ECE 595: Machine Learning I
Lecture 8.1: Hand-Crafted and Deep Features - Convolution

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Stanley Chan

School of Electrical and Computer Engineering
Purdue University

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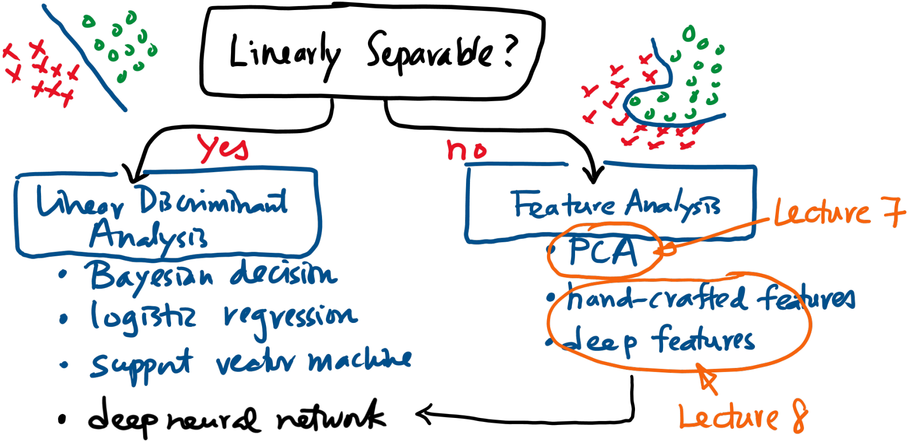
Overview

Supervised Learning for Classification



Outline

Feature Analysis

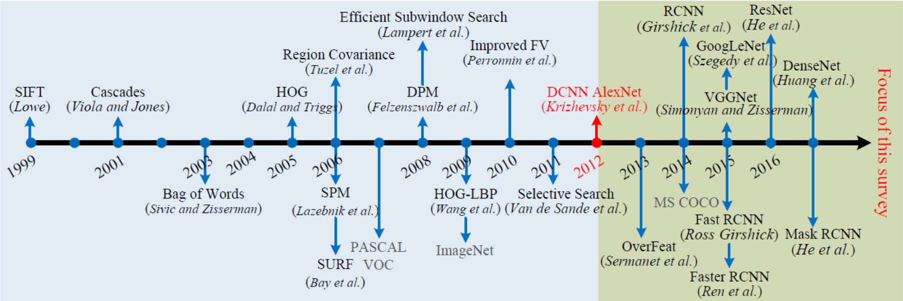
- Lecture 7 Principal Component Analysis (PCA)
- Lecture 8 Hand-Crafted and Deep Features

This Lecture

- Little History of Feature Extractions
- Convolution
  - What is convolution (if you don't know what it is yet)?
  - Some interesting facts about convolution
- SIFT and HOG
  - Gaussian derivatives
  - Pyramid
  - Histogram of oriented gradients
- Deep Features
  - What are they?
  - How to use them?

A Rough History of Feature Extraction

Deep Learning for Generic Object Detection: A Survey, `https://arxiv.org/pdf/1809.02165.pdf`



- PCA: Statistical analysis. Content agnostic.
- SIFT: Image specific. Non-training.
- Deep Features: Image specific. Training.

Convolution

Convolution (2D) is between two functions f and h:

- An **input** function $f(\boldsymbol{x})$, indexed by spatial coordinate $\boldsymbol{x} = [x_1,\ x_2]^T$
- A **filter** $h(\boldsymbol{x})$

The **output** is of the convolution is

$$\begin{aligned} g(\boldsymbol{x}) &= f(\boldsymbol{x}) * h(\boldsymbol{x}) \\ &= \int f(\boldsymbol{x} - \boldsymbol{\xi}) h(\boldsymbol{\xi}) d\boldsymbol{\xi} \end{aligned}$$

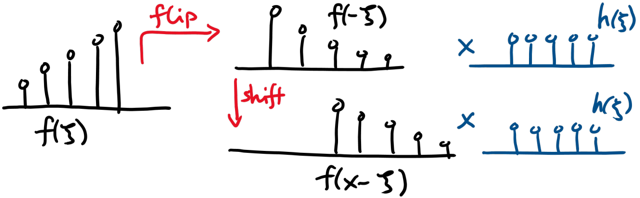
Do not be confused with **correlation**:

$$\begin{aligned} g(\boldsymbol{x}) &= f(\boldsymbol{x}) \circledast h(\boldsymbol{x}) \\ &= \int f(\boldsymbol{x} + \boldsymbol{\xi}) h(\boldsymbol{\xi}) d\boldsymbol{\xi} \end{aligned}$$

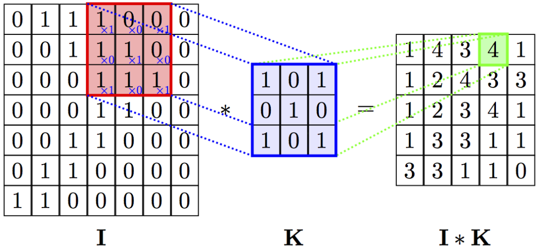
Convolution **flips** the filter, whereas correlation does not.

Pictorial Illustration

A convolution operation always involves 3 steps: flip-shift-add.



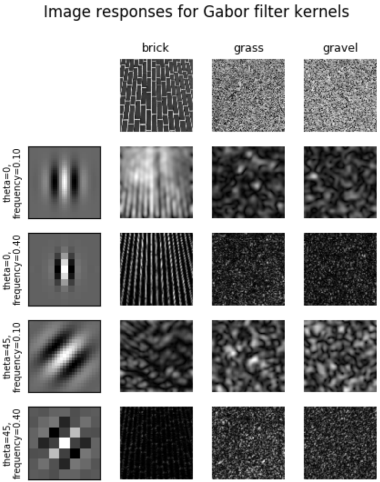
Most tutorials you see on the internet are correlations.



A Few Things You Need to Know about Convolution

- The core of convolution is the concept of **linear shift invariant** (LSI).
- A system $\mathcal{T}$ is LSI if
  - $\mathcal{T}(af_1(\boldsymbol{x}) + bf_2(\boldsymbol{x})) = a\mathcal{T}(f_1(\boldsymbol{x})) + b\mathcal{T}(f_2(\boldsymbol{x}))$
  - Let $g(\boldsymbol{x}) = \mathcal{T}(f(\boldsymbol{x}))$. Then for any $\boldsymbol{\xi}$, $f(\boldsymbol{x} + \boldsymbol{\xi}) \mapsto g(\boldsymbol{x} + \boldsymbol{\xi})$.
- Convolution is the only operation that allows LSI.
- **Eigen-functions** of a convolution operation are the Fourier series.
- The flip operation is necessary to define the Fourier series.
- This can be dated back to Pierre-Simon Laplace (1749-1827) and Joseph Fourier (1768-1830), with 200 years of work in real / functional analysis.
- Convolution with large filters are always implemented by **Fast Fourier Transforms**.
- Convolution can be performed at the **speed of light**! Put a mask at the focal plane of the lens. It will give you the convolution of the mask and the image (in the Fourier domain).
  `https://www.youtube.com/watch?v=4Eg0Tbk601s`

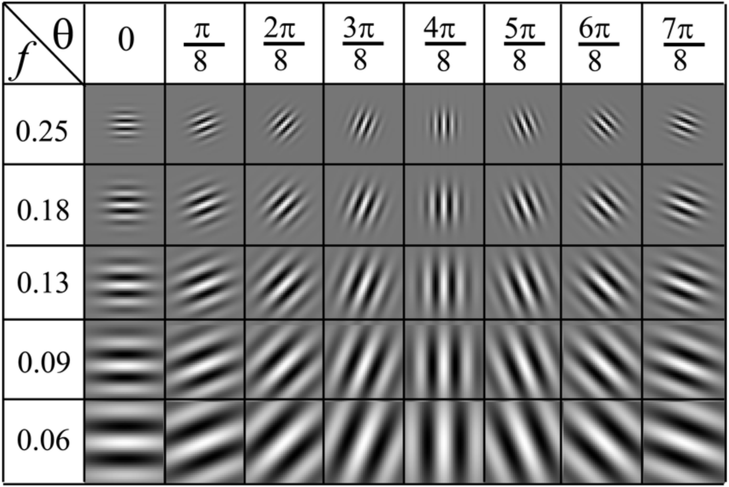
Effect of Convolution / Correlation



https://scikit-image.org/docs/dev/auto_examples/features_detection/plot_gabor.html

Examples of Filters

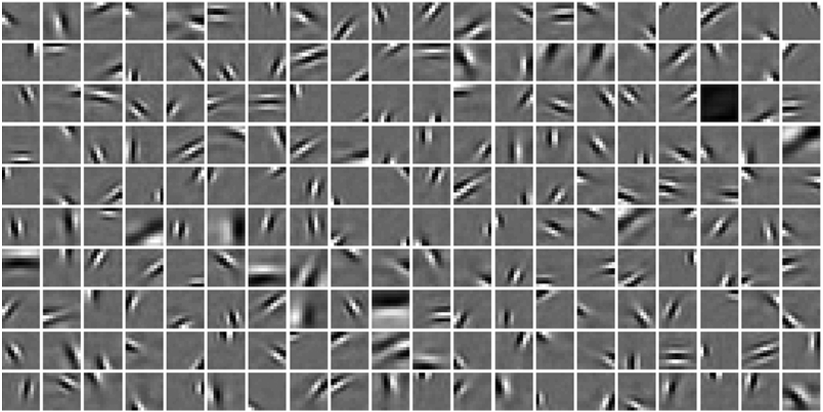
Gabor Filter



https://www.researchgate.net/figure/
Real-parts-of-the-Gabor-filter-bank-Generated-for-different-combinations-of-th-in_fig5_292671765

Examples of Filters

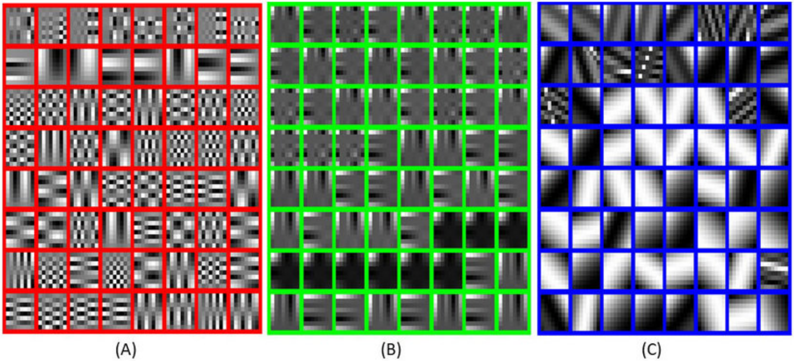
Another Gabor Filter



https://www.quora.com/How-are-Gabor-filters-implemented-in-visual-area-V1-in-the-brain

Examples of Filters

KSVD Filters



https://www.researchgate.net/figure/
Basis-functions-used-by-a-KSVD-The-KSVD-based-dictionary-elements-or-atoms-mostly_fig6_336133323