The Magic of Intelligent Coherent Optical Processing

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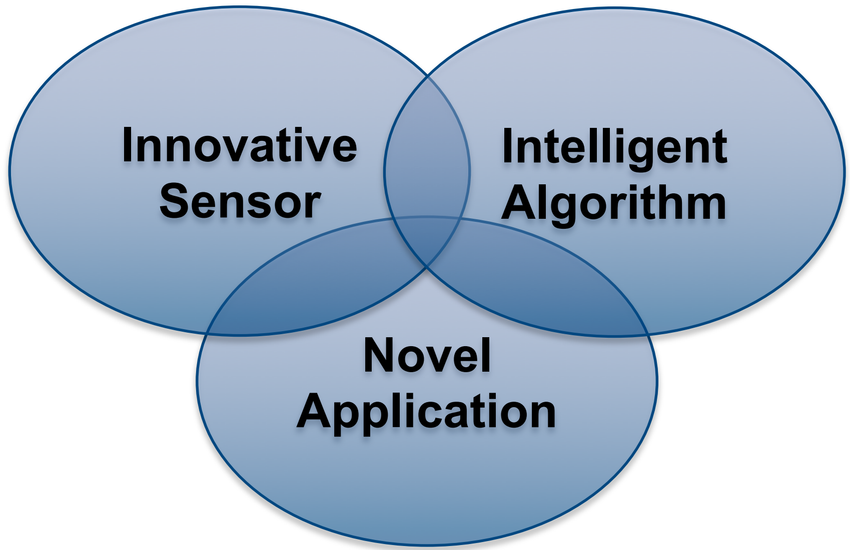
Outline

1. What is Computational Imaging?

2. My view of phase recovery…

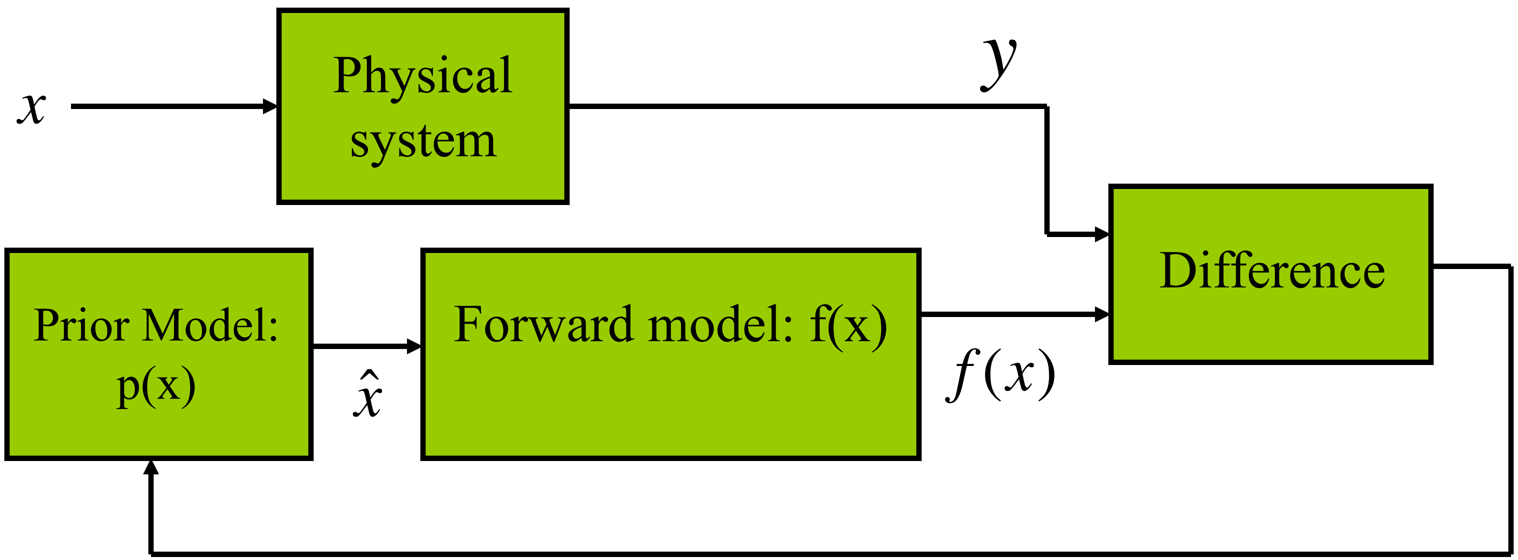
3. Digital holographic "single shot" imaging through deep turbulence
   - Leverages advanced signal processing techniques
   - Integrates physics and machine learning models
   - Is fast and effective

What is Computational Imaging? (Integrated Imaging)



- Traditional sensor design is reaching its limits
- Make the **most informative** measurement, rather than the **"purest"** measurement.
- Mick Jagger's Theorem: You can't always get what you want, but if you try sometimes, you might get what you need.

Model Based Iterative Reconstruction (MBIR): A General Framework for Solving Inverse Problems

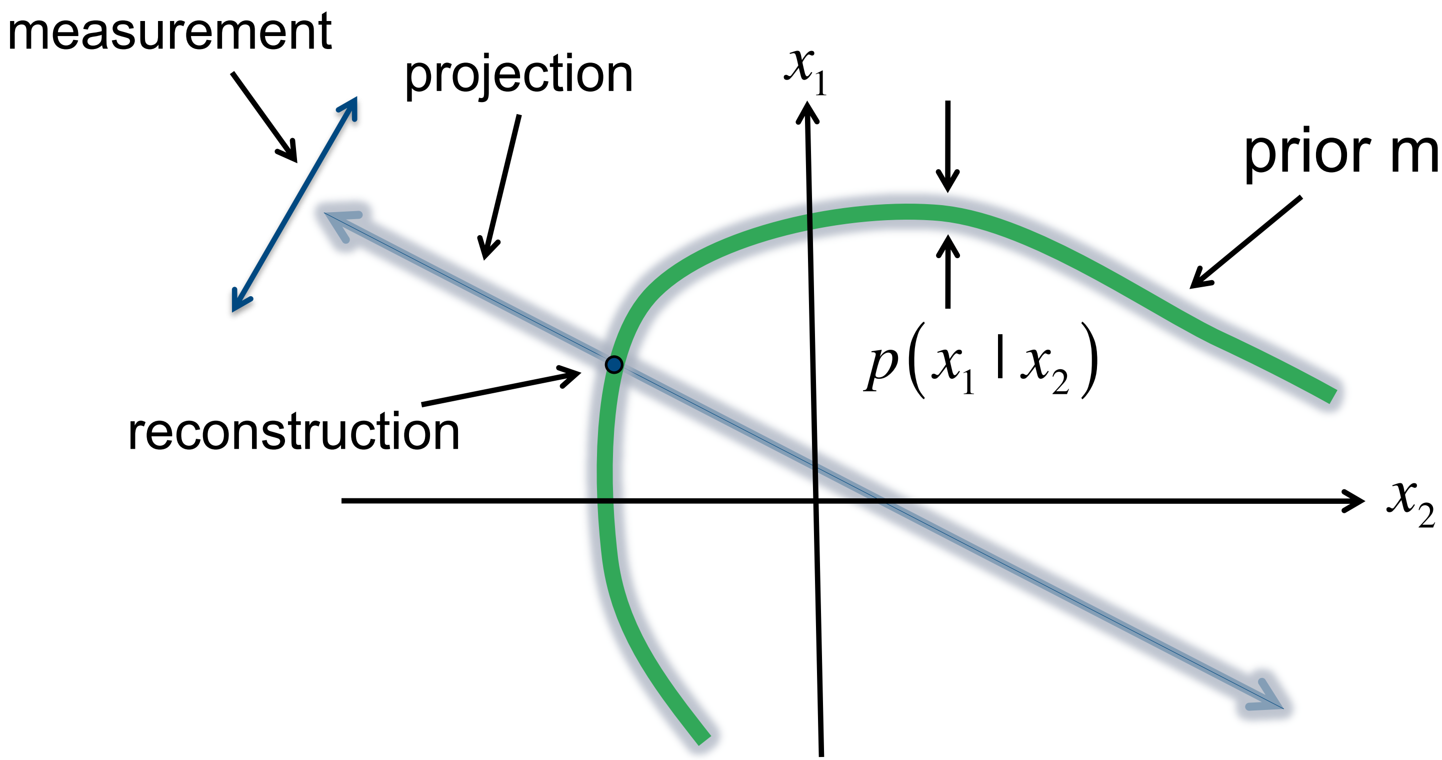


$$\hat{x} \leftarrow \underset{x}{\arg\max} \{ \underbrace{\log p(y \mid x)}_{\text{forward model}} + \underbrace{\log p(x)}_{\text{prior model}} \}$$

$\hat{x}$ – Reconstructed object

y – Measurements from physical system

"Thin Manifold" View of Prior Models



- Notice that prior manifold fills the space but…
  - Not a linear manifold
  - PCA can not effectively reduce dimension
- But it has thickness
- Dimension of measurement > dimension of manifold

Phase Recovery for Complex Signals

- If you can only measure energy, then…

$$x(n) \longrightarrow \boxed{|\cdot|^2} \longrightarrow y(n)$$

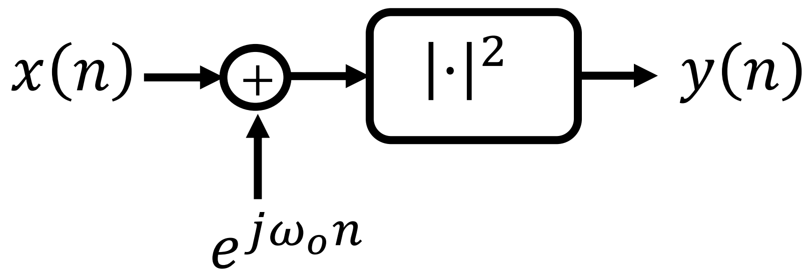
  - So you only know that:

$$|x(n)| = \sqrt{y(n)}$$

$$\angle x(n) = ??$$

Phase Recovery with Heterodyne Demodulation

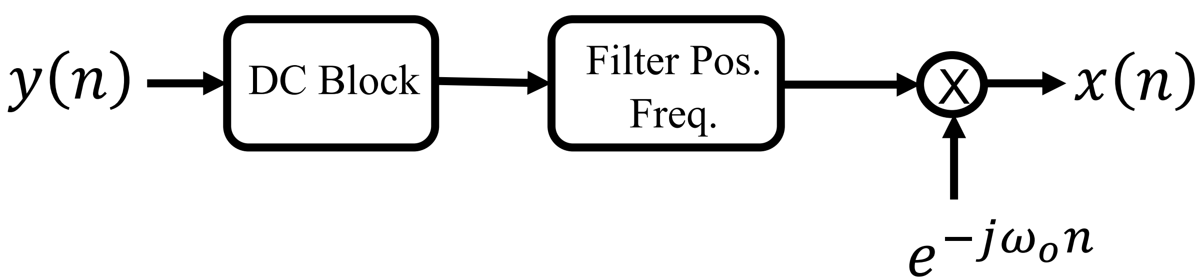
- Heterodyne



small DC

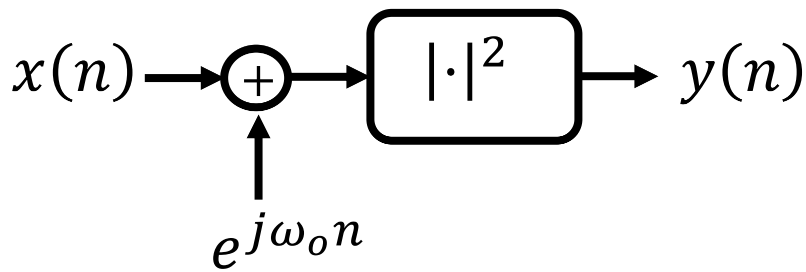
$$y(n) = 2\,Re\{\,x(n)e^{j\omega_o n}\} + |x(n)|^2 + 1$$

- Complex signal recovery

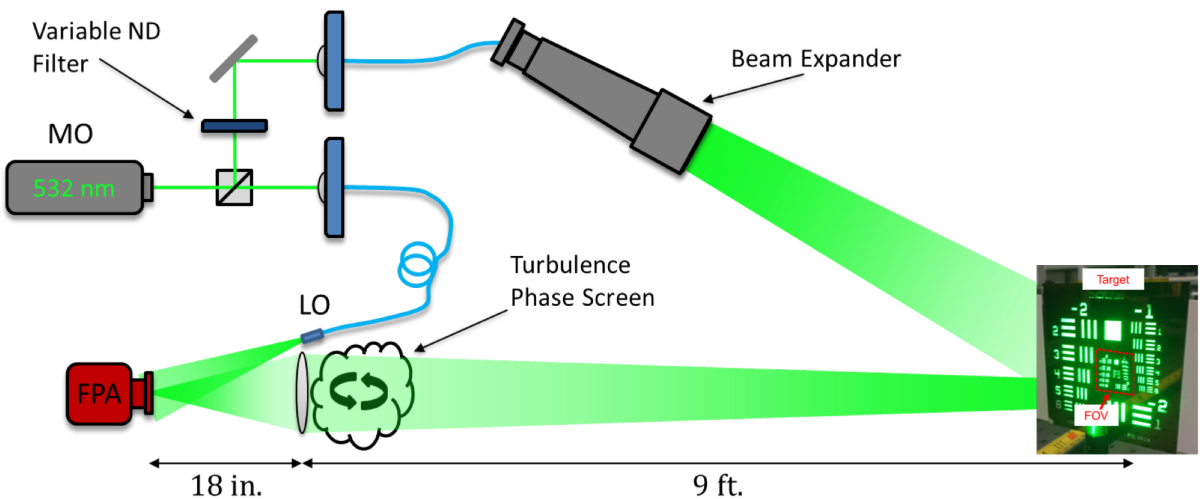


Digital Holography: Math versus Experiment

Math

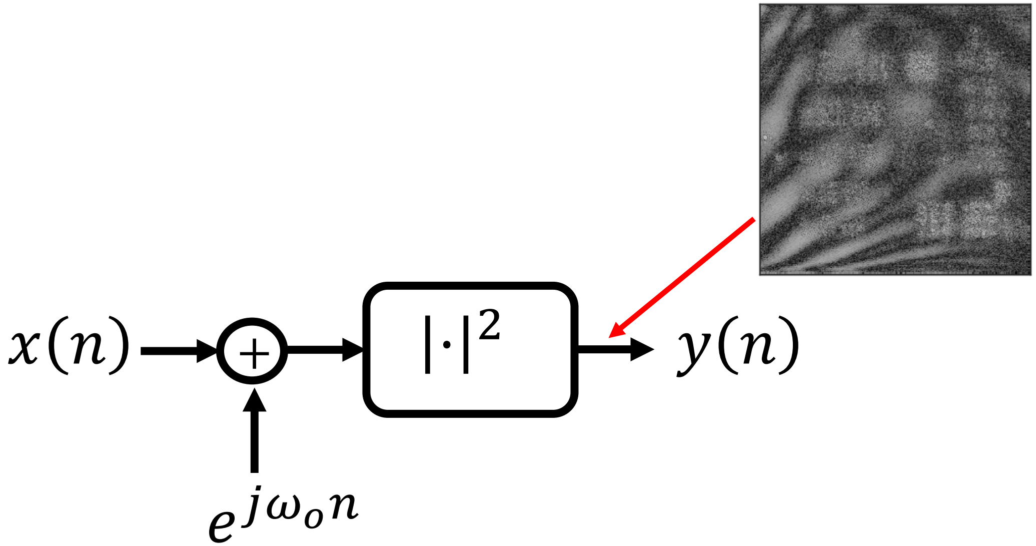


Experiment

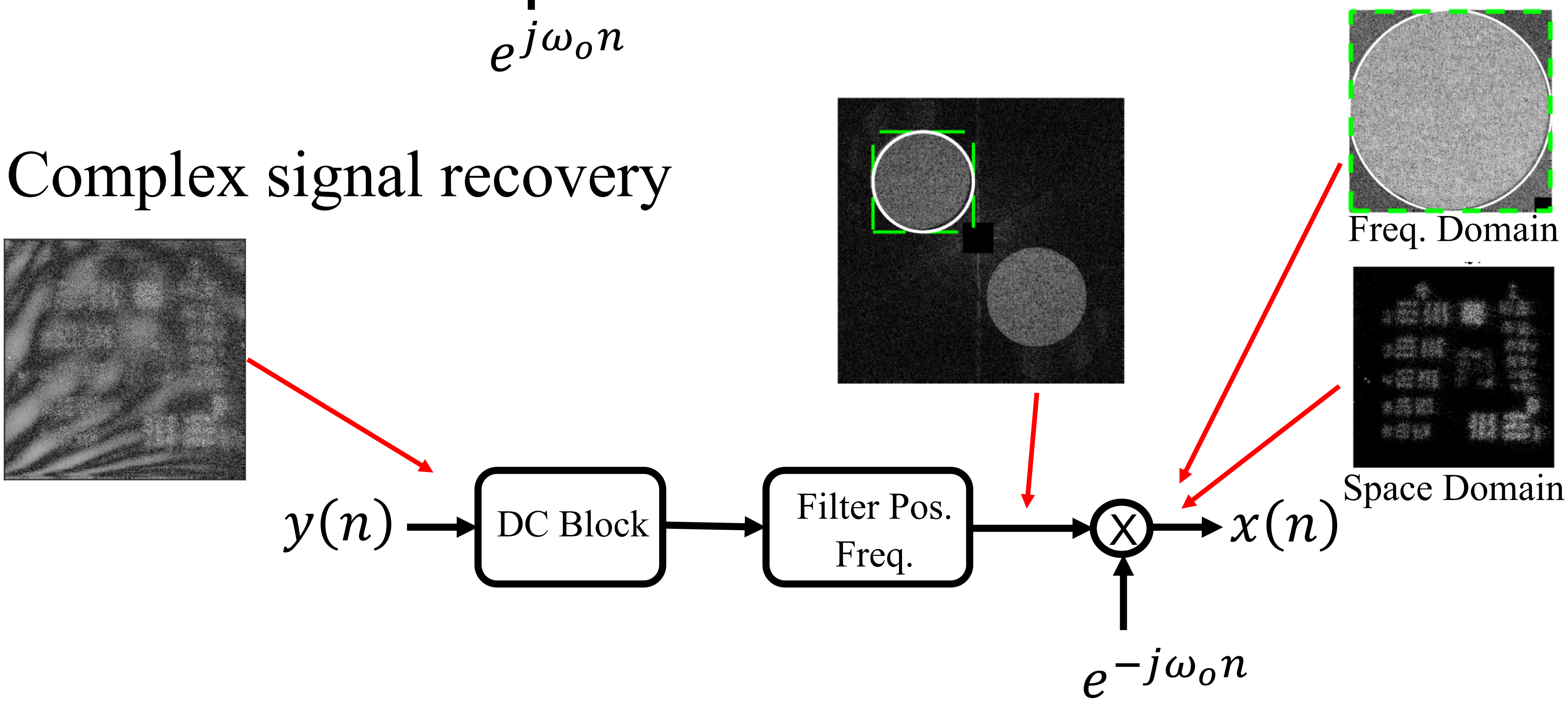


Digital Holography: Graphical

- Heterodyne

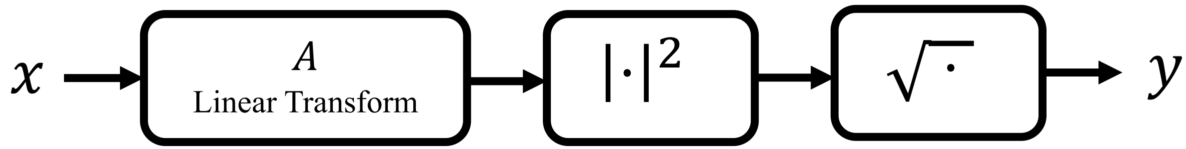


- Complex signal recovery



General Phase Recovery

- Detect magnitude of linear transform $A \in \Re^{M \times N}$



  – Can we recover x?

Answer: Mostly "yes" if $M \geq 2N$.
Sometimes "yes" when $M < 2N$.

  – Alternating minimization:

$$\mathit{Repeat}\ \{$$

$$\hat{x} \leftarrow \arg\min_{x} \left\{ \left\| y.* e^{j\hat{\theta}} - Ax \right\|^2 \right\} \quad \textit{LS inversion}$$

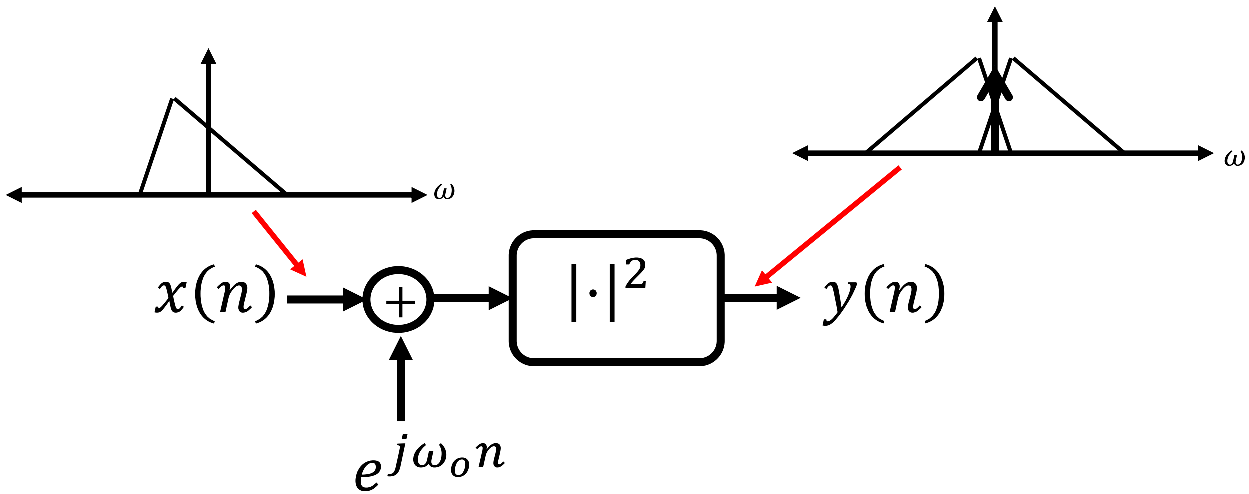
$$\hat{\theta} \leftarrow \arg\min_{\theta} \left\{ \left\| y.* e^{j\theta} - Ax \right\|^2 \right\} \quad \textit{Super easy}$$

$$\}$$

Example: Phase Recovery with Aliasing

with Dennis Lee and Andy Weiner

- What if there is aliasing? (ω_o too small)

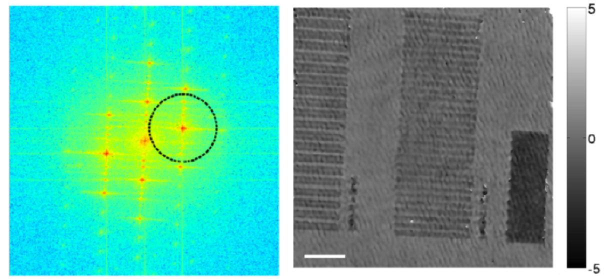


- Then heterodyning doesn't work, right???

Answer: Yes, but we can still recover x using regularized iterative phase recovery!

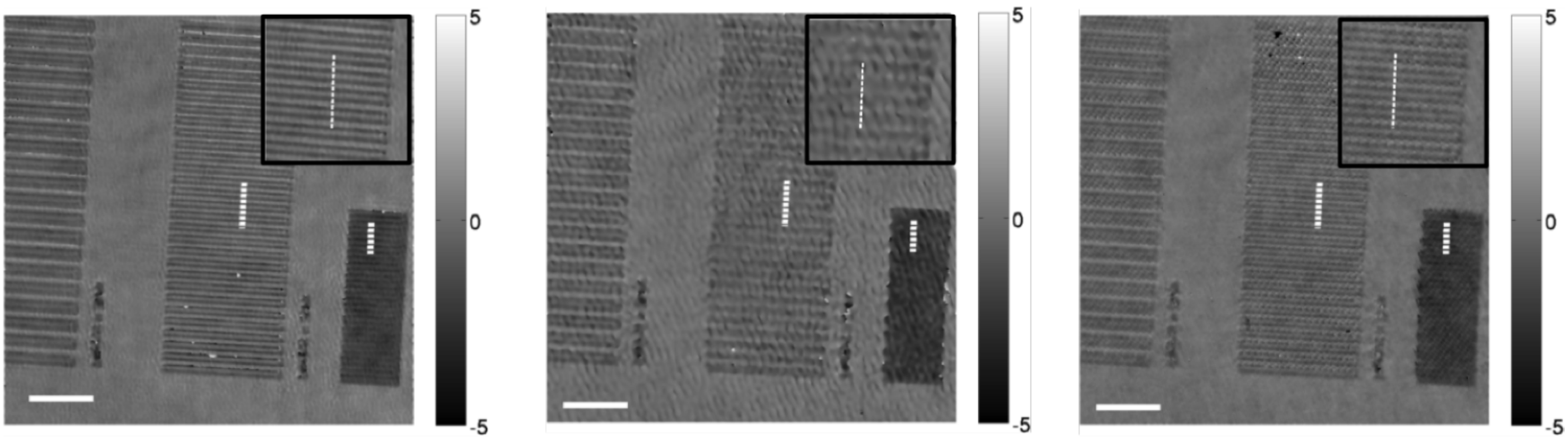
Example: Phase Recovery with Aliasing*

- Coherently imaged phase grating with aliasing:



(b) Power Spectrum of (a) (c) Fourier Filtering Applied to (a)

- Regularized iterative reconstruction recovers phase!

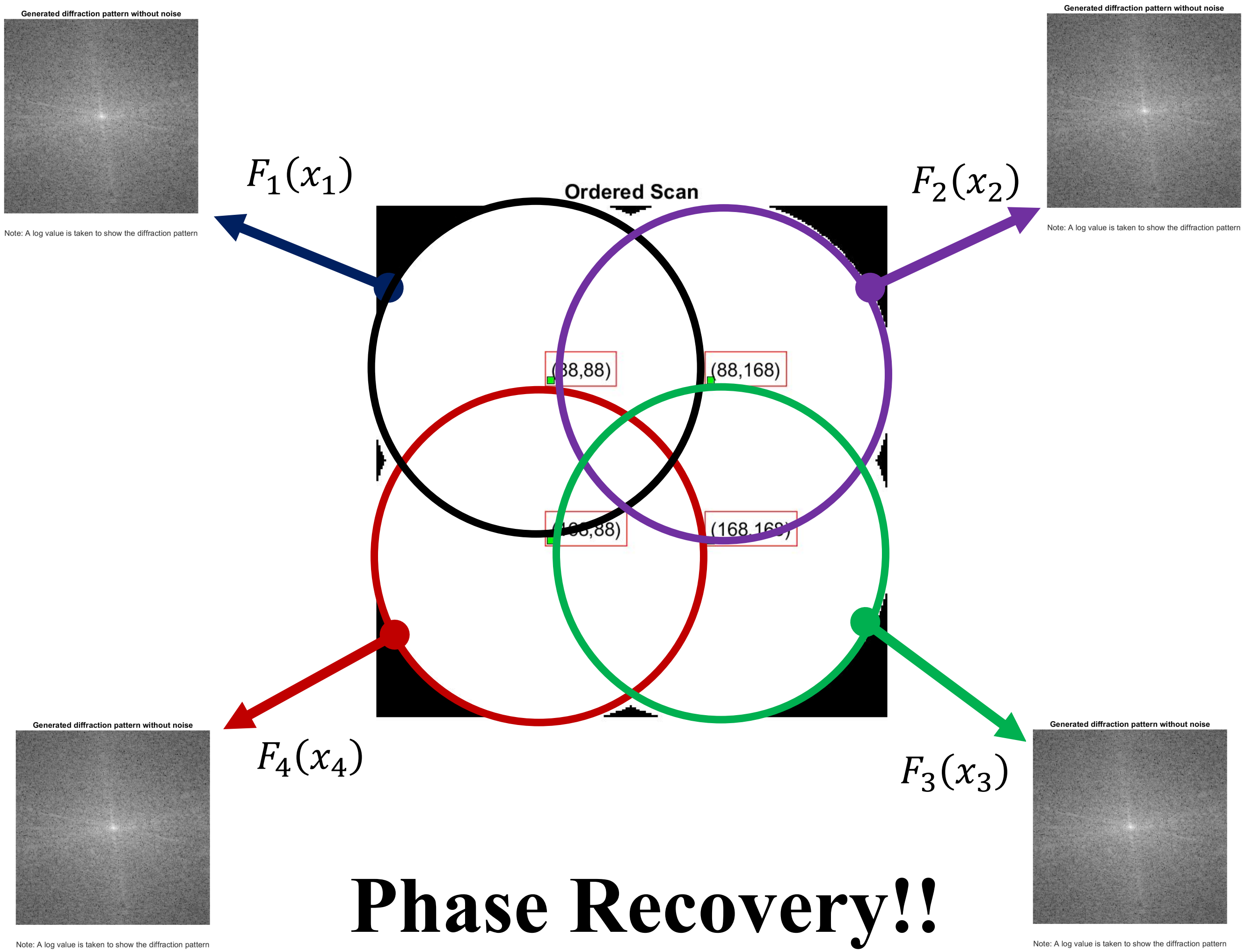


(a) Ground Truth (b) Fourier Filtering (c) Iterative Reconstruction

*D. J. Lee, C. A. Bouman, and A. M. Weiner, "Single Shot Digital Holography Using Iterative Reconstruction with Alternating Updates of Amplitude and Phase," *Computational Imaging Conference* 2016

Example: Ptychography

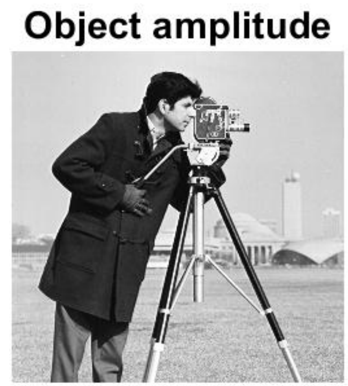
with Qiuchen Zhai and Greg Buzzard



Phase Recovery!!

Ptychographic Reconstruction

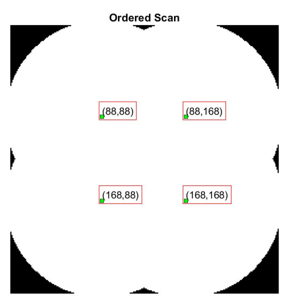
Mag and Phase images



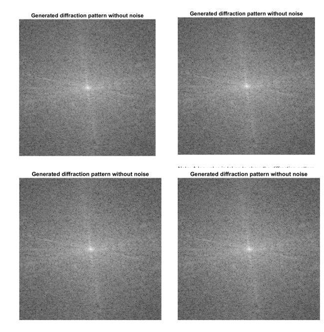
256x256

256x256

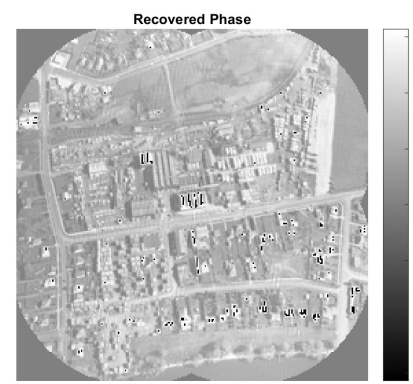
Diffraction measurements



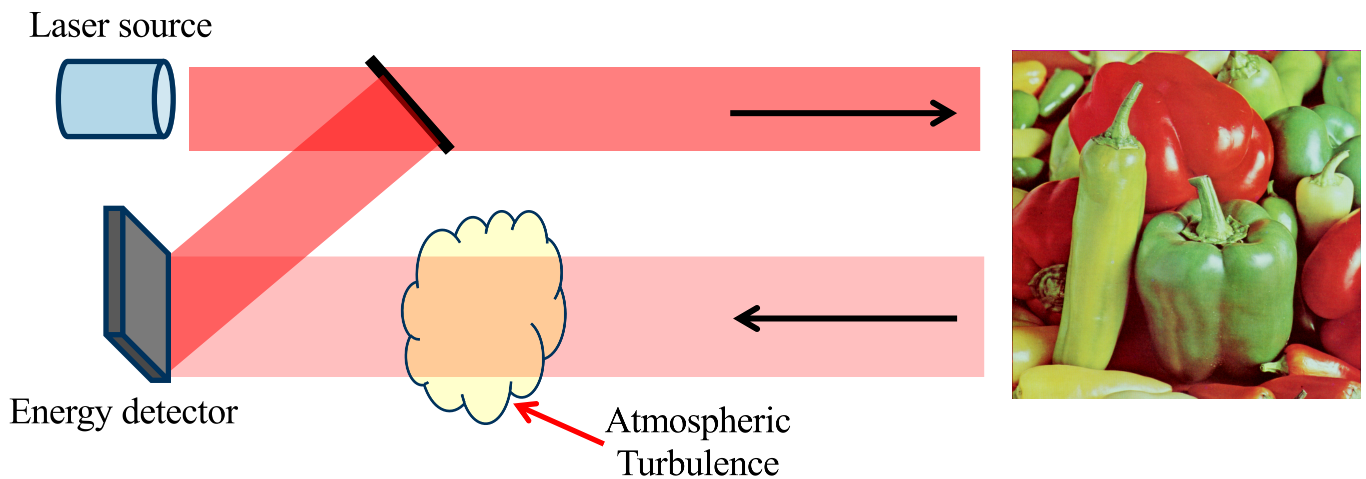
Fourier Transform



Reconstruction



Imaging through Turbulence
with Casey Pellizzari and Mark Spencer



$$\boldsymbol{y} = \boldsymbol{A}_{\boldsymbol{\phi}}\boldsymbol{g} + \boldsymbol{w}$$

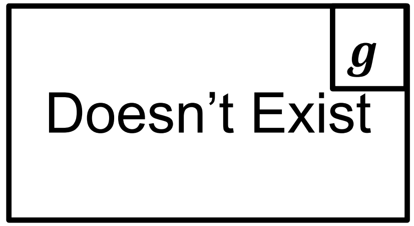
Assumes Fourier demodulation

contains speckle

$\boldsymbol{y}$ – Complex measurement
$\boldsymbol{g}$ – Complex reflectance coefficient
$\boldsymbol{w}$ – Complex noise
$\boldsymbol{A}_{\boldsymbol{\phi}}$ – Linear propagation model
$\boldsymbol{\phi}$ – Unknown phase distortion

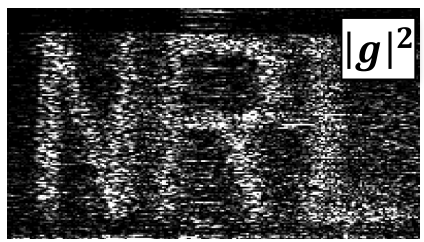
Variables to remember…

$\boldsymbol{g}$ – reflection coefficient



complex valued image

$|\boldsymbol{g}|^{2}$– mag^2 of reflection coefficient



speckly image

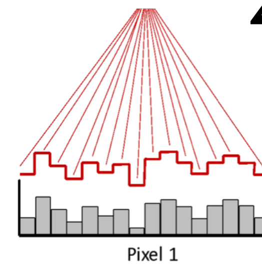
$\boldsymbol{r}$ – reflectance

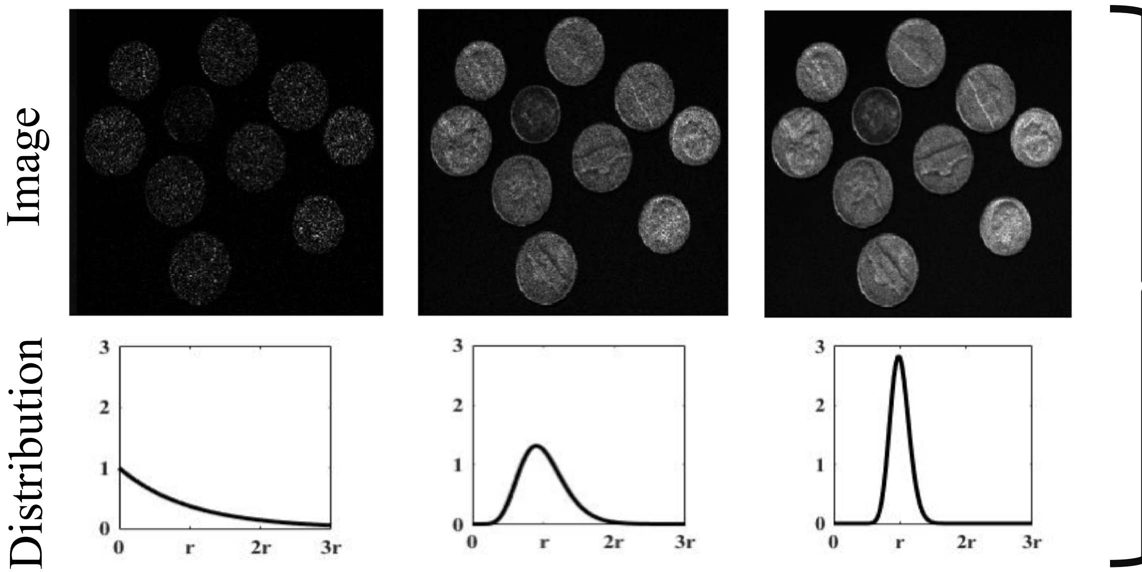
de-speckled image

Conventional Estimation of r

- Speckle averaging
  - Average many “independent shots”
  - Depends on fact that $\boldsymbol{E}\left[|\boldsymbol{g}|^2|\boldsymbol{r}\right] = \boldsymbol{r}$

$$g = \sum_i s_i$$





$\boldsymbol{N = 1}$ $\boldsymbol{N = 10}$ $\boldsymbol{N = 50}$

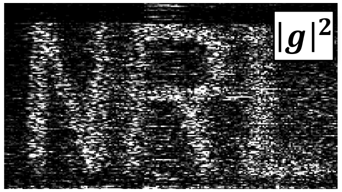
As number of realizations, *N*, increases, the distribution converges to *r*.

$$\sim \chi^2_{2N}$$

Huge Advantage of Estimating r

- Estimating $\boldsymbol{r}$ is much better because…
  - $\boldsymbol{r}$ lives in a lower dimensional space

    $\left(\frac{measurements}{unknowns}\right)$ >> large $\Rightarrow$ easer phase recovery
  - Results in less noisy image



$\boldsymbol{g}$ – speckly (high dimension)

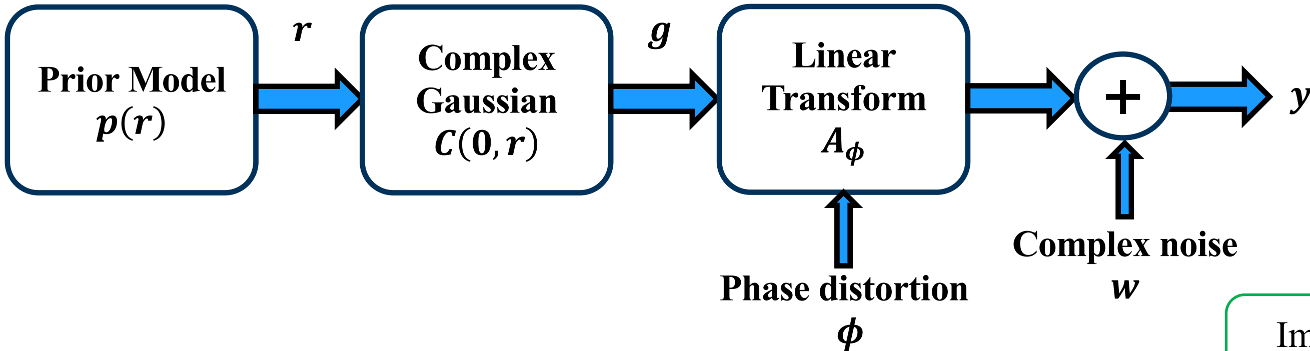
$\boldsymbol{r}$ – good (low dimension)

Problem: Estimating r is difficult!

MBIR (Model-Based Iterative Reconstruction)

- Forward Model

stochastic and nonlinear



- MBIR estimation of (r, ϕ) given y

Image and phase priors

$$(\hat{r}, \hat{\phi}) = \arg\max_{(r,\phi)} \{\log p(y|r,\phi) + \log p(r) + \log p(\phi)\}$$

where $$p(y|r) = \int p(y|g,\phi)p(g|r)dg$$

Intractable integral

Oh no! … Is MBIR impossible?

The Magic: EM Algorithm to the Rescue

- Define $\boldsymbol{Q}$ function (E-step)

$$Q(r,\phi;r',\phi') = E[\log p(y|g,\phi) + \log p(g|r) + \log p(r) + \log p(\phi)\,|y,r',\phi']$$

E-step

- EM algorithm for MAP estimation (local min)

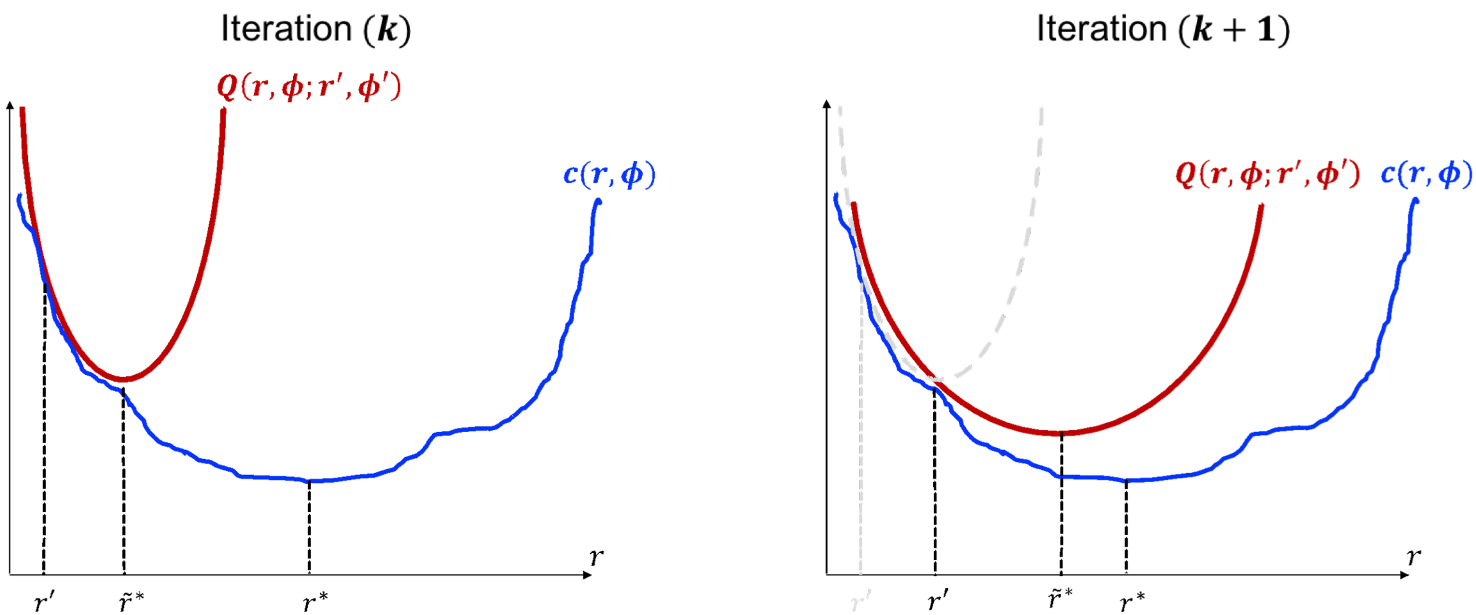
Initialize (r,ϕ)
Repeat {

$$(r',\phi') \leftarrow \arg\max_{(r,\phi)} Q(r,\phi;r',\phi')$$

}
Return (r',ϕ')

M-step

Iterative EM Optimization



The beauty of the EM algorithm is that Q is crafted in such a way that it upper bounds c. Therefore, by minimizing Q, we converge to a minima of c

Magical Closed Form for Q Function!

- When $A_\phi^H A_\phi \approx I$, then

Only function of ϕ

$$Q(r,\phi;r',\phi') \approx -\frac{2}{\sigma_w^2} Re\{y^H A_\phi \mu\} + p(\phi)$$

Pixel-wise function of r

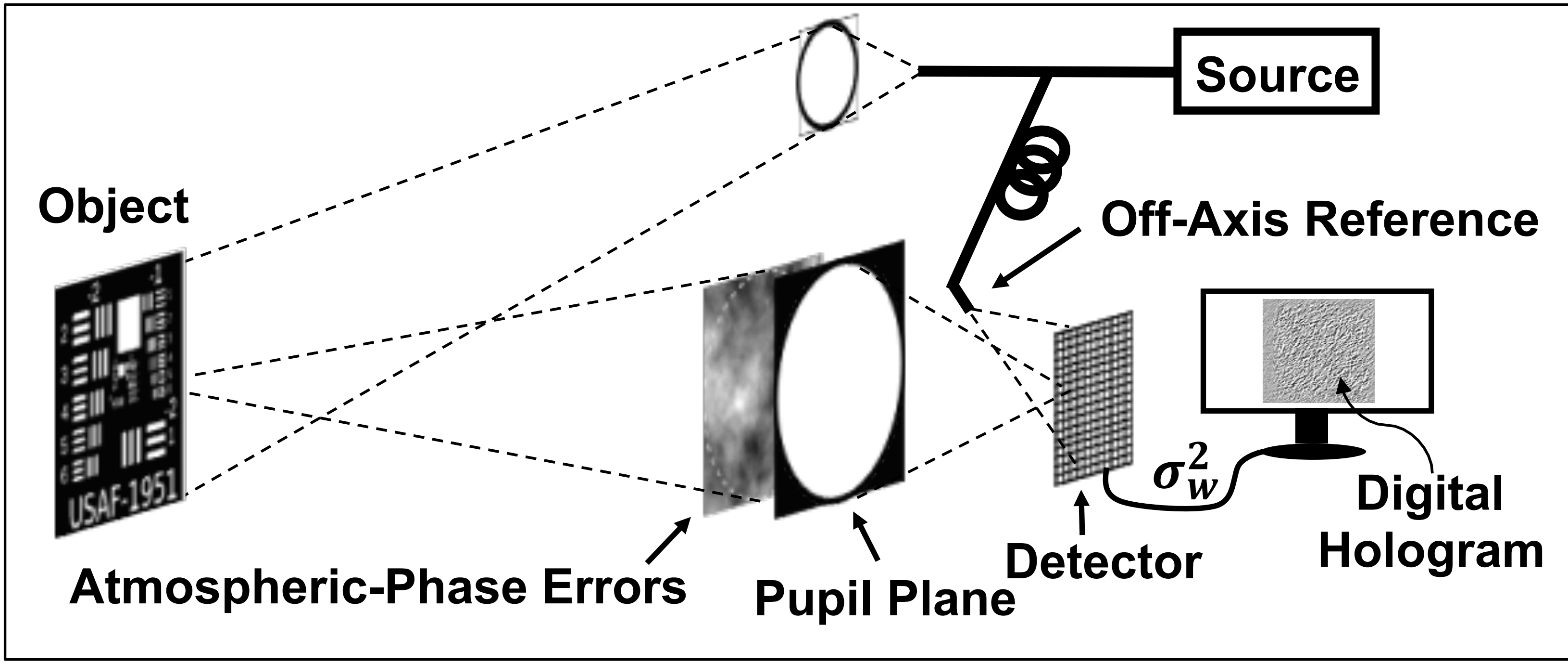
$$+\log|\mathcal{D}(r)| + \sum_{i=1}^{N} \frac{1}{r_i}\left(C_{i,i} + |\mu_i|^2\right) + p(r)$$

where

$$C = \mathcal{D}\left(\frac{r'}{r'/\sigma_w^2 + 1}\right) \qquad \mu = C\,\frac{1}{\sigma_w^2} A_{\phi'}^H y$$

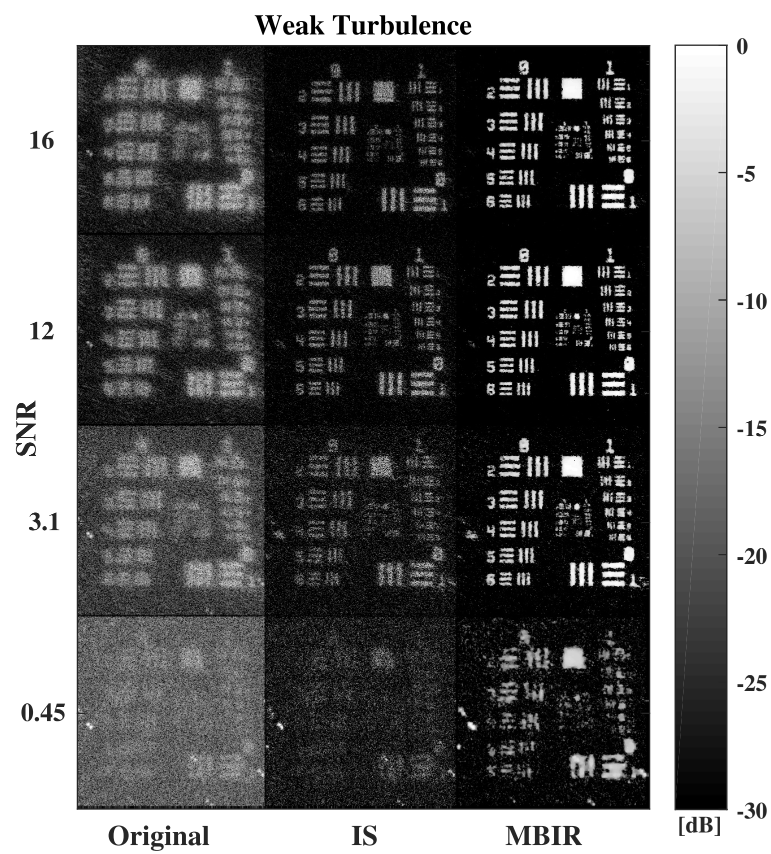
diagonal matrix

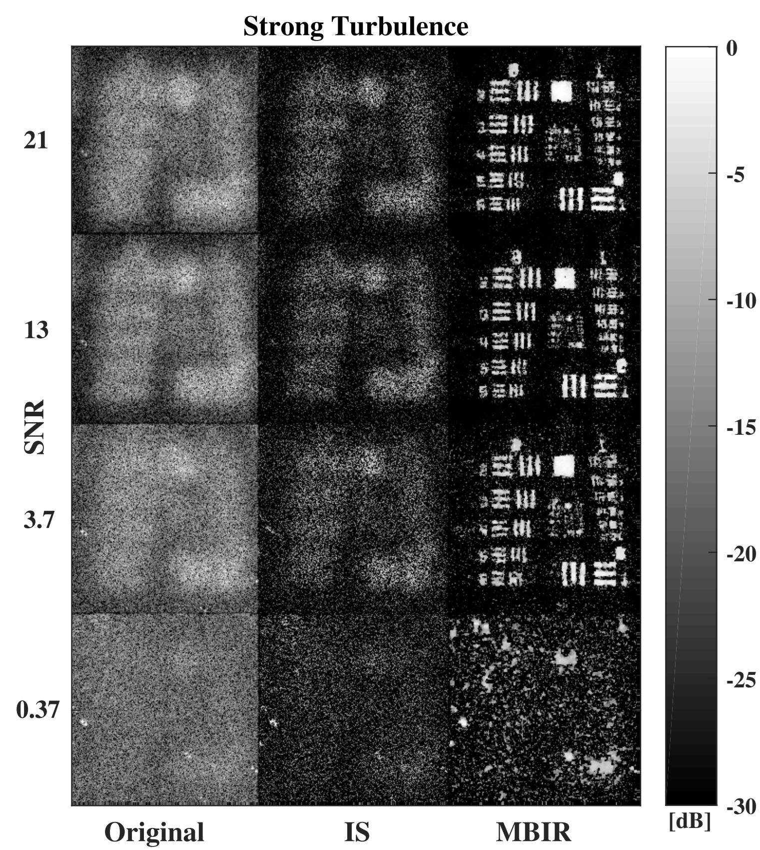
Isoplanatic Experiments



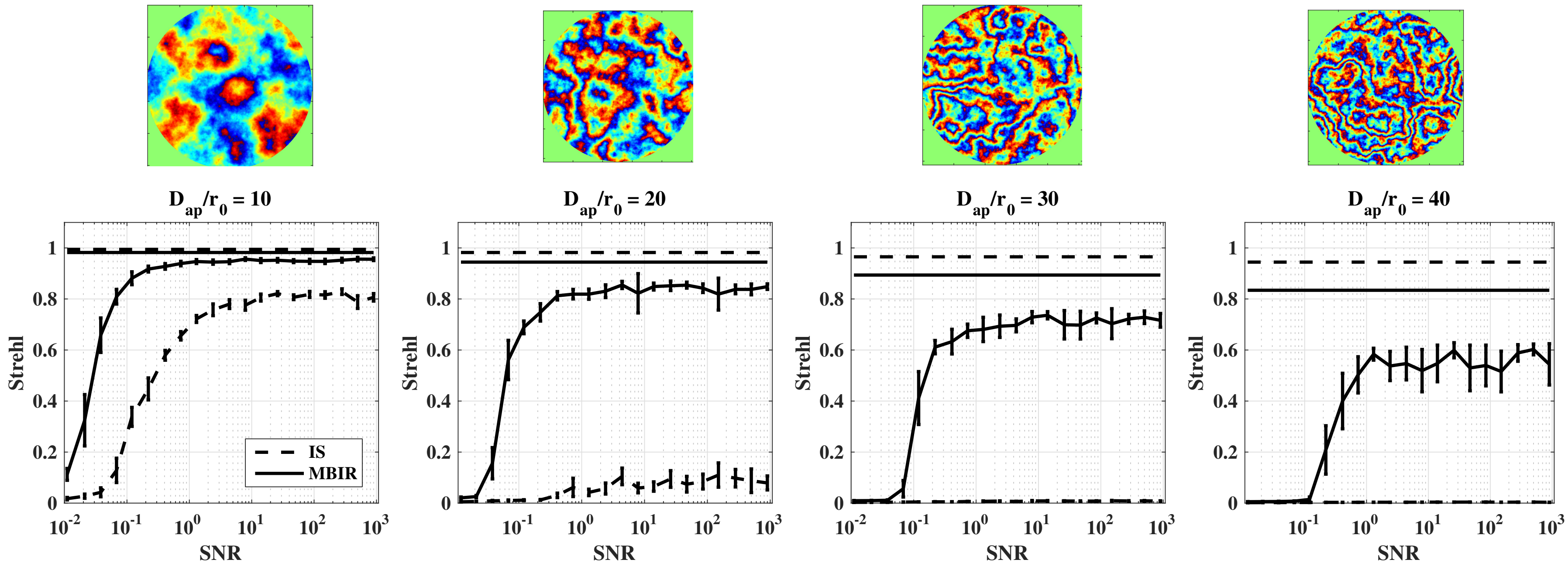
Isoplanatic ⇒ shift-invariant PSF

DH-MBIR: Isoplanatic Result (Experimental)



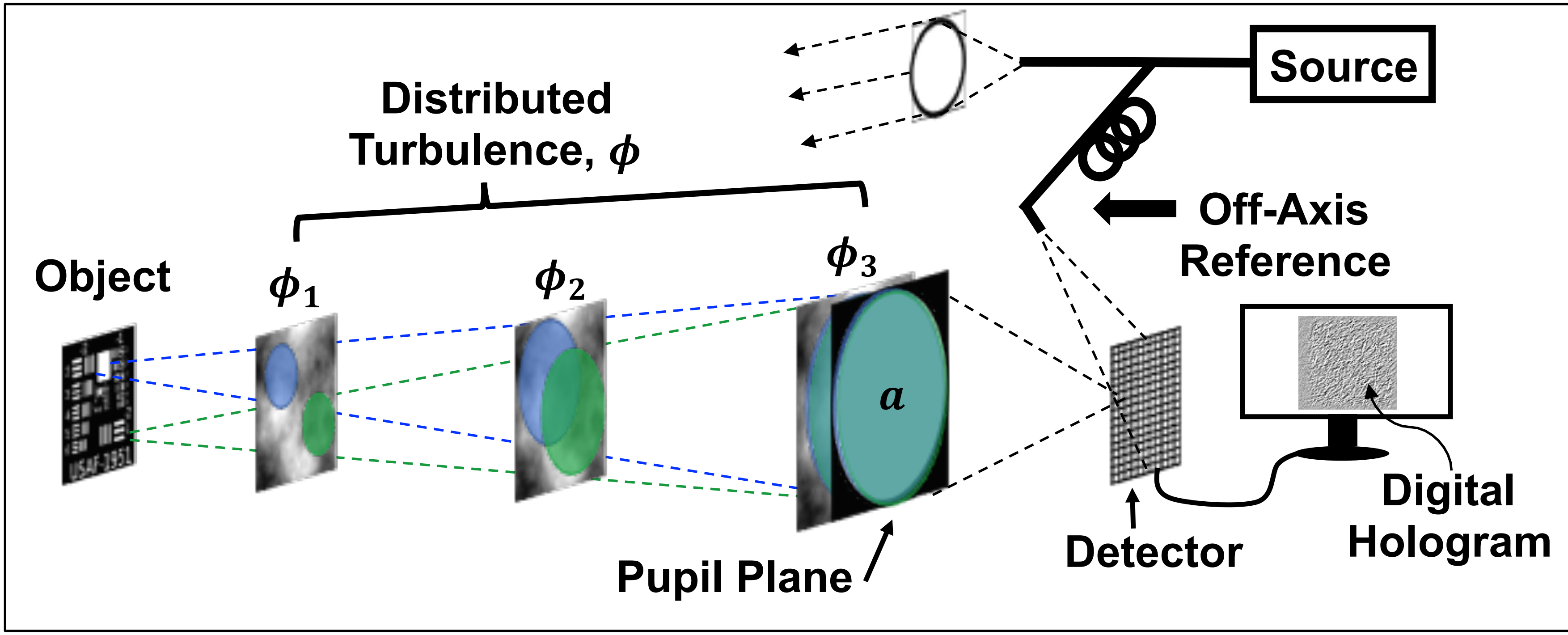


DH-MBIR: Strehl Ratio vs SNR (Simulated)



MBIR produces more accurate phase-error estimates than Image Sharpening at low SNRs and/or strong phase errors

Anisoplanatic Experiments

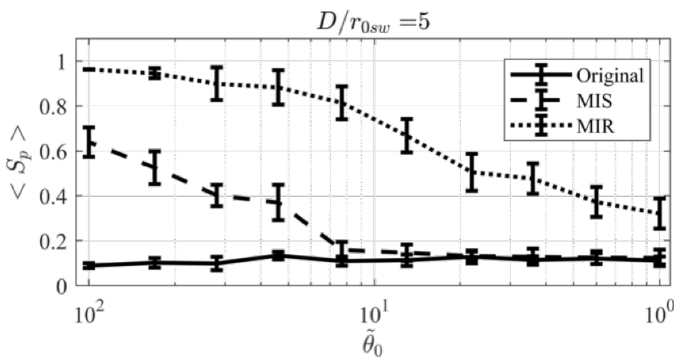


$$\tilde{\theta}_0 = \frac{\theta_0}{\left(\frac{\lambda}{D}\right)} = \frac{isoplanatic\ angle}{diffraction\ limit}$$

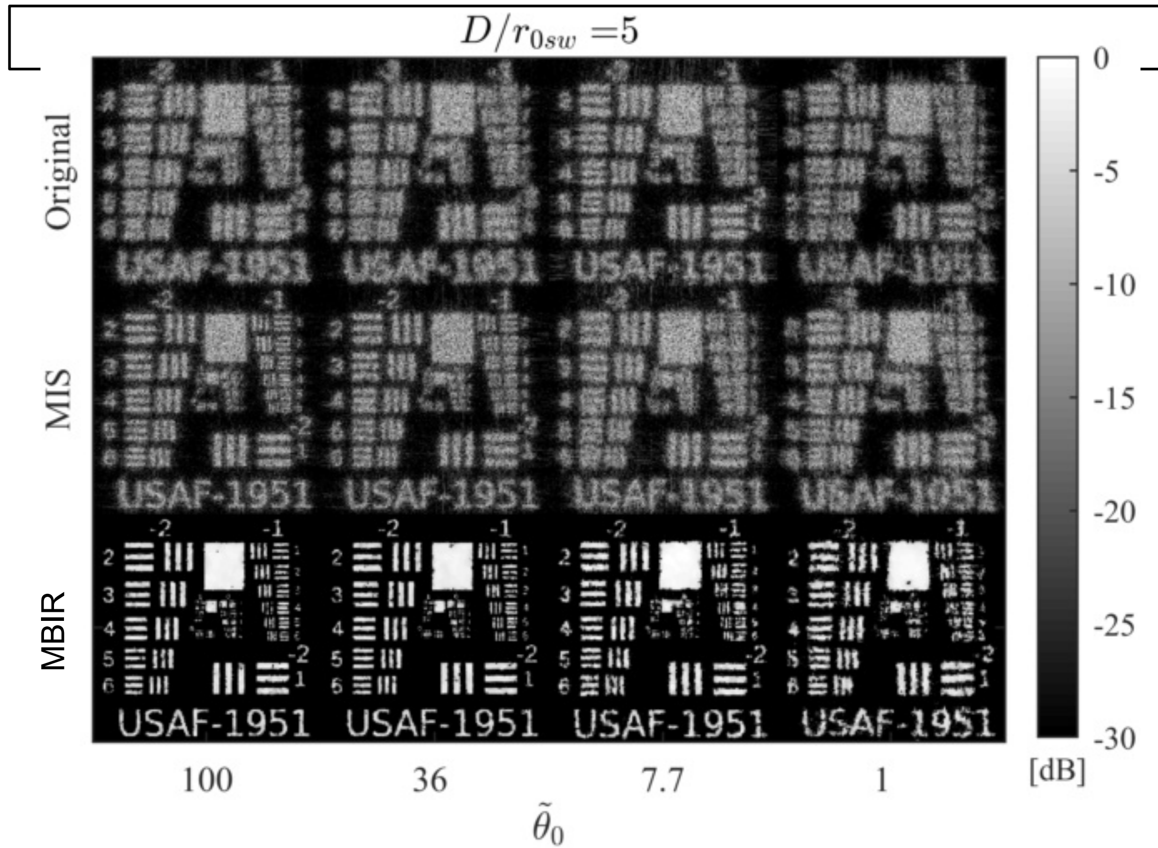
We set $\left(\frac{\lambda}{D}\right) = 1\ pixel$

$\tilde{\theta}_0$***=1*** $\Rightarrow$ ***more PSFs than pixels in the image***

DH-MBIR: Anisoplanatic Results (Simulated)



Peak Strehl ratio vs. $\tilde{\theta}_0$, averaged over 10 i.i.d. realizations



Simulation Parameters:

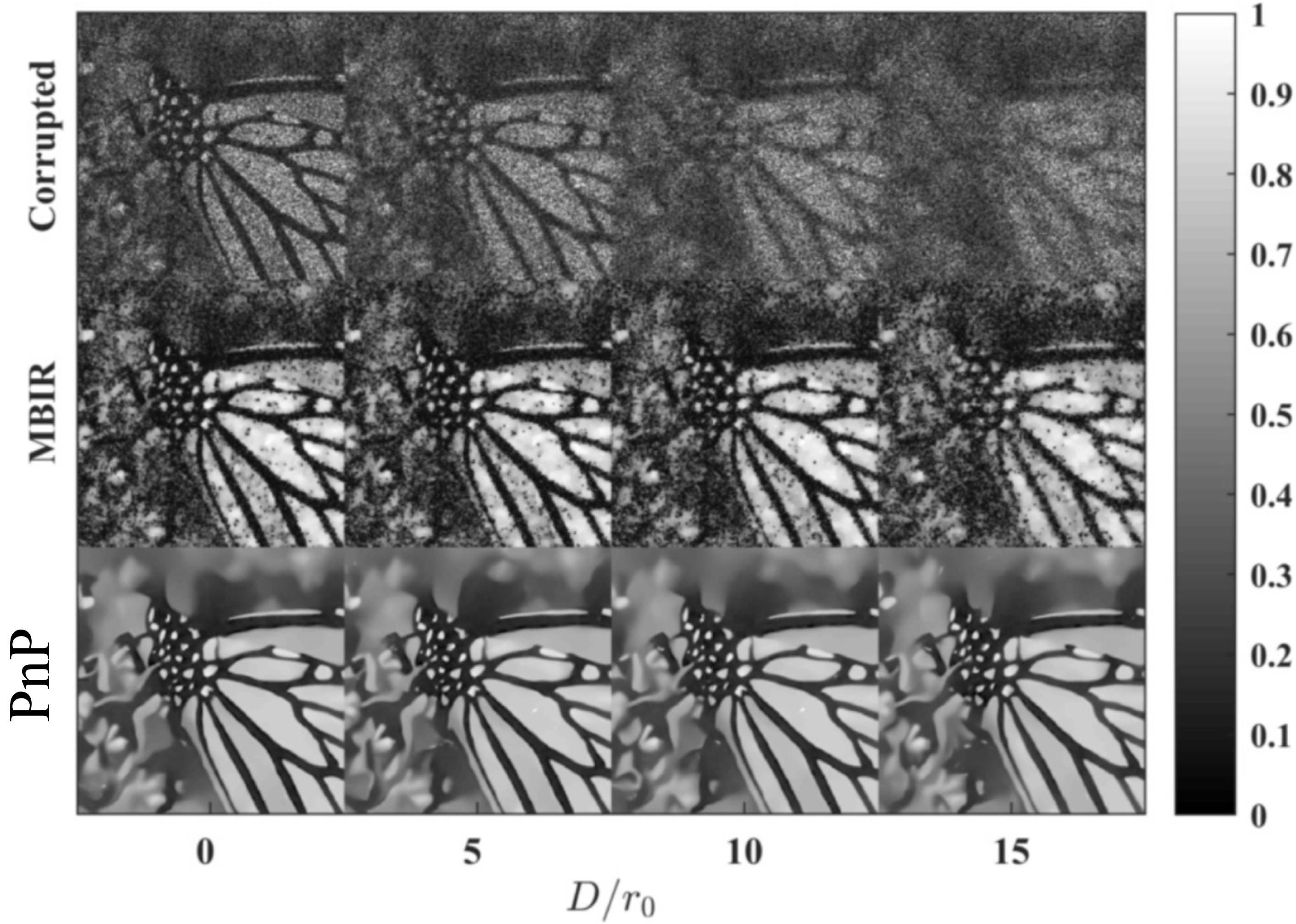
- 256x256 images
- 3 phase screens
- $\left(\frac{\lambda}{D}\right) = 1\, pixel$

Plug and Play/Consensus Equilibrium Approach

- PnP/CE is a framework for integrating models:
  - Physics based model of DH sensor
  - Machine learning model of images and phase errors

- Approach:
  - Build an "agent" for forward model and prior model
  - Forward model uses EM algorithm
  - Prior model is a convolutional neural network (CNN) denoiser

PnP Reconstruction (Simulation Data)



"It's the power of the deep neural network…,"

— Dong Hye Ye

Takeaways…

- Phase recovery maybe easier then you thought
- Computational Imaging offers a new perspective to optical sensing problems
  - Regularized iterative inversion (MBIR)
  - The EM Algorithm
  - Plug-and-Play methods
  - Convolutional neural networks