Sensitivity & Resolution of Quantum-Enhanced Imaging: Decoherence, Coherence, and Ancillas

Bahaa Saleh

CREOL, The College of Optics & Photonics

University of Central Florida

Outline

1 Brief overview of quantum-enhanced sensing & imaging

2 Quantum limits on precision of phase measurement: combatting decoherence with ancilla

3 Quantum limits on precision of phase-gradient measurement: Role of beam size

4 Quantum limits on two-point resolution: coherence resurrects Rayleigh's curse

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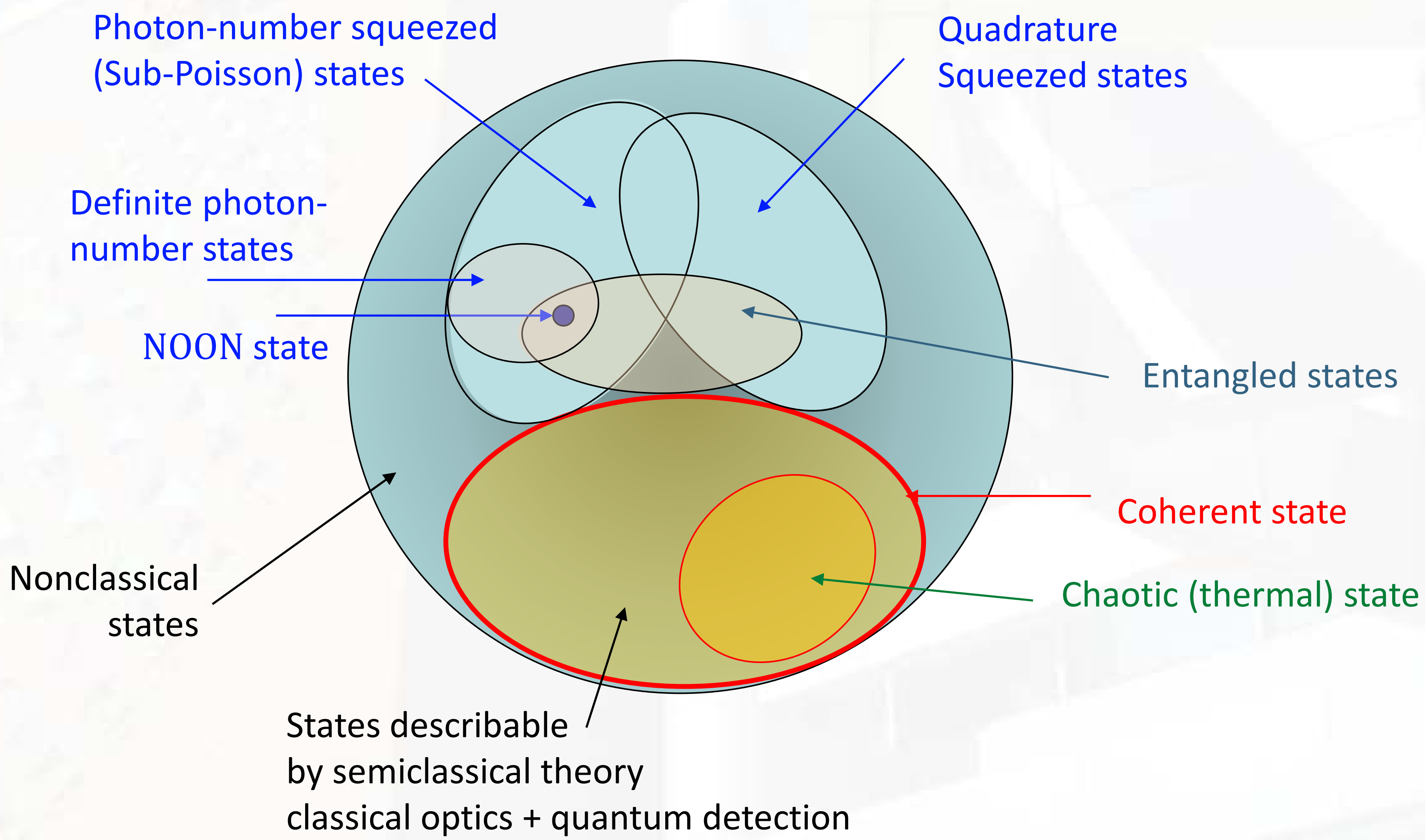
Quantum Sensing and Imaging

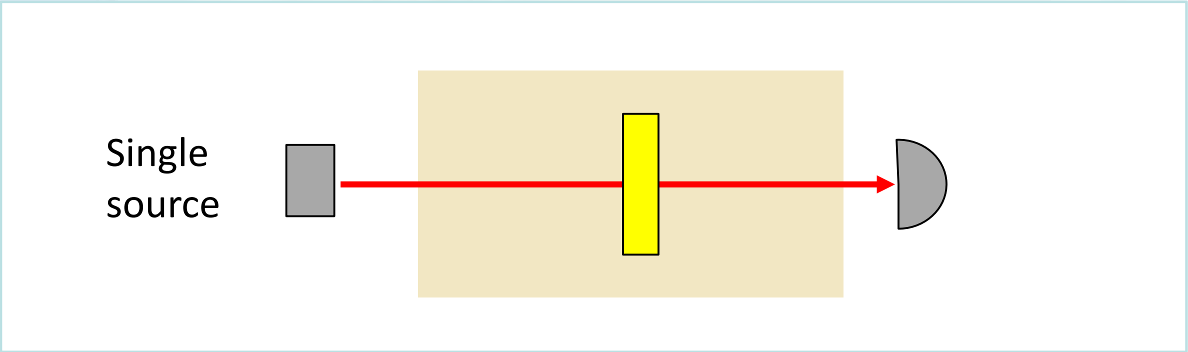
= Measurement of a physical parameter (sensing)
or the spatial distribution of a physical parameter (imaging)
by use of light in a nonclassical state
with precision exceeding that attained by classical means.

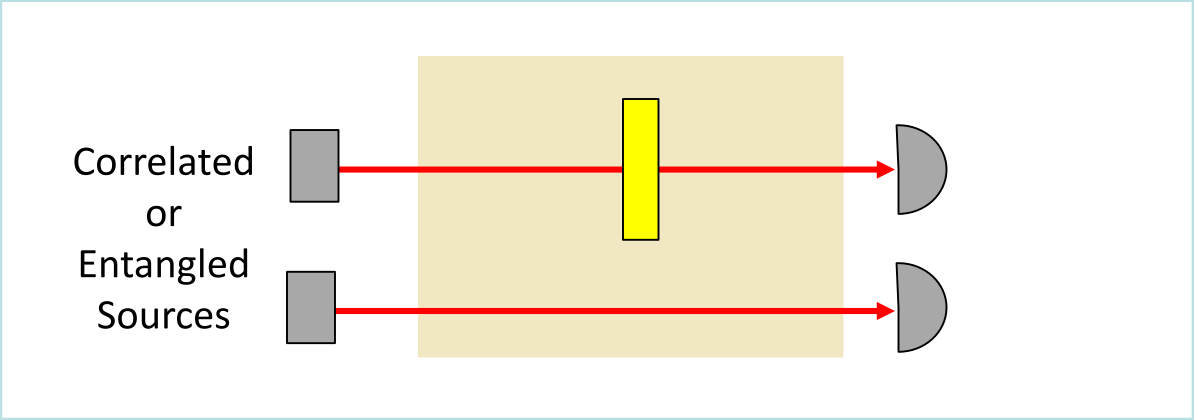
Fundamental limits on the precision are dictated by the quantum state of the probing field

Taxonomy of Quantum states of light

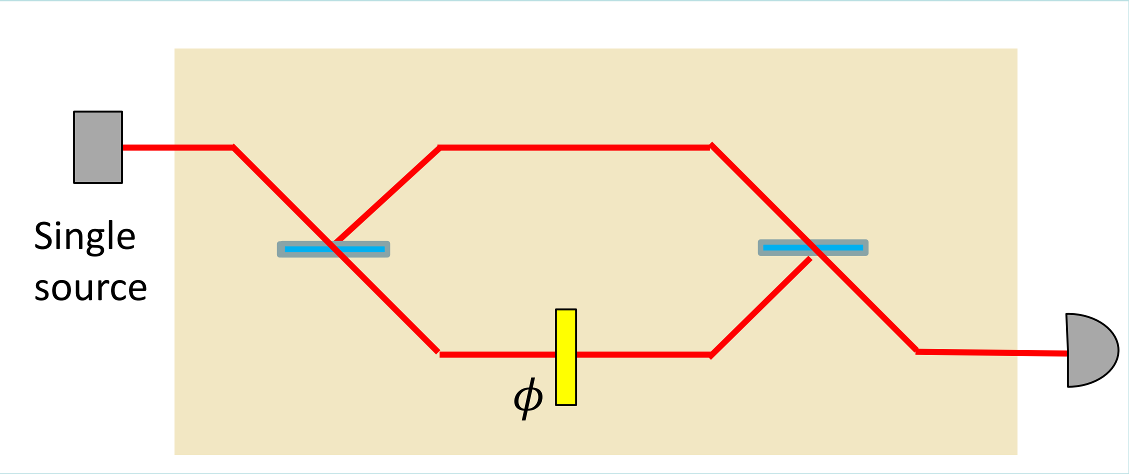


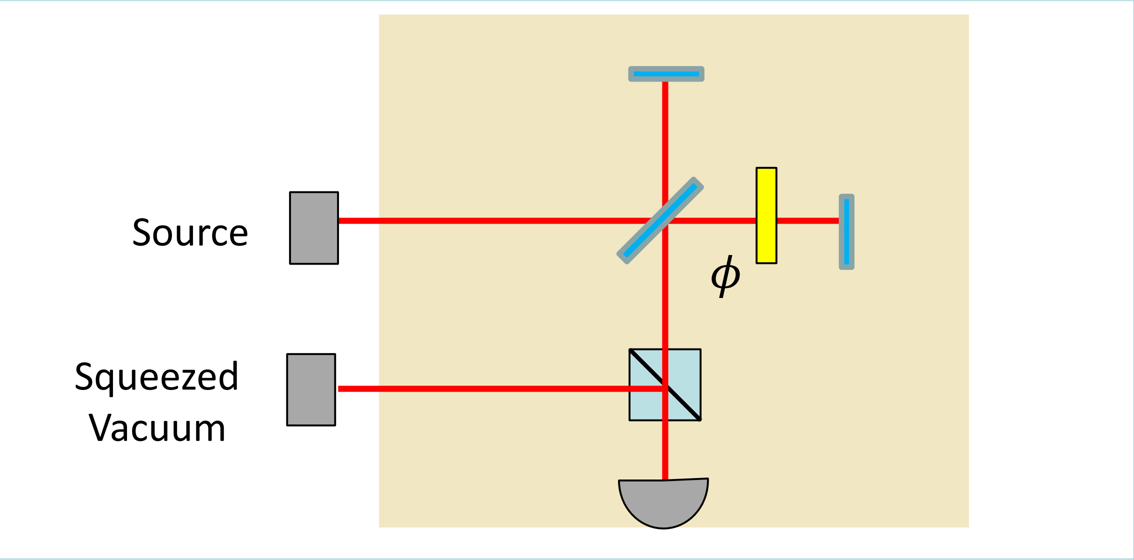
Amplitude Sensing

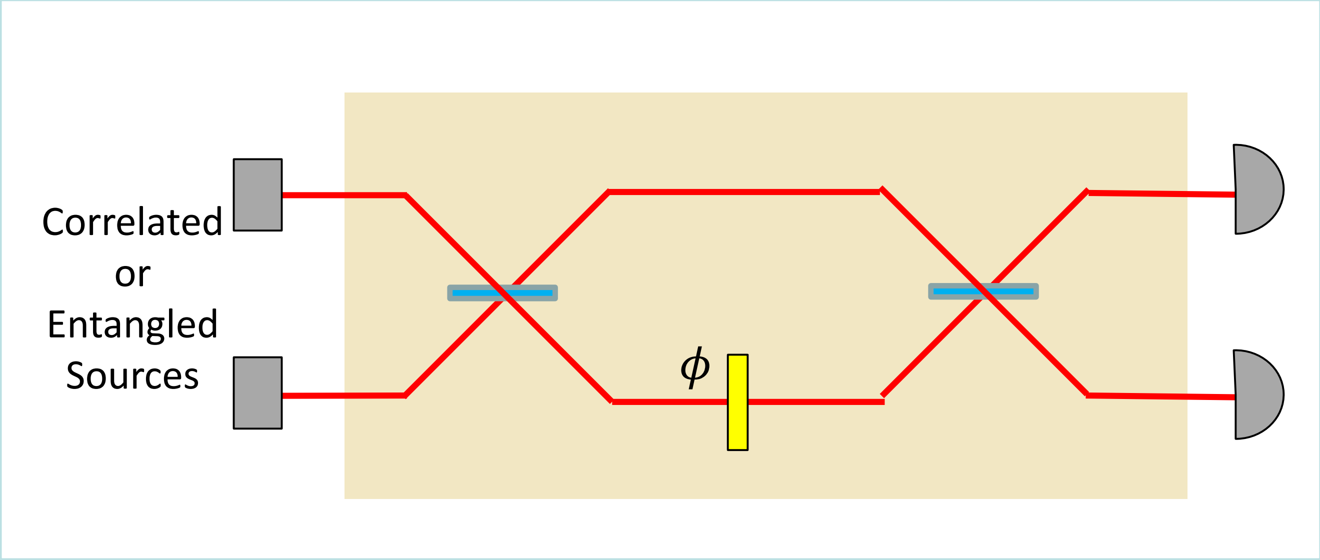




Phase Sensing



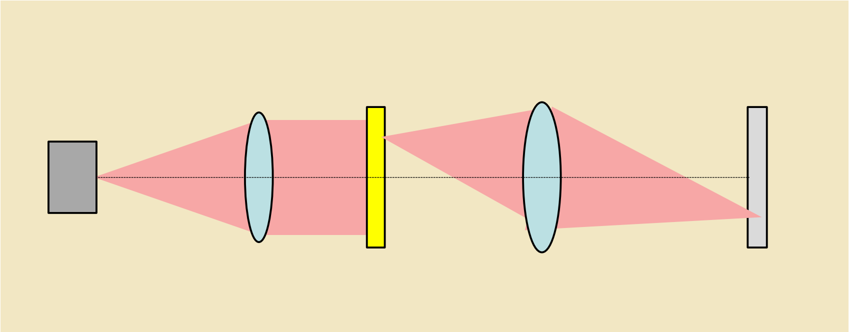




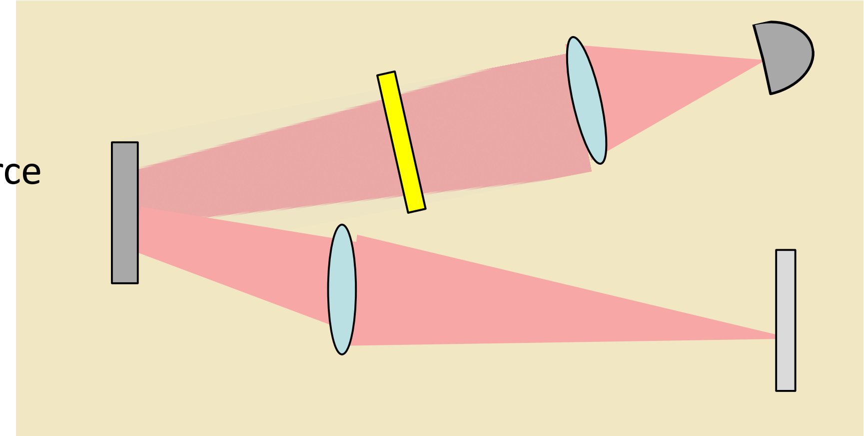
Taxonomy of Imaging Configurations

Amplitude Imaging

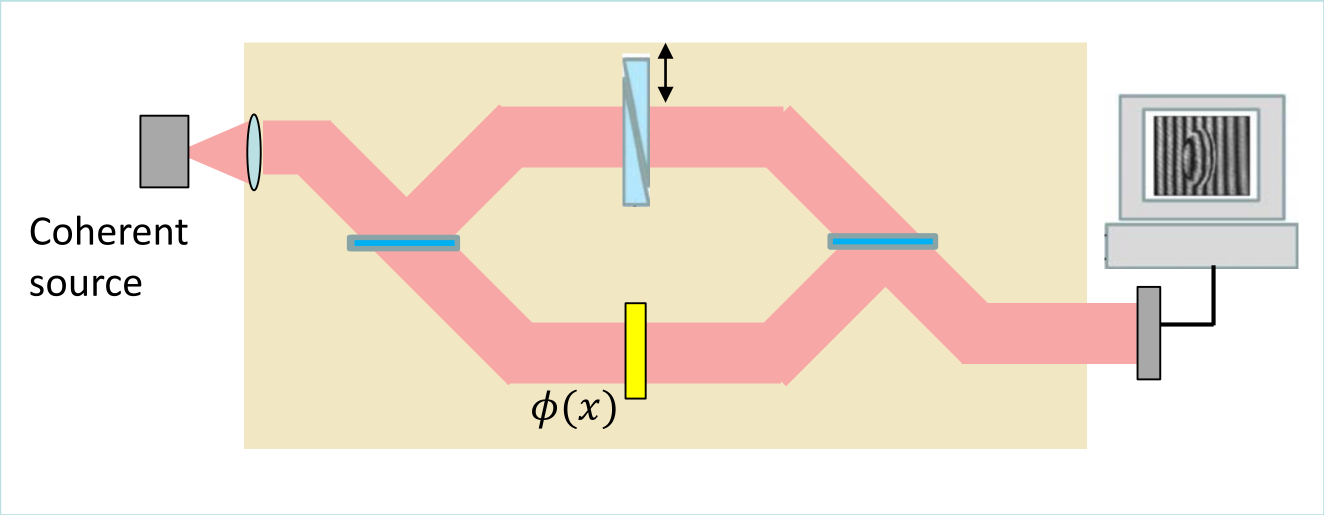
Single source

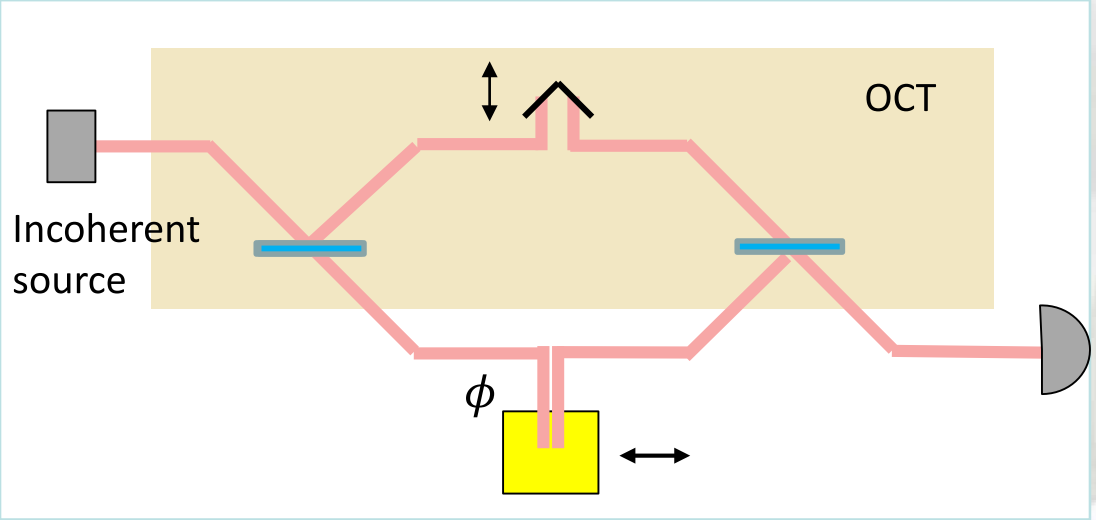


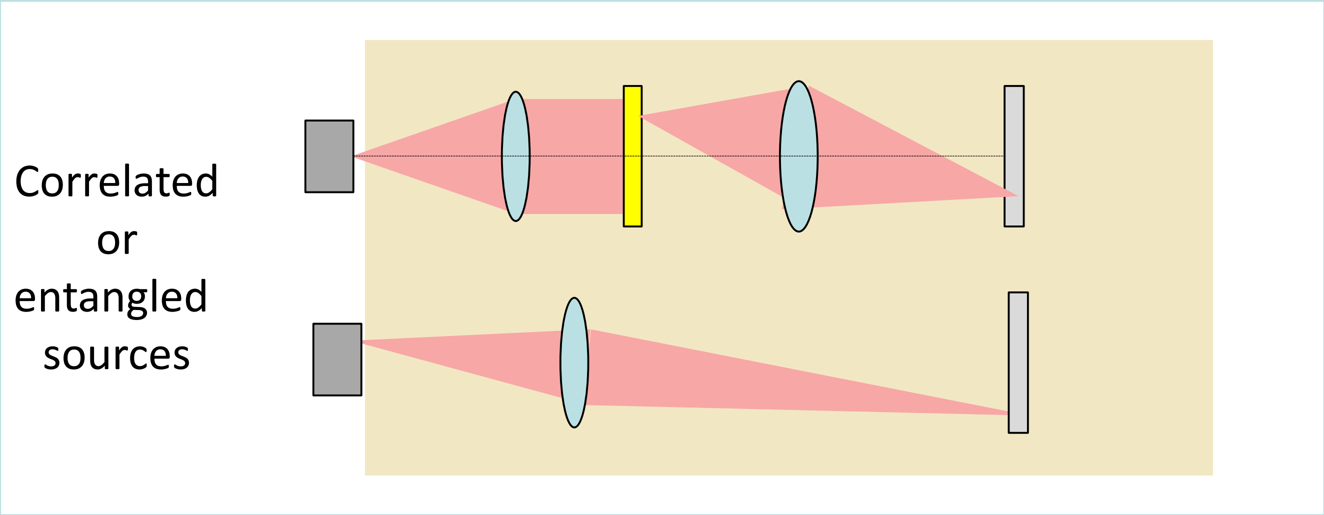
Extended Incoherent Source or Entangled Photon pairs

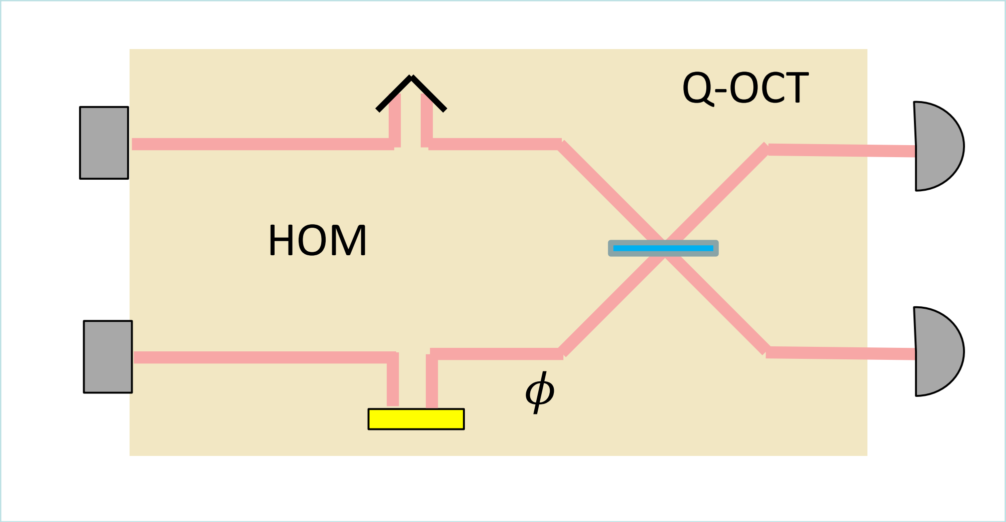


Phase Imaging

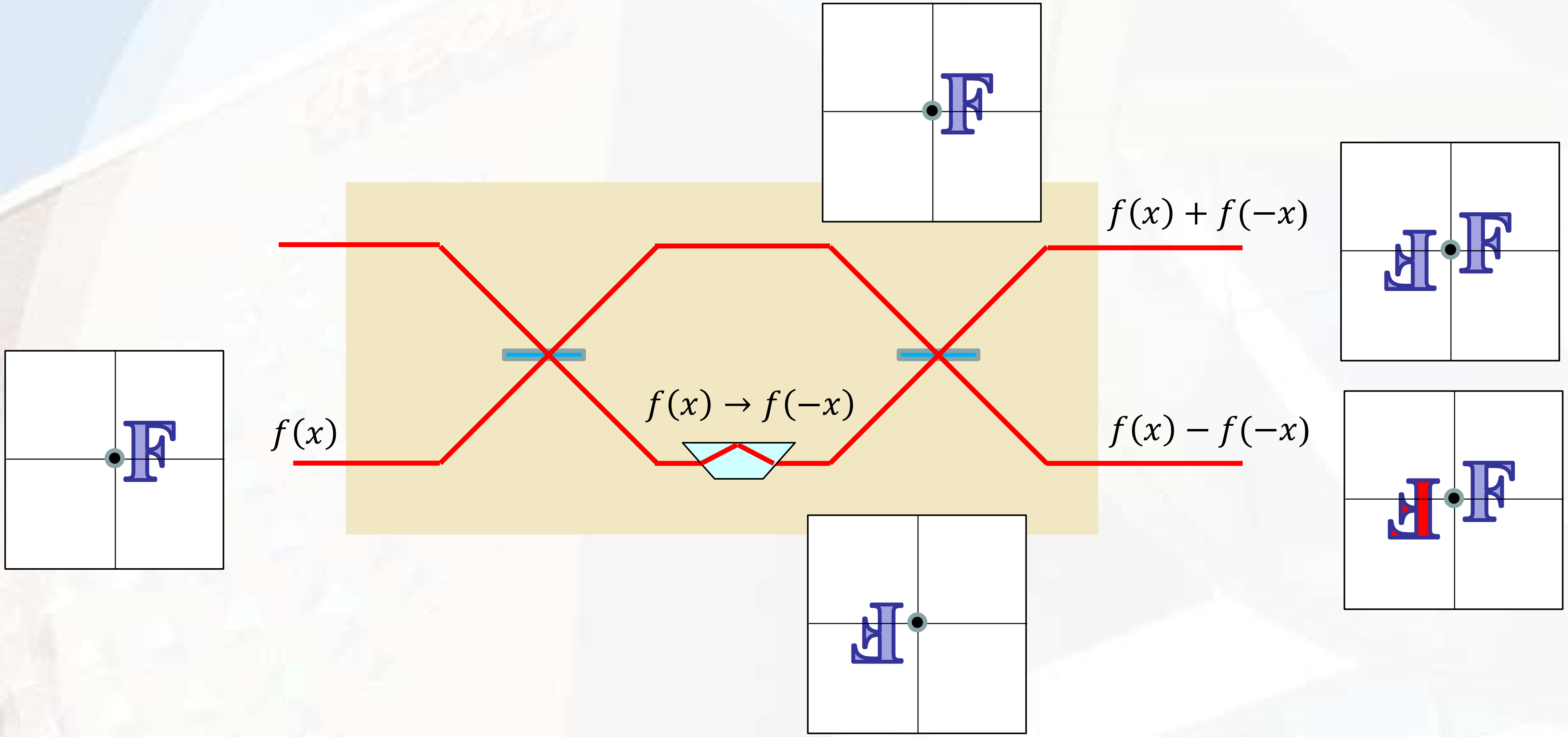








The image-inversion interferometer



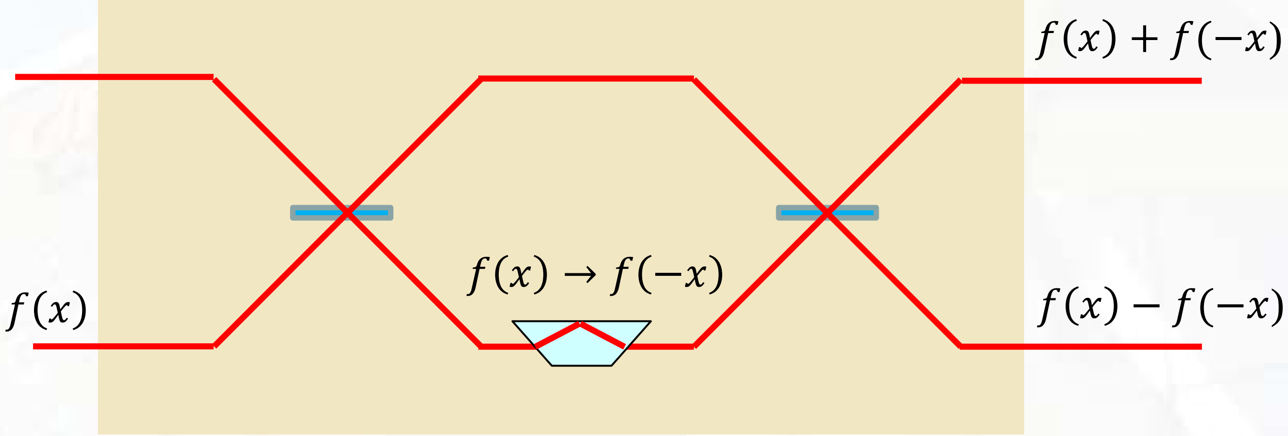
Physical implementations using common-path polarization-dependent lenses (± 1 magnification for R & L polarizations

Choudhary, . . ., Boyd, Op Exp. 43, 6101 (Dec 2018)

Larson, Tabiryan, Saleh, Opt Exp 27, 5685 (Feb 2019)

Larson, Tabiryan, Saleh, Opt Exp 27, 5685 (2019)

App 1 Parity analysis



Quantum spatial parity modes

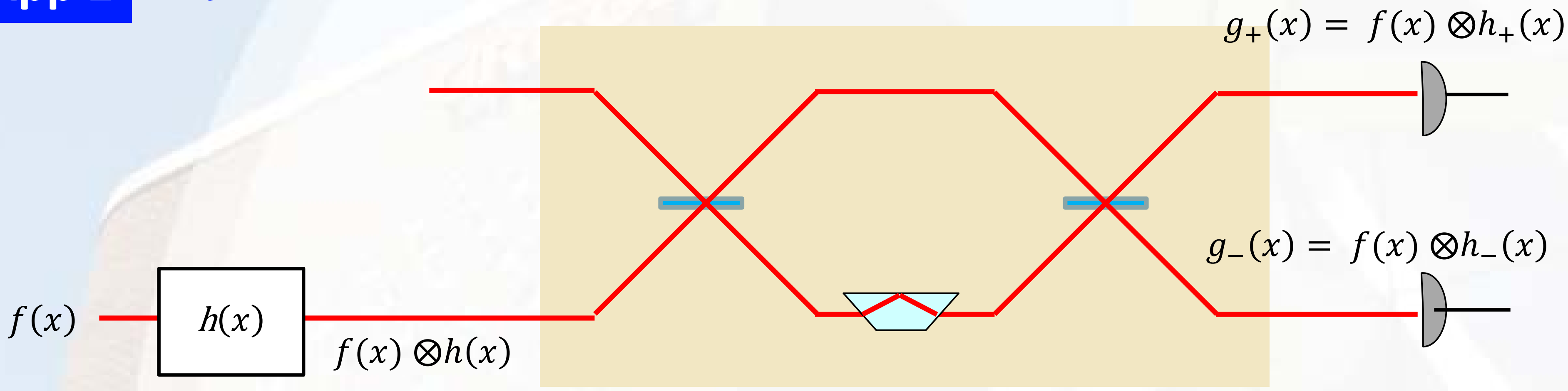
Classical & quantum spatial mode Mux/DeMux

Abouraddy, Yarnal, Saleh, Teich , PRA 75, 052114 (2007)

. . .

. . .

App 2 Projection on even and odd functions



$$g_+(0) = \int dx\, h_+(x) f(x)$$

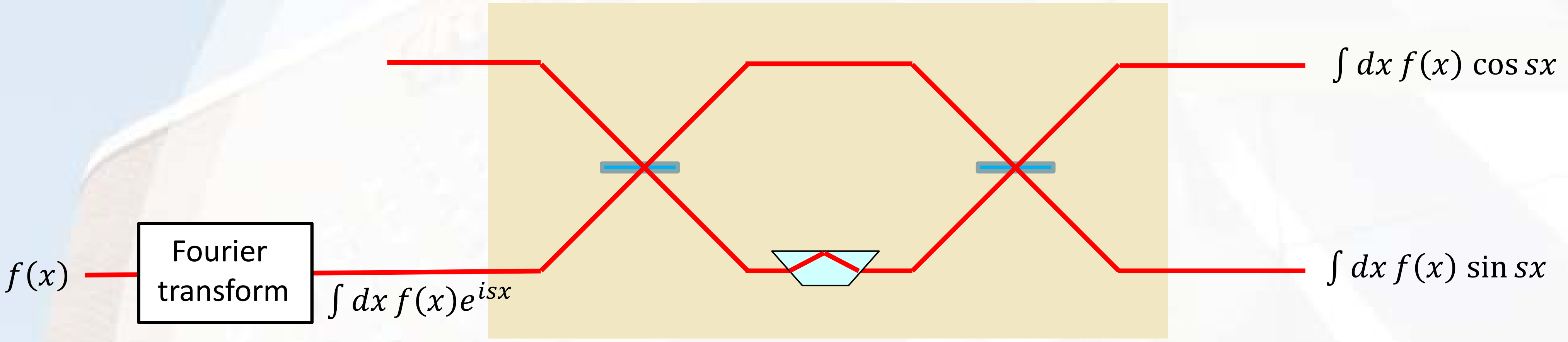
$$g_-(0) = \int dx\, h_-(x) f(x)$$

$$h_\pm(x) = h(x) \pm h(-x)$$

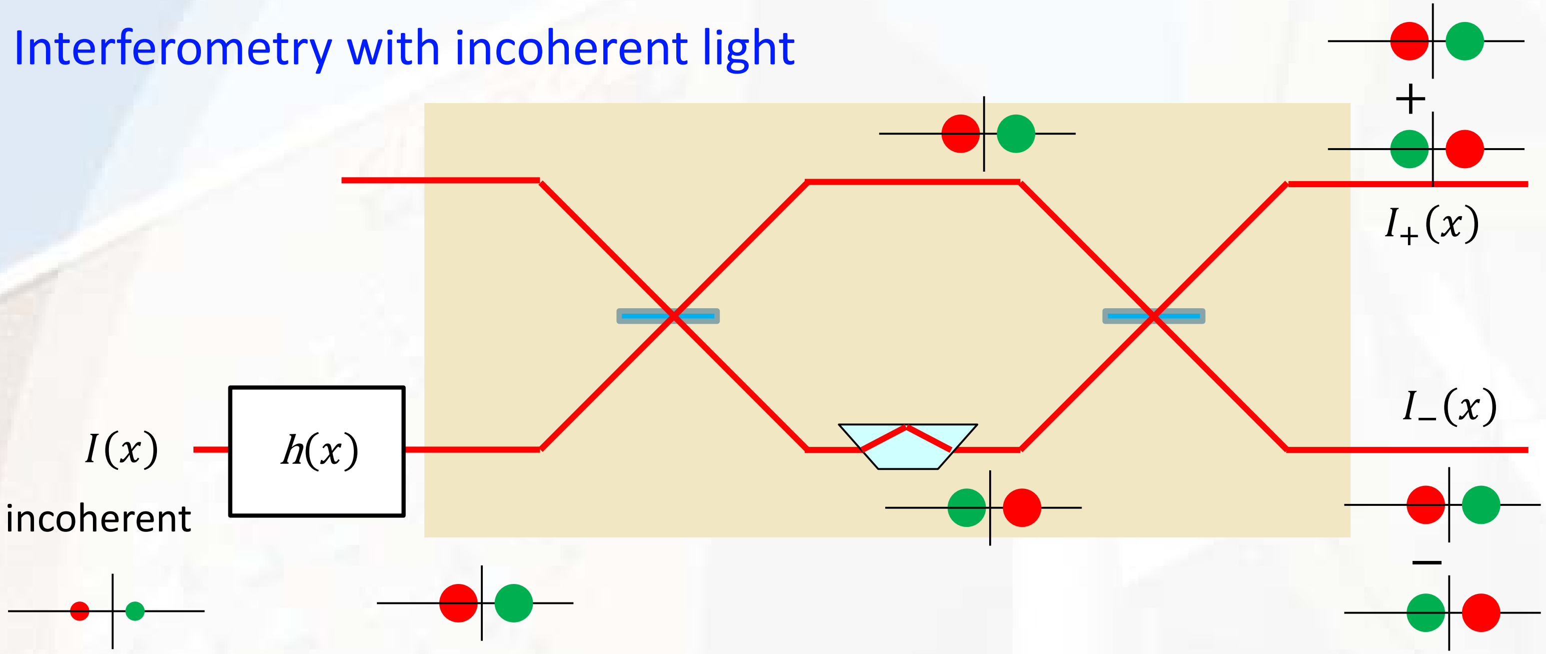
Super-resolution

Nair et al. Opt. Express ***24,*** 3684 (2016)
Tsang et al. PRX 6, 031033 (2016)

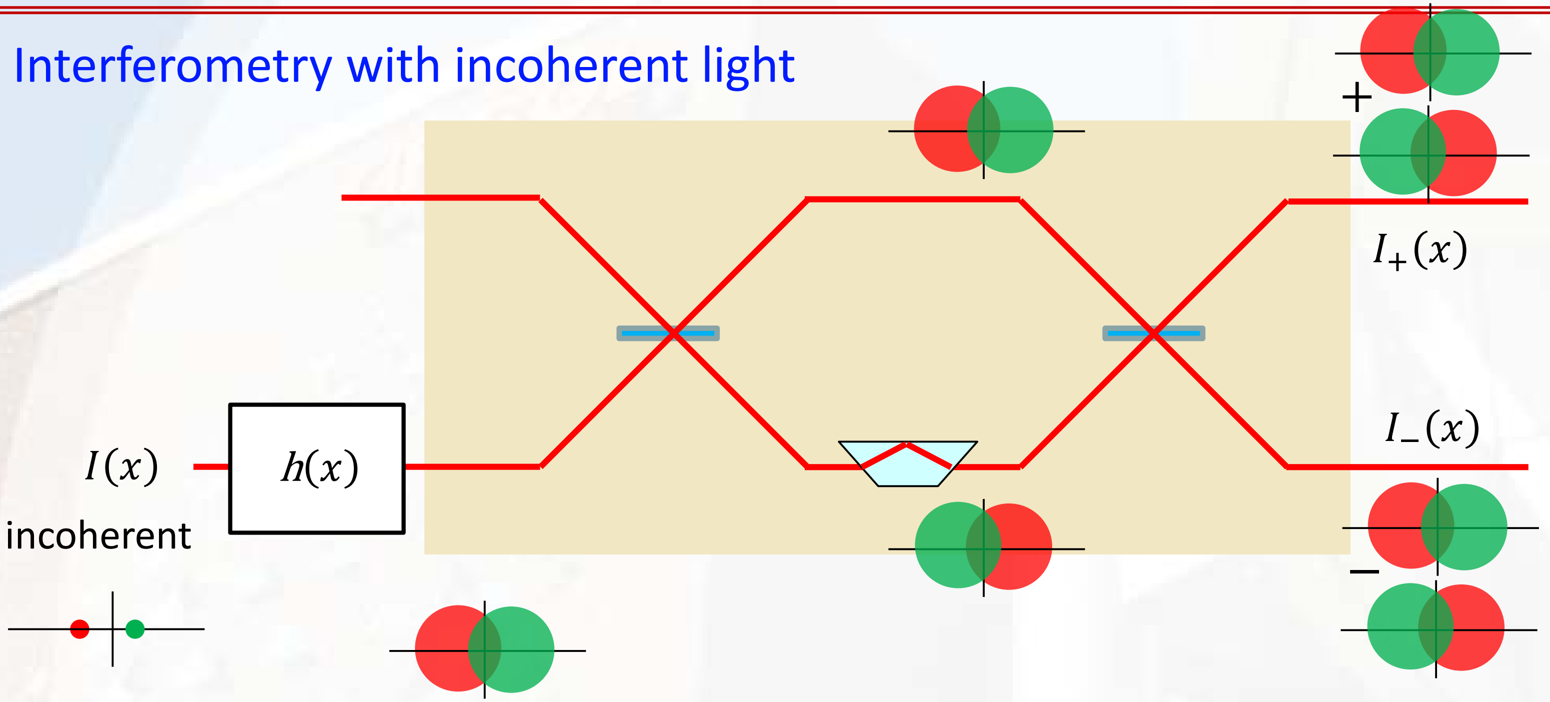
App 3 Cosine & sine Fourier transforms (concurrent)



App 4 Interferometry with incoherent light



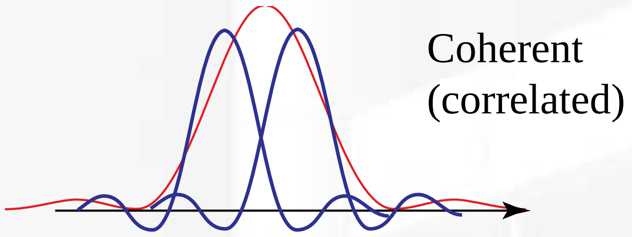
App 4 Interferometry with incoherent light

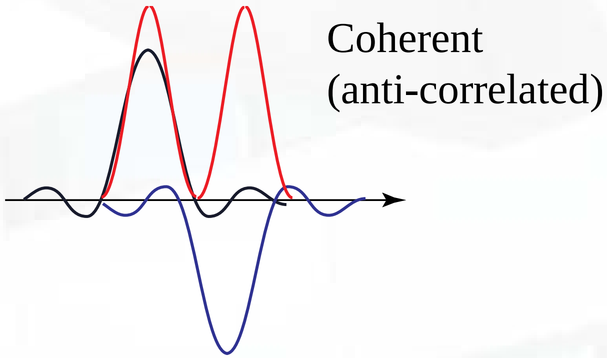


An impulse at $\delta(x-s)$ generates outputs $h(x-s)$ and $h(-x-s)$ and signals

$$h_\pm(x,s) = h(x-s) \pm h(-x-s)$$

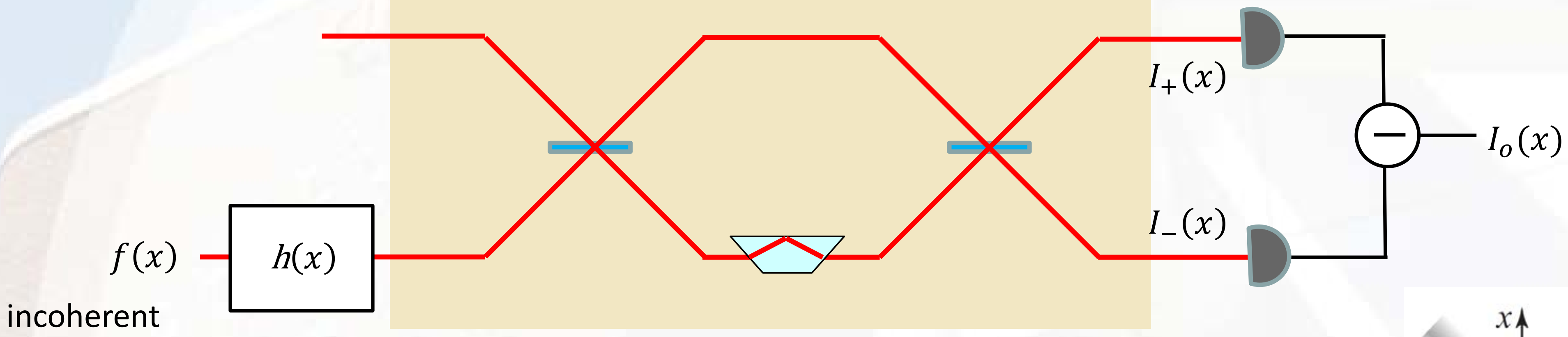
$$\mathrm{PSF}_\pm(x,s) = |h(x-s) \pm h(x+s)|^2$$

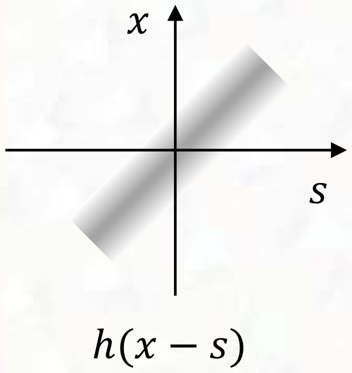




Yarnal, Abouraddy, Saleh, Teich. Opt Exp 16, 7634 (2008)

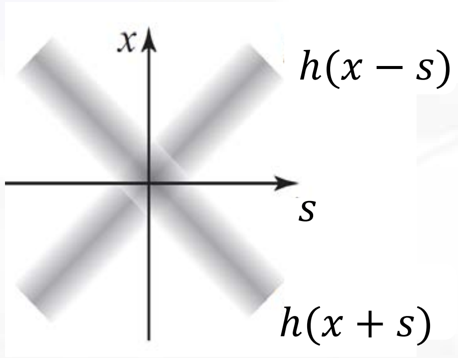
App 5 Laser scanning fluorescence microscopy

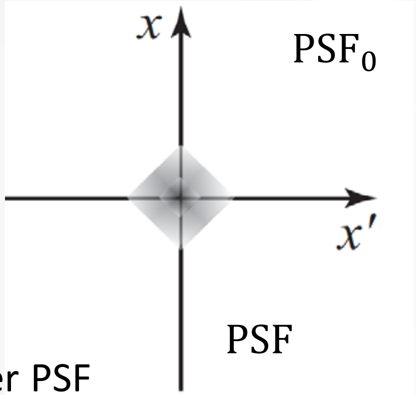




$$\mathrm{PSF}_{\pm}(x,s) = |h(x-s) \pm h(x+s)|^2$$

$$\mathrm{PSF}_0 = \mathrm{PSF}_+ - \mathrm{PSF}_- \propto \mathrm{Re}\{h^*(x-s)h(x+s)\}$$



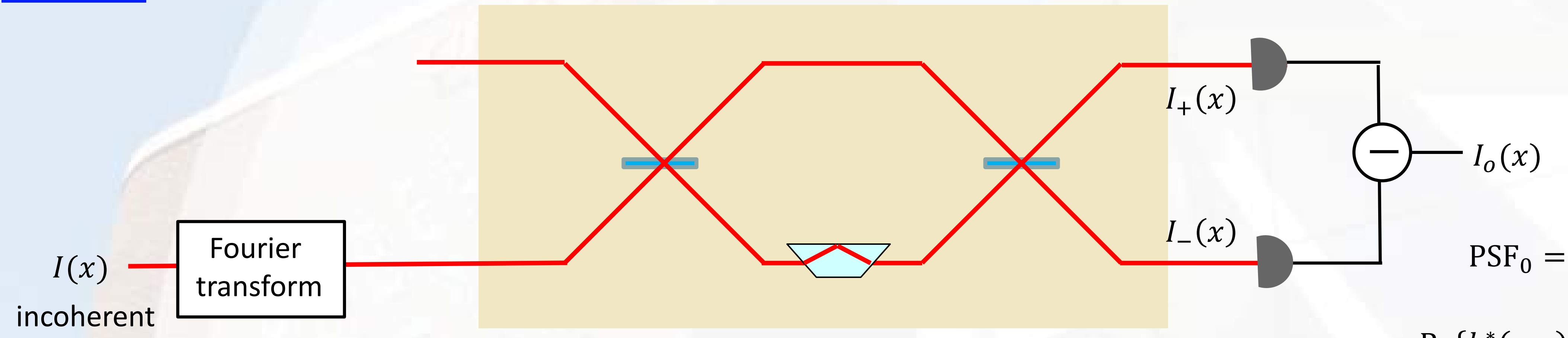


Narrower PSF
Better resolution

Murty, et al., Appl. Opt. 5, 615 (1966)

Weigel et al., Opt. Commun. 332, 301 (2014) & Opt. Commun 342, 102-108 (2015) & Opt Exp 23, 20505 (2015)

App 6 Cosine transform with incoherent light



$$\mathrm{PSF}_0 = \mathrm{PSF}_+ - \mathrm{PSF}_-$$

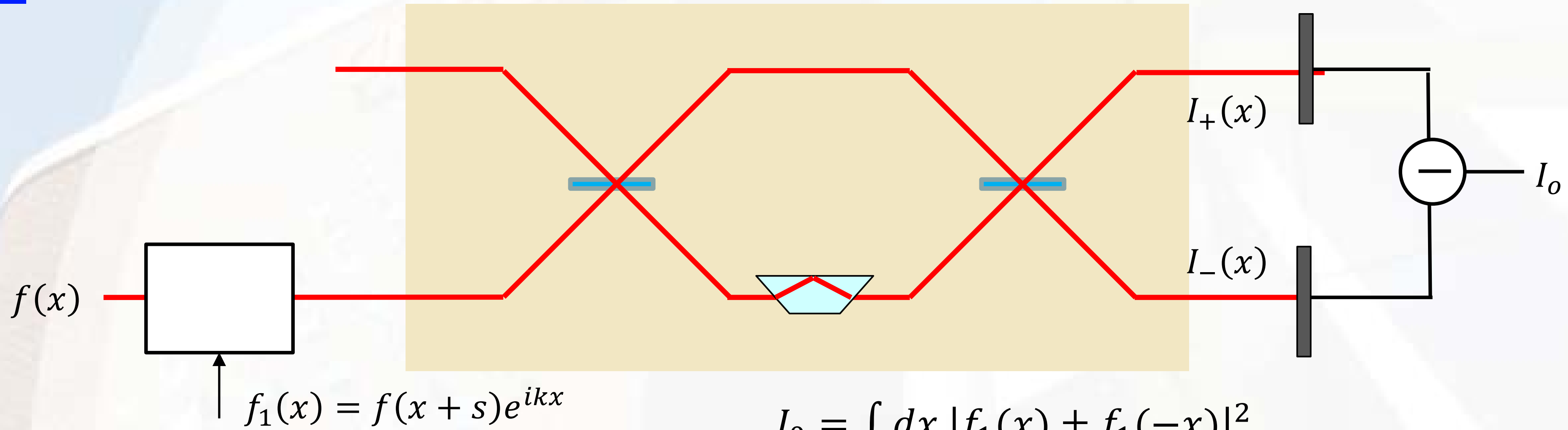
$$= \mathrm{Re}\{h^*(s,x)h(-s,x)\} \propto \cos 2sx$$

Impulse response function

$$h(s,x) = e^{isx}$$

Mendlovic, Zalevsky, Konforti, Dorsch, Lohmann, Appl. Opt. 34, 7615 (1995)

App 7 Wigner Distribution Function



$f_1(x) = f(x+s)e^{ikx}$

Displacement $-s$
Rotation k

$$I_0 = \int dx\, |f_1(x) \pm f_1(-x)|^2$$

$$\mathrm{PSF}_0 = \mathrm{PSF}_+ - \mathrm{PSF}_- \propto \mathrm{Re}\{\int dx\, f_1(x) f_1^*(-x)\}$$

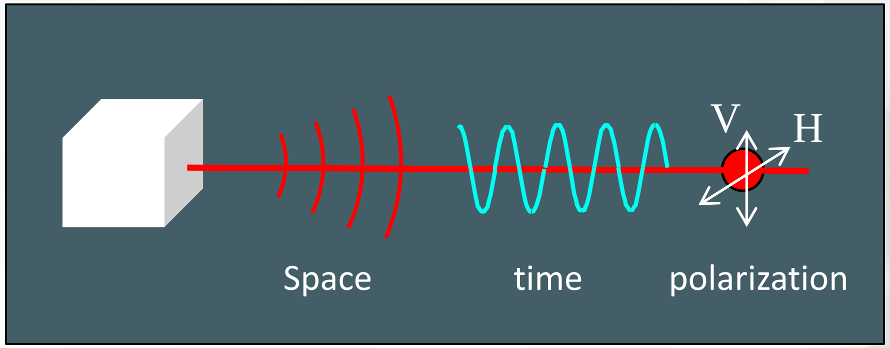
$$\downarrow$$

$$\int dx\, f(x+s)e^{ikx}\left[f(-x+s)e^{-ikx}\right]^*$$

$$= \int dx e^{i2kx} f^*(s-x) f(s+x) = W(s,k)$$

Mukamel, Banaszek, Walmsley, Dorrer, Opt Lett. 28, 1317 (2003)

Single-Photon Optics



A photon is described by a wave function $\psi(\mathbf{r}, t)$ obeying the same classical laws of propagation as the optical field $E(\mathbf{r}, t)$ (wave equation)

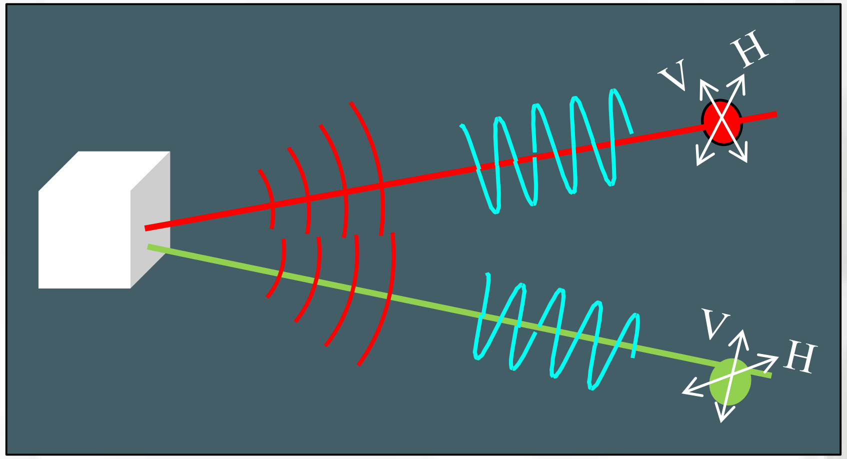
$|\psi(\mathbf{r}, t)|^2$ = probability density of detecting the photon at position $\mathbf{r}$ and time t

Same applies to polarization: A single photon is described by a Jones vector $\begin{bmatrix}\alpha_H \\ \alpha_V\end{bmatrix}$ or the state $\alpha_H|H\rangle + \alpha_V|V\rangle$

In sensing/imaging application, a single photon (repeated) sensing/imaging offers:

- Better accuracy (no shot noise!)
- Security (QKD)
- But imperfect quantum efficiency of detectors introduces deletion noise

Two-Photon Optics



Two-photon light is described by a wave function of two positions and two times $\psi(\mathbf{r}_1, t_1; \mathbf{r}_1, t_2)$

$|\psi(\mathbf{r}_1, t_1; \mathbf{r}_1, t_2)|^2$ = probability density that the photons are detected at times t_1 and t_2

ψ obeys the same classical laws of propagation of the correlation function $G^{(2,0)}(\mathbf{r}_1, t_1; \mathbf{r}_1, t_2) = \langle E(\mathbf{r}_1, t_1) E(\mathbf{r}_1, t_2) \rangle$ of coherence theory (Wolf equation)

For polarization $\alpha_{HH}|HH\rangle + \alpha_{HV}|HV\rangle + \alpha_{VH}|VH\rangle + \alpha_{VV}|VV\rangle$

$|\alpha_{ij}|^2$ = probability density that the photons are detected with polarization i and j

Entanglement = non-separability of the two-photon wavefunction $\psi(\mathbf{r}_1, t_1; \mathbf{r}_1, t_2)$

↕

Coherence = separability of the coherence function $G^{(2,0)}(\mathbf{r}_1, t_1; \mathbf{r}_1, t_2)$

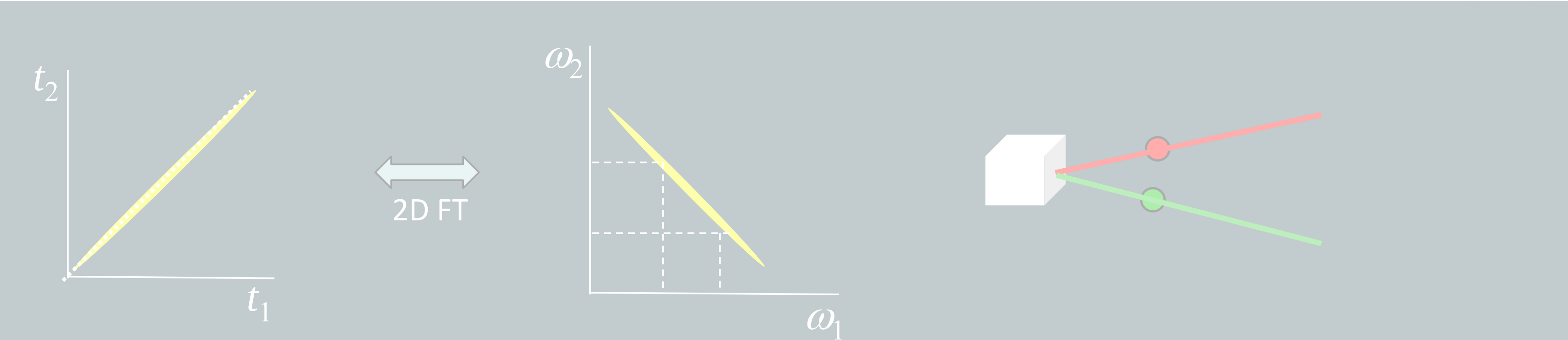
Saleh, Teich, Sergienko, PRL 94, 223601 (2005)

Entanglement: a resource for quantum sensing & imaging

Temporal/spectral entanglement

Arrival time of each photon is totally uncertain, but the photons always arrive together.

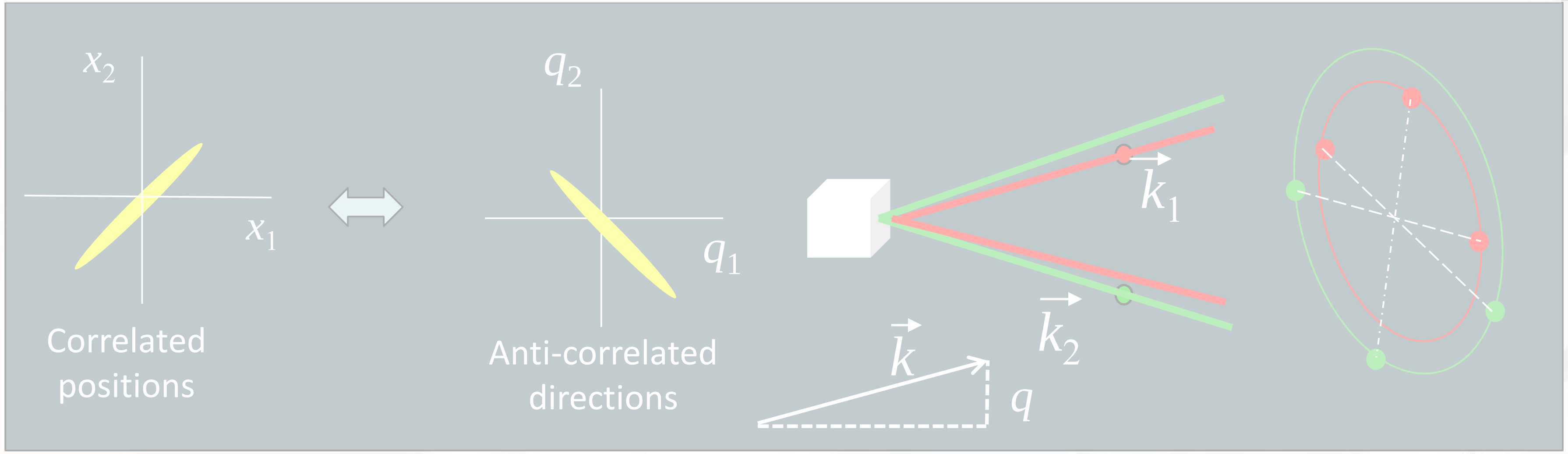
Each of photon is broadband, but the sum $\omega_1 + \omega_2$ is monochromatic



Spatial/angular entanglement

Photons are emitted from the same random position

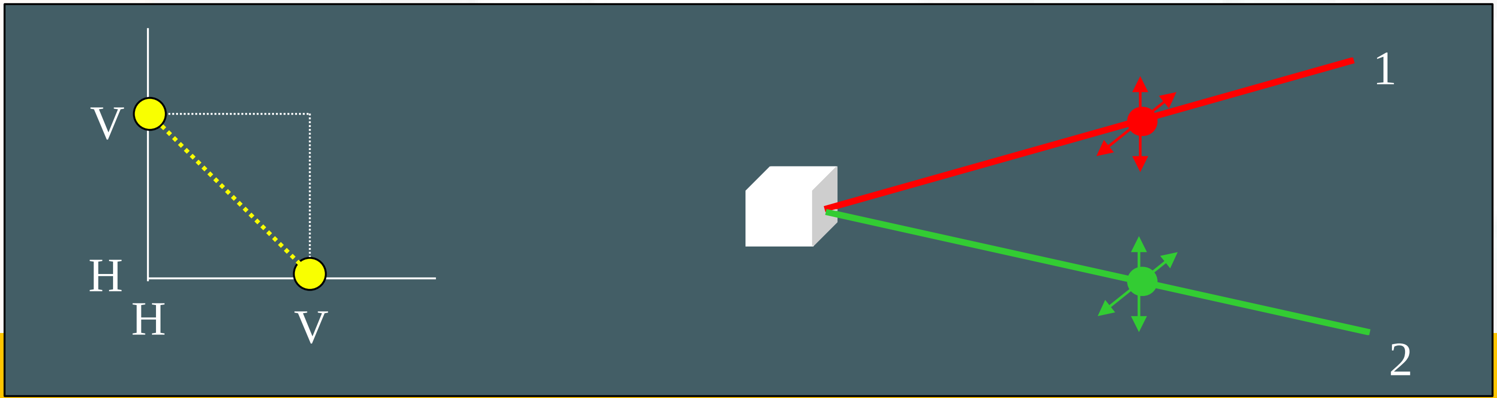
Direction of each photon is random, but perfectly correlated with direction of the other photon



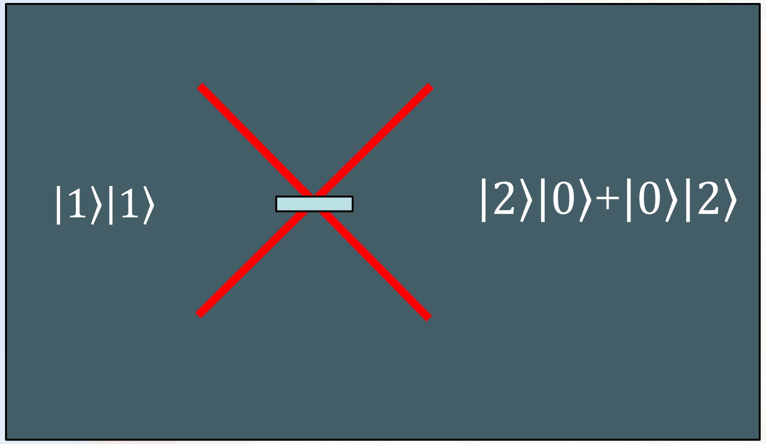
Polarization entanglement

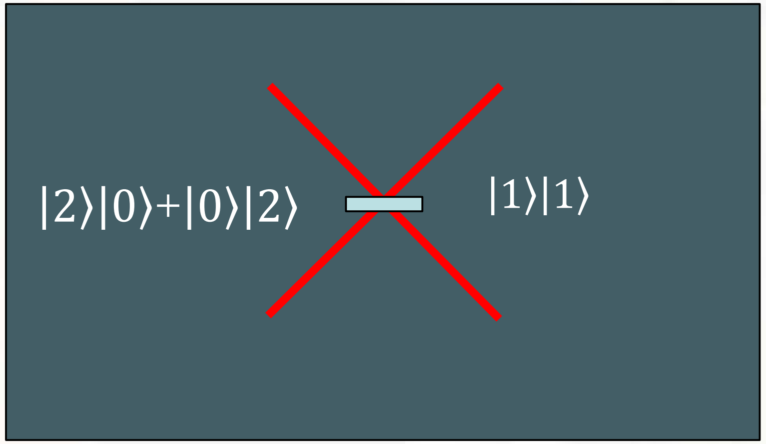
Each photon is unpolarized,

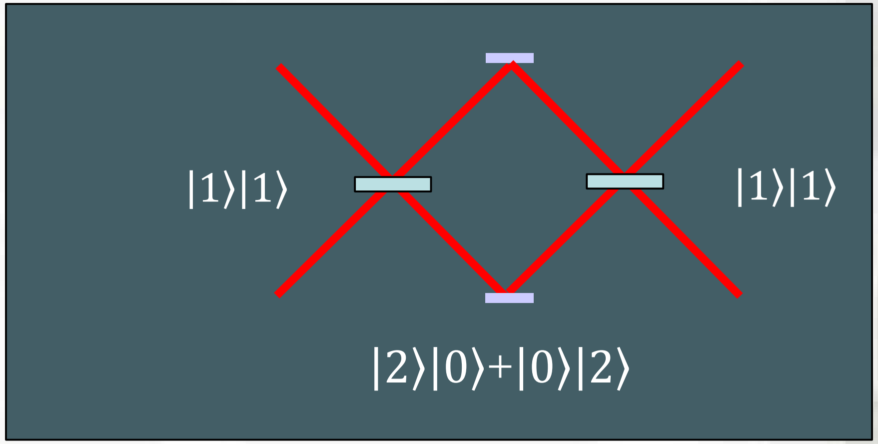
but the polarizations are always orthogonal

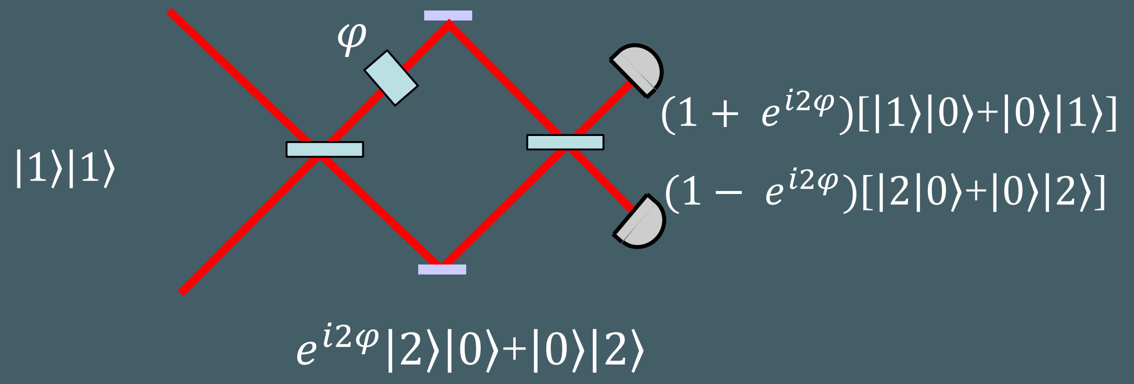


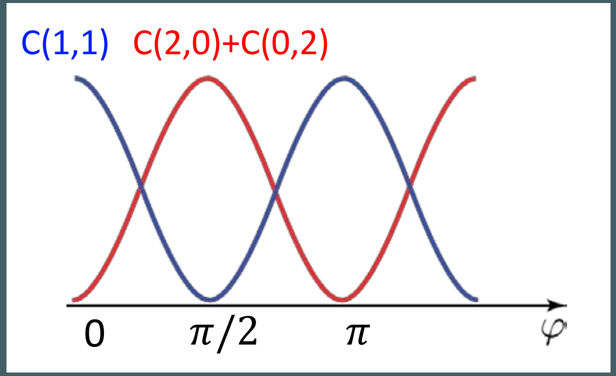
Two-photon optics of the beam splitter





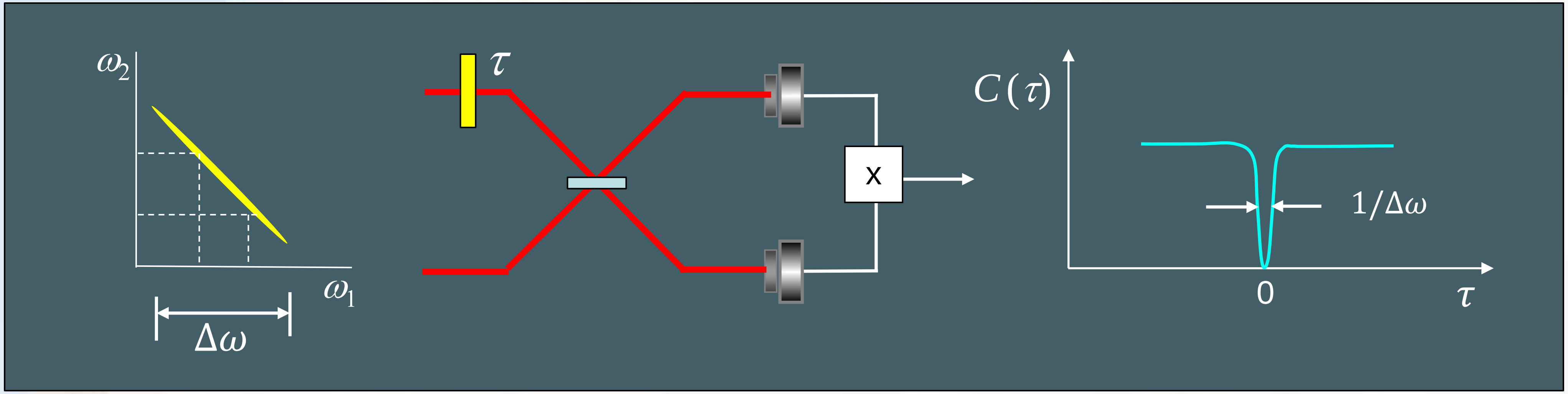






Factor of 2 more sensitive measurement of phase φ than a classical MZI.

Hong-Ou-Mandel Interferometer



Measurement of time delay with fs precision Hong, Ou, Mandel 87

A few principles from classical & quantum estimation theory

Classical

If a parameter ϕ is to be estimated based on measured outcome m,

and $p(m|\phi) =$ conditional probability of measuring m given ϕ, then

the variance of σ_ϕ^2 of any unbiased estimate $\hat{\phi}$ of ϕ satisfies:

$$\sigma_\phi^2 \geq \frac{1}{F(\phi)}$$

Cramér-Rao (CR) bound

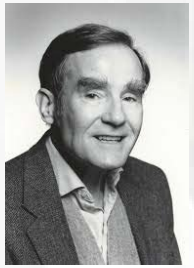
$$F(\phi) = \sum_m \frac{1}{p(m|\phi)} \left[\frac{\partial}{\partial\phi} p(m|\phi)\right]^2$$

Fisher information

Quantum

For a quantum state dependent on an unknown parameter ϕ,

$$\sigma_\phi^2 \geq \frac{1}{F_Q(\phi)}$$ Quantum Cramer-Rao (QCR) bound



Carl W. Helstrom

$F_Q(\phi) =$ Quantum Fisher Information (QFI)

The QCR bound is the absolute limit on precision of estimates of ϕ for any quantum measurement

The QFI

For a pure state $|\psi\rangle$

$$F_Q(\phi) = 4\langle\psi'|\psi'\rangle - 4|\langle\psi|\psi'\rangle|^2$$

$$|\psi'\rangle = \frac{d}{d\phi}|\psi\rangle$$

For a mixed state ρ

$$F_Q(\phi) = \mathrm{Tr}\,(\rho\mathcal{L}^2)$$

$\mathcal{L}$ = symmetric logarithmic derivative (SLD) satisfies:

$$d\rho/d\phi = \tfrac{1}{2}(\mathcal{L}\rho + \rho\mathcal{L})$$

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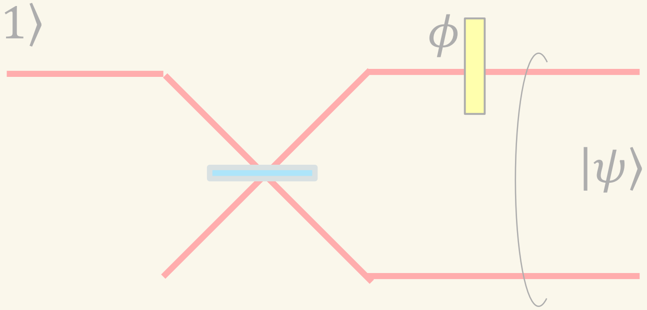
Single photon (or coherent state with an average of one photon)

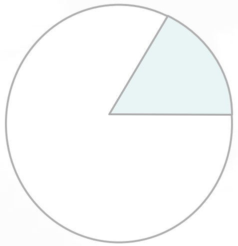
$|\psi\rangle = |1\rangle$



$|\psi\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi}|1{,}0\rangle + |0{,}1\rangle\right)$

$F_Q^{(1p)}(\phi) = 1$

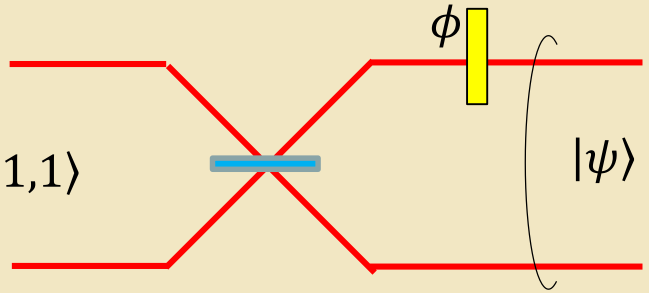
$\sigma_\phi = 1$



Classical Limit (CL)
Or Standard Quantum Limit (SQL)

Two photons

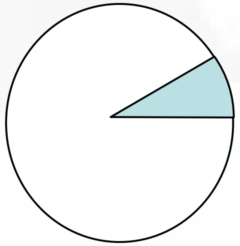
$|\psi\rangle = |1{,}1\rangle$



$|\psi\rangle = \frac{1}{\sqrt{2}}\left(e^{i2\phi}|2{,}0\rangle + |0{,}2\rangle\right)$

$F_Q^{(2p)}(\phi) = 4$

$\sigma_\phi = \frac{1}{2}$



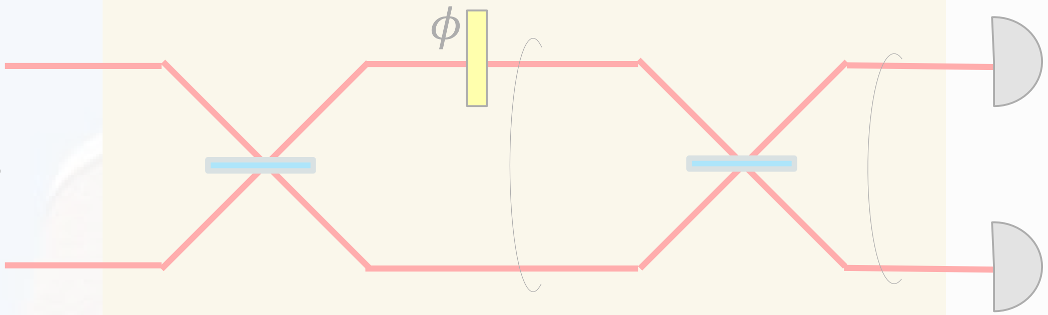
Heisenberg Limit (HL)

Factor of 2 more sensitive

Phase estimation by use of an MZ interferometer

Single photon

$|1,0\rangle$



$p_{\mathrm{up}} = \cos^2\phi$

$p_{\mathrm{down}} = \sin^2\phi$

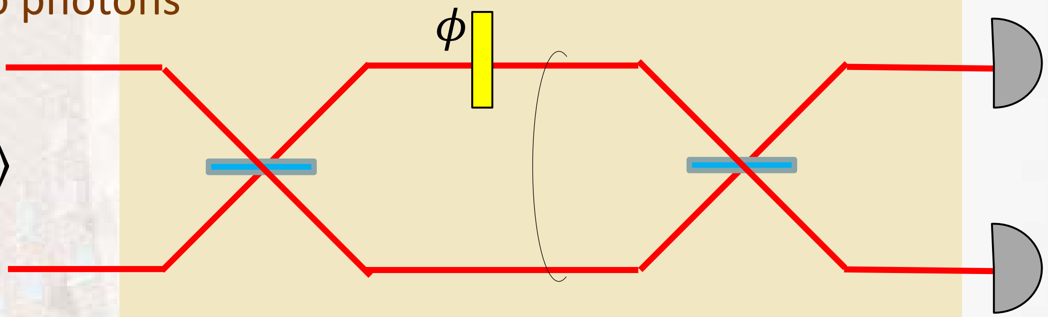
$\frac{1}{\sqrt{2}}\left(e^{i\phi}|1,0\rangle + |0,1\rangle\right)$ $\quad\frac{1}{\sqrt{2}}\left(\cos\phi\ |1,0\rangle + \sin\phi\ |0,1\rangle\right)$

$$F^{(1p)}(\phi) = 1 = F_Q^{(1p)}(\phi)$$

$$\sigma_\phi = 1$$

Two photons

$|1,1\rangle$



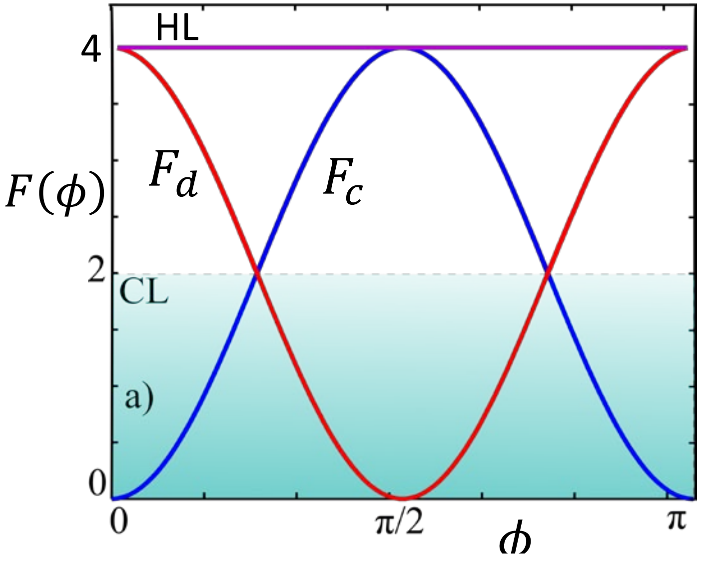
$$p_d = p(2,0) + p(0,2) = \sin^2 2\phi$$

$$p_c = p(1,1) = \cos^2 2\phi$$

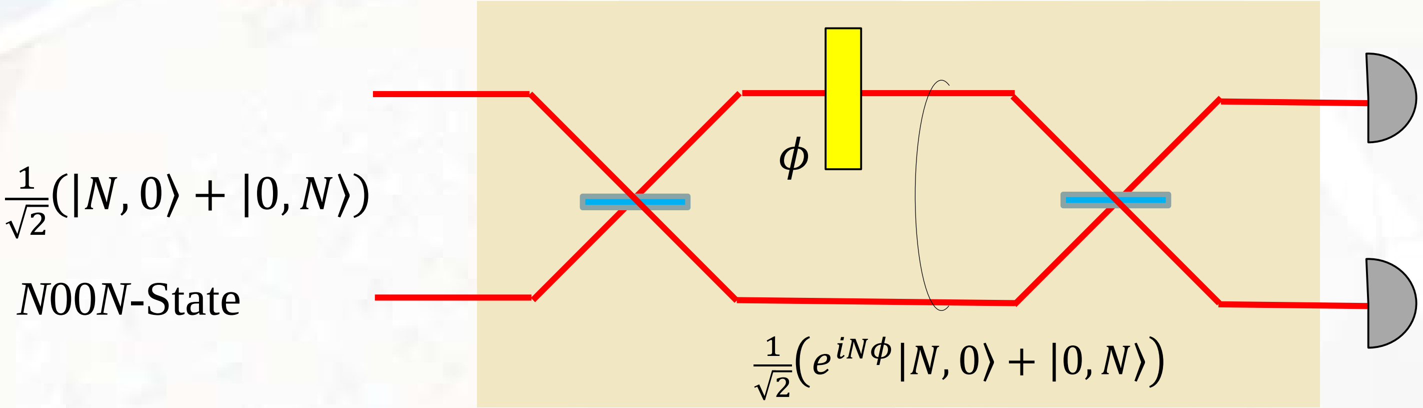
$\frac{1}{\sqrt{2}}\left(e^{i2\phi}|2,0\rangle + |0,2\rangle\right)$

$$F^{(2p)}(\phi) = 4 = F_Q^{(2p)}(\phi)$$

$$\sigma_\phi = \frac{1}{2}$$



N photons in $N00N$ State

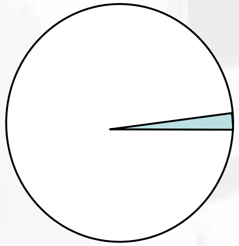


Measure $p(N,0)$ and $p(0,N)$

with N-photon absorber

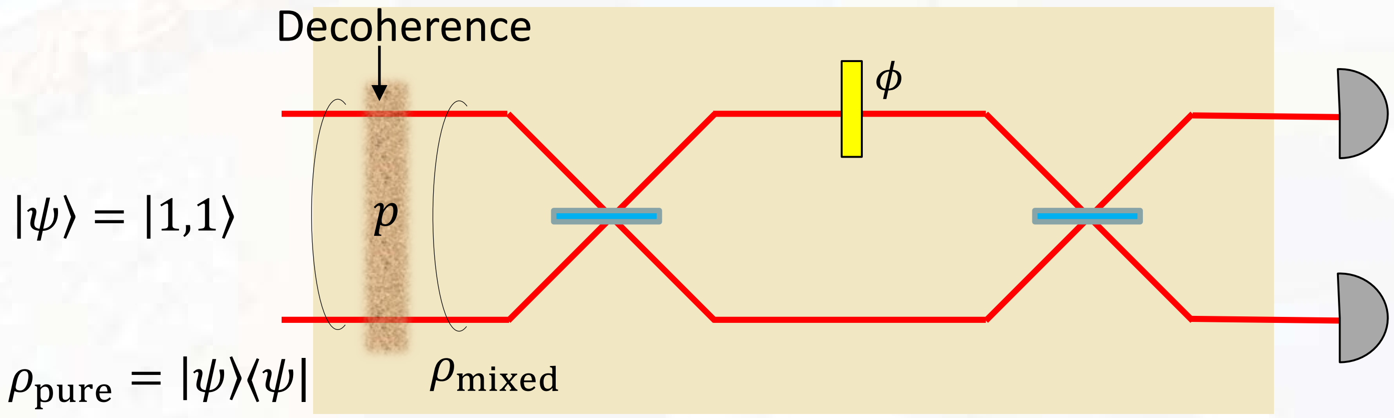
$$F^{(Np)}(\phi) = F_Q^{(Np)}(\phi) = N^2$$

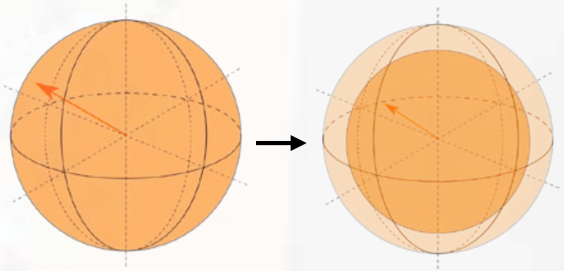
$$\sigma_\varphi = \frac{1}{N}$$



Factor of N more sensitive to phase φ than a classical MZI

Super-sensitivity diminishes in the presence of decoherence



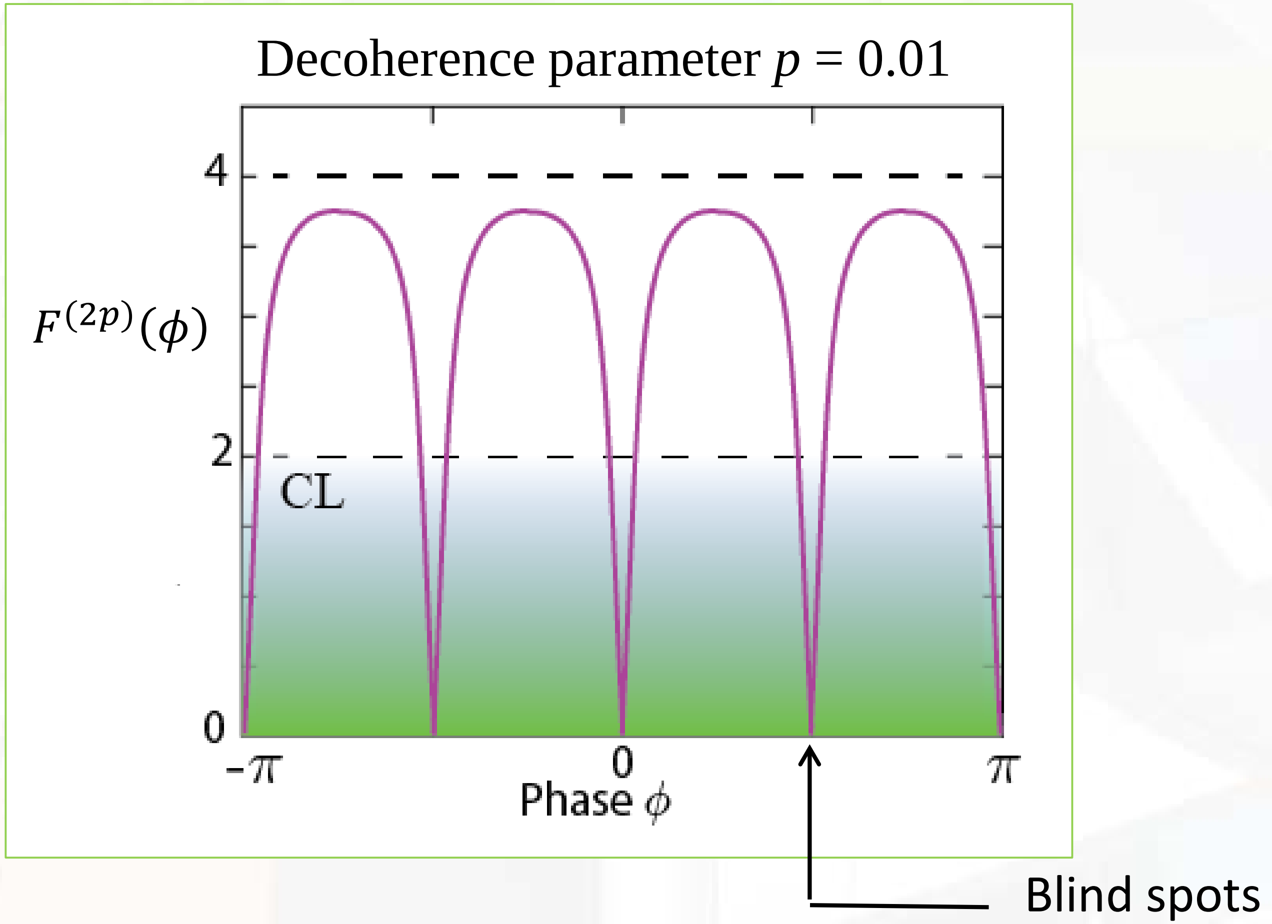


The "Depolarizing" channel model of decoherence

$$\rho_{\text{mixed}} = \sum_{i=1}^{4} E_i\, \rho_{\text{pure}} E_i^{\dagger}$$

$$E_1 = \sqrt{1 - \tfrac{3}{4} p}\ \mathrm{I} \qquad E_2 = \tfrac{1}{2}\sqrt{p}\ \sigma_x \qquad E_3 = \tfrac{1}{2}\sqrt{p}\ \sigma_y \qquad E_4 = \tfrac{1}{2}\sqrt{p}\ \sigma_z$$

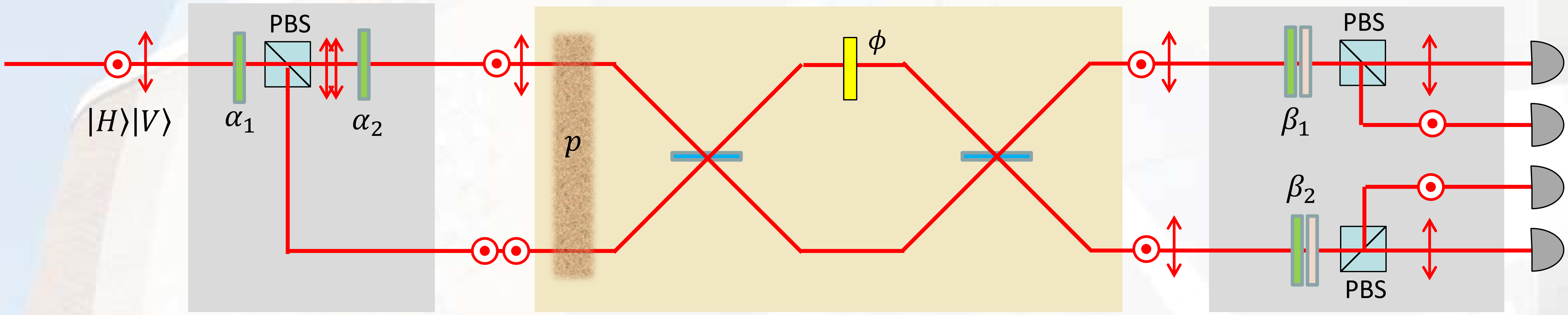
I = identity matrix, σ_i = Pauli matrices p = measure of level of decoherence



Even very weak decoherence destroys sensitivity at certain phases (blind spots)

We can use a reference phase that shifts the measurement to a more sensitive phase difference, but this requires prior knowledge of the phase and an adaptive estimation procedure.

Restoration of 2-photon super-sensitivity by use of a polarization ancilla (Hilbert space of dimension 4 instead of 2)



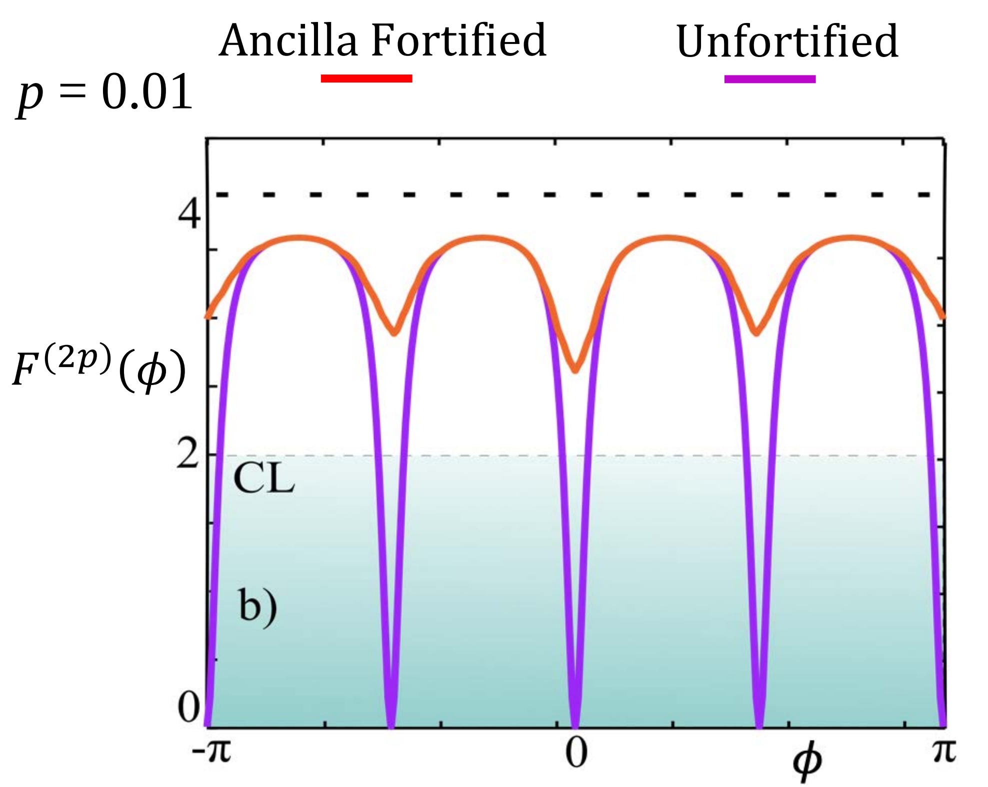
Two photons *entangled* in path & polarization controlled by waveplates α_1, α_2

Waveplates at angles β_1, β_2 + polarization analyzers

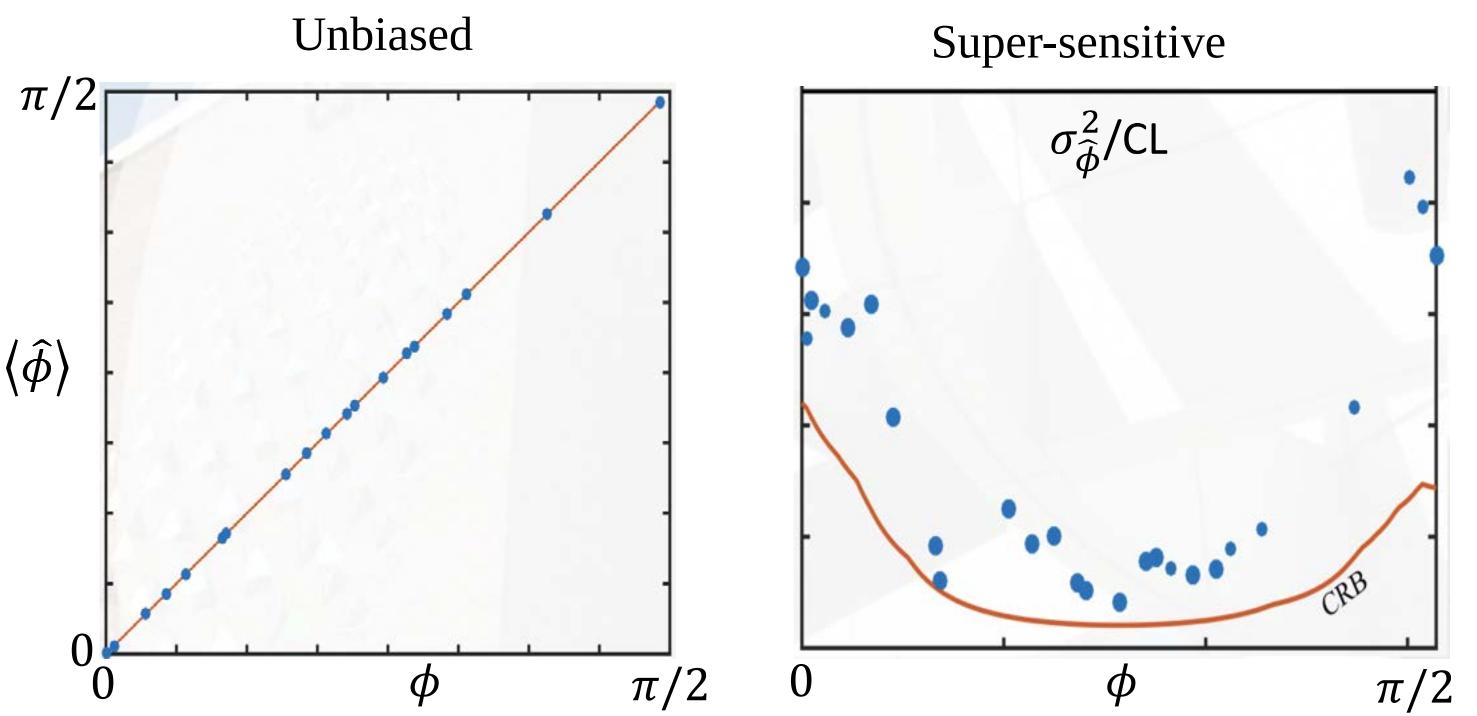
Estimation is based on measurement of:
– coincident photons at pairs of detectors, and
– double photons at each detector

Precision at each ϕ is maximized over waveplate parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$

Result



Monte Carlo simulations without using a reference phase



Ancillary DoF helps. Photon number is not the only commodity in quantum-optical parameter estimation!

Larson, Saleh, PRA 96, 042110 (2017)

Outline

1 Brief overview of quantum-enhanced sensing & imaging

2 Quantum limits on precision of phase measurement: combatting decoherence with ancilla

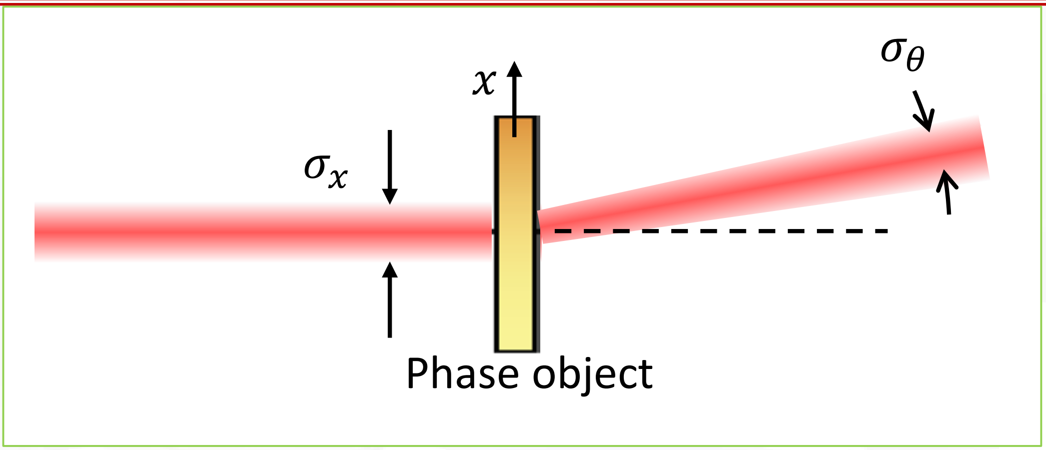
3 Quantum limits on precision of phase-gradient measurement: Role of beam size

4 Quantum limits on two-point resolution: coherence resurrects Rayleigh's curse

Precision bounds for measurement of phase gradient

Quantum Fisher information for estimating the phase Gradient (wavefront tilt) θ

$$\varphi(x) = \varphi_0 + \theta x$$



Single-photon beam

$$|\psi\rangle = \int dx\, e^{-i\theta x}\psi_0(x)\,|x\rangle$$

$$F_Q^{(1p)}(\theta) = 4\sigma_x^2 \qquad \sigma_\theta = \frac{1}{2\sigma_x} \qquad \sigma_x\sigma_\theta = \frac{1}{2}$$

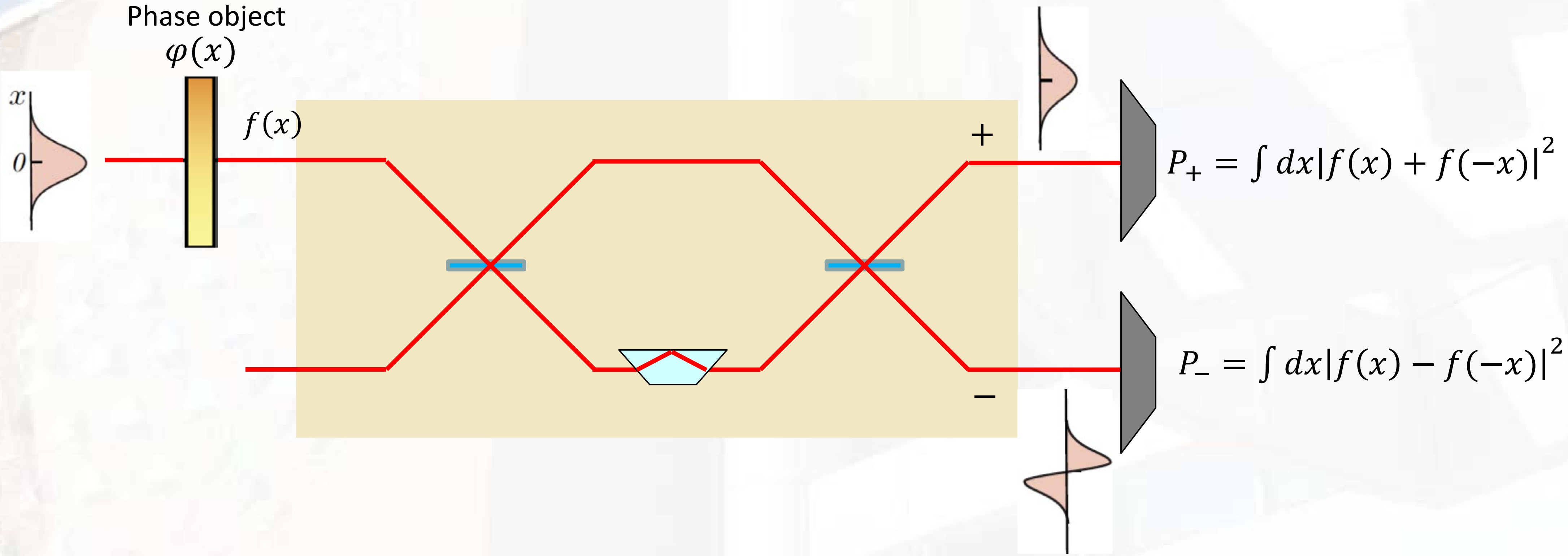
Two-photon beam (entangled)

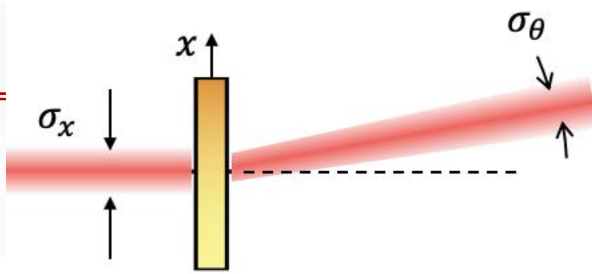
$$|\psi\rangle = \iint dx_1\, dx_2\, \underbrace{\psi_0(x_1, x_2)}_{f_0(x_1)\delta(x_1 - x_2)}\, e^{-i\theta(x_1+x_2)}\,|x_1, x_2\rangle$$

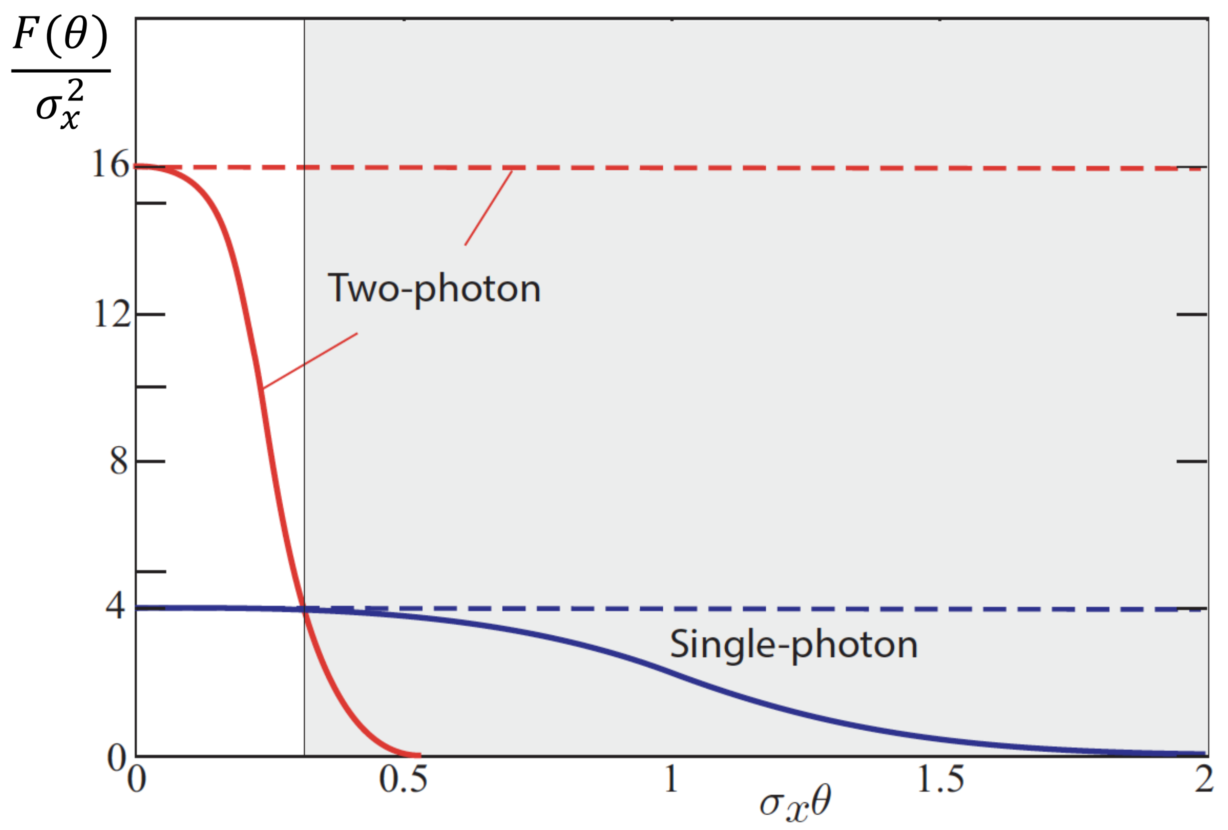
$$F_Q^{(2p)}(\theta) = 16\,\sigma_x^2 \qquad \sigma_\theta = \frac{1}{4\sigma_x} \qquad \sigma_x\sigma_\theta = \frac{1}{4}$$

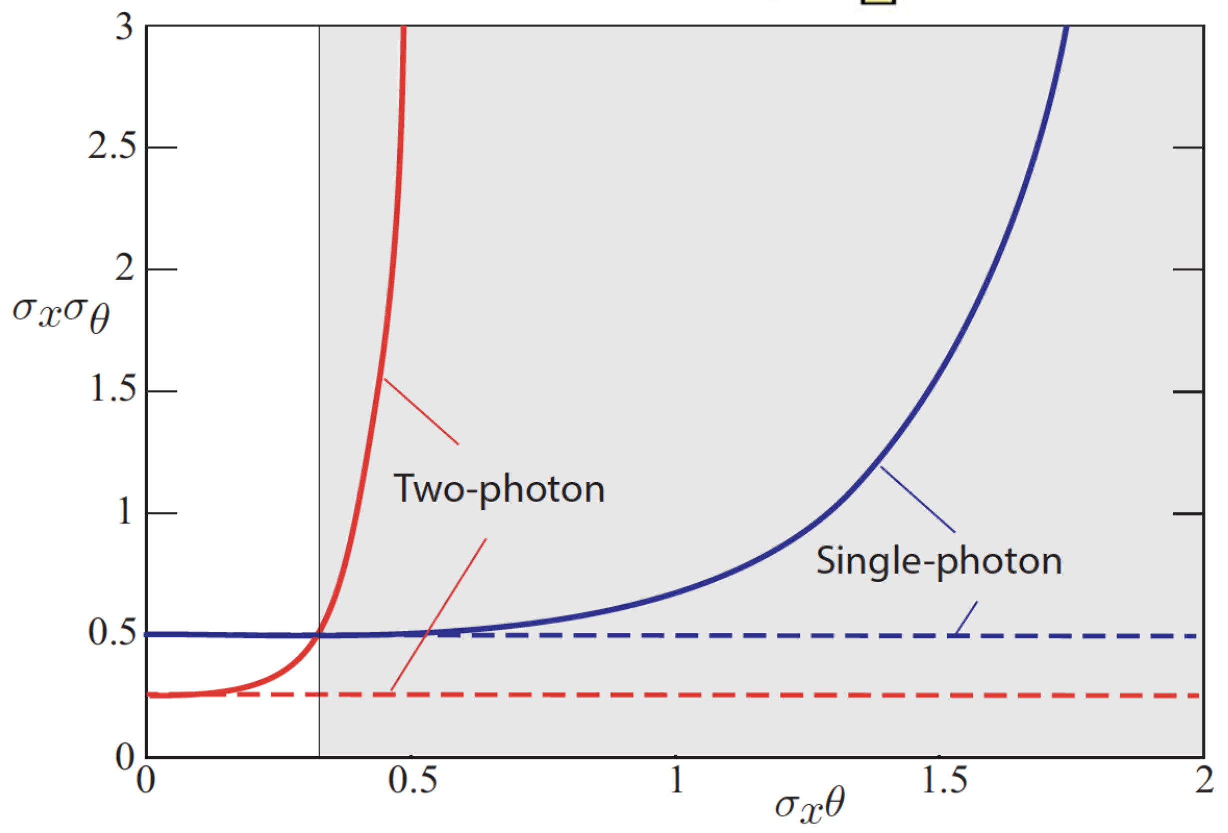
Factor of 2 advantage

Fisher information for measurement of θ using an image-inversion interferometer & binary measurement



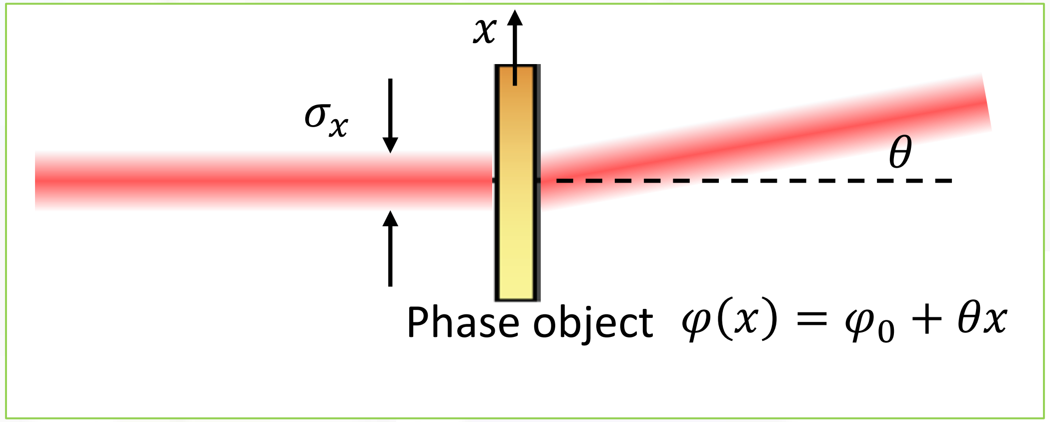






Factor of 2 advantage is lost for large beam width

Larson, Saleh ArXiv paper 1908.03145 (2019)



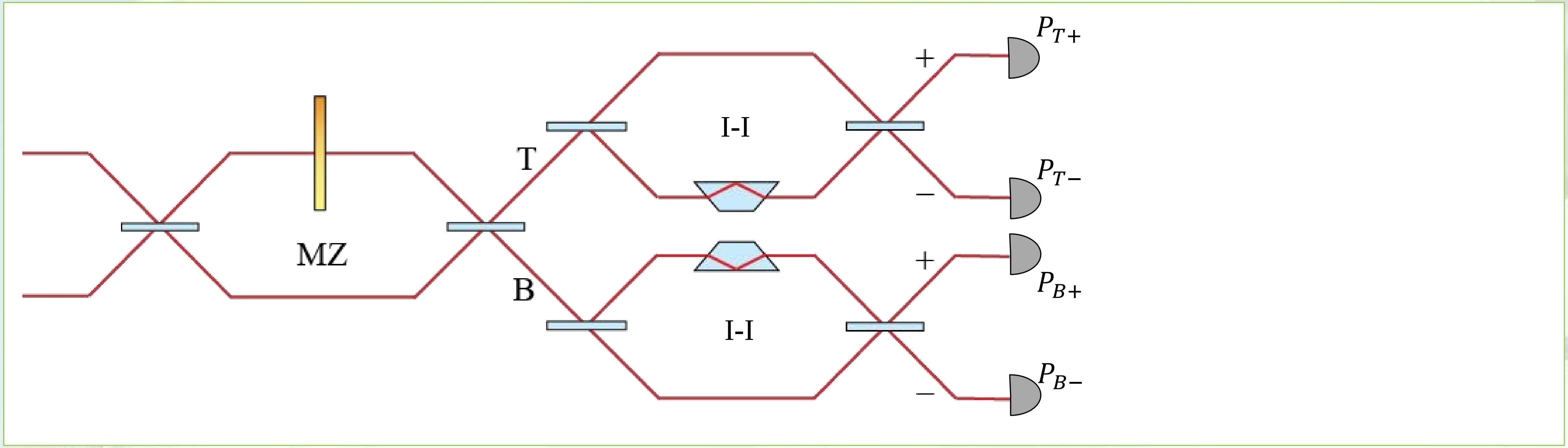
Quantum Fisher information

Single photon $F_Q^{(1p)}(\varphi_0) = 1$ $F_Q^{(1p)}(\theta) = \sigma_x^2$

Two photons $F_Q^{(2p)}(\varphi_0) = 4$ $F_Q^{(2p)}(\theta) = 16\sigma_x^2$

Concurrent estimation of phase φ_0 and phase gradient θ

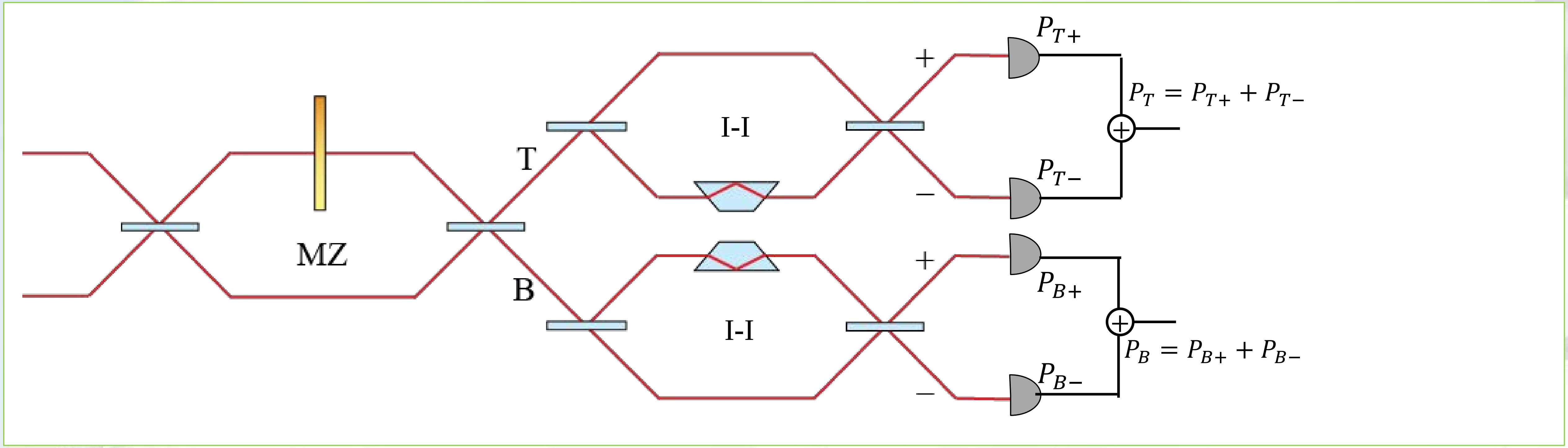
Single photon



Single photon

Estimation of φ_0

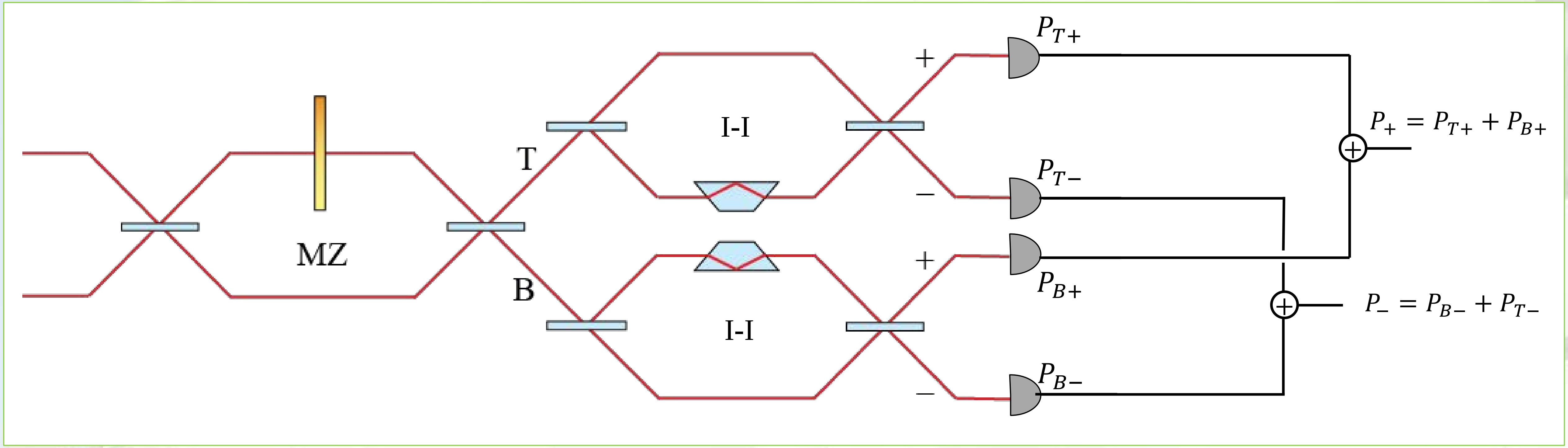
$$F^{(1p)}(\varphi_0) = 1$$



Single photon

Estimation of θ

$$F^{(1p)}(\theta) = \sigma_x^2$$



Concurrent estimation of phase φ_0 and phase gradient θ

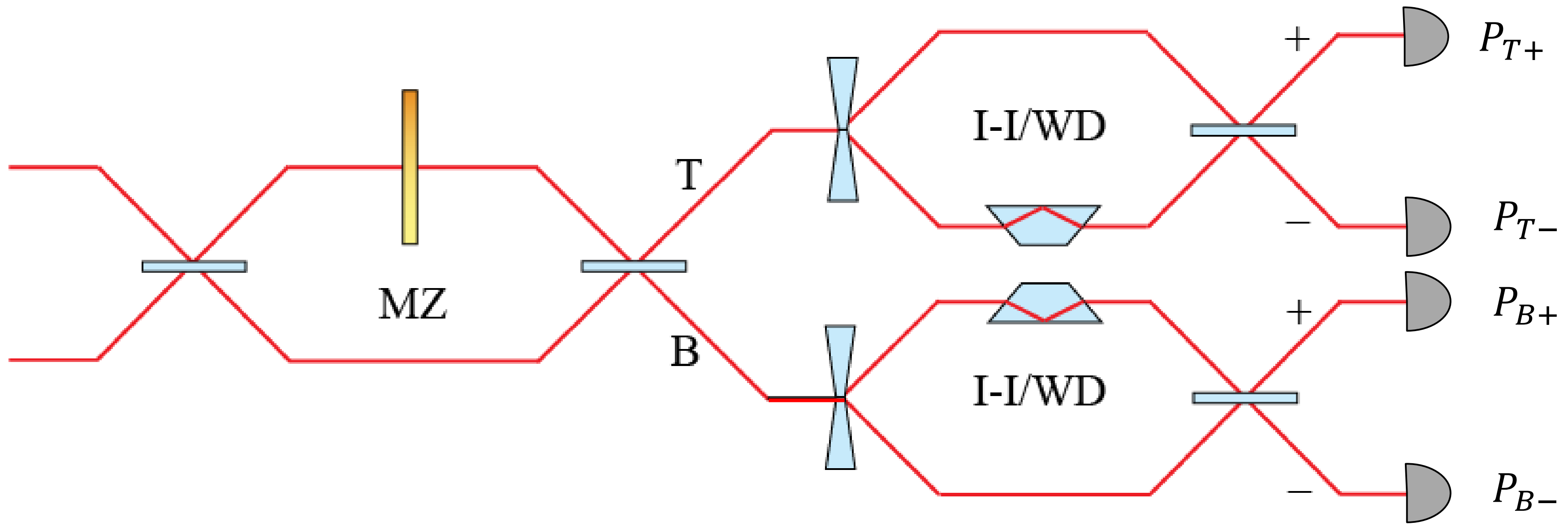
Two photons

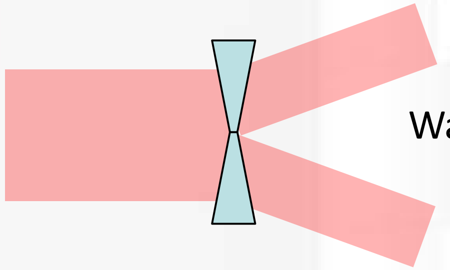
Estimation of φ_0

$$F^{(2p)}(\varphi_0) = 4$$

Estimation of θ

$$F^{(2p)}(\theta) = 16\sigma_x^2$$





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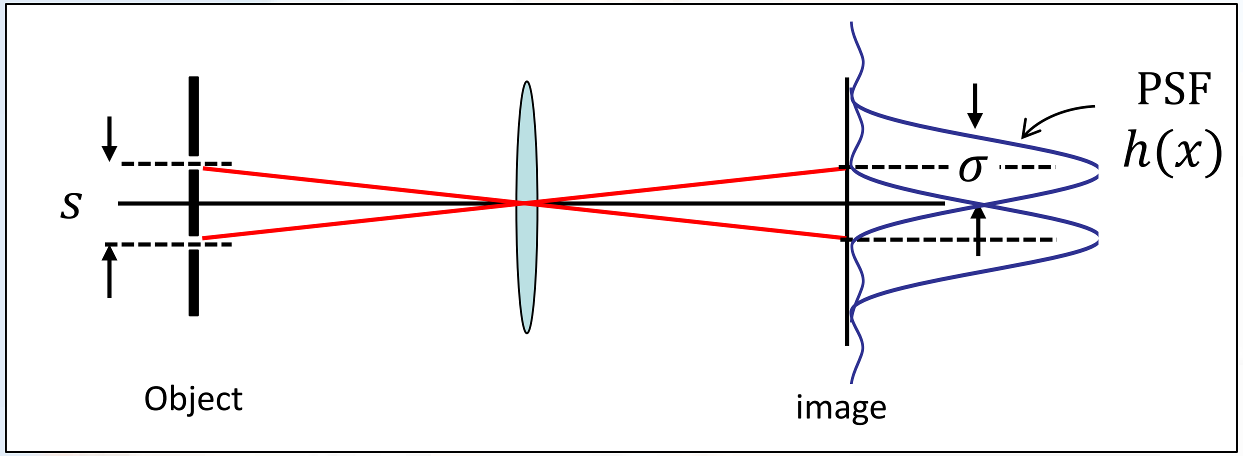
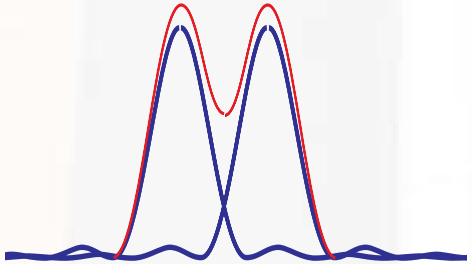
Best precision of estimates of s based on measurement of the intensity or the photon probability distribution



Assuming classical light with an average of one photon.

$$I(x) = \tfrac{1}{2}(|h_+(x)|^2 + |h_-(x)|^2)$$

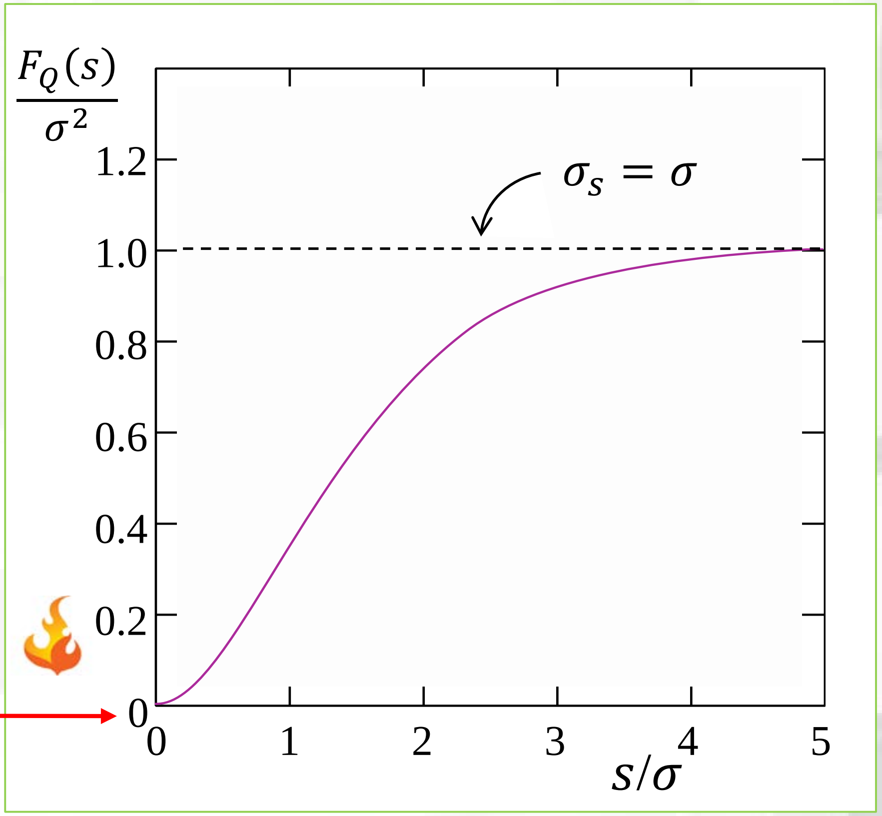
$$h_\pm(x) = h\left(x \pm \tfrac{s}{2}\right)$$



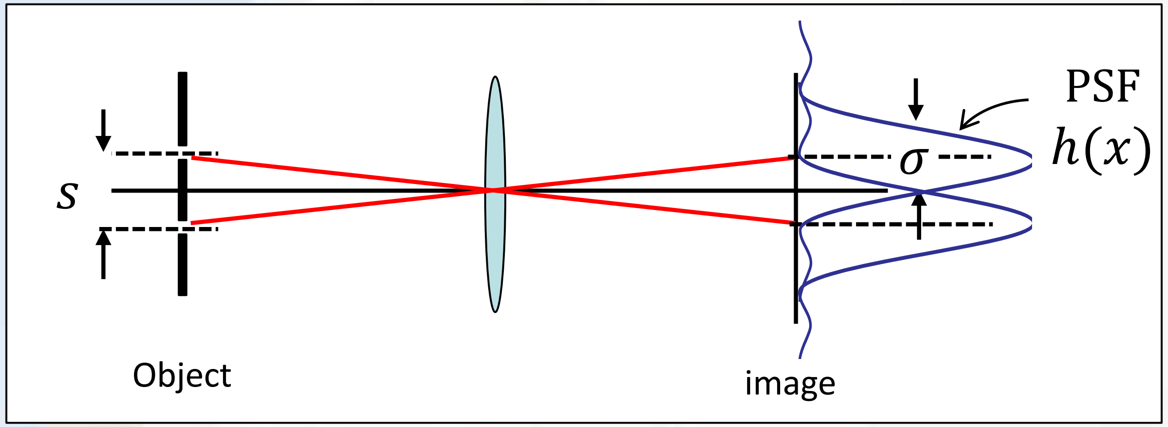
Or one-photon quantum state

$$\rho(s) = \tfrac{1}{2}\int dx[|h_+(x)|^2 + |h_-(x)|^2]|x\rangle\langle x|$$

Rayleigh's Curse

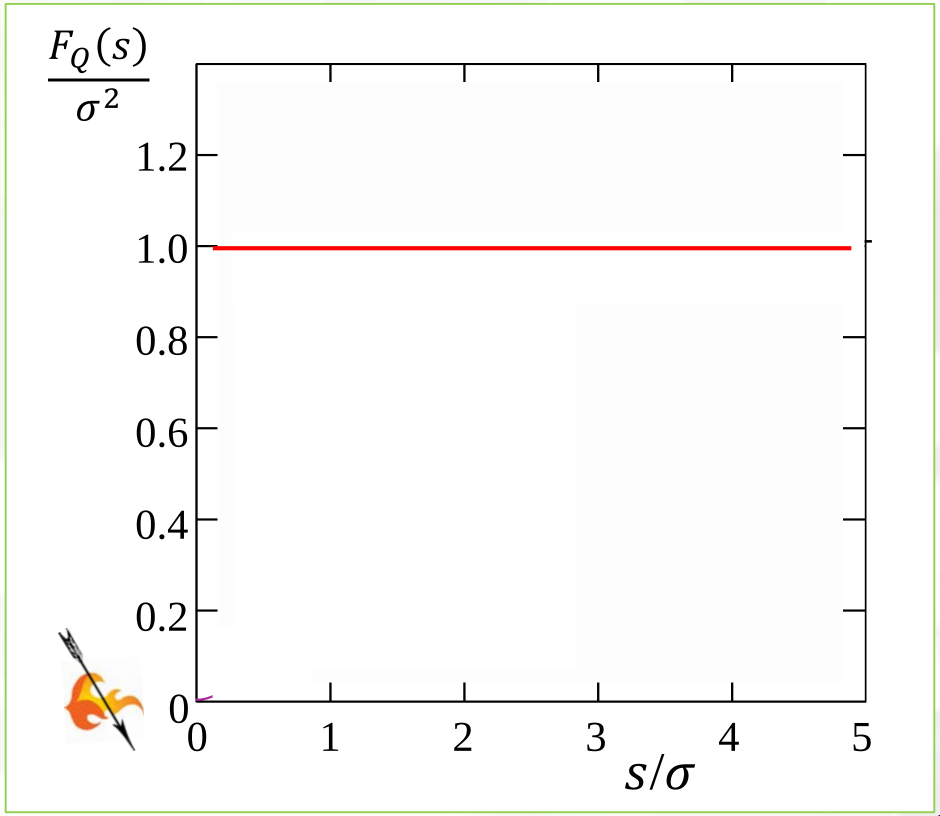


Best precision of estimates of the distance between two incoherent point sources from optimal quantum measurement of the image-plane field

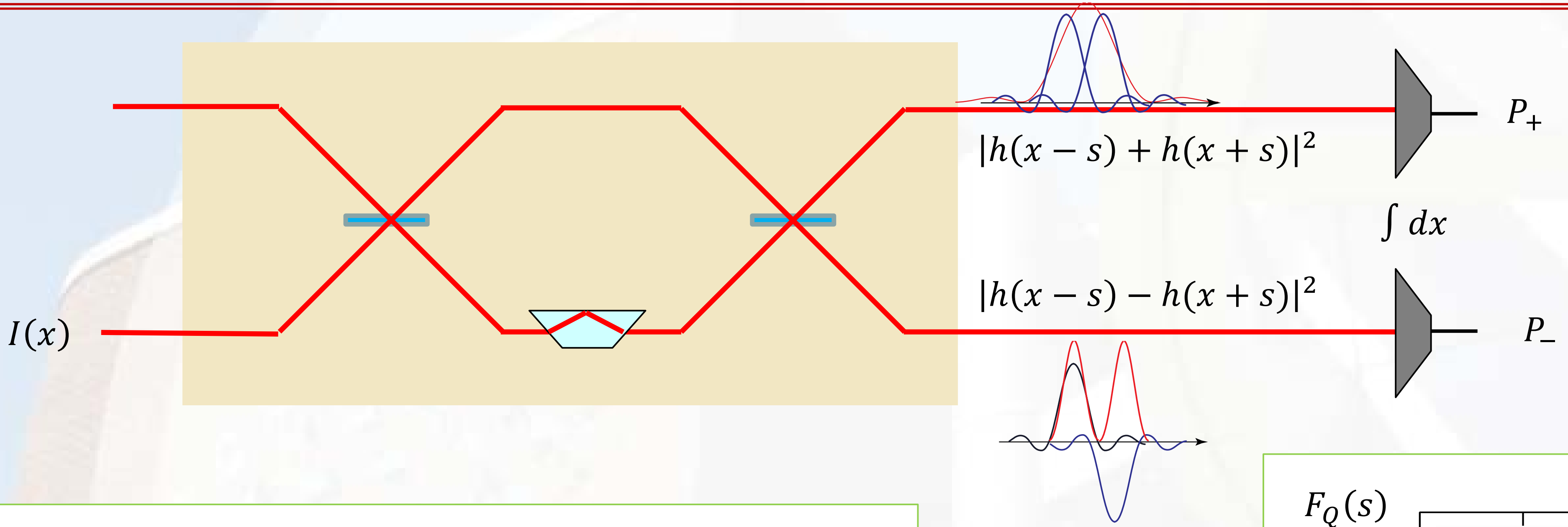


One-photon state $\rho(s) = \frac{1}{2}\int dx[|h_+(x)|^2 + |h_-(x)|^2]|x\rangle\langle x|$

$$h_\pm(x) = h\left(x \pm \frac{s}{2}\right)$$



Configuration using an image inversion interferometer and binary measurement



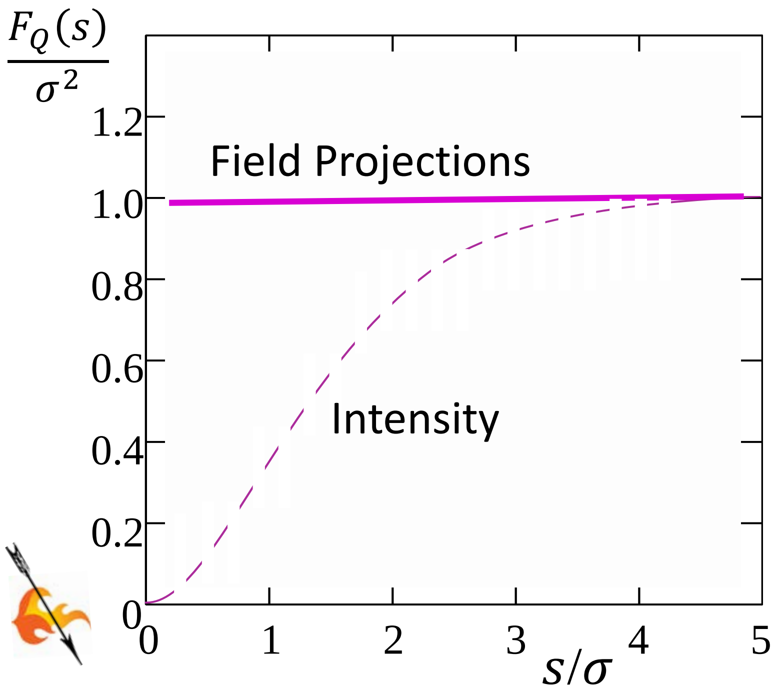
Projections on correlated & anticorrelated coherent images

Media Coverage

Physicists dispel Rayleigh's curse

The Fall of Rayleigh's Curse

Physicists break the Rayleigh limit on resolution



Nair, Tsang. *Opt. Express* 24, 3684 (2016).

Tham, Ferretti, Steinberg, PRL 118, 1-6 (2017).

Classical

$$f(x) = \frac{1}{2}\left(|h_+(x)|^2 + |h_-(x)|^2 + \gamma \mathrm{Re}\{h_+^*(x)h_-(x)\}\right.$$

$$h_\pm(x) = h\left(x \pm \frac{s}{2}\right)$$

$\gamma =$ Degree of coherence

Quantum

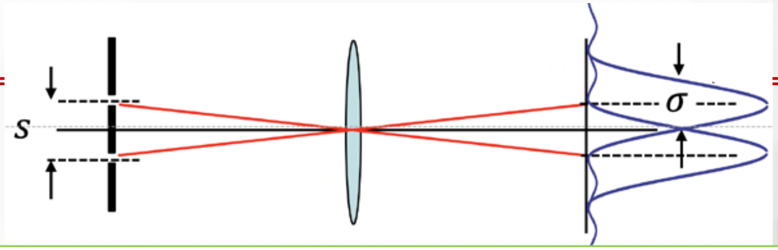
$$\hat{\rho} = p\hat{\rho}_c + (1-p)\hat{\rho}_i$$

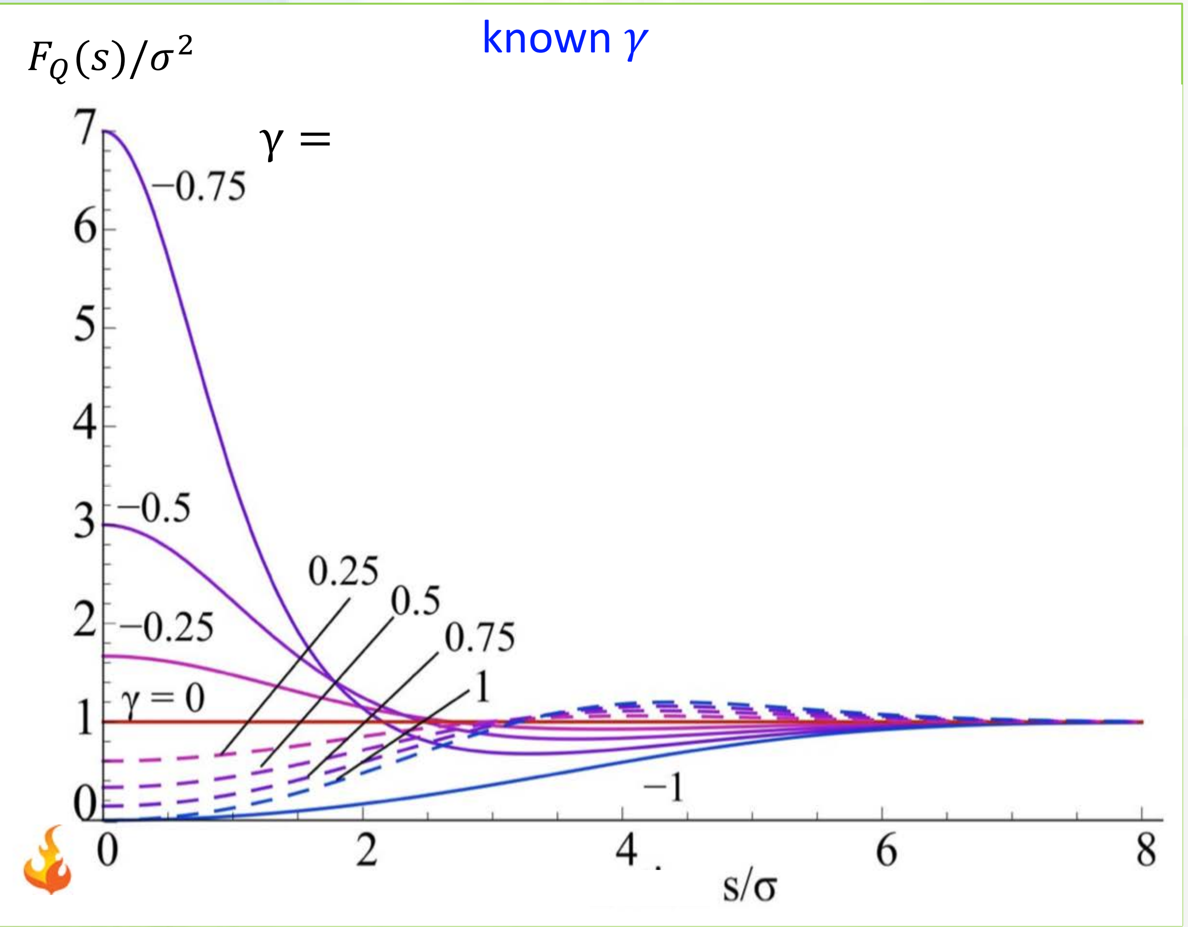
$$\hat{\rho}_c = |\psi_c\rangle\langle\psi_c| \qquad \hat{\rho}_i = \frac{1}{2}\left(|\psi_+\rangle\langle\psi_+| + |\psi_-\rangle\langle\psi_-|\right)$$

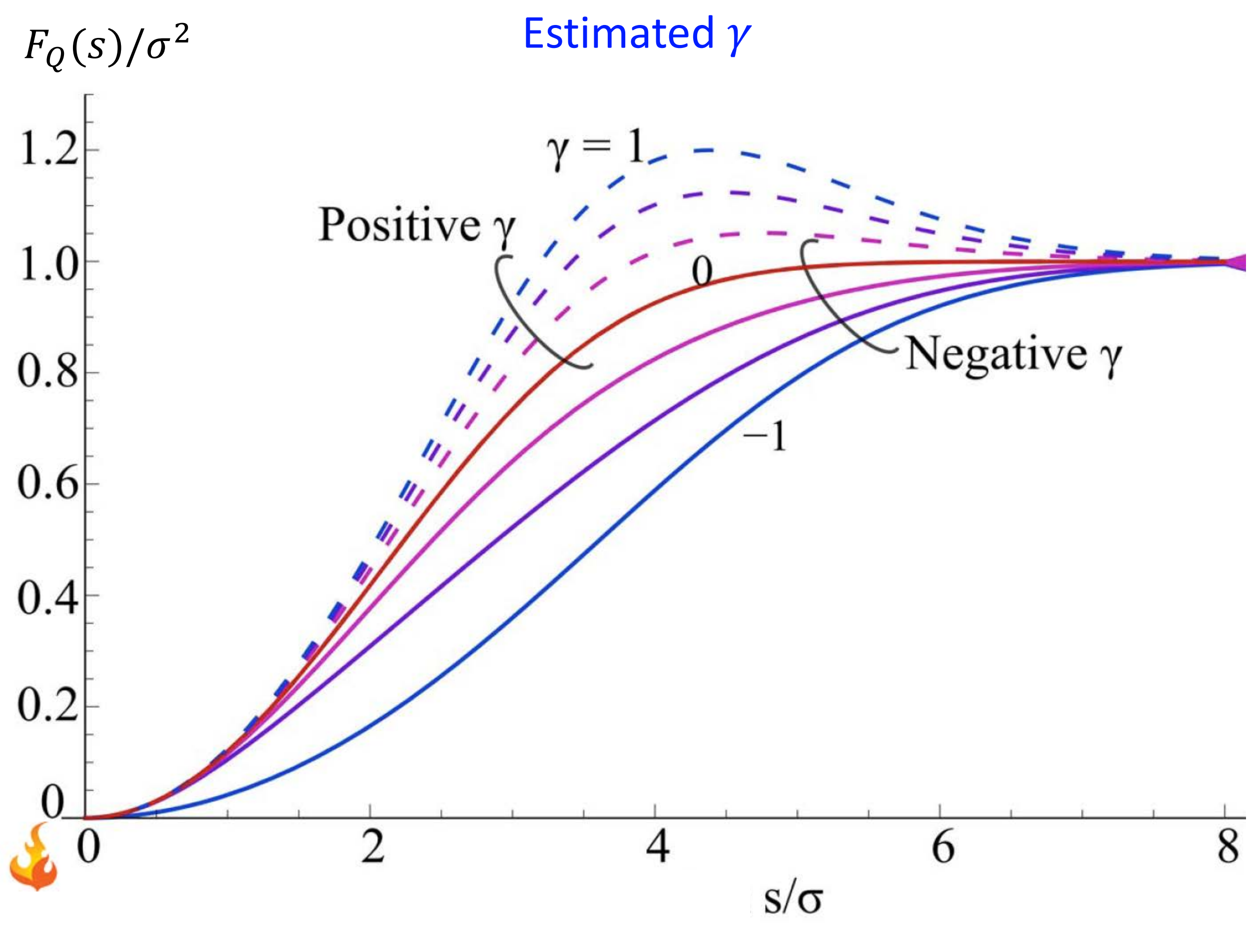
$$|\psi_c\rangle \propto |\psi_+\rangle \pm |\psi_-\rangle \qquad |\psi_\pm\rangle = \int dx\, h_\pm(x)\,|x\rangle \qquad h_\pm(x) = h\left(x \pm \frac{s}{2}\right)$$

$$\gamma = \pm\frac{p}{p + (1-p)(1 \pm d)} \qquad d = \langle\psi_-|\psi_+\rangle$$

Quantum precision bound (Optimal field measurement)







If the correlation coefficient γ is precisely known, then positive correlation reduces the precision, while anti-correlation enhances the precision.

If γ is to be estimated concurrently with s, then the curse resurges.

Concluding Remarks

Optical fields in nonclassical states enable enhanced-precision sensing/imaging

- Quadrature squeezed, photon-number squeezed (sub-Poisson)
- Fixed photon number (sub-shot noise)
- Fixed photon number with entanglement in one or more degrees of freedom (DoF)
  - Spatial (ghost imaging)
  - Spatial/path (phase imaging with N00N state)
  - Temporal (HOM for dispersion-cancelled imaging)
  - Polarization (quantum ellipsometry)

An ancillary degrees of freedom can be useful

- Decoherence in one DoF may be combated by entanglement with an ancillary DoF (larger Hilbert space)
- An ancillary DoF enables decoherence-free communication
- An ancillary DoF enables concurrent quantum estimation of non-commuting variables

Optimal quantum estimation is highly sensitive to accuracy of other parameters of the model

- If other parameters of the model are not known exactly, use of multiparameter estimation theory is necessary.
- Multiparameter estimation may or may not constitute independent estimation problems.

Optimal quantum estimation may be achieved with simple binary projections!

- Estimation of distance between two incoherent sources (transversely or axially displaced)
- Estimation of phase and phase gradient

Thank you!

besaleh@creol.ucf.edu