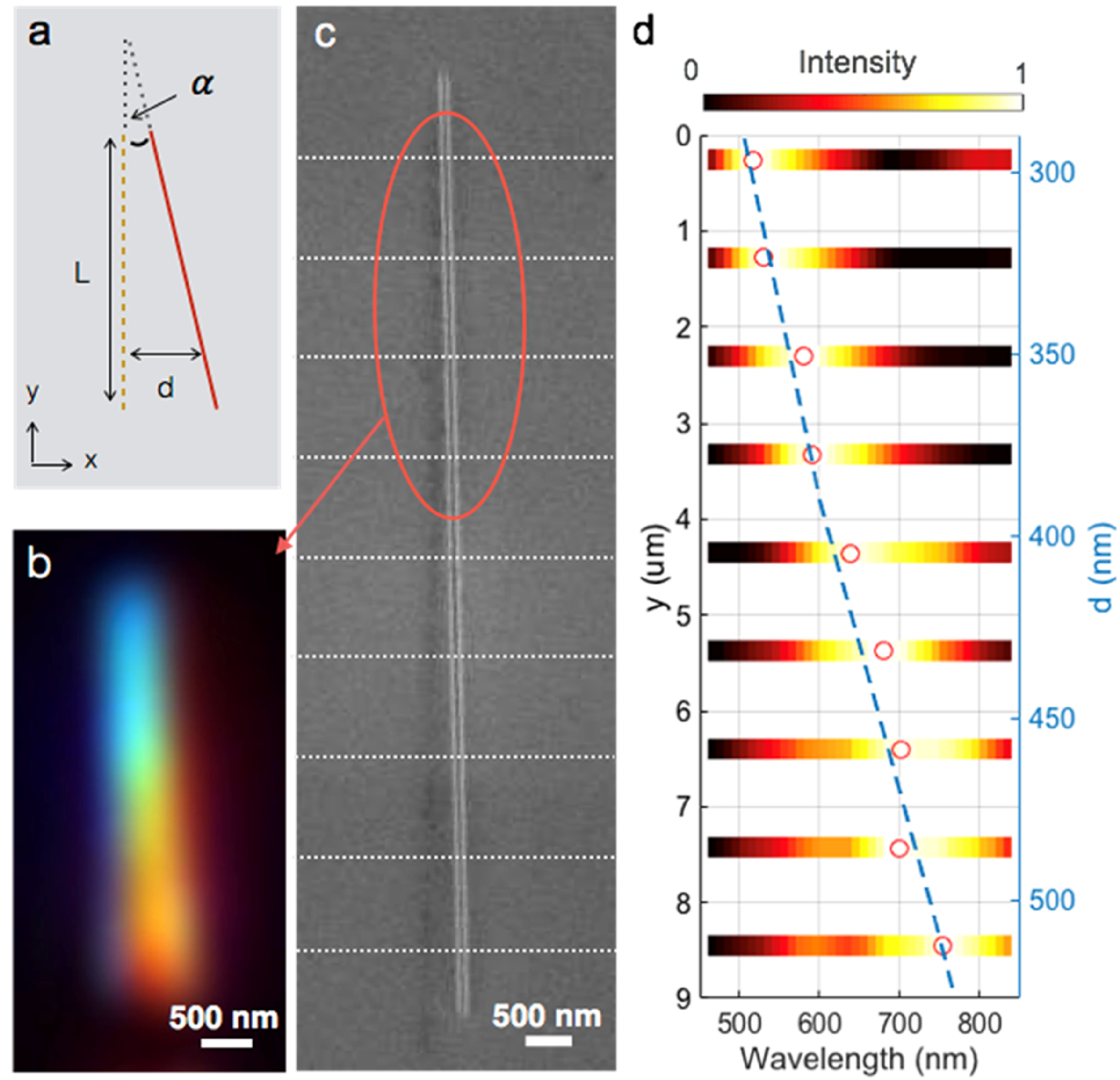
ACS NANO
Cite This: ACS Nano XXXX, XXX, XXX−XXX www.acsnano.org

Experimental Demonstration of Hyperbolic Metamaterial Assisted Illumination Nanoscopy

Qian Ma,[†] Haoliang Qian,[†] Sergio Montoya,[†] Wei Bao,[‡] Lorenzo Ferrari,[§,∥] Huan Hu,[†] Emroz Khan,[⊥] Yuan Wang,[‡] Eric E. Fullerton,[†,§] Evgenii E. Narimanov,[⊥] Xiang Zhang,[‡] and Zhaowei Liu*[,†,§,∥]

Single "line" object





ACS NANO Cite This: ACS Nano XXXX, XXX, XXX−XXX www.acsnano.org

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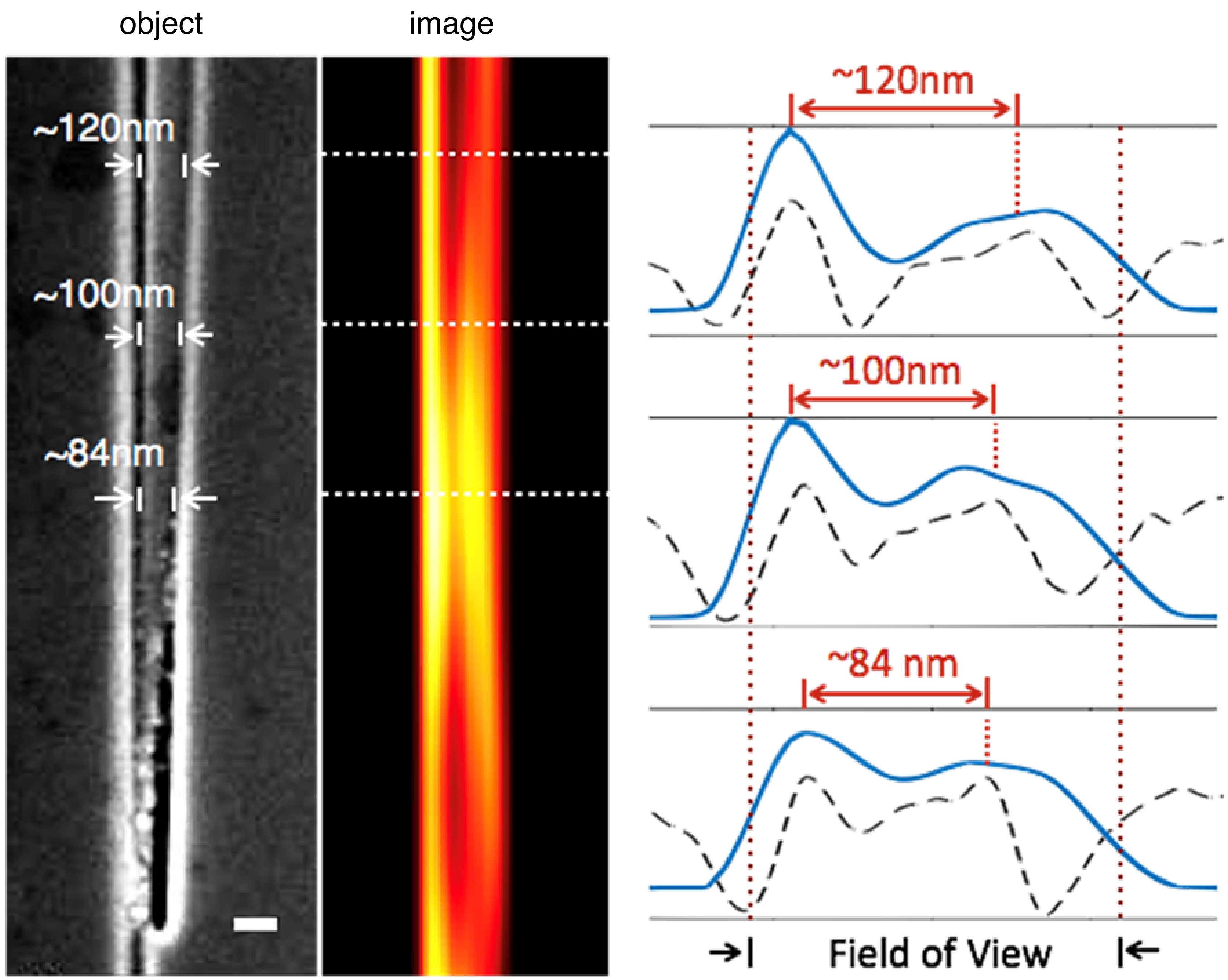
“Double line” object (“V”) :



Optical

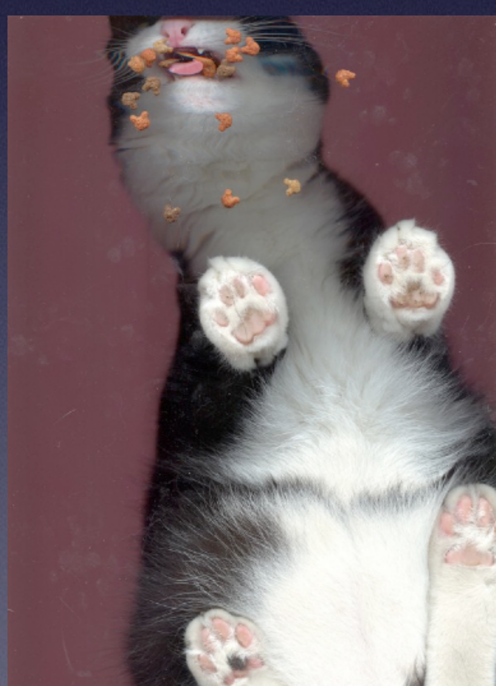
SEM

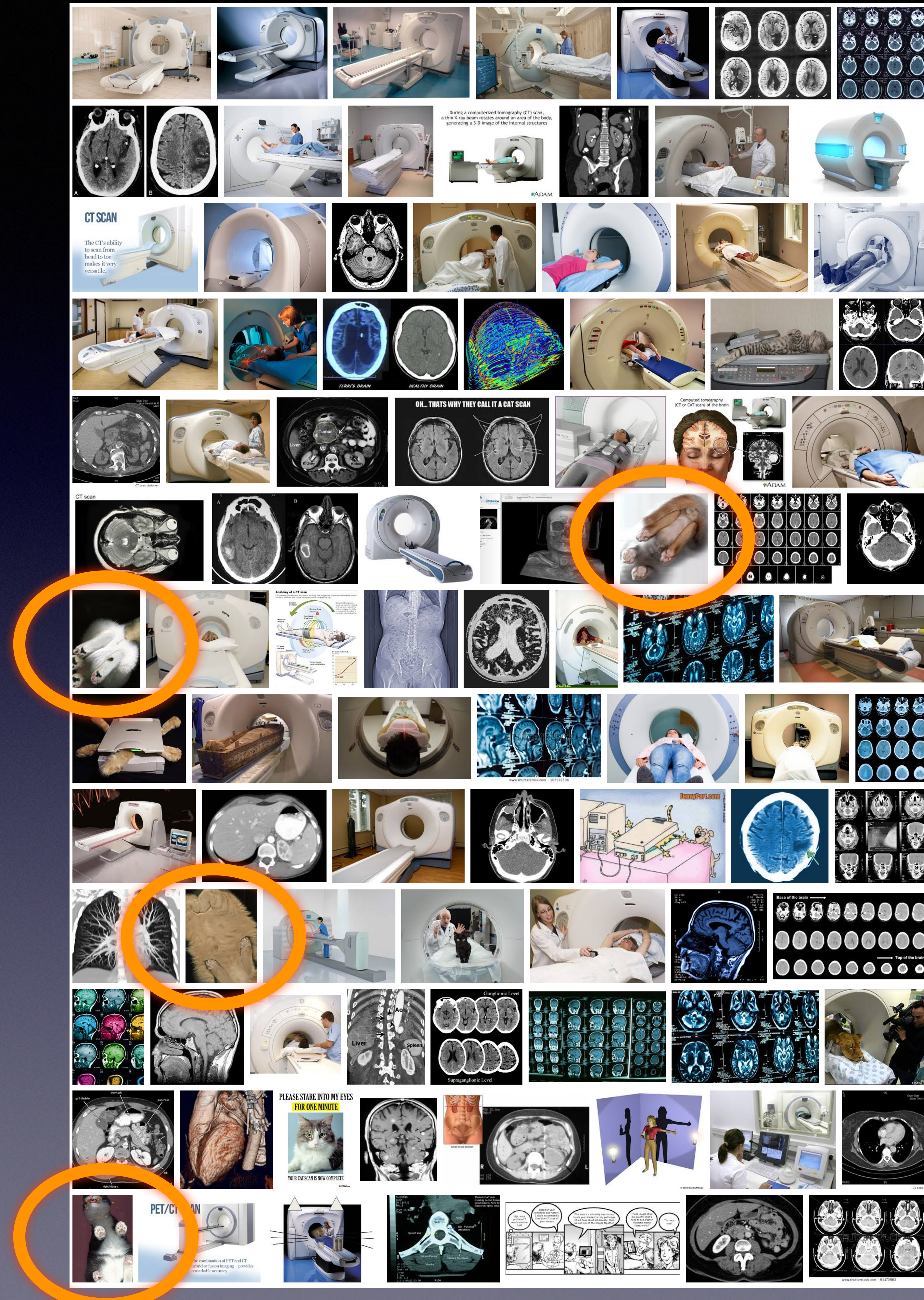
Google "Cat-scan" :

Google "Cat-scan" :

Google “Cat-scan” :

Google “Cat-scan” :

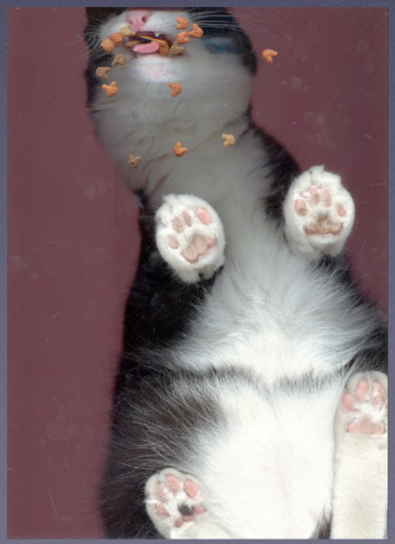




Hyper-Structured Illumination High and Low

(Can) learn a lot:

vs.



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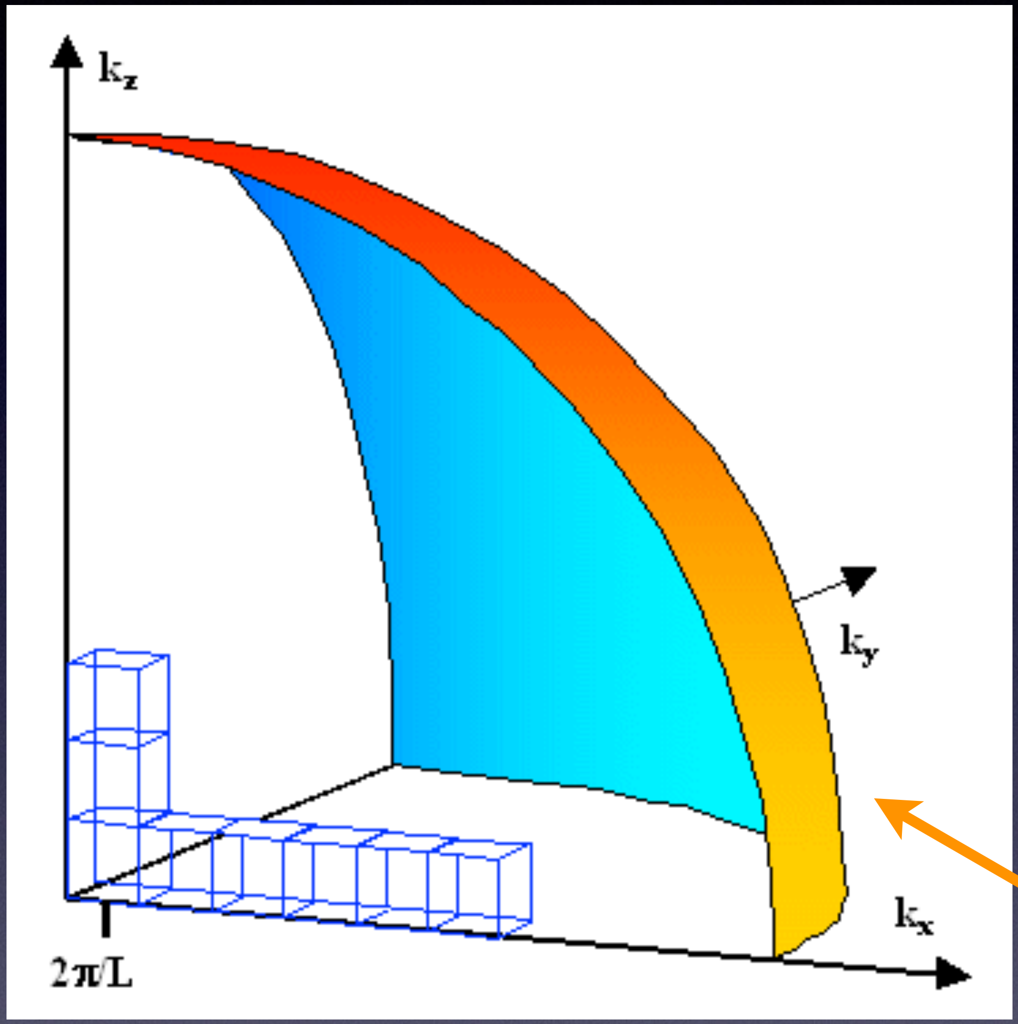
Photonic Density of States



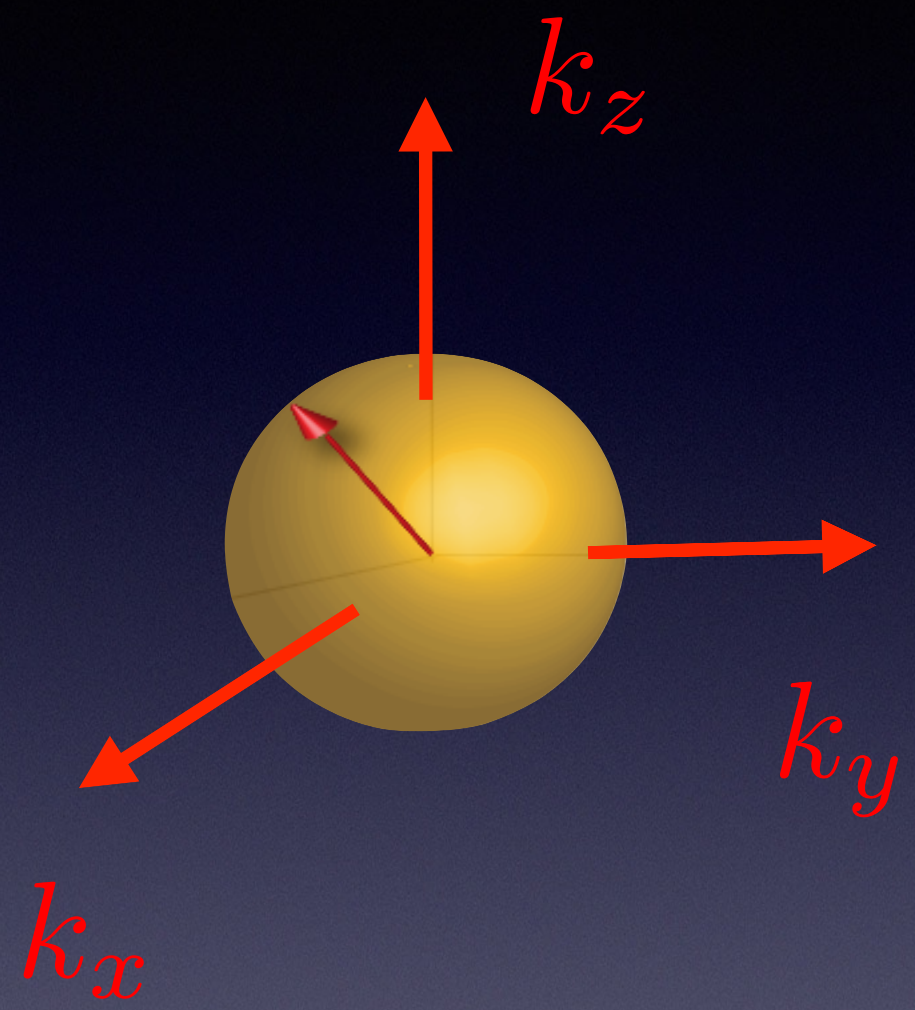
$$\rho(\omega) = \frac{dN}{d\omega}$$

N(ω) - number of states with frequency below *ω*

$$k_x^2 + k_y^2 + k_z^2 = \epsilon \frac{\omega^2}{c^2}$$

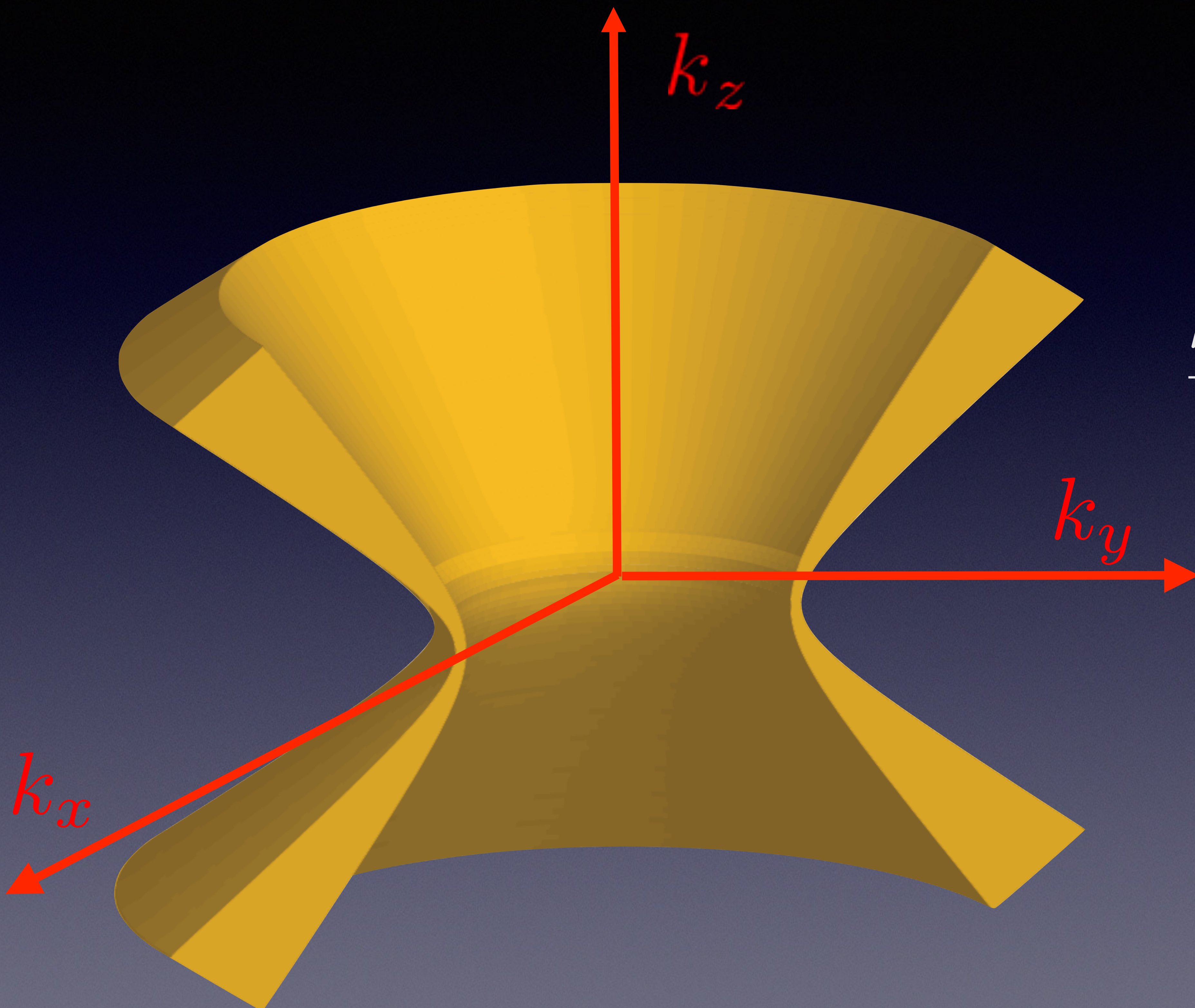
Free space : $\rho\left(\omega\right) \sim \omega^2$

Uniaxial Medium: $\varepsilon_x, \varepsilon_y > 0, \varepsilon_z > 0$



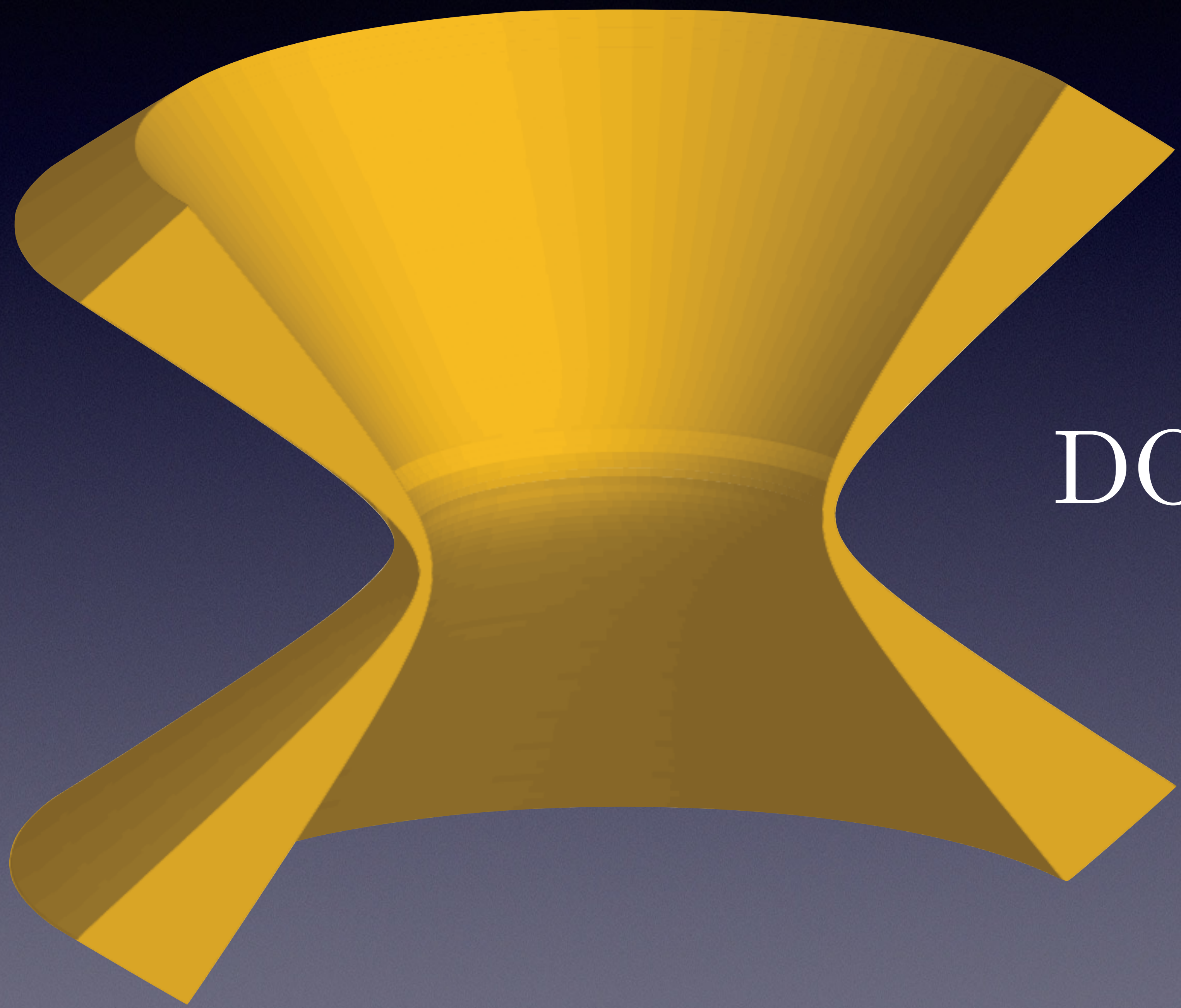
$$\frac{k_x^2 + k_y^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

Uniaxial Medium: $\varepsilon_x, \varepsilon_y < 0$, $\varepsilon_z > 0$



$$\frac{k_x^2 + k_y^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

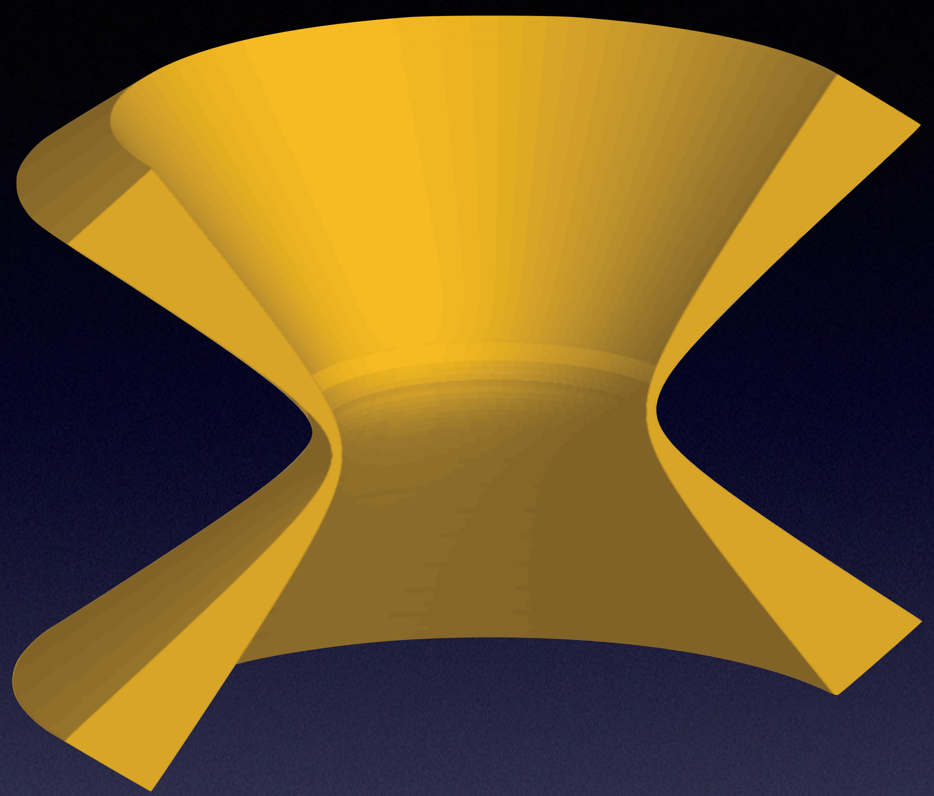
Photonic DOS in a Hyperbolic Medium:
"Super"- Singularity !



$$\mathrm{DOS} = \infty, \ \forall \omega \ !$$

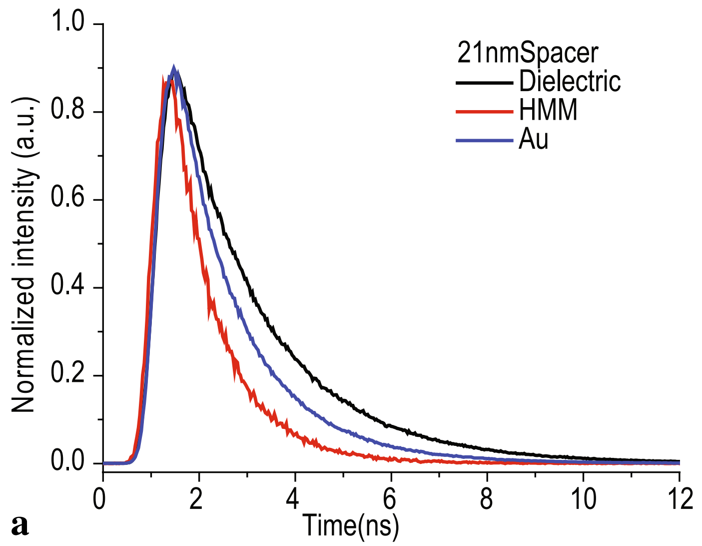
I.Smolyaninov & *EN*, PRL 105, 067402 (2010)

Electromagnetic Energy in a Hyperbolic Medium



$$U_T \propto \int d\omega \, \frac{\rho\left(\omega\right) \, \hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}$$

$$\rho\left(\omega\right) = \infty, \;\; \forall\omega$$

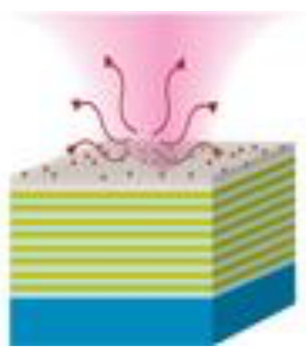


Appl Phys B (2010) 100: 215–218
DOI 10.1007/s00340-010-4096-5

Applied Physics B
Lasers and Optics

Engineering photonic density of states using metamaterials

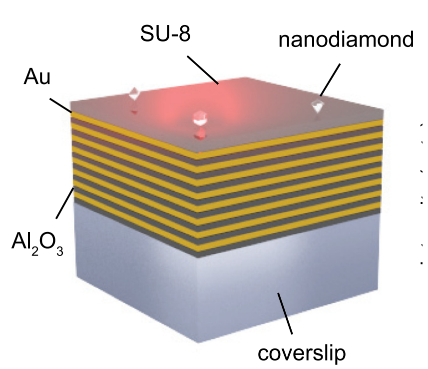
**Z. Jacob · J.-Y. Kim · G.V. Naik · A. Boltasseva ·
E.E. Narimanov · V.M. Shalaev**



#161992 - $15.00 USD Received 23 Jan 2012; revised 2 Mar 2012; accepted 2 Mar 2012; published 22 Mar 2012
(C) 2012 OSA 26 March 2012 / Vol. 20, No. 7 / OPTICS EXPRESS 8100

Improving the radiative decay rate for dye molecules with hyperbolic metamaterials

J. Kim,[1] V. P. Drachev,[1*] Z. Jacob,[1,2] G. V. Naik,[1] A. Boltasseva,[1] E. E. Narimanov,[1] and V. M. Shalaev[1]

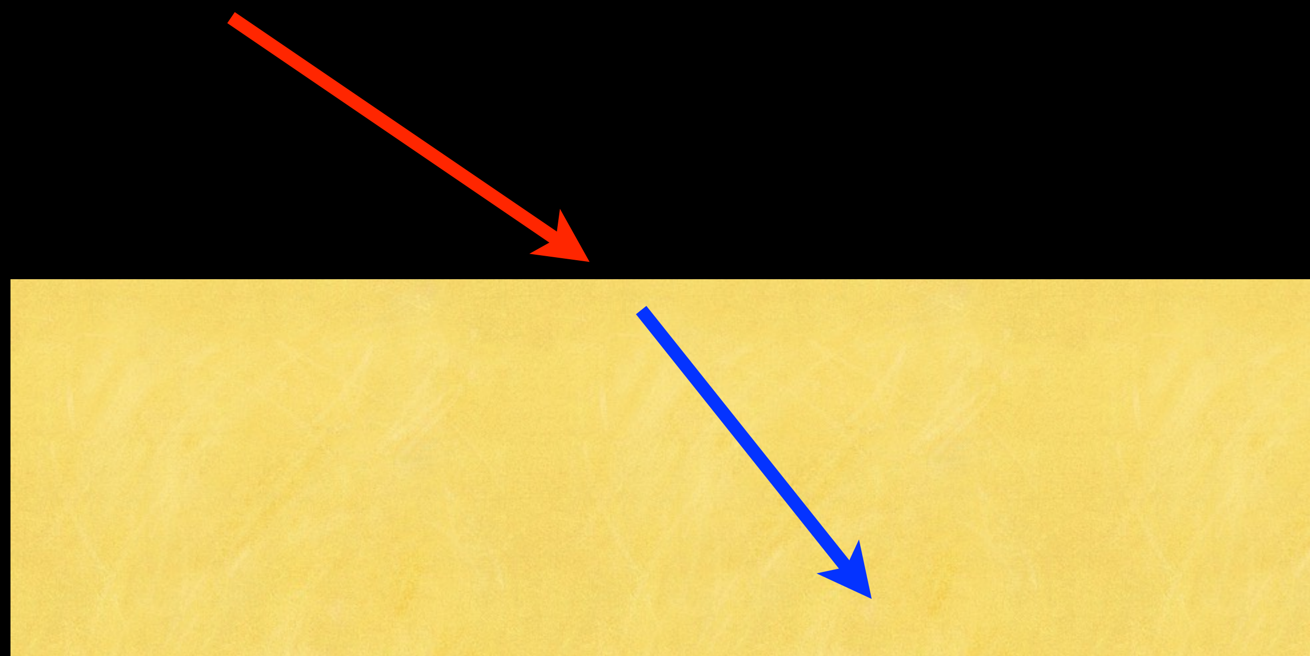


APPLIED PHYSICS LETTERS **102**, 173114 (2013)

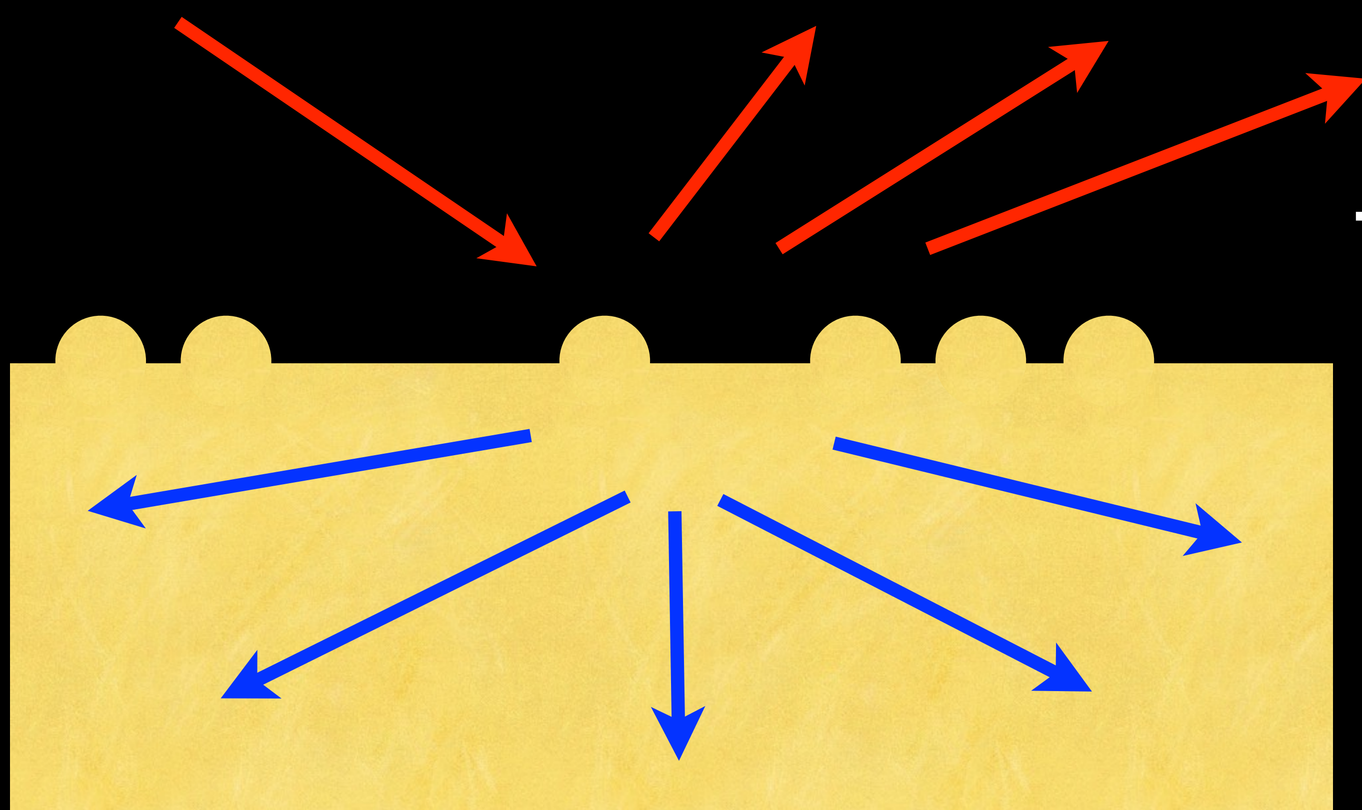
Broadband enhancement of spontaneous emission from nitrogen-vacancy centers in nanodiamonds by hyperbolic metamaterials

M. Y. Shalaginov,[1,2] S. Ishii,[1,2,3] J. Liu,[1,2] J. Liu,[4] J. Irudayaraj,[4] A. Lagutchev,[2]
A. V. Kildishev,[1,2] and V. M. Shalaev[1,2,a)]

"Darker than Black" Materials :



- impedance-matching
- antireflection coatings
- resonant elements
- ...



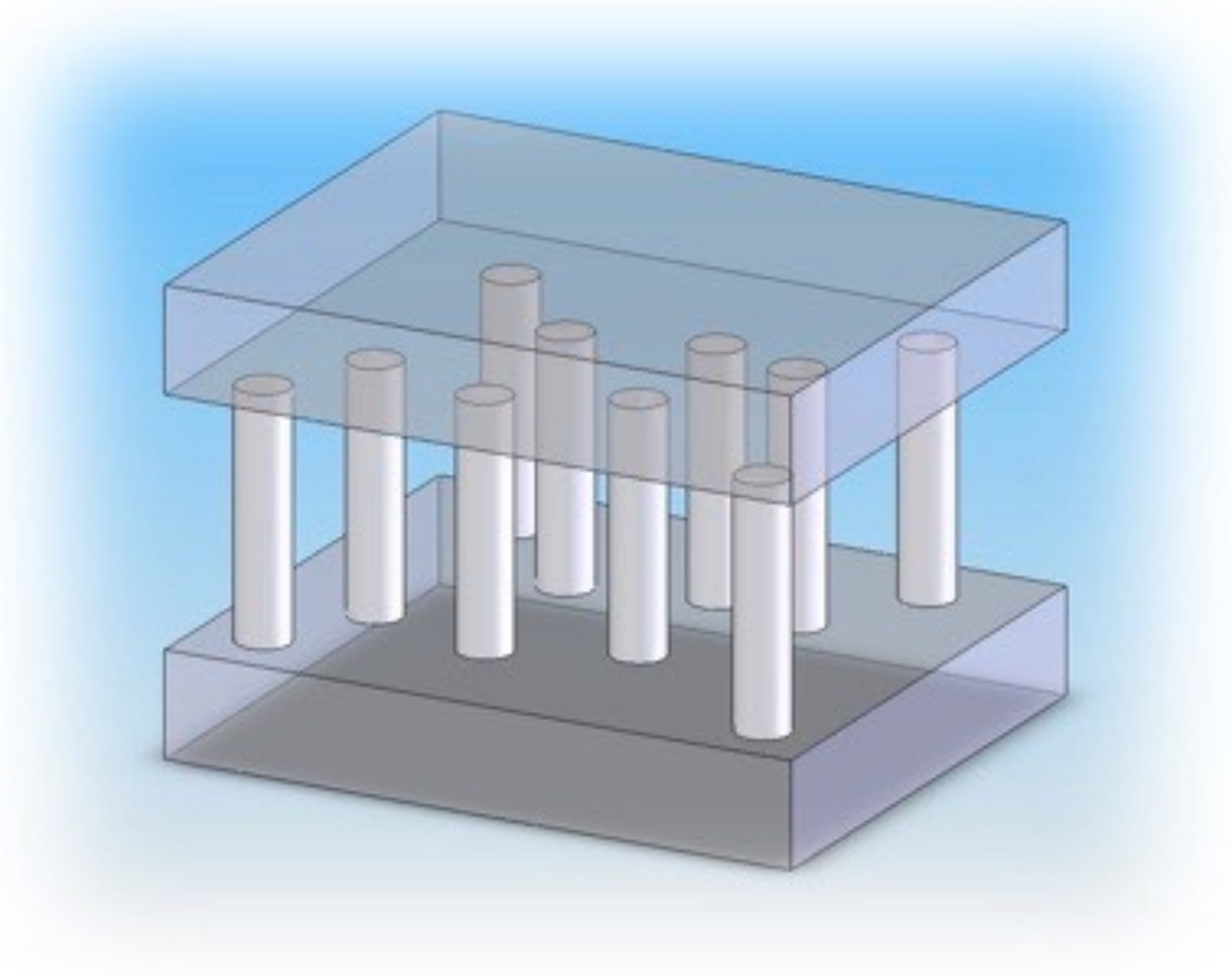
The scattering rate $W \propto \rho(\omega)$

$$W_\downarrow \propto \rho_\downarrow(\omega)$$

$$W_\uparrow \propto \rho_\uparrow(\omega)$$

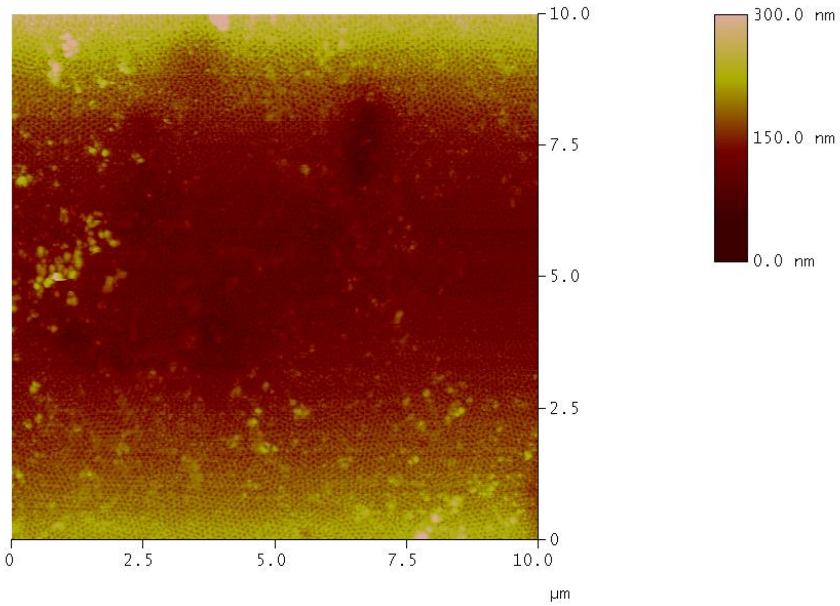
$$\text{Need} \quad \rho_\downarrow \longrightarrow \infty \ !$$

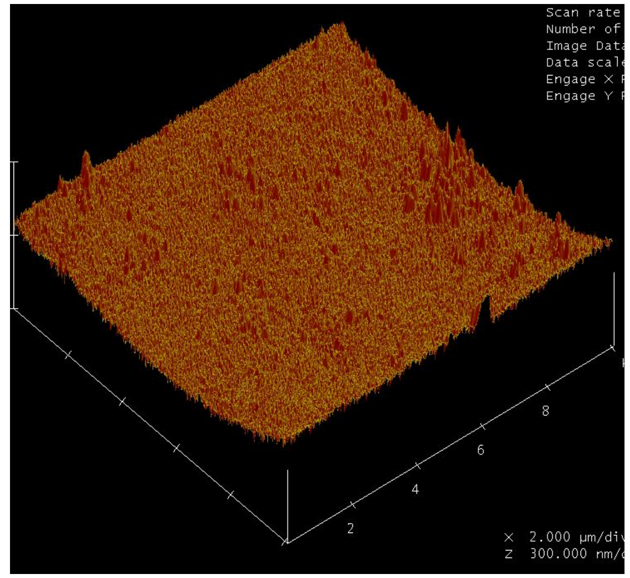
Start with a hyperbolic metamaterial (e.g. wire-based):



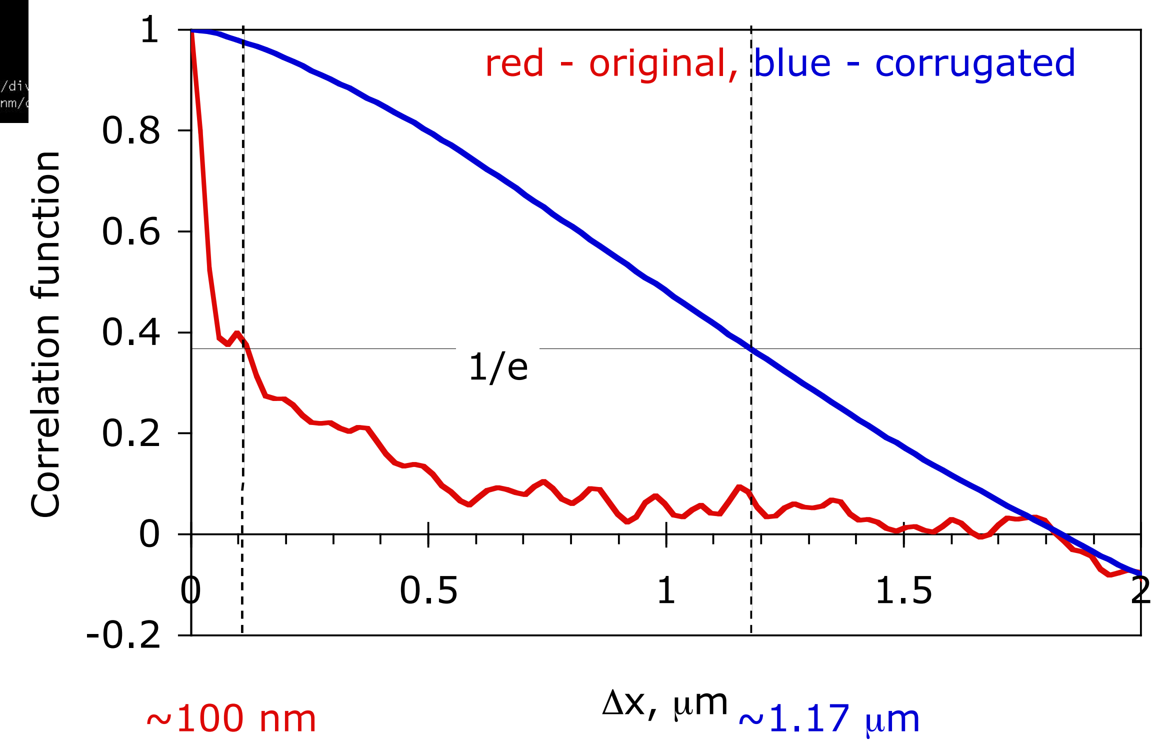
... and introduce roughness to its surface

Original sample

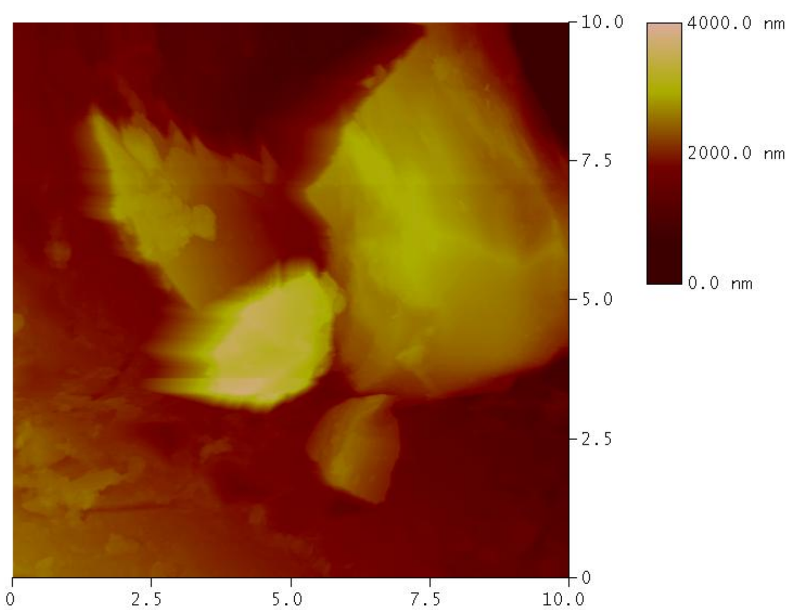


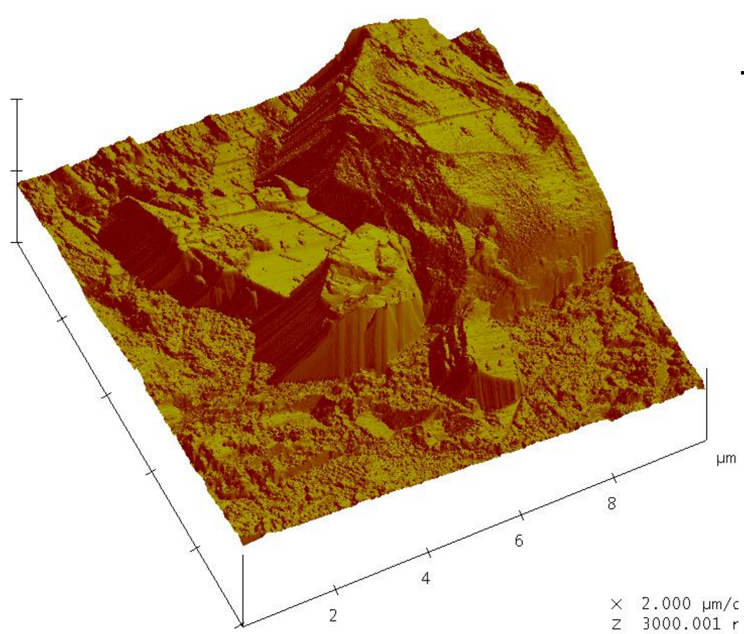


vertical rms=40 nm



Corrugated sample

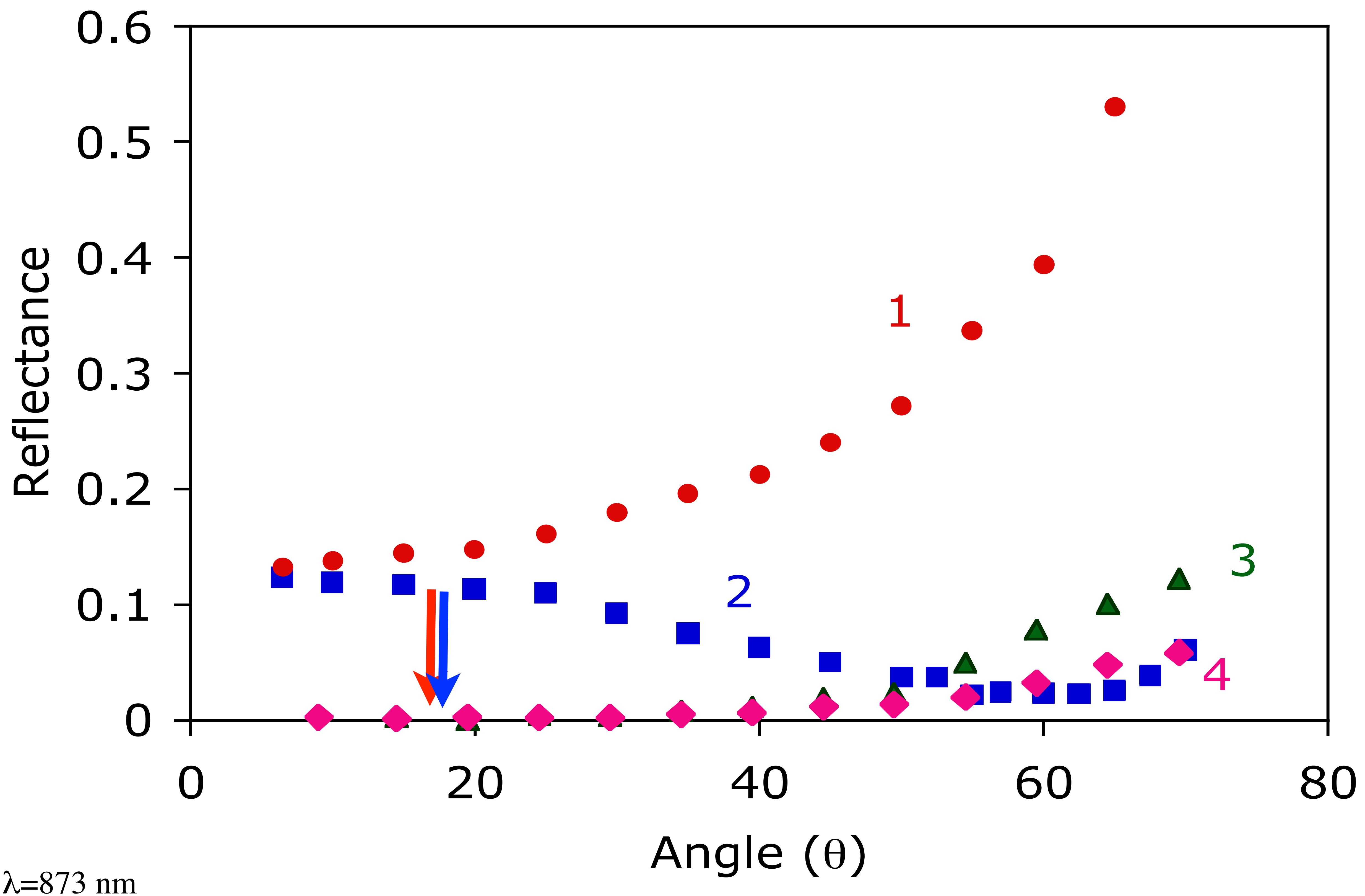




vertical rms=600 nm

Angular reflectance curves before (1,2) and after (3,4) corrugation.

S-polarization: 1,3 P-polarization: 2,4



λ=873 nm

- resolution of a “hyperbolic” microscope is *not* limited by the (free-space) wavelength

- light (in a hyperbolic medium) *can* be focused to arbitrarily* small spot

- forbidden optical transitions are *not* forbidden

- LED modulation speed is *not* limited by the spontaneous emission rate of its active medium

- and a lot more (negative refraction, “darker-than-black” materials, etc.)

* subject to the atom size constraint

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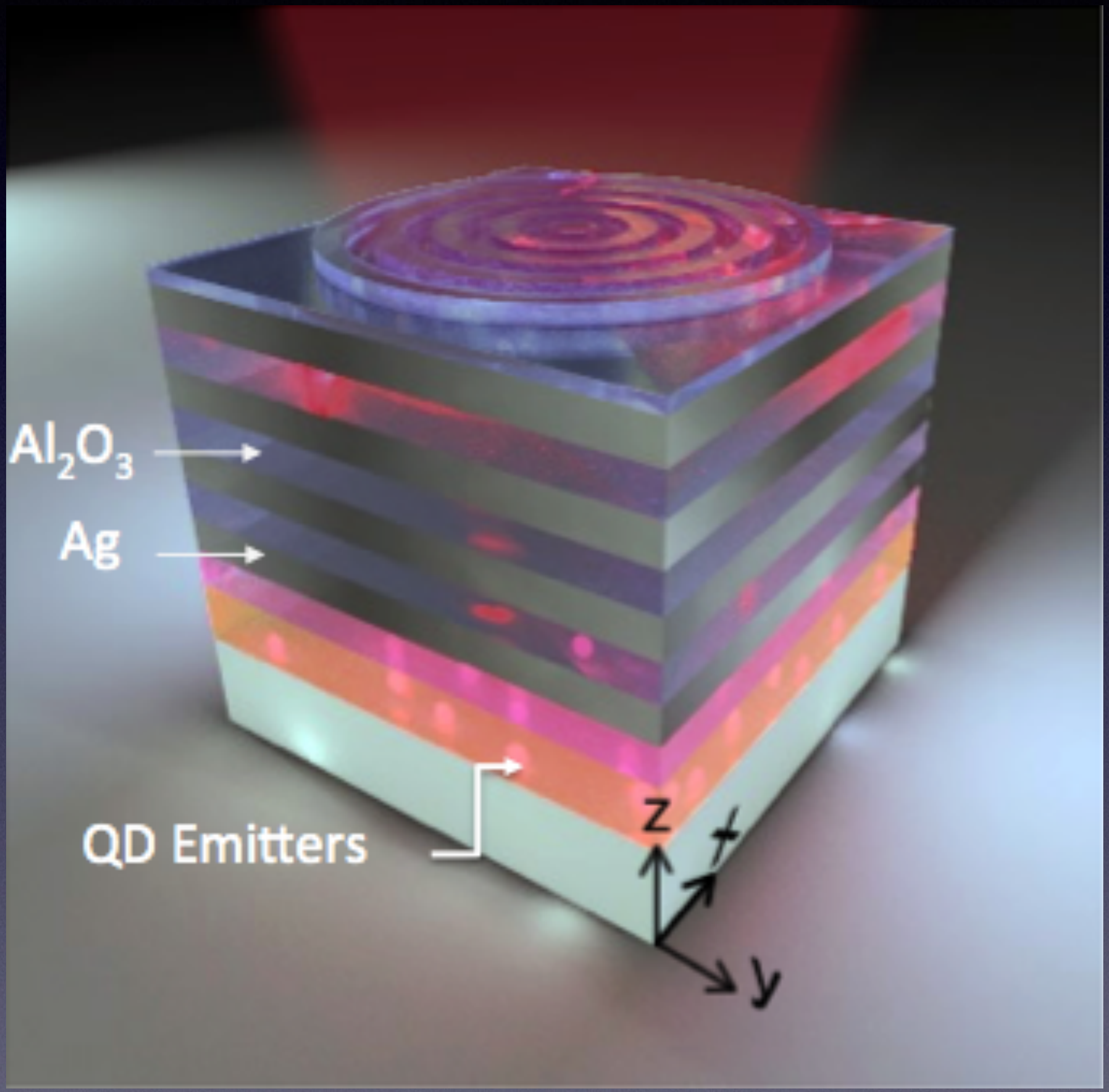
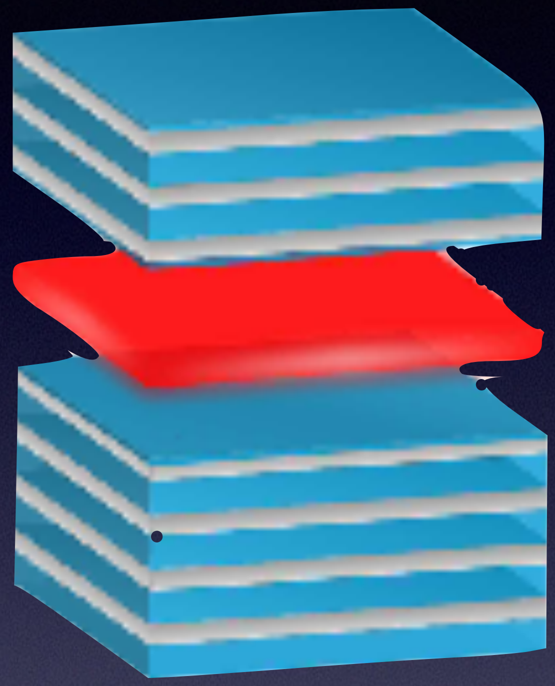
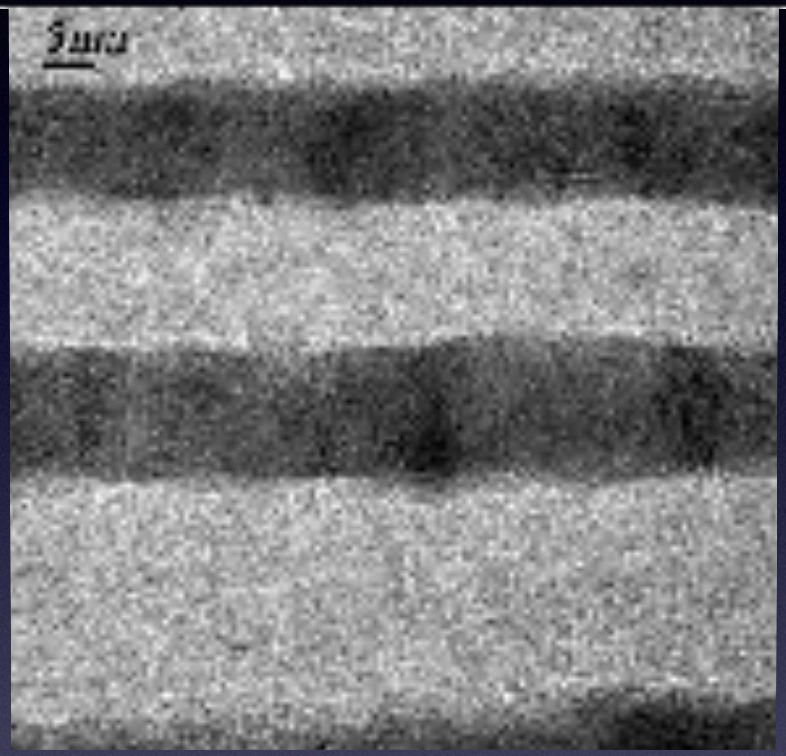
Active Photonic Hypercrystals

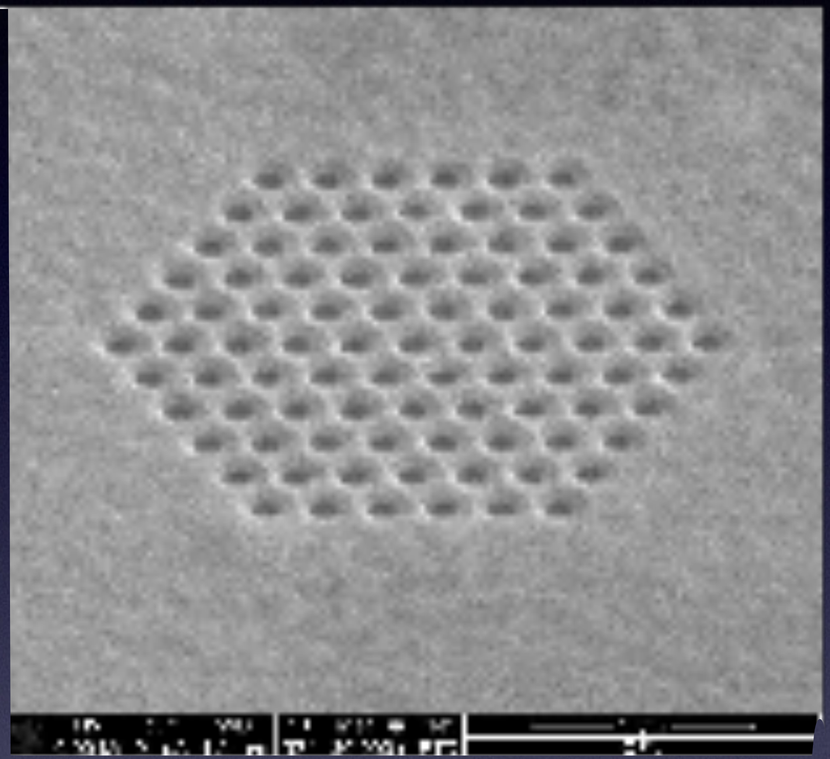


T. Galfsky, H. N. S. Krishnamorhy, W. Newman, EN, Z. Jacob & V. Menon, Optica 2015

Active Photonic Hypercrystals

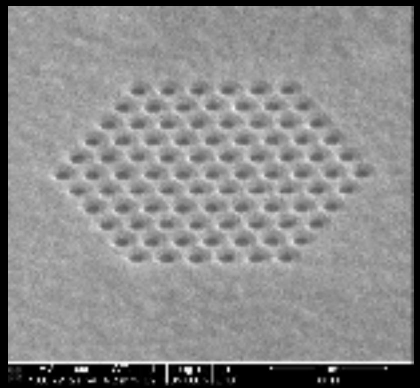


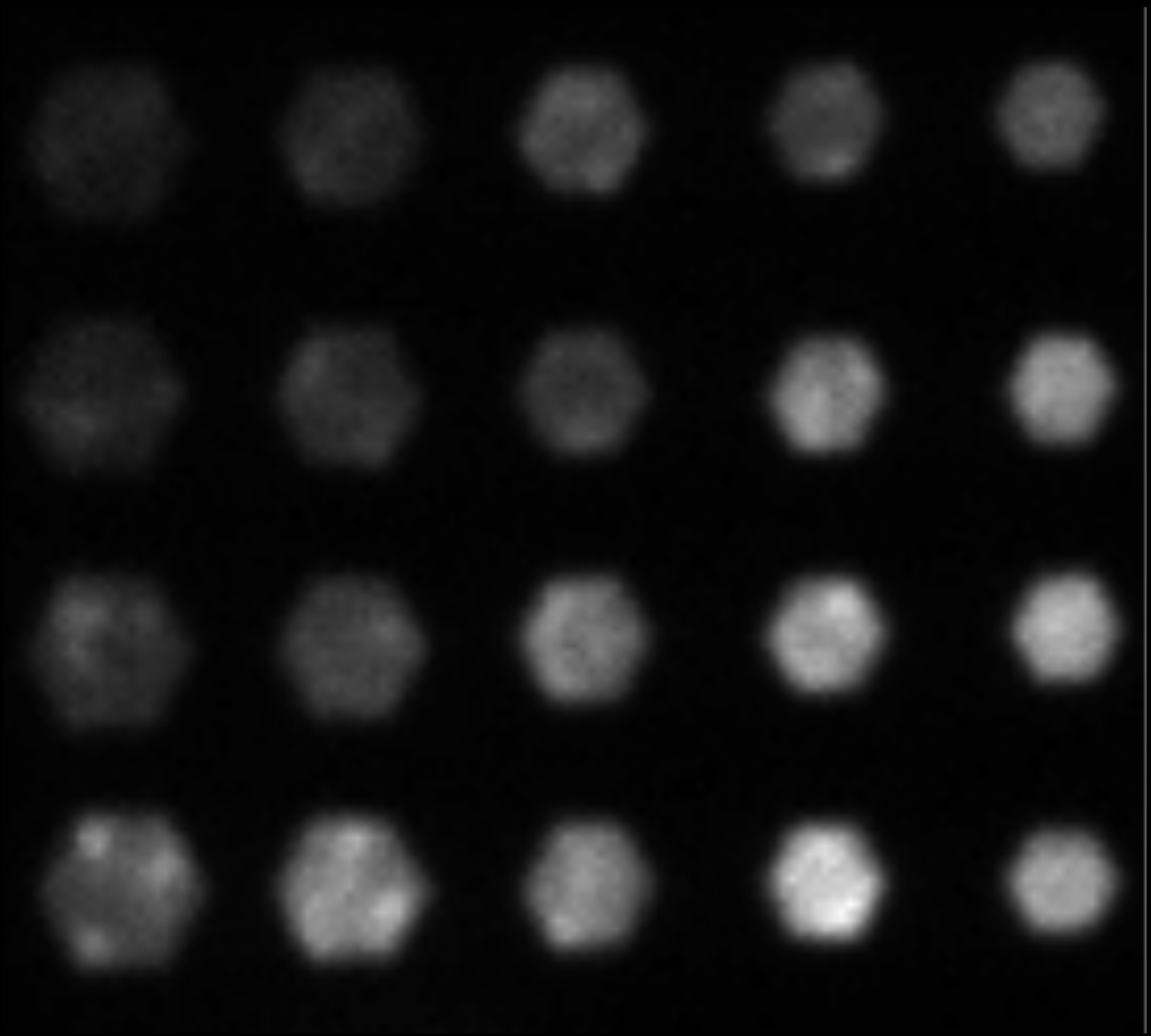




T. Galfsky, J. Gu, EN & V. Menon,
PNAS 114 (20), 5125 (2017)

Intensity x 100 ! Speed x 20 !





T. Galfsky, J. Gu, EN & V. Menon,
PNAS 114 (20), 5125 (2017)

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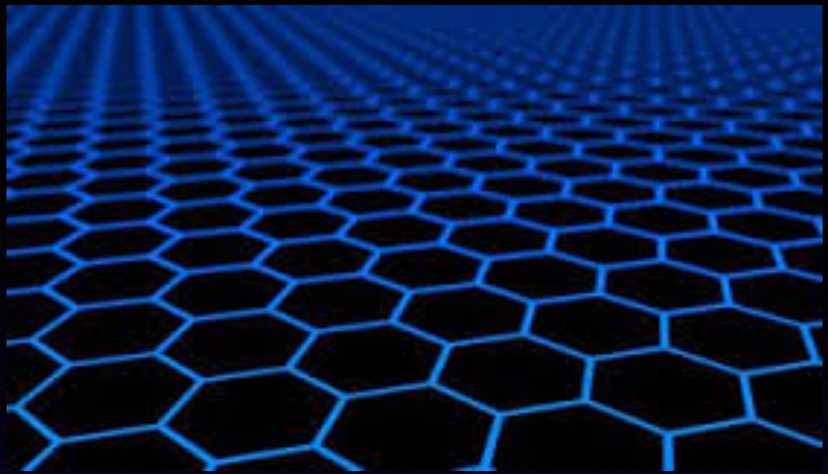
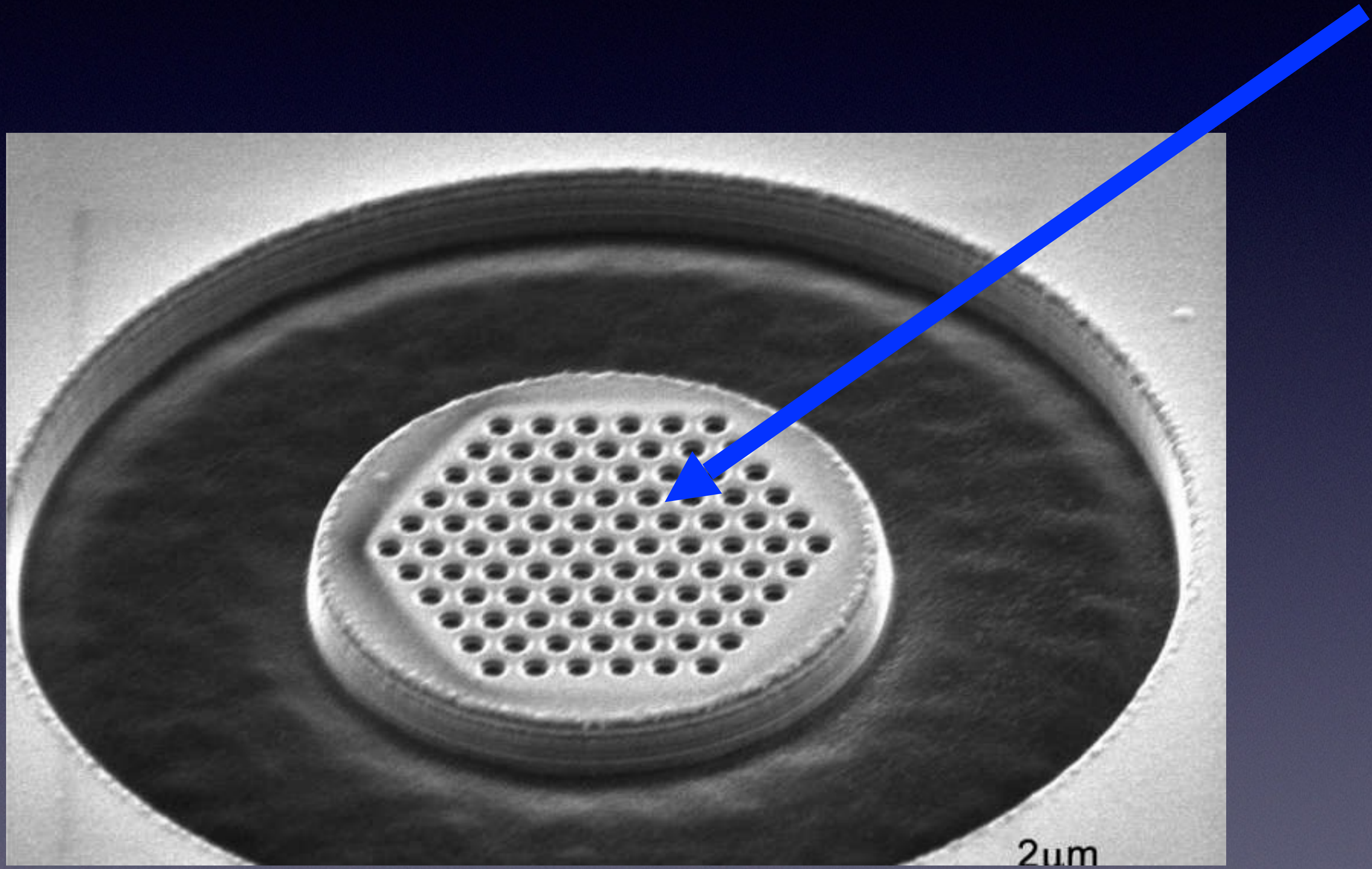
Hyperbolic Materials
as lab tools / instruments

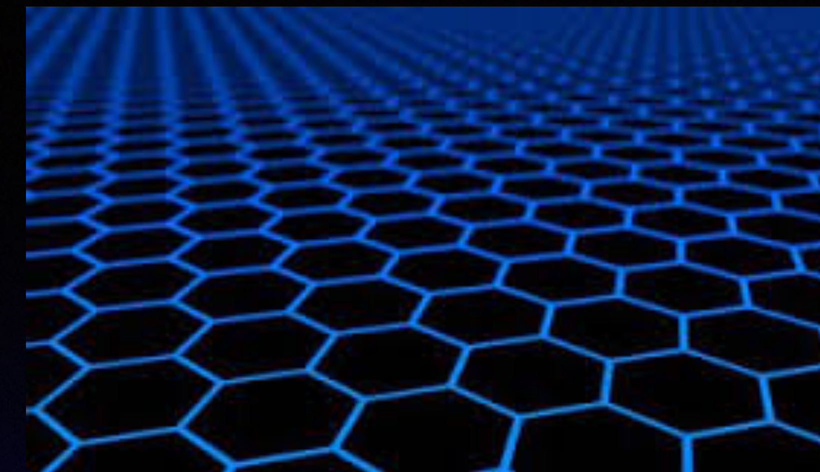


WS_2

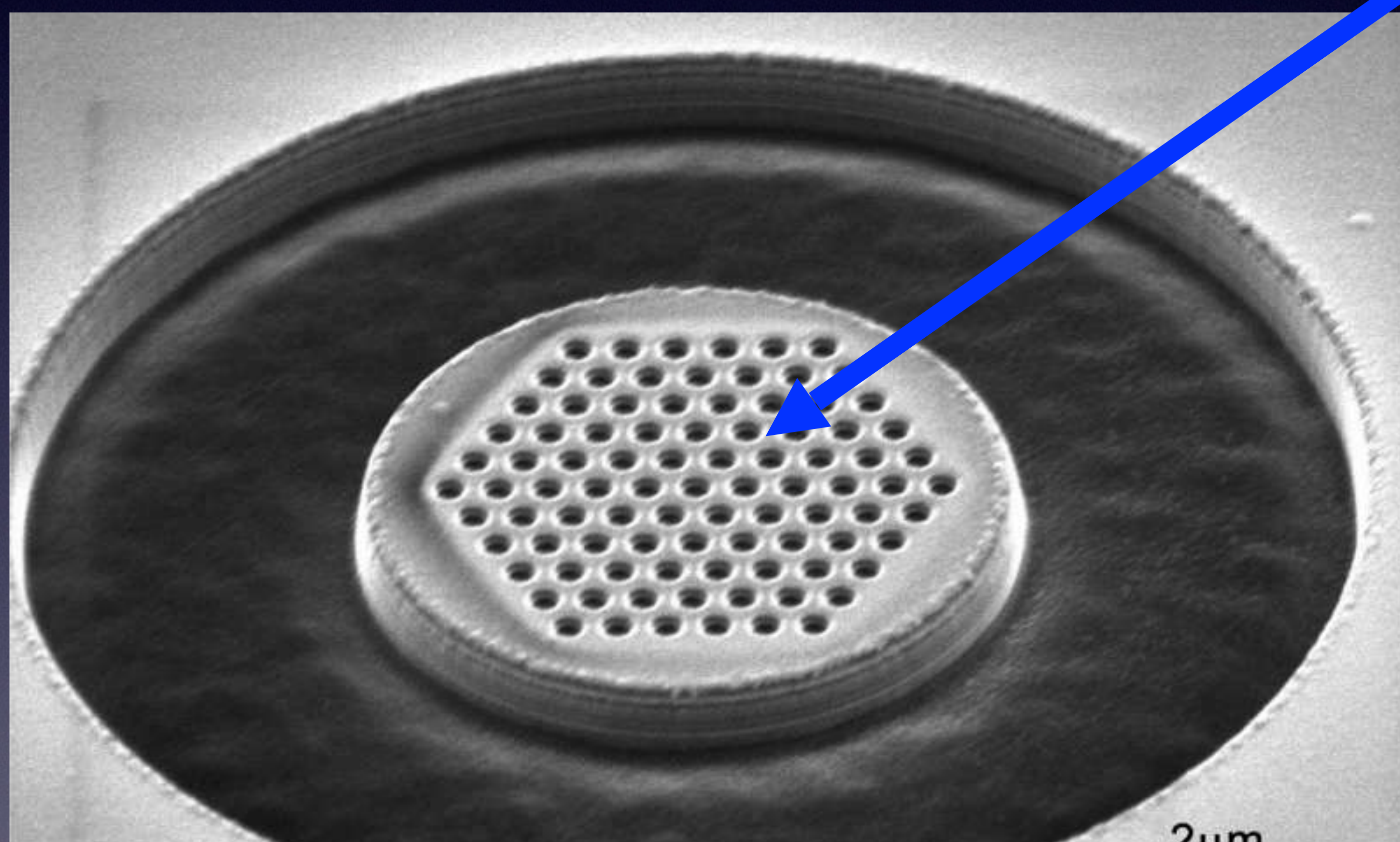


T. Galfsky, Z. Sun, C. R. Constantine, C. T. Chou, W.C. Ko, Y. H. Lee, EN & V. Menon , NanoLetters 16 (8), 4940 (2016)

Spontaneous Emission Enhancement from 2D semiconductors



WS_2



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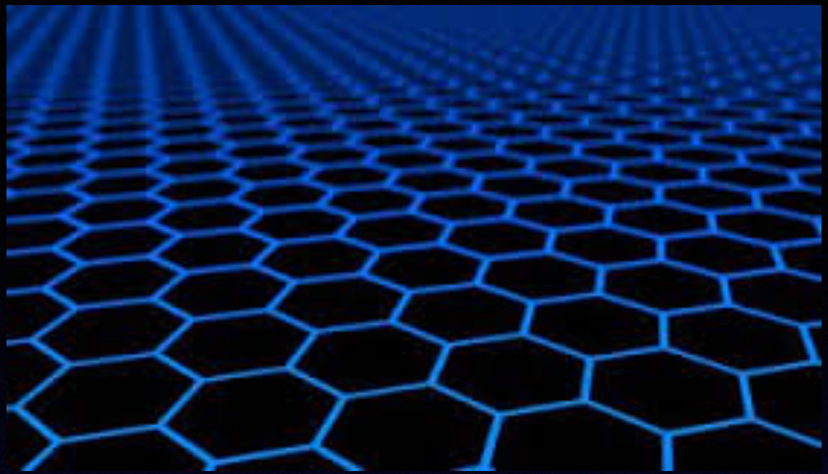
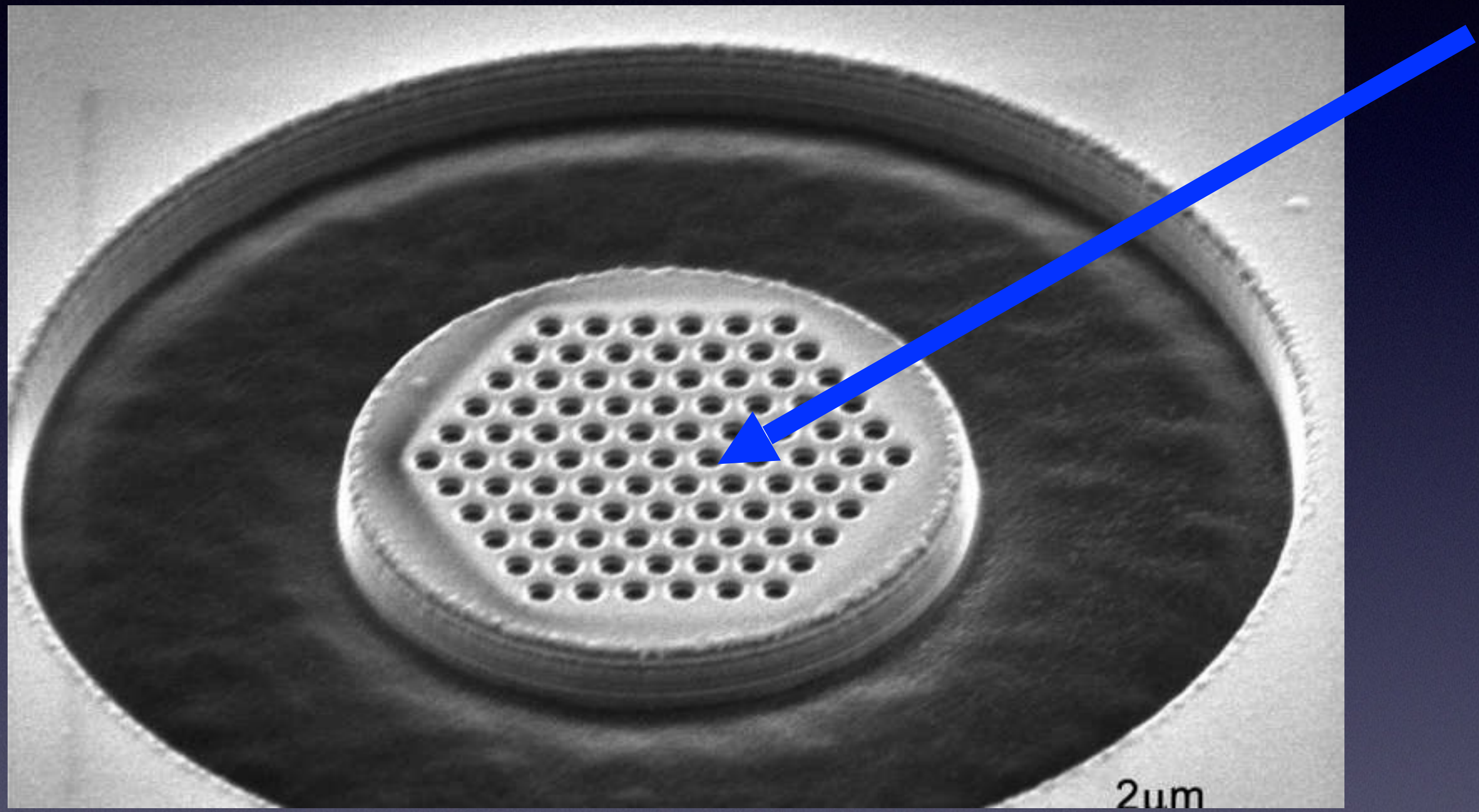
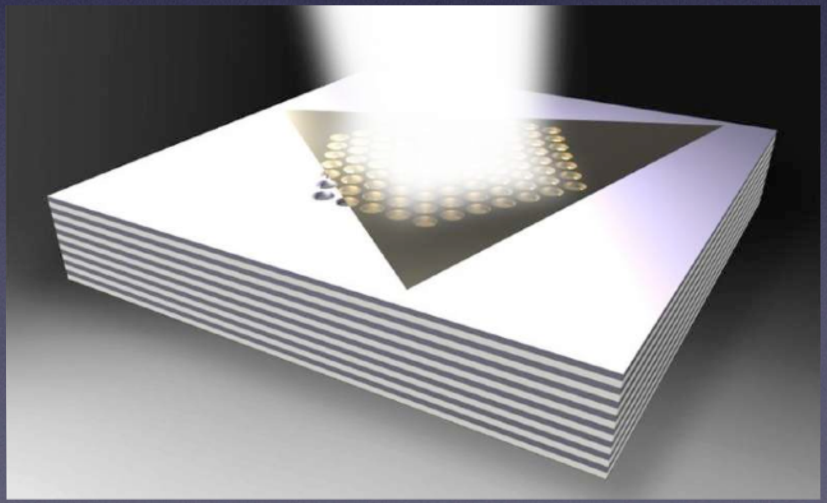


WS_2





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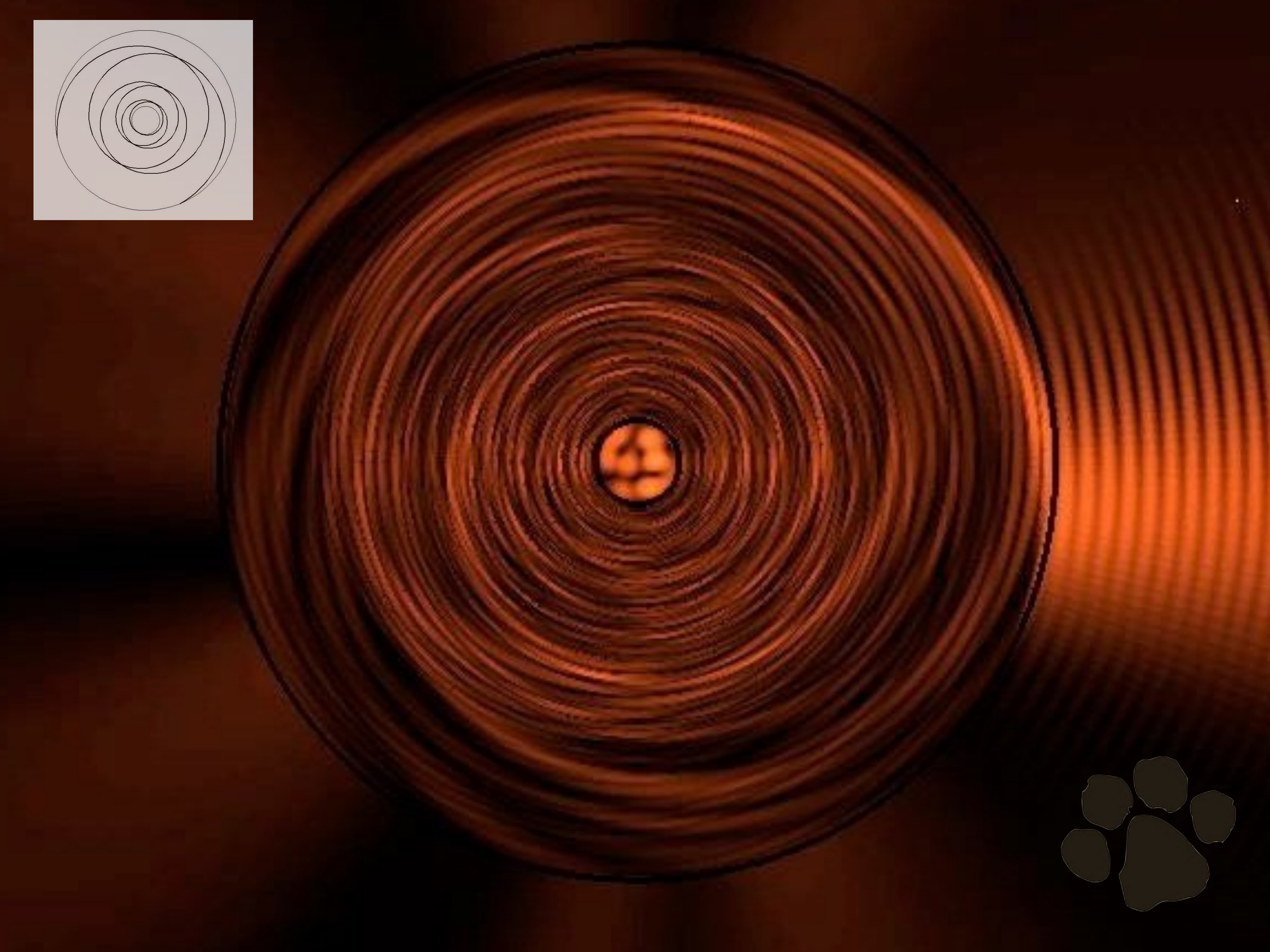
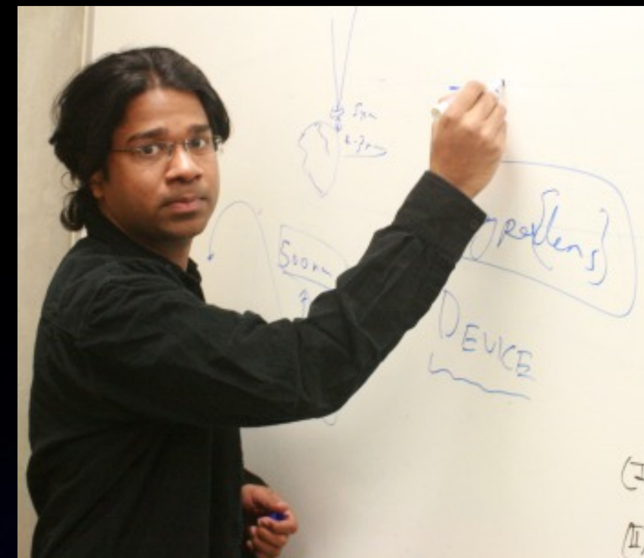


Viktor Podolskiy



Zubin Jacob

Leo Alekseyev

V. Shalaev

C. Gmachl

X. Zhang

M. Noginov

V. Menon